

SOLUTION 1. In Section 5.3, it is shown that the power spectral density is

$$S_X(f) = \frac{|\psi_{\mathcal{F}}(f)|^2}{T} \sum_k K_X[k] \exp(-j2\pi k f T),$$

where  $K_X[k]$  is the auto-covariance of  $X_i$  and  $\psi_{\mathcal{F}}(f)$  is the Fourier transform of  $\psi(t)$ .

Because  $\{X_i\}_{i=-\infty}^{\infty}$  are i.i.d. and have zero-mean,

$$K_X[k] = \mathbb{E}[X_{i+k}X_i^*] = \mathcal{E}\mathbb{1}\{k = 0\},$$

so

$$S_X(f) = \mathcal{E} \frac{|\psi_{\mathcal{F}}(f)|^2}{T}.$$

Moreover,

$$\begin{aligned} \psi_{\mathcal{F}}(f) &= \int_{-\infty}^{\infty} \psi(t) e^{-j2\pi f t} dt \\ &= \frac{1}{\sqrt{T}} \int_0^{\frac{T}{2}} e^{-j2\pi f t} dt - \frac{1}{\sqrt{T}} \int_{\frac{T}{2}}^T e^{-j2\pi f t} dt \\ &= \frac{j}{2\pi f \sqrt{T}} \left( e^{-j2\pi f \frac{T}{2}} - 1 - e^{-j2\pi f T} + e^{-j2\pi f \frac{T}{2}} \right) \\ &= \frac{j}{2\pi f \sqrt{T}} e^{-j2\pi f \frac{T}{2}} \left( 2 - e^{j2\pi f \frac{T}{2}} - e^{-j2\pi f \frac{T}{2}} \right) \\ &= \frac{j}{2\pi f \sqrt{T}} e^{-j2\pi f \frac{T}{2}} (2 - 2 \cos(\pi f T)) \\ &= \frac{j}{2\pi f \sqrt{T}} e^{-j2\pi f \frac{T}{2}} 4 \sin^2 \left( \pi f \frac{T}{2} \right). \end{aligned}$$

Therefore,

$$\begin{aligned} S_X(f) &= \mathcal{E} \frac{16 \sin^4(\pi f \frac{T}{2})}{4\pi^2 f^2 T^2} \\ &= \mathcal{E} \left( \pi f \frac{T}{2} \right)^2 \operatorname{sinc}^4 \left( f \frac{T}{2} \right). \end{aligned}$$

SOLUTION 2.

(a) When  $i = j$ ,  $\mathbb{E}[X_i X_j]$  equals

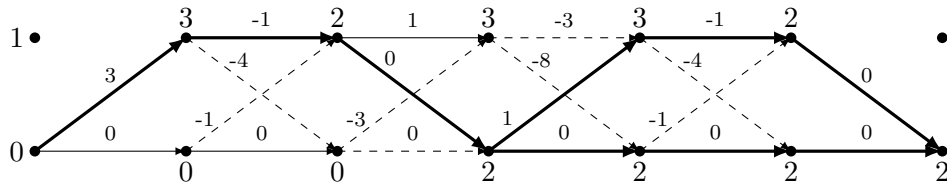
$$\mathbb{E}[X_i^2] = \mathbb{E}[1] = 1.$$

Remember that the  $B_i$  are i.i.d. Bernoulli( $\frac{1}{2}$ ) random variables. Hence, we find immediately

$$\begin{aligned} K_X[1] &= \mathbb{E}[X_{2n} X_{2n+1}] = \mathbb{E}[B_n B_{n-2} B_n B_{n-1} B_{n-2}] \\ &= \mathbb{E}[B_n^2 B_{n-1} B_{n-2}^2] \\ &= \mathbb{E}[B_{n-1}] = 0, \end{aligned}$$

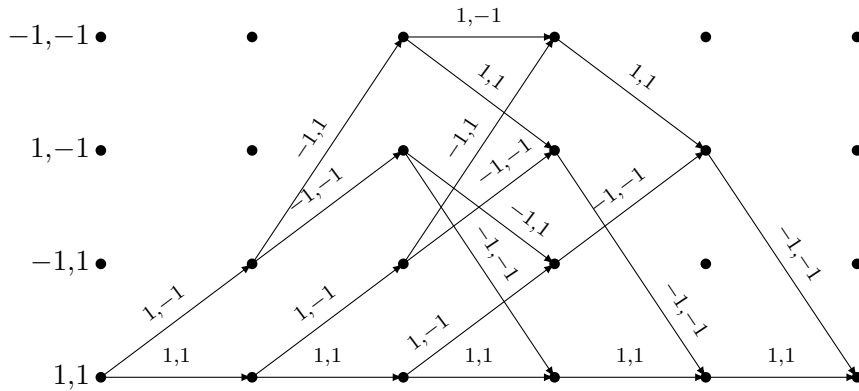


(c) We label our trellis with the edge metric  $2y_i x_i - x_i^2$  and then trace back the decoding path.

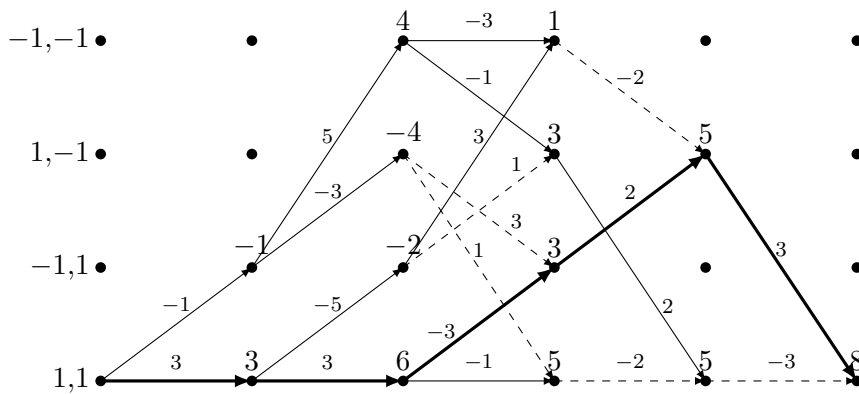


We see that the two sequences 1,1,0,0,0 and 1,1,0,1,1 are equally likely, so the decoder would choose either of the two.

SOLUTION 4. The trellis representing the encoder is shown below:



We display the diagram labeled with edge-metric according to the received sequence and state-metric of the survivor path. We also indicate the survivor paths and the decoding path.



From the figure we can read the decoded sequence 1, 1, -1, 1, 1.

SOLUTION 5. The output of encoder (a) is

$$T(\bar{x}_{2j-1}) = T(\bar{b}_j + \bar{b}_{j-2}) = T(\bar{b}_j)T(\bar{b}_{j-2}) = b_j b_{j-2}$$

$$T(\bar{x}_{2j}) = T(\bar{b}_j + \bar{b}_{j-1} + \bar{b}_{j-2}) = T(\bar{b}_j)T(\bar{b}_{j-1})T(\bar{b}_{j-2}) = b_j b_{j-1} b_{j-2},$$

which is identical to the output of encoder (b).