UBS

Modelling Counterparty Exposure and CVA
An Integrated Approach

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- **Basic Concepts**
- CVA Computation
- Underlying Models
- Modelling Framework: AMC
- CVA: C-CDS approach
- Next Steps

Basic Concepts

Section 1

What is Counterparty Credit Exposure?

Exposure to loss due to failure by a counterparty to perform

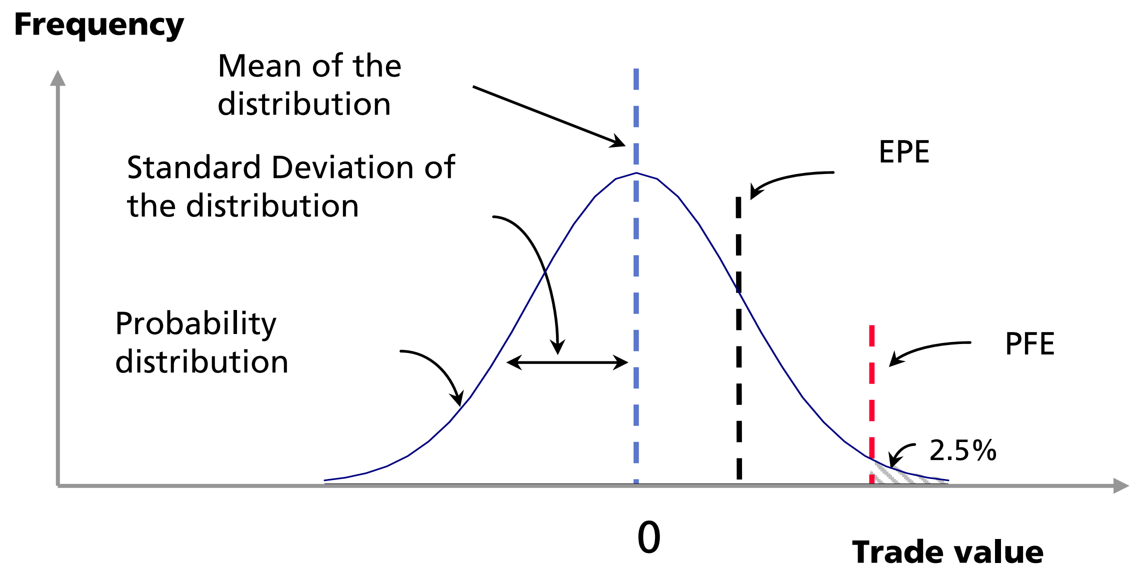
- Counterparty Credit Exposure: exposure to loss due to failure by a counterparty to perform
- Counterparty risk is at the root of traditional banking
  - Historically, the first form of financial instruments were bonds
  - Value driven by the perceived credit worthiness
- Financial transactions typically involves cash flows to other institutions or individual
- If any of these counterparty should fail to fulfill their obligation there will be a replacement cost incurred
- Take-and-hold exposure
  - Lending products – loans, commitments
  - Trading products – OTC products / SFTs

We focus on OTC!

