LIBOR Market Models with Stochastic Basis

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- Extension of LMM model accounting for spread-widening between discount rates and LIBOR rates
- Related papers
  - Schoenbucher (1999)
  - Mercurio (2009)
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- Formulae for IRS, caps, swaptions and guideline modeling different tenors simultaneously
- Very tractable application using SABR and shifted Lognormal models
- Fit to data is extraordinarily good

Critique I

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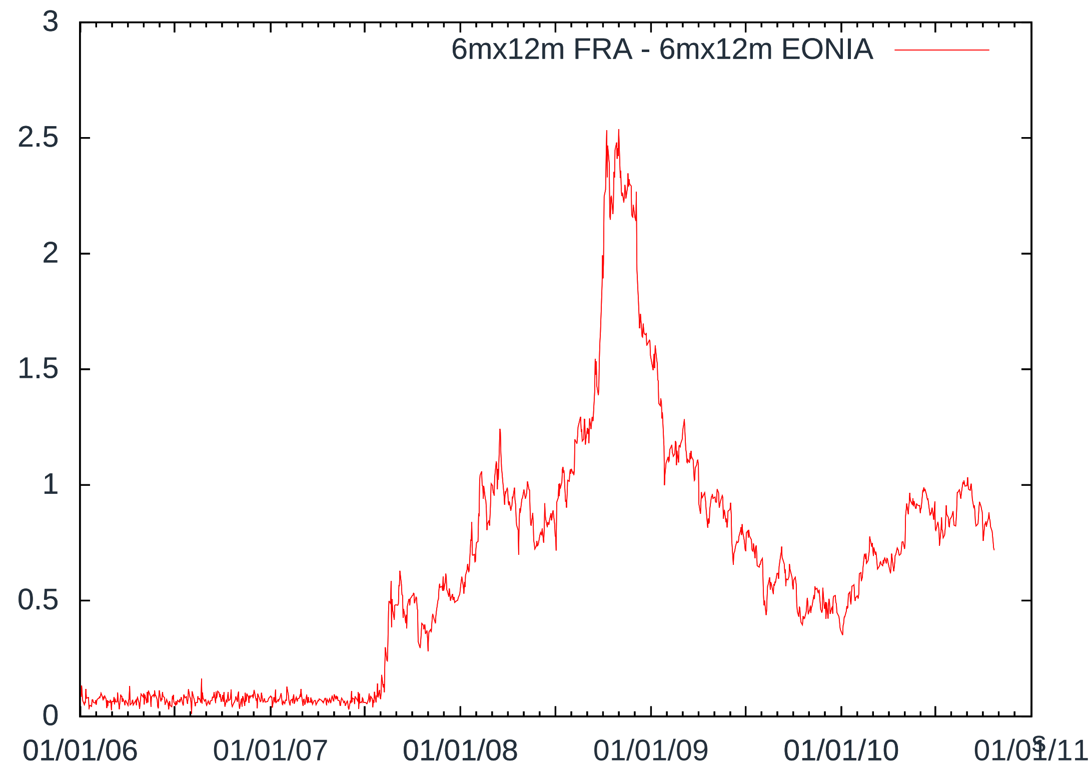
- The framework models OIS forward rates $F_k^x(t)$ and spreads $S_k^x(t) := L_k^x(t) - F_k^x(t)$ as independent processes
- But F and S are dependent by construction
- Also conditional variance of the two is likely related



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- Linear dependence regression $\Delta F = \theta_0 + \theta_1 \Delta S + \varepsilon$

Coefficient	Est.	Prob.
$\widehat{\theta}_0$	-0.001	0.1672
$\widehat{\theta}_1$	-0.971	0.0000

- Variance dependence regression $\Delta F^2 = \eta_0 + \eta_1 \Delta S^2 + \varepsilon$

Coefficient	Est.	Prob.
$\widehat{\eta}_0$	10.4962	0.0000
$\widehat{\eta}_1$	-1.6587	0.0000

