

On the relative pricing of long maturity S&P 500 index options and CDX tranches

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Swissquote Conference on Interest Rate and Credit Risk

October 2010

Securitized Credit Markets Crisis

- ▶ Pre-crisis saw large growth in securitized credit markets (CDO).
- ▶ Pooling and tranching used to create 'virtually risk-free' AAA securities, in response to high demand for highly rated securities.
- ▶ During the crisis **all** AAA markets were hit hard:
 - ▶ Home equity loan CDO prices fell (ABX.HE AAA < 60%).
 - ▶ Super Senior (30-100) tranche spreads > 100bps.
 - ▶ CMBX.AAA (super duper) > 750bps.
- ▶ Raises several questions:
 - Q? Were ratings incorrect (ex-ante default probability higher than expected)?
 - Q? Are ratings sufficient statistics (risk \neq expected loss)?
 - Q? Were AAA tranches mis-priced (relative to option prices)?
- ▶ Many other surprises:
 - ▶ Corporate Credit spreads widened (CDX-IG > 200bps).
 - ▶ Cash-CDS basis **negative** (-200 bps for IG; -700bps for HY).
 - ▶ LIBOR-Treasury and LIBOR-OIS widened (> 400bps).
 - ▶ Long term Swap spreads became negative (30 year swap over Treasury < -50 bps).
 - ▶ Defaults on the rise (Bear Stearns, Lehman).

Evidence from ABX markets

- ▶ ABX.HE (subprime) AAA and BBB spreads widened dramatically (prices dropped)

J.P.Morgan DataQuery

J.P.Morgan Inc.

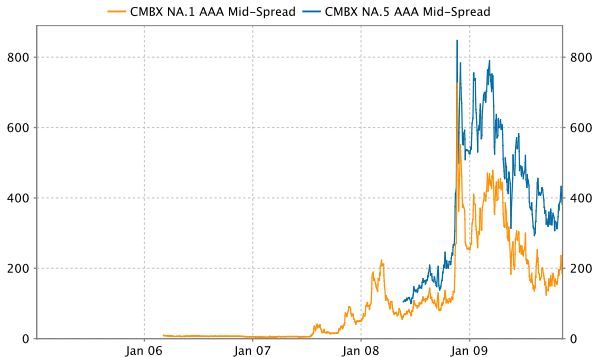


Evidence from CMBX markets

- ▶ CMBX (commercial real estate) AAA spreads widened even more dramatically

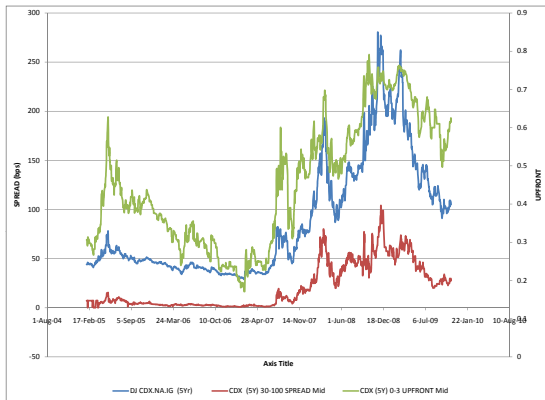
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Corporate IG CDX Tranche spreads

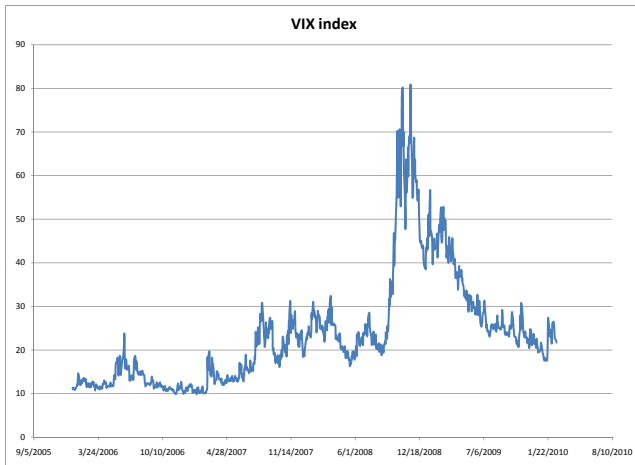
- The impact on tranche prices was dramatic



