Implementation on the composer : (i) Deutsch-Josza algorithm ; (ii) entanglement creation of two distant qubits by swapping.

Exercise 1 *Deutsch-Josza algorithm implementations*

(a) Implement the DJ algorithm for the single-bit functions $f(x) = 0$, $f(x) = 1$, $f(x) = x$ and $f(x) = x \oplus 1$. For each function you have to implement U_f in some way. Use ibmqx4 and qubits $q[3]$ and $q[4]$ (you will find results in the cache if you test 1024 shot runs).

(b) How many constant or balanced two-bit functions exist ? Implement the DJ algorithm for two-bit balanced functions $f(x_1, x_2) = x_1 \oplus x_2$ and $f(x_1, x_2) = \bar{x}_1 \oplus x_2$. How do you implement U_f ? Use ibmqx4 and the qubits $q[0]$, $q[1]$, $q[2]$ were $q[1]$, $q[2]$ carry the entries x_1, x_2 (you will find results in the cache if you test 1024 shot runs).

Exercise 2 *Entanglement swapping*

The entanglement swapping protocol is the following sequence of operations :

1) Create two Bell pairs, say

$$(|00\rangle_{01} + |11\rangle_{01}) \otimes (|00\rangle_{23} + |11\rangle_{23})$$

2) Make a measurement of qubits (12) in the Bell basis.

3) There are four possible equiprobable outcomes :

$$(|00\rangle_{12} + |11\rangle_{12}) \otimes (|00\rangle_{03} + |11\rangle_{03})$$

$$(|01\rangle_{12} + |10\rangle_{12}) \otimes (|01\rangle_{03} + |10\rangle_{03})$$

$$(|01\rangle_{12} - |10\rangle_{12}) \otimes (|01\rangle_{03} - |10\rangle_{03})$$

$$(|00\rangle_{12} - |11\rangle_{12}) \otimes (|00\rangle_{03} - |11\rangle_{03})$$

The entanglement has been swapped. Say that particles 0 and 3 are far away and particles 1 and 2 are close together. The local measurement in the (12) lab entangles the distant particles 0 and 3 !

You are asked to implement a *related* sequence of operations on ibmqx4.

(a) Create two Bell pairs $(|00\rangle_{01} + |11\rangle_{01}) \otimes (|00\rangle_{23} + |11\rangle_{23})$. Then, make measurements of the four qubits involved and observe the histograms. Simulate and run (with 1024 shots).
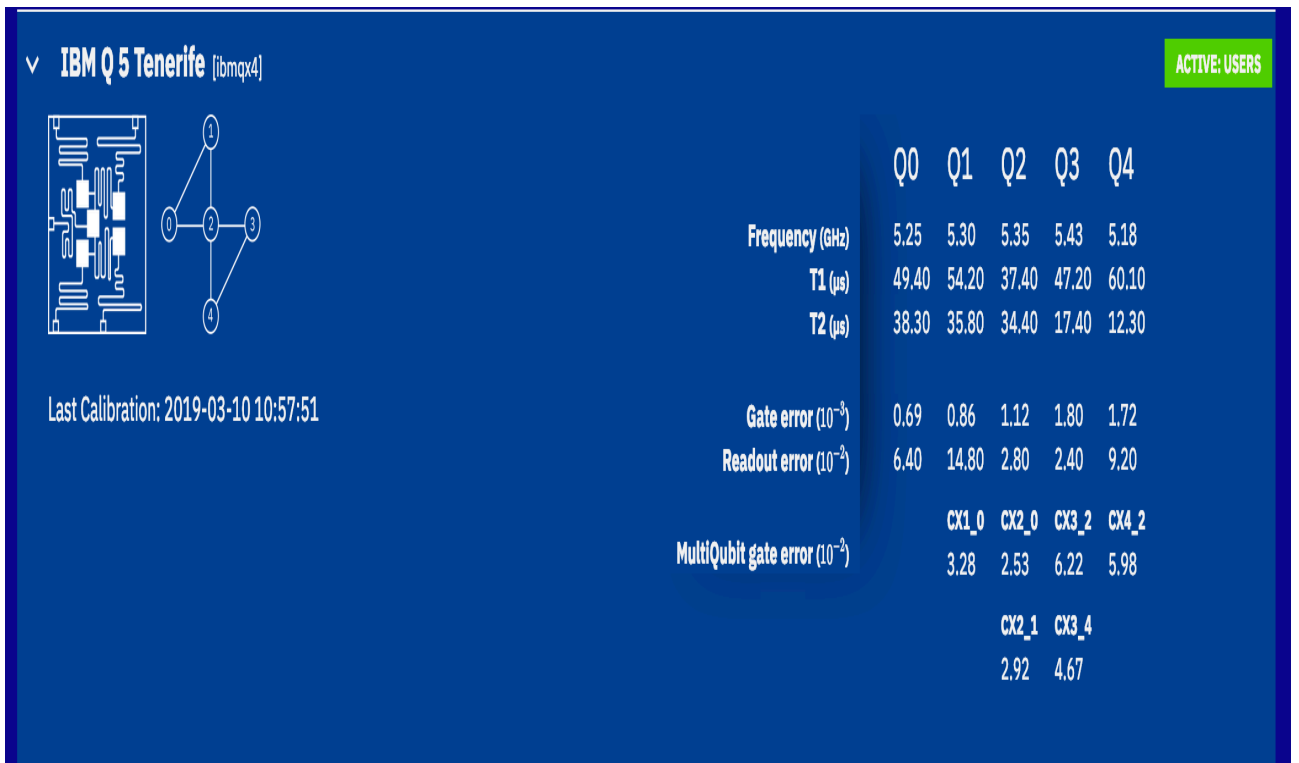
FIGURE 1 – 5 qubit ibmqx4 Tenerife device - parameters of March 10, 2019

(b) Create the state $|\Psi\rangle = (|00\rangle_{01} + |11\rangle_{01}) \otimes (|00\rangle_{23} + |11\rangle_{23})$ as in the previous question. Then implement the operation

$$H_2\left(CX_{2-1}\right)|\Psi\rangle$$

and measure the resulting state in the computational basis. Simulate and observe the histogram. You are asked to interpret the simulated histogram. In order to understand the interpretation it is useful to prove with pencil and paper that the result of a single measurement is one of the four equiprobable outcomes :

$$|00\rangle_{12} \otimes (|00\rangle_{03} + |11\rangle_{03})$$

$$|01\rangle_{12} \otimes (|00\rangle_{03} - |11\rangle_{03})$$
$$|10\rangle_{12} \otimes (|01\rangle_{03} + |10\rangle_{03})$$
$$|11\rangle_{12} \otimes (|10\rangle_{03} - |01\rangle_{03})$$

You will find the result of true experiments with 1024 shots in the cache.