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- Suppose that F and L are both driven by the same variance factor

$$d\ln F_k^x(t) = -\frac{1}{2}V_k(t)dt + \sqrt{V_k(t)}dZ_k^F(t)$$
$$d\ln S_k^x(t) = -\frac{1}{2}V_k(t)dt + \sqrt{V_k(t)}dZ_k^S(t)$$
$$dV_k(t) = (b + \beta V_k(t))dt + \sqrt{\alpha V_k(t)}dZ_k^V(t).$$

  They are instantaneously uncorrelated. For fixed time $T > 0$ correlation proportional to V

- Note that conditional on

$$IV_k(t,T) := \int_t^T V_k(s)ds$$

  $\log F_k^x(T_{k-1}^x) \mid IV_k(t, T_{k-1}^x)$ and $\log S_k^x(T_{k-1}^x) \mid IV_k(t, T_{k-1}^x)$ are independently normally distributed

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- Condition on $IV_k(t, T_{k-1}^x)$ instead of $S_k^x(T_{k-1}^x)$

$$\mathbf{Cplt}(t, K; T_{k-1}^x, T_k^x) = \tau_k^x P_D(t, T_k^x)$$
$$\cdot\, \mathbb{E}_D^{T_k^x}\left[\mathbb{E}_D^{T_k^x}\left[\left[L_k^x(T_{k-1}^x) - K\right]^+ \mid \mathcal{F}_t \vee IV_k(t, T_{k-1}^x)\right] \mid \mathcal{F}_t\right]$$

- Inner expectation is an integration against Lognormal convolution

$$f(z) := \mathbb{E}_D^{T_k^x}\left[\left[L_k^x(T_{k-1}^x) - K\right]^+ \mid \mathcal{F}_t \vee IV_k(t, T_{k-1}^x) = z\right]$$

- Denote with $g(z)$ the conditional density of $IV_k(t, T_{k-1}^x) \mid V_k(t)$. We need to solve

$$\int_0^\infty f(z) g(z) dz$$

Closed-form Density Expansion

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- Approximate the conditional density of $IV_k(T) \mid V_k(t)$ using Filipović, Mayerhofer, and Schneider (2010) likelihood expansions

- Generate affine Markov process through embedding $V_k(t) \longrightarrow (V_k(t), \int_0^t V_k(s)ds) =: IV_k(t))$

$$dV_k(t) = (b + \beta V_k(t))dt + \sqrt{\alpha V_k(t)}dZ_k^V(t)$$
$$dIV_k(t) = V_k(t)dt$$

- This process is polynomial and polynomial moments can be computed in closed-form

- Approximate the marginal distribution of $IV_k(T) \mid V_k(t)$ through polynomial expansion in a weighted $\mathcal{L}^2$ space

- Expansion performs very accurately.