- Implied correlation on equity tranche hit $> 40\%$
- Correlation on Super-Senior tranches $> 100\%$ (!) with standard recovery assumption
- Relative importance of expected loss in senior tranche versus in equity tranche indicates increased crash risk.

Evidence from S&P500 Option markets

- Implied volatility index widened dramatically: increased market and crash risk.



CDX Index & CDX Tranche Markets

- ▶ Credit Default Swaps (CDS)
 - ▶ Buyer of protection makes regular (quarterly) payments = CDS spread
 - ▶ Seller of protection makes buyer whole if underlying bond defaults
 - ▶ CDS spread \approx corporate bond spread ($y - r_f$)
- ▶ CDX Investment Grade (IG) Index
 - ▶ portfolio of 125 IG credits
 - ▶ Buyer of protection makes regular payments on **remaining portfolio notional**
 - ▶ Seller of protection makes buyer whole at time of each bond default
 - ▶ CDX index spread \approx weighted average of CDS spreads
- ▶ CDX (IG) Tranches written on same portfolio
 - ▶ Associated with standard attachment/detachment points (subordination levels):
 - ▶ 0-3% (Equity tranche)
 - ▶ 3-7% (Mezzanine tranche)
 - ▶ 7-10%
 - ▶ 10-15%
 - ▶ 15-30% (Senior tranche)
 - ▶ 30-100% (Super-senior tranche)
 - ▶ Buyer of protection makes regular payments on **remaining tranche notional**
 - ▶ Seller of protection makes buyer whole for each bond default which reduces tranche notional
- ▶ CDS, CDX index spreads determined from **marginal** default probabilities.
- ▶ CDX tranche spreads need entire **joint** distribution (correlation market).

Relation Between SP500 Index Option Prices and CDX Tranche Spreads

- ▶ Given the Arrow-Debreu (or state) prices for every date and every state of nature, one can determine the arbitrage-free price of any (derivative) security
- ▶ Given option prices across all strikes (and dates) of SP500 index options, one can back out the A/D prices
 - ▶ Breeden and Litzenberger (1978)
- ▶ Due to diversification effects of 125 firms composing CDX index, CF's associated with CDX tranche positions closely tied to overall market performance
 - ⇒ Identifying state prices from option prices should be useful for estimating tranche spreads
- ▶ In practice, strikes typically limited to (70% - 130%) of current index levels
- ▶ Can we extrapolate state prices from SP500 option prices to price credit derivatives?
 - ▶ Payoffs of most senior tranches associated with losses well below 70% of current levels
 - ▶ Need to extrapolate well beyond observable prices

Structural/Copula Models of Default

- Specify market (S&P500) value dynamics as:

$$\frac{dM}{M} = (r - \delta_M) dt + \sigma_M dz_M^Q$$

- Specify firm asset value dynamics via CAPM (market plus idiosyncratic risks):

$$\frac{dA_i}{A_i} = (r - \delta_i) dt + \beta_i \sigma_M dz_M^Q + \sigma_i dz_i^Q$$

Note: total variance is sum of market variance plus idiosyncratic variance

$$v_i^2 = (\beta_i \sigma_M)^2 + \sigma_i^2$$

- Default occurs if $A(t) \leq B$ for $t < T$
- From Black/Scholes/Merton, to determine CDS spread, only need to know v^2
 - To determine CDX index spread on 2 (or 125) identical firms, only need to know v^2
- Consider insurance contract (\sim CDX tranches) that pays iff exactly 1 firm defaults
 - If $v^2 = (\beta \sigma_M)^2$, returns perfectly correlated: either zero firms or all firms will default
 - value of insurance on exactly one default is zero
 - If $v^2 > (\beta \sigma_M)^2$, returns are imperfectly correlated: a single default is possible
 - value of insurance on exactly one default is positive