Typical Counterparty Exposure Risk Measures

PFE and EPE are the key statistical measures

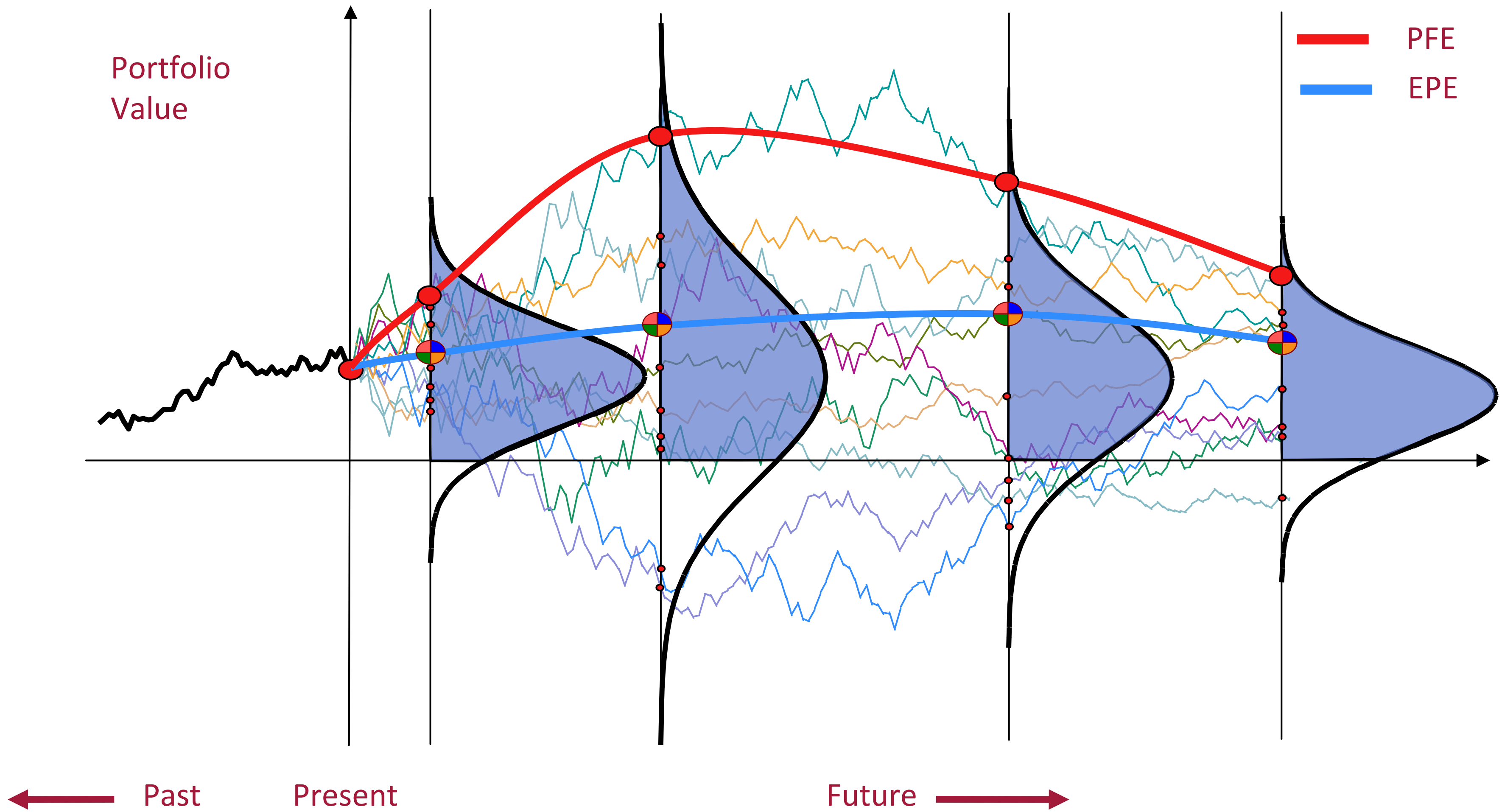
- Compute price distributions at different times in the future
- Statistical measures are then calculated on this price distribution
  - Potential Future Exposure (PFE), usually a quantile measure at 97.5% or 99%
  - Expected Positive Exposure (EPE), the mean of the positive part of the distribution
  - Mean Exposure



We will see that these measures have different meanings depending on the context

Computing Exposure by Simulation

Example: Vanilla Swap



What is CVA?

