IBM Q practice 2 March 26 2020
Hmw Quantum Computation

Implementations on the composer : (i) Deutsch-Josza algorithm ; (ii) entanglement creation of two distant qubits by swapping.

Exercise 1 *Deutsch-Josza algorithm implementations*

(a) Implement the DJ algorithm for the single-bit functions $f(x) = 0$, $f(x) = 1$, $f(x) = x$ and $f(x) = x \oplus 1$. For each function you have to implement U_f in some way. Make tests with one shot (one run of the circuit) and many shots as well (e.g. 1024 shots). Observe the effect of noise when you compare the results of the simulator with the real experiment.

(b) How many constant or balanced two-bit functions exist ? Implement the DJ algorithm for two-bit balanced functions $f(x_1, x_2) = x_1 \oplus x_2$ and $f(x_1, x_2) = \bar{x}_1 \oplus x_2$. How do you implement U_f ?

Exercise 2 *Entanglement swapping*

The entanglement swapping protocol is the following sequence of operations :

1) *Create two Bell pairs, say*

$$(|00\rangle_{01} + |11\rangle_{01}) \otimes (|00\rangle_{23} + |11\rangle_{23})$$

2) *Make a measurement of qubits* (12) *in the Bell basis.*

3) *There are four possible equiprobable outcomes :*

$$(|00\rangle_{12} + |11\rangle_{12}) \otimes (|00\rangle_{03} + |11\rangle_{03})$$

$$(|01\rangle_{12} + |10\rangle_{12}) \otimes (|01\rangle_{03} + |10\rangle_{03})$$

$$(|01\rangle_{12} - |10\rangle_{12}) \otimes (|01\rangle_{03} - |10\rangle_{03})$$

$$(|00\rangle_{12} - |11\rangle_{12}) \otimes (|00\rangle_{03} - |11\rangle_{03})$$

The entanglement has been swapped. Say that particles 0 *and* 3 *are far away and particles* 1 *and* 2 *are close together. The local measurement in the* (12) *lab entangles the distant particles* 0 *and* 3 *!*

You are asked to implement a *related* sequence of operations on the simulator and on a real machine.

(a) Create two Bell pairs $(|00\rangle_{01} + |11\rangle_{01}) \otimes (|00\rangle_{23} + |11\rangle_{23})$. Then, make measurements of the four qubits involved and observe the histograms. Simulate and run (with 1024 shots).

(b) Create the state $|\Psi\rangle = (|00\rangle_{01} + |11\rangle_{01}) \otimes (|00\rangle_{23} + |11\rangle_{23})$ as in the previous question. Then implement the operation

$$H_2\,(CX_{2-1})|\Psi\rangle$$

where H_2 is the Hadamard gate on qubit 2 and CX_{2-1} is the control-not gate on qubits 2 and 1 with 2 the control qubit and 1 the target qubit.

Measure the resulting state in the computational basis. Simulate and observe the histogram. You are asked to interpret the simulated histogram. In order to understand the interpretation it is useful to prove with pencil and paper that the result of a single measurement is one of the four equiprobable outcomes :

$$|00\rangle_{12} \otimes (|00\rangle_{03} + |11\rangle_{03})$$

$$|01\rangle_{12} \otimes (|00\rangle_{03} - |11\rangle_{03})$$

$$|10\rangle_{12} \otimes (|01\rangle_{03} + |10\rangle_{03})$$

$$|11\rangle_{12} \otimes (|10\rangle_{03} - |01\rangle_{03})$$