1. Consider a field theory with Weyl fermion fields ($\psi, \psi_c, \lambda$) with electric charges respectively equal to (1, $-1$, 0), with scalars ($\phi, \phi_c$) with charges equal to (1, $-1$) and lagrangian

$$L = L_{\text{kin}} + \left[ g_1 \psi \lambda \phi_c + g_2 \psi_c \lambda \phi + \frac{m}{2} \lambda \lambda + m_3 \phi \phi_c + \text{h.c.} \right]$$

$$+ \ m_1^2 |\phi|^2 + m_2^2 |\phi_c|^2 + (g_3 (\phi \phi_c)^2 + \text{h.c.}) + g_4 |\phi|^4 + g_5 |\phi_c|^4 + g_6 |\phi \phi_c|^2$$ (0.1)

Is the fermion $\psi$ naturally massless in such a theory? Explain the result using symmetries. What about the case $m_3 = 0$. (Assume the scalar potential is positive definite so that no scalar acquires a vacuum expectation value).

2. Consider now adding a neutral scalar $\eta$ to the above model, modifying the lagrangian to

$$L = L_{\text{kin}} + [g_1 \psi \lambda \phi_c + g_2 \psi_c \lambda \phi + g_7 \eta \lambda \lambda + m_3 \eta^* \phi \phi_c + \text{h.c.}]$$

$$+ \ m_1^2 |\phi|^2 + m_2^2 |\phi_c|^2 + m_3^2 |\eta|^2 + g_4 |\phi|^4 + g_5 |\phi_c|^4 + g_6 |\phi \phi_c|^2$$ (0.3)

Are fermions naturally massless in the above field theory? Explain the result as before (again the scalar potential is assumed positive definite).