Homework 4 (Graded Homework 1): due Tuesday, March 24 2020
CS-526 Learning Theory

Instructions

- The deadline is *Tuesday, March 24 2020*.
- You can discuss and solve the exercises with other people but you have to write down your own solution.
- Please hand in your homework during either the lecture (March 23) or the exercise session (March 24). You can also send it by email to **both** teaching assistants: *clement.luneau[at]epfl.ch* and *andreas.maggiori[at]epfl.ch*.
- **No scan of handwritten homework is accepted**.

Exercises

Exercises from *Understanding Machine Learning: From Theory to Algorithms* by Shai Shalev-Shwartz and Shai Ben-David:

- Exercises 2, 8 and 9 of Chapter 6.
- Exercise 5, question 2 of Chapter 7.

Exercise *"VC dimension of circles"*. In this problem we consider hypothesis functions from $\mathbb{R}^2$ to $\{0,1\}$. The plane $\mathbb{R}^2$ is equipped with the usual Euclidean norm $\|\cdot\|$. We denote $B(\mathbf{y},r) = \left\{\mathbf{x} \in \mathbb{R}^2 : \|\mathbf{x}-\mathbf{y}\| \leq r\right\}$ the closed disk of radius r centered in $\mathbf{y} \in \mathbb{R}^2$. Let $\mathcal{H} = \left\{\mathbb{1}_{B(\mathbf{y},r)} : r \geq 0 \text{ and } \mathbf{y} \in \mathbb{R}^2\right\}$ be the hypothesis class made of the indicator functions of all possible closed disks in the plane. Call d the VC dimension of $\mathcal{H}$.

1. Show that for any $n \leq d$ there exist n distinct points in the plane shattered by $\mathcal{H}$.
   Hint: You can propose an instance of d points and for each labeling draw the valid circle.
2. Show that no set of n distinct points with $n > d$ can be shattered by $\mathcal{H}$.
   Hint: You should consider two cases: 1) one of the points is in the convex hull of the other points, and 2) none of the points is in the convex hull of the other points. A formal proof might be difficult. It will suffice if you give us a *convincing* argument.