Coval, Jurek and Stafford (CJS, 2009)

- ▶ Model Specification (\sim standard copula with Option-implied market factor)
 - ▶ Estimate 5-year state prices using 5-year SP500 option prices (\sim local vol model)
 - ▶ Specify idiosyncratic risk as Gaussian diffusion
 - ▶ Calibrate model to match the 5-year CDX index spread
 - ▶ Have only 5-year state prices; estimating PV[CF's] (0-5 years)
- ▶ Findings: Observed spreads on
 - ▶ equity tranche too high compared to model predictions
 - ▶ other tranches (except super-senior) too low compared to model predictions

	0-3%	3-7%	7-10%	10-15%	15-30%	30-100%
data	1472	135	37	17	8	4
CJS	914	267	150	87	28	1

- ▶ Interpretation:
 - ▶ sellers of insurance on senior tranches naive:
 - ▶ focused on high credit ratings/low probability of payout
 - ▶ did not properly account for the level of systematic risk exposure

Our Approach

► Methodology:

- Specify several (jump-diffusion-SV) structural model for both market (S&P500) and individual (CDX) firm dynamics.
- Price options (closed-form) and tranches (Monte-carlo simulations).
- Calibrate market dynamics to match **all maturities** and strikes of SP500 options.
- Calibrate idiosyncratic dynamics to match all maturities of CDX index spreads.
- Calibrate to beta and total variance (estimated from CRSP/Compustat for constituents of CDX index).

► Main Findings:

- Spread on super-senior tranche too far out of the money to estimate using option prices
- Taking Super Senior spreads as input, other tranche spreads well estimated by any model

► Interpretation:

- sellers of insurance on senior tranches sophisticated:
 - Required fair (relative) compensation for risks involved
 - May have enjoyed the “window dressing” associated with highly rated securities (\sim rating ‘arbitrage’).

A structural model for pricing long-dated S&P500 options

- ▶ The market model is the **Stochastic Volatility Common Jump (SVCJ)** model of Broadie, Chernov, Johannes (2009):

$$\frac{dM_t}{M_t} = (r - \delta) dt + \sqrt{V_t} dw_1^Q + (e^y - 1) dq - \bar{\mu}_y \lambda^Q dt + (e^{y_c} - 1) (dq_c - \lambda_c^Q dt)$$

$$dV_t = \kappa_V (\bar{V} - V_t) dt + \sigma_V \sqrt{V_t} (\rho dw_1^Q + \sqrt{1 - \rho^2} dw_2^Q) + y_V dq$$

$$d\delta_t = \kappa_\delta (\bar{\delta} - \delta_t) dt + \sigma_\delta \sqrt{V_t} (\rho_1 dw_1^Q + \rho_2 dw_2^Q + \sqrt{1 - \rho_1^2 - \rho_2^2} dw_3^Q) + y_\delta dq.$$

- ▶ We add stochastic dividend yield (**SVDCJ**) to help fit long-dated options as well.
- ▶ The parameters of the model are calibrated to 5-year index option prices obtained from CJS.
- ▶ State variables are extracted given parameters from time-series of short maturity options (obtained from OptionMetrics).
- ▶ Advantage of using structural model: Arbitrage-free extrapolation into lower strikes (needed for senior tranches).

