Dynamic counterparty risk valuation

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Thank you Damir...

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and **congratulations on the happy event!**.

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  - Some of *the factors that influence your counterpart's default might also influence the pay-off of the contract or the value of the underlying at the time of default*.
- To evaluate exposure (CVA) one needs **both** the distribution of the default time and the distribution of all state variables conditional on the occurrence of default.

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- Another example: CDS on a name whose credit quality is positively correlated to that of the protection seller.

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- **Goal(s)**: calibrate the barrier and/or dynamics to observed spreads and compute the distribution of $(X_t)_{t\geq 0}$ conditional on default occurring at some given point during the life of the contract.

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- **This paper**: choose ν, $\sigma(t)$ and the distribution of Y_0 so that the hitting time of zero is distributed according to H.
- The authors derive a **very neat solution**. In particular:

$$\sigma(t)^2 \propto \frac{H'(t)}{1 - H(t)}$$

so that the implied credit index becomes more volatile as the hazard rate of the default distribution increases.

2. Conditional distributions

- The theory of $h-$transforms states that

$$\mathbb{P}\left[A|\tau = s\right] = \mathbb{E}\left[1_{\{A\}}\frac{h_s(t, Y_t)}{h_s(0, Y_0)}\right] \equiv \widehat{\mathbb{P}}_s\left[A\right]$$

for all $t < s$ and $A \in \mathscr{F}_t$ where

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- Together with Girsanov's theorem this **allows to compute/simulate dynamics conditional on** $\{\tau = s\}$

2. Conditional distributions (cont'd)

- In particular, the dynamics of the credit index conditional on default occurring at time s are given by

$$\mathrm{d}Y_t = \left(\frac{1}{Y_t} - \frac{Y_t}{v(t,s)}\right)\sigma(t)^2\mathrm{d}t + \sigma(t)\mathrm{d}\widehat{B}_{s,t}$$

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- If $\sigma \equiv 1$ then the conditioned credit index is a 3d–Bessel bridge from the point $(0, Y_0)$ to the point $(s, 0)$.
- There are close connections to the theory of conditioned SDEs and to initial enlargements of filtrations.

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4. Can the modified IFPT be solved in a multidimensional setting to match the joint distribution of default times?
5. Can a similar approach be used in reduced–form models?