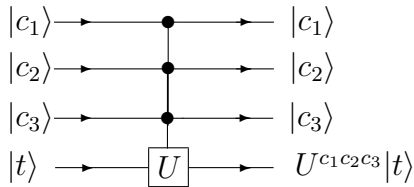
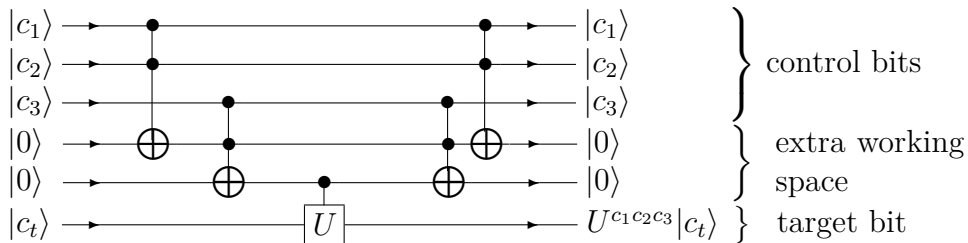


Exercise 1 *Construction of a multi-control- U .*

Verify that the multi-control- U :



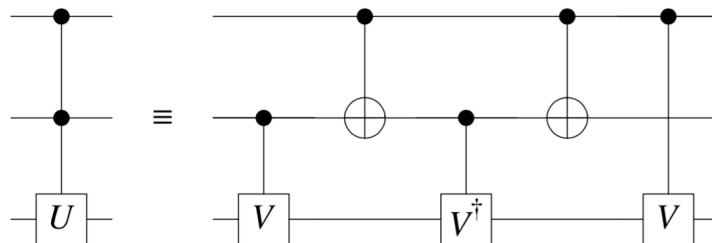
can be realized with the Toffoli gate (control-control-NOT) and a simple control- U .



Exercise 2 *Controlled-controlled- U*

In the last exercise, we have seen the construction of multi-controlled- U gate with the Toffoli gates and a simple controlled- U gate. This time, we are going to see a different construction for controlled-controlled- U gate.

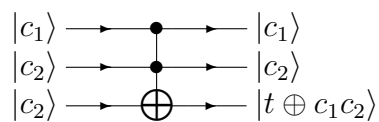
Let V be any quantum gate such that $V^2 = U$. Prove the following circuit identity.



Explain in what sense this second construction is "not universal", whereas the one of the previous exercise is "universal" (for a given fixed U).

Exercise 3 Construction of the Toffoli gate from a control-NOT (hint : long calculation).

Verify that the control-control-NOT also called Toffoli gate :



is equivalent to the following circuit made of *CNOT*, *H*, *T* and *S*