A structural model of individual firm's default

- ▶ Given market dynamics, we assume individual firm i dynamics:

$$\begin{aligned} \frac{dA_i(t)}{A_i(t)} + \delta_A dt - rdt &= \beta_i \left(\sqrt{V_t} dw_1^Q + (e^y - 1) dq - \bar{\mu}_y \lambda^Q dt \right) + \sigma_i dw_i \\ &\quad + (e^{y_C} - 1) (dq_C - \lambda_C^Q dt) + (e^{y_i} - 1) (dq_i - \lambda_i^Q dt). \end{aligned}$$

- ▶ Note

- ▶ β : exposure to market excess return (i.e., systematic diffusion **and** jumps).
- ▶ dq_C : 'catastrophic' market wide jumps.
- ▶ dq_i : idiosyncratic firm specific jumps.
- ▶ dw_i : idiosyncratic diffusion risks.
- ▶ Default occurs the first time firm value falls below a default barrier B_i (Black (1976)):

$$\tau_i = \inf\{t : A_i(t) \leq B_i\}. \quad (1)$$

- ▶ Recovery upon default is a fraction $(1 - \ell)$ of the remaining asset value.

Pricing of the CDX index via Monte-Carlo

- ▶ The running spread on the CDX index is closely related to a weighted average of CDS spreads.
- ▶ Determined such that the present value of the **protection leg** ($V_{idx, prot}$) equals the PV of the **premium leg** ($V_{idx, prem}$):

$$V_{idx, prem}(S) = S E \left[\sum_{m=1}^M e^{-rt_m} (1 - n(t_m)) \Delta + \int_{t_{m-1}}^{t_m} du e^{-ru} (u - t_{m-1}) dn_u \right]$$

$$V_{idx, prot} = E \left[\int_0^T e^{-rt} dL_t \right].$$

- ▶ We have defined:
 - ▶ The (percentage) defaulted notional in the portfolio: $n(t) = \frac{1}{N} \sum_i \mathbf{1}_{\{\tau_i \leq t\}}$,
 - ▶ The cumulative (percentage) loss in the portfolio: $L(t) = \frac{1}{N} \sum_i \mathbf{1}_{\{\tau_i \leq t\}} (1 - R_i(\tau_i))$

Pricing of the CDX Tranches via Monte-Carlo

- ▶ The tranche loss as a function of portfolio loss is

$$T_j(L(t)) = \max [L(t) - K_{j-1}, 0] - \max [L(t) - K_j, 0] .$$

- ▶ The initial value of the protection leg on tranche- j is

$$Prot_j(0, T) = E^Q \left[\int_0^T e^{-rt} dT_j(L(t)) \right]$$

- ▶ For a tranche spread S_j , the initial value of the premium leg on tranche- j is

$$Prem_j(0, T) = S_j E^Q \left[\sum_{m=1}^M e^{-rt_m} \int_{t_{m-1}}^{t_m} du (K_j - K_{j-1} - T_j(L(u))) \right] .$$

- ▶ Appropriate modifications to the cash-flows
 - ▶ Equity tranche (upfront payment),
 - ▶ Super-senior tranche (recovery accounting).

Calibration of firms' asset value processes

- ▶ Calibrate 7 (unlevered) asset value parameters ($\beta, \sigma, B, \lambda_1, \lambda_2, \lambda_3, \lambda_4$) to match median CDX-series firm's:
 - ▶ Market beta
 - ▶ Idiosyncratic risk (estimated from rolling regressions for CDX series constituents using CRSP-Compustat)
 - ▶ Term structure of CDX spreads (1 to 5 year)
- ▶ Set jump size to -2 (\sim jump to default).
- ▶ When present, calibrate catastrophic jump intensity to match super-senior ($\lambda_C < 1$ event per 1000 years).
- ▶ Set loss given default $1 - \ell$ to 40% (\sim match historical average) in normal times.
- ▶ Set $1 - \ell = 20\%$ if catastrophe jump occurs (\sim Altman et al.).
- ▶ Market volatility, jump-risk, dividend-yield all estimated from S&P500 option data in previous step.

