Exercise 1 *The Young double slit experiment (1803)*

In this exercise we want to calculate the form of the Young interference fringes. A beam of monochromatic light of wavelength $\lambda$ is sent through a double slit, and the light is reflected on a screen at a distance $D$. The distance between the two slits is $d$.

We assume that the waves diffracted by each slit have a spherical shape ($\lambda$ the wavelength and $\nu$ the frequency):

$$
\phi_B(\vec{r}) = Ae^{i\left(\frac{2\pi}{\lambda}|\vec{r}_B-\vec{r}| - 2\pi\nu t\right)} , \quad \phi_C(\vec{r}) = Ae^{i\left(\frac{2\pi}{\lambda}|\vec{r}_C-\vec{r}| - 2\pi\nu t\right)}. 
$$

The total wave function at $P$ on the screen is

$$
\psi(\vec{r}_P) = \phi_B(\vec{r}_P) + \phi_C(\vec{r}_P).
$$

We will use the plane wave approximation for $D >> d$:

$$
\psi(\vec{r}_P) \simeq \frac{A}{D}e^{-2\pi i\nu t} \left(e^{\frac{2\pi i}{\lambda}|\vec{r}_B-\vec{r}_P|} + e^{\frac{2\pi i}{\lambda}|\vec{r}_C-\vec{r}_P|}\right).
$$

1) Show that the intensity at $P$ on the screen is equal to

$$
|\psi(\vec{r}_P)|^2 \approx \frac{4A^2}{D^2} \cos^2 \left(\frac{\pi d}{\lambda \sin \theta}\right).
$$

**Hint:** evaluate first the path difference $|\vec{r}_C - \vec{r}_P| - |\vec{r}_B - \vec{r}_P|$ for $D >> d$.

2) Find the condition on $\sin \theta$ which leads to minima and maxima of the intensity on the screen.
3) Let $\rho$ be the coordinate on the screen measured from $O$. We have $\tan \theta \approx \frac{\rho}{D}$ and since $\theta$ is small $\theta \approx \frac{\rho}{D}$. Compute the distance between two successive minima of the intensity pattern on the screen.

Let $d = 0.25\text{mm}$, $D = 10\text{m}$ and $\lambda = 652\text{nm}$ (red light). What is the distance between two successive minima?

**Exercise 2 Modern Young’s experiment**

Young’s double slit experiment has been reformed with Carbon 60 molecules, $C_{60}$, in 1999. Surprisingly, these molecules behave like waves when they are well isolated from their environment. The more recent experiments have evidenced such a wave-like behavior for bigger molecules with 400 to 1000 atoms.

The diameter of $C_{60}$ (this molecule has the form of a sphere and contains 60 carbon atoms) is approximately 0.7 nm and a mole containing $N_A = 6.022 \times 10^{23}$ carbon atoms weights 12 grams.

1) Compute the De Broglie wavelength of molecules produced in an oven which have an average velocity of 220m/s. Compare with the size of individual molecules.

2) We perform a Young’s experiment with $d = 100\text{nm}$ and $D = 1.25\text{m}$. What do we observe on the screen assuming a wave like behavior?

3) A football weights approximately 450g and the initial velocity of a professional shoot can attain 100km/h. Estimate the De Broglie wavelength.

**Exercise 3 Photoelectric effect**

The maximal wavelength to extract a photoelectron from tungsten is 230nm (ultraviolet). What is the necessary wavelength of light to extract electrons with kinetic energy 1.5eV? What is the speed of these electrons?

**Useful Constant**

- $c = 2.997 \times 10^8 \text{ m/s}$ (speed of light)
- $h = 1.054 \times 10^{-34} \text{ J} \cdot \text{s}$ (where $h$ is Planck’s constant)
- $m = 9.109 \times 10^{-34} \text{kg}$ (mass of an electron)
- $1\text{eV} = 1.6 \times 10^{-19} \text{J}$