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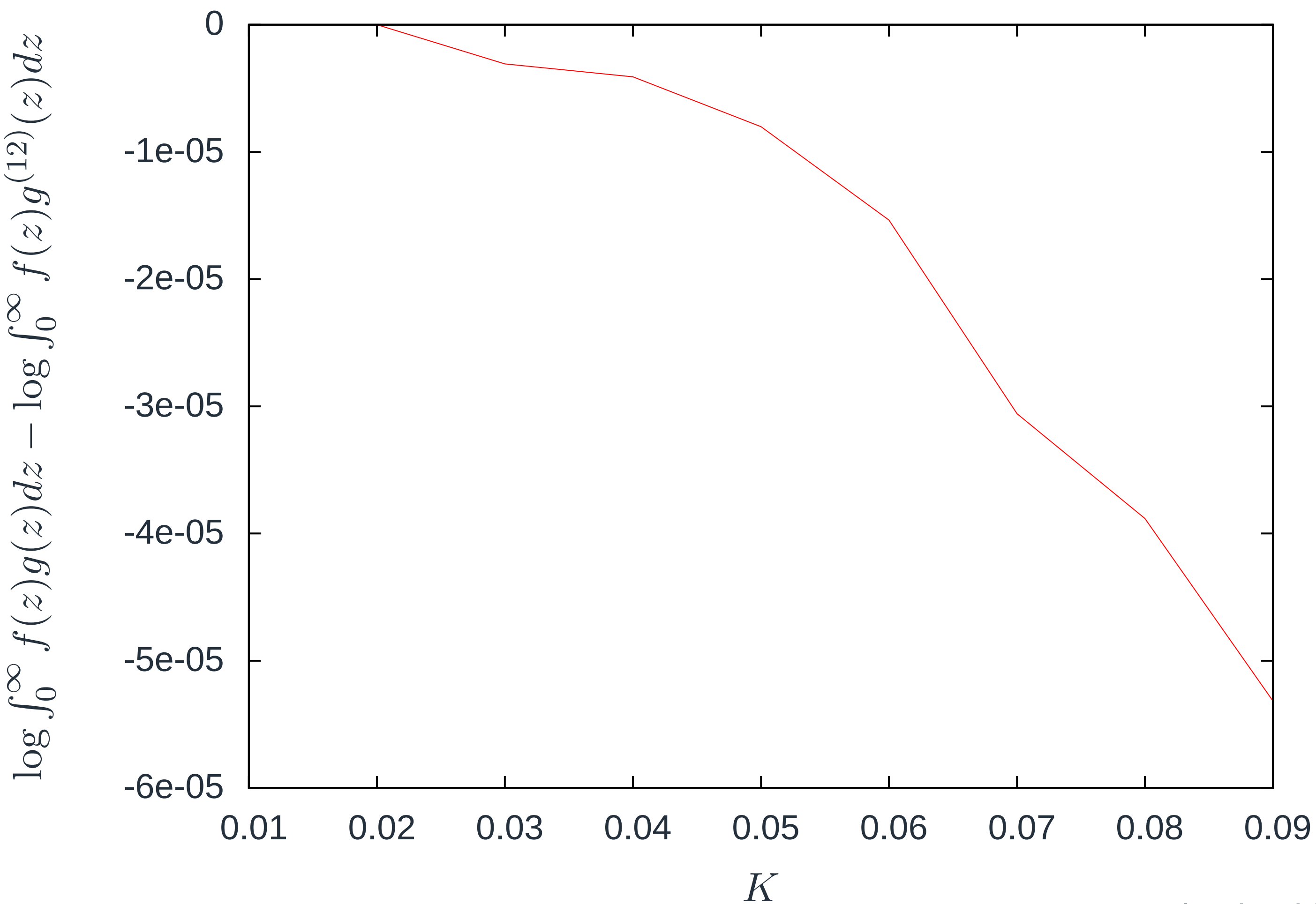
The Picture shows the percentage deviation of the conditional expectation obtained from true density (Fourier inversion) and Filipović, Mayerhofer, and Schneider (2010) expansion



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Conclusion

- Praise
  - Easy to use LMM adapted to current economic environment
  - Formulae for IRS, caps, swaptions
  - Guideline for modeling different tenors simultaneously
- Future Topics
  - Where does the spread come from?
  - How can we make the model more realistic while maintaining tractability?

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Consider

$$d\ln F_k^x(t) = -\frac{1}{2}V_k(t)dt + \sqrt{V_k(t)}\left(\rho dZ_k^V(t) + \sqrt{1-\rho^2}dZ_k^F(t)\right)$$
$$d\ln S_k^x(t) = -\frac{1}{2}V_k(t)dt + \sqrt{V_k(t)}\left(\eta dZ_k^V(t) + \sqrt{1-\eta^2}dZ_k^S(t)\right)$$
$$dV_k(t) = (b + \beta V_k(t))dt + \sqrt{\alpha V_k(t)}dZ_k^V(t).$$

Since

$$\int_t^T \sqrt{V_k(s)}dZ_k^V(s) = -\frac{b(T-t) + \beta IV_k(T) + V_k(t) - V_k(T)}{\sqrt{\alpha}},$$
$$\ln F(T) - \ln F(t) \mid V(T), IV(T)$$
$$\sim N\left(-\frac{1}{2}IV(T) + \rho\int_t^T \sqrt{V(s)}dZ^V(s), (1-\rho^2)IV(T)\right)$$

By approximating $V_k(T), IV_k(T) \mid V_k(t)$ we could also induce instantaneous correlation