Average tranche spreads predicted for pre-crisis period

- ▶ We report six tranche spreads averaged over the pre-crisis period Sep 04 - Sep 07:
 - ▶ The historical values;
 - ▶ Benchmark model: Catastrophic jumps calibrated to match the super-senior tranche; Idiosyncratic jumps and default boundary calibrated to match the 1 to 5 year CDX index.
 - ▶ $\lambda_C^Q = 0$: No catastrophic jumps; Idiosyncratic jumps and default boundary calibrated to match 1 to 5 year CDX index;
 - ▶ $\lambda_i^Q = 0$: Catastrophic jumps calibrated to match the super-senior tranche; No idiosyncratic jumps; Default boundary calibrated to match only the 5Y CDX index.
 - ▶ $\lambda_C^Q = 0, \lambda_i^Q = 0$: No catastrophic jumps; No idiosyncratic jumps; Default boundary calibrated to match only the 5Y CDX index;
 - ▶ The results reported by CJS

	0-3%	3-7%	7-10%	10-15%	15-30%	30-100%	0-3% Upftr
data	1472	135	37	17	8	4	0.34
benchmark	1449	113	25	13	8	4	0.33
$\lambda_C^Q = 0$	1669	133	21	6	1	0	0.40
$\lambda_i^Q = 0$	1077	206	70	32	12	4	0.22
$\lambda_C^Q = 0, \lambda_i^Q = 0$	1184	238	79	31	6	0	0.26
CJS	914	267	150	87	28	1	na
$\frac{CJS - Data}{Benchmark - Data}$	24.3	6	9.4	17.5	∞	∞	

Interpretation

- ▶ Errors are an order of magnitude smaller than those reported by CJS.
- ▶ However, model without jumps ($\lambda_C^Q = 0$, $\lambda_i^Q = 0$) generates similar predictions to CJS.
- ▶ Why? Problem is two-fold:
 - ▶ **Backloading** of defaults in standard diffusion model:

Average CDX index spreads for different models					
	1 year	2 year	3 year	4 year	5 year
Data	13	20	28	36	45
Benchmark	13	20	28	36	45
$\lambda_C^Q = 0$	13	20	28	36	45
$\lambda_i^Q = 0$	6	7	16	29	45
$(\lambda_C^Q = 0, \lambda_i^Q = 0)$	0	3	13	28	45

- ▶ Idiosyncratic jumps generates a five-year loss distribution that is **more peaked** around the risk-neutral expected losses of 2.4%.
(loss distribution with $\lambda_C^Q = 0$, $\lambda_i^Q = 0$ has std dev of 2.9%, whereas loss distribution with $(\lambda_i^Q > 0, \lambda_C^Q = 0)$ has std dev of 1.7%).

More Generally....

- ▶ We claim that if:
 - ▶ Take any “reasonable” dynamic model of market returns to match SP500 option prices
 - ▶ Specify idiosyncratic dynamics as a diffusion process
 - ▶ Calibrate the model to match the 5-year CDX index
- ▶ Then model will generate:
 - ▶ **Short term credit spreads that are well below observed levels**
 - ▶ Tranche spreads similar to those found by CJS

	1 year	2 year	3 year	4 year	5 year
data	13	20	28	36	45
$E^Q[\#def]$	0.27	0.83	1.75	3.00	4.69
our model	0	3	13	28	45
SVCJ	0	3	14	29	45
Heston	0	2	12	28	45
$E^Q[\#def]$	0.01	0.13	0.81	2.33	4.69

More Generally....

- ▶ We claim that if:
 - ▶ Take any “reasonable” dynamic model of market returns to match SP500 option prices
 - ▶ Specify idiosyncratic dynamics as a diffusion process
 - ▶ Calibrate the model to match the 5-year CDX index
- ▶ Then model will generate:
 - ▶ Short term credit spreads that are well below observed levels
 - ▶ **Tranche spreads similar to those found by CJS**

	0-3% Upftr	0-3%	3-7%	7-10%	10-15%	15-30%	30-100%
data	0.34	1472	135	37	17	8	4
our model	0.26	1184	238	79	31	6	0
SVCJ	0.22	1078	243	96	44	11	0
Heston	0.23	1097	230	83	39	10	0
CJS	na	914	267	150	87	28	1

