

PROBLEM 1. (a) Suppose  $U$  and  $V$  are binary random variables. The joint distribution induced on  $(U, V)$  is given as

$$p_{UV}(u, v) = \begin{cases} 1/3, & (u, v) = (0, 0) \\ 1/3, & (u, v) = (1, 0) \\ 1/3, & (u, v) = (1, 1) \\ 0, & (u, v) = (0, 1) \end{cases}.$$

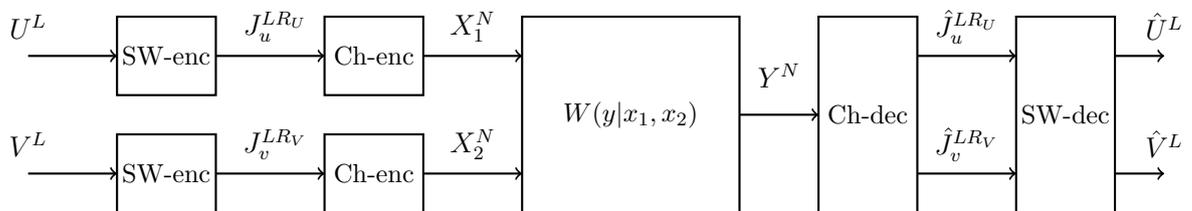
Find the Slepian-Wolf rate region for  $(U, V)$  pair.

(b) Now suppose we have a binary additive MAC channel with inputs  $X_1, X_2$  and output  $Y$ . The random variables  $X_1$  and  $X_2$  can take values in the set  $\{0, 1\}$  and  $Y$  can take values in the set  $\{0, 1, 2\}$ . The relationship between  $X_1, X_2$  and  $Y$  is given as

$$Y = X_1 + X_2.$$

Find the capacity region for this MAC.

(c) Now, the aim is to design a communication system that first compresses the source into a bitstream and then employs some channel coding technique to achieve reliable communication. The scheme is given as follows.



Here, SW-enc represents Slepian-Wolf encoder for the source  $U, V$  of length  $L$  which outputs a bitstream of length  $LR_U, LR_V$  respectively. Later, the bitstreams  $J_u$  and  $J_v$  are encoded by channel encoders (Ch-enc) and then passed through the multiple access channel. As usual, from  $Y^N$ ; bitstreams  $\hat{J}_u$  and  $\hat{J}_v$  are estimated by a channel decoder. Finally, the estimated bitstreams are decoded by Slepian-Wolf decoders to obtain  $U^L$  and  $V^L$ .

For the sources described in part (a) and channel described in part (b), what is the maximum value that  $L/N$  can take for a reliable communication?

(d) Consider now an uncoded scheme with the same sources and same channel where  $X_1 = U$  and  $X_2 = V$ . Note that in this scheme,  $L = N = 1$ . Can  $(U, V)$  be recovered from  $Y$ ? Can the value  $L/N$  of this scheme be achieved by schemes as in part (c)?

PROBLEM 2. Consider the multiplicative multiple access channel  $Y = X_1X_2$ . Find the capacity region when

(a)  $X_1 \in \{0, 1\}$ ,  $X_2 \in \{1, 2\}$ .

(b)  $X_1 \in \{0, 1\}$ ,  $X_2 \in \{1, 2, 3\}$ .

(c)  $X_1 \in \{1, 2\}$ ,  $X_2 \in \{1, 2\}$ .