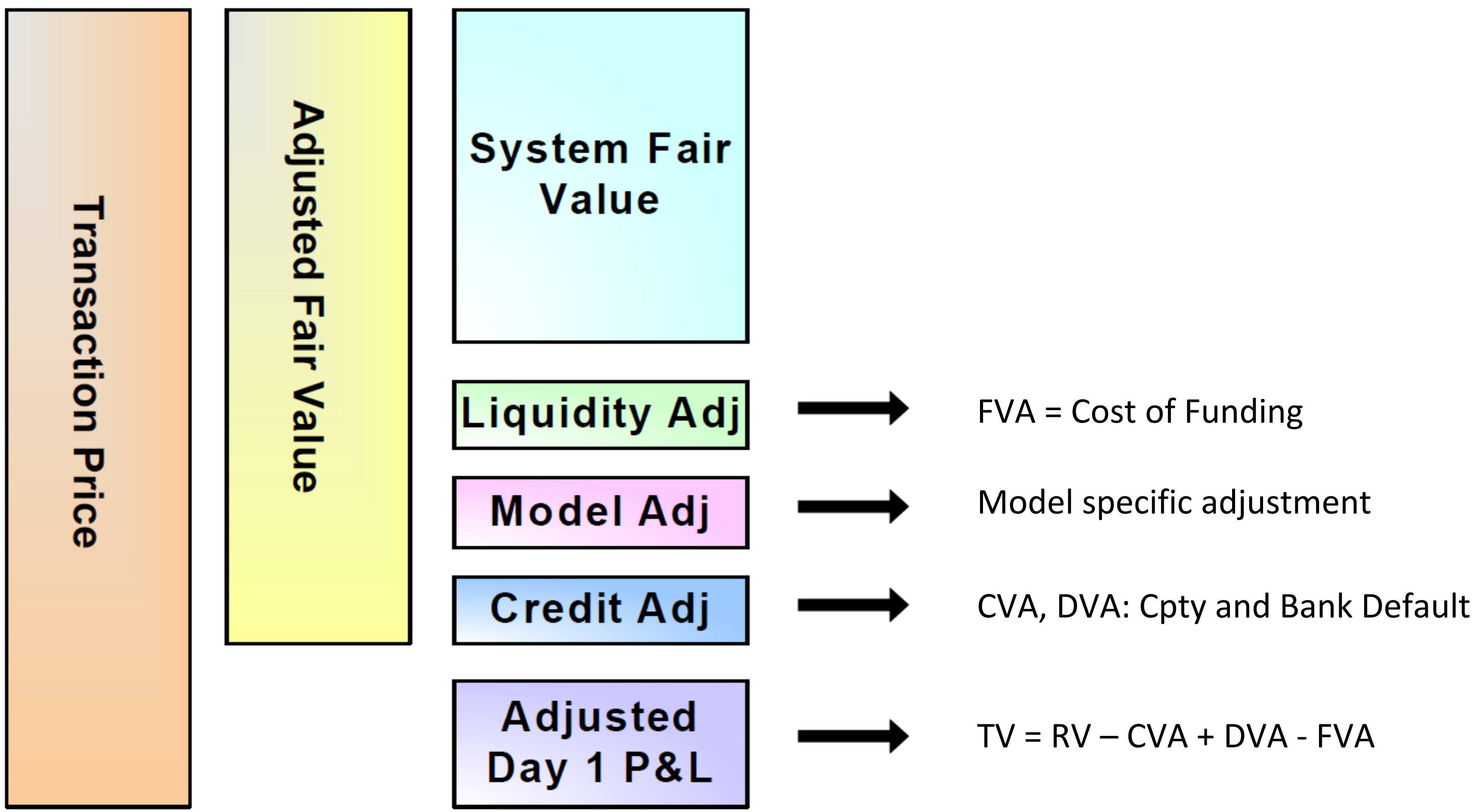
Counterparty exposure from a pricing perspective

- CVA - Credit Value Adjustment
- It is the price of counterparty credit exposure
- It is an adjustment to the price of a derivative to take into account counterparty credit exposure
  - It is not the only adjustment that we need to make however…

Risk Free Derivative = **Risky Derivative** + **CVA**

Fair Value of a Financial Instrument

There are several adjustments required to adjust Mark-To-Market value



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CVA Computation

Section 2

CVA Computation

CVA is a pricing measure: some details

- In case of default at time τ we pay the positive part of the value of the portfolio Max[V,0]

$$\mathrm{Payoff}_t = \underbrace{(1 - R_V)}_{\text{Recovery on portfolio}} \; \underbrace{\mathbb{1}_{t=\tau}}_{\text{We pay if a default occurs}} \; V_\tau^+$$