Intuition for Findings

- ▶ Diffusion-based structural models can't explain short maturity spreads for IG debt
 - ▶ Some level of jumps captured in market dynamics implied from options
 - ▶ However, most risk at individual firm level is idiosyncratic
 - ▶ Need to specify idiosyncratic dynamics with jumps to capture short term spreads
- ▶ By calibrating model to 5Y CDX index, all models agree on 5Y expected loss
- ▶ By calibrating model to observed term structure of spreads, defaults occur earlier
 - ▶ eliminate “backloading” of defaults
 - ▶ crucial for pricing equity tranche spreads
 - ▶ first default associated with $\approx 16\%$ drop in insurance premium payments
 - ▶ timing of defaults so crucial that equity tranche typically priced with an up-front premium
 - ▶ Agents willing to pay more initially if future payments expected to drop more quickly
 - ▶ “Backloading” biases equity tranche spreads downward
 - ▶ Downward bias on equity tranche generates an upward bias on senior tranches
- ▶ In addition, calibrating model to short maturity spreads increases proportion of idiosyncratic risk to systematic risk
 - ▶ Tends to make loss distribution more peaked
 - ▶ Also tends to increase spreads on equity tranche/decrease spreads on senior tranches

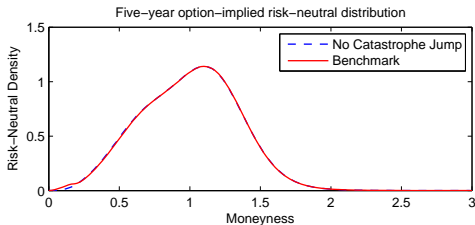
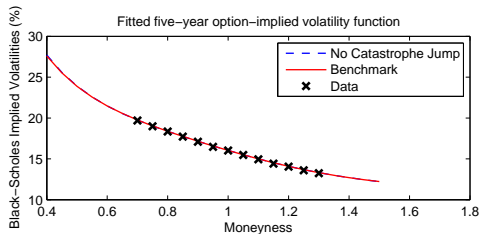
Calibrating Model to Term Structure of CDX Index Spreads

- ▶ When models are calibrated to match short term credit spreads, the results of CJS disappear, and sometimes are even reversed!!
- ▶ Predicted super-senior tranche spreads ≈ 0

	0-3% Upftr	0-3%	3-7%	7-10%	10-15%	15-30%	30-100%
data	0.34	1472	135	37	17	8	4
our model	0.40	1669	133	21	6	1	0
SVCJ	0.35	1505	166	45	19	4	0
Heston	0.34	1500	157	42	18	5	0

Calibrating Model to Term Structure of CDX Index Spreads and SS Spread

- ▶ However, can add a “catastrophic jump” to market dynamics
 - ▶ Rietz (1988), Barro (2006)
 - ▶ has negligible impact on observed option prices
 - ▶ has large impact on SS spreads.



Five-year option-implied risk-neutral distribution

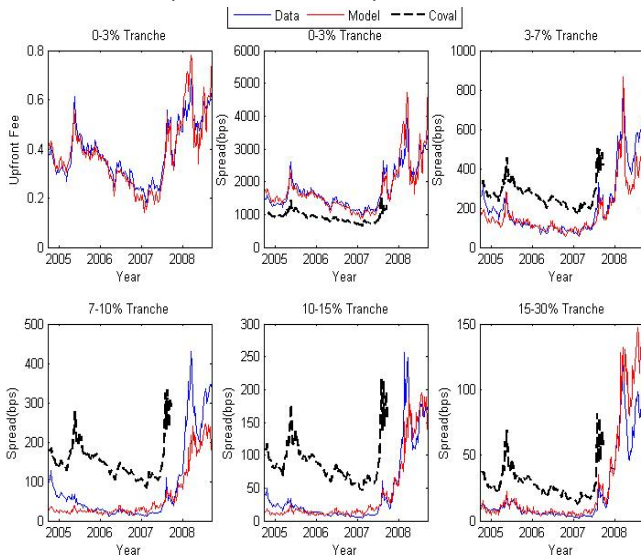
Calibrating Model to Term Structure of CDX Index Spreads and SS Spread

