ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

**Handout 31** | Information Theory and Coding
Homework 13 | Dec. 17, 2019

PROBLEM 1.

(a) Suppose $U$ and $V$ are binary random variables. The joint distribution induced on $(U, V)$ is given as

$$p_{UV}(u,v) = \begin{cases} 1/3, & (u,v) = (0,0) \\ 1/3, & (u,v) = (1,0) \\ 1/3, & (u,v) = (1,1) \\ 0, & (u,v) = (0,1) \end{cases}.$$

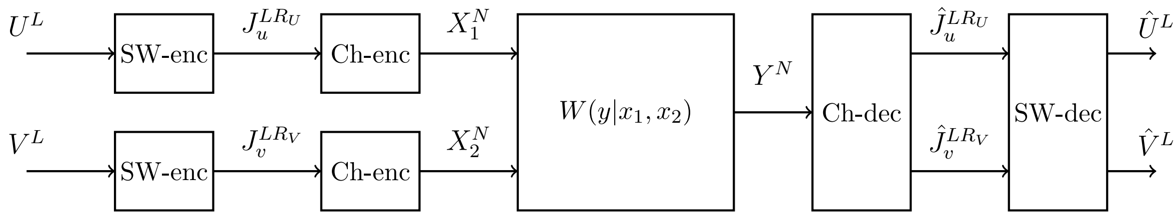
Find the Slepian-Wolf rate region for $(U, V)$ pair.

(b) Now suppose we have a binary additive MAC channel with inputs $X_1$, $X_2$ and output $Y$. The random variables $X_1$ and $X_2$ can take values in the set $\{0, 1\}$ and $Y$ can take values in the set $\{0, 1, 2\}$. The relationship between $X_1$, $X_2$ and $Y$ is given as

$$Y = X_1 + X_2.$$

Find the capacity region for this MAC.

(c) Now, the aim is to design a communication system that first compresses the source into a bitstream and then employs some channel coding technique to achieve reliable communication. The scheme is given as follows.




Here, SW-enc represents Slepian-Wolf encoder for the source $U$,$V$ of length $L$ which outputs a bitstream of length $LR_U$, $LR_V$ respectively. Later, the bitstreams $J_u$ and $J_v$ are encoded by channel encoders (Ch-enc) and then passed through the multiple access channel. As usual, from $Y^N$; bitstreams $\hat{J}_u$ and $\hat{J}_v$ are estimated by a channel decoder. Finally, the estimated bitstreams are decoded by Slepian-Wolf decoders to obtain $U^L$ and $V^L$.

For the sources described in part (a) and channel described in part (b), what is the maximum value that $L/N$ can take for a reliable communication?

(d) Consider now an uncoded scheme with the same sources and same channel where $X_1 = U$ and $X_2 = V$. Note that in this scheme, $L = N = 1$. Can $(U, V)$ be recovered from $Y$? Can the value $L/N$ of this scheme be achieved by schemes as in part (c)?

Problem 2.

Consider the multiplicative multiple access channel $Y = X_1 X_2$. Find the capacity region when

(a) $X_1 \in \{0, 1\}$, $X_2 \in \{1, 2\}$.

(b) $X_1 \in \{0, 1\}$, $X_2 \in \{1, 2, 3\}$.

(c) $X_1 \in \{1, 2\}$, $X_2 \in \{1, 2\}$.