- ▶ However, can add a “catastrophic jump” to market dynamics
 - ▶ Rietz (1988), Barro (2006)
 - ▶ has negligible impact on observed option prices
 - ▶ has large impact on SS spreads.
 - ▶ Can improve fit further by taking tranche spreads in-sample
 - ▶ Mortensen (2006), Longstaff and Rajan (2008), Eckner (2009)

	0-3% Upftr	0-3%	3-7%	7-10%	10-15%	15-30%	30-100%
data	0.34	1472	135	37	17	8	4
our model	0.33	1449	113	25	13	8	4
SVCJ	0.30	1330	138	47	26	12	4
Heston	0.29	1301	142	46	24	12	4
CJS	na	914	267	150	87	28	1

Time Series Performance

- Model fits data well, both pre-crisis and crisis periods



Conclusion

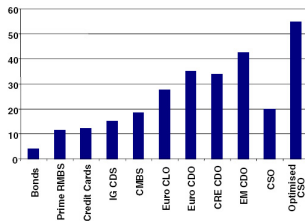
- ▶ CF's associated with CDX tranche spreads occur throughout 5 year horizon
 - ▶ need dynamic model of market and idiosyncratic dynamics to price consistently
- ▶ Market dynamics (mostly) extracted from option prices
- ▶ idiosyncratic dynamics extracted from term structure of credit spreads
 - ▶ need idiosyncratic jumps to explain short maturity spreads
- ▶ without these jumps:
 - ▶ default events are “backloaded”
 - ▶ ratio of idiosyncratic to market risk is off
 - ▶ CDX equity tranche spreads biased downward
 - ▶ CDX senior tranche spreads biased upward
- ▶ Super senior tranche spreads cannot be estimated via extrapolation
 - ▶ Instead, need to take them as input
 - ▶ Other tranche spreads well-predicted by any model that also matches option prices, CDS spreads
- ▶ Calibrating model to term structure of credit spreads imposes more structure/ less freedom
 - ▶ We used “HJM approach”
 - ▶ More consistently, can add state variables driving idiosyncratic jump processes

Are senior tranches priced inefficiently by naive investors?

- ▶ Investors care only about expected losses (\sim ratings) and not about covariance (ironic since they trade in correlation markets!).

⇒ Spreads across AAA assets should be equalized. Are they?

AAA spreads by asset
5y generic, bp



Source: Citi

- ⇒ All spreads should converge to **Physical** measure expected loss.
 - ▶ We observe large risk-premium across the board ($\lambda^Q/\lambda^P > 6$.)
 - ▶ Large time-variation in that risk-premium.
- ⇒ Time-variation in spreads should be similar to that of rating changes (smoother?).
 - ▶ Evidence seems inconsistent with marginal price setters caring only about expected loss (\sim ratings).

What drives differences between structured AAA spreads?

- ▶ 'Reaching for yield' by rating constrained investors who **want** to take more risk because their incentives (limited liability) and **can** because ratings simply do not reflect risk and/or expected loss.
- ▶ Taking more risk by loading on systematic risk was the name of the game (agency conflicts).
- ▶ Possible that excess 'liquidity'/leverage lead to spreads being 'too' narrow **in all** markets, but little evidence that markets were ex-ante mis-priced on a relative basis.
- ▶ Ex-post (during the crisis) other issues, such as availability of collateral and funding costs, seem more relevant to explain cross-section of spreads across markets.
- ▶ Indeed, how to explain negative and persistent:
 - ▶ swap spreads?
 - ▶ cds basis?