Positive part of portfolio value

Recovery on portfolio

We pay if a default occurs

τ is the default time

t< T (maturity)

- Pricing is done via Risk Neutral Valuation

$$\mathrm{CVA}_{0,T} \;=\; (1 - R_V) \int_0^T \mathbb{E}\left[N_u^{-1} V_u^+ \; \mathbb{1}_{\tau=u} \; du\right]$$

Integral: we sum over all possible time intervals

Expectation is in the measure N

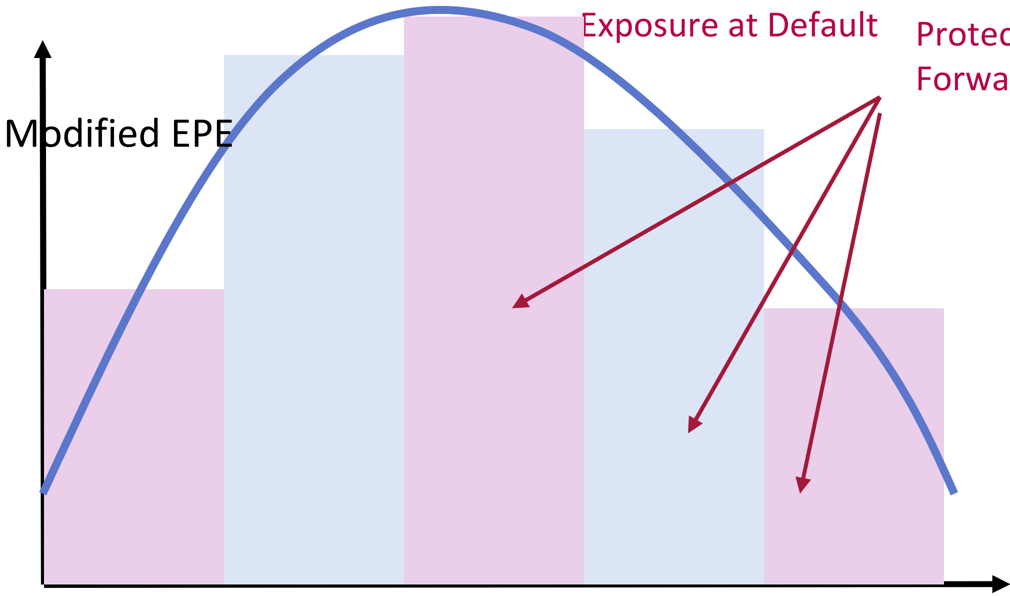
Numeraire: Risk neutral discounting

CVA Computation

The EPE x Spread approach

- We can now discretize the interval to compute the integral and assume spread constant over the interval: this approach has some deficiencies

$$\begin{aligned} \mathrm{CVA}_{0,T} &= \frac{1-R_V}{1-R}\int_0^T \mathrm{EPE}_u^{\mathrm{mod}}\, d\mathrm{CDS}_u \\ &\approx \frac{1-R_V}{1-R}\sum_i \underbrace{\mathrm{EPE}_{T_{i-1},T_i}^{\mathrm{mod}}}_{\text{Exposure at Default}}\ \underbrace{\mathrm{CDS}_{T_{i-1},T_i}}_{\text{Protection Leg of Forward starting CDS}} \end{aligned}$$



Expectation in the measure N

Exposure at de

$$\mathrm{EPE}_u^{mod} := D_{0,u}^{-1}\,\mathbb{E}\underbrace{\left[N_u^{-1} V_u^+ \middle| \tau = u\right]}_{\text{Discounted exposure}}$$

CVA vs Counterparty Exposure: Fundamental Differences

Both compute price distributions at different times in the future, but…

- Counterparty Exposure
  - Statistical measures
  - Potential Future Exposure (PFE), usually a quantile measure at 97.5% or 99%
  - Expected Positive Exposure (EPE), the mean of the positive part of the distribution
  - PFE is used against limits
  - EPE is used for RWA and capital

- CVA
  - CVA is the cost of buying protection on the counterparty that pays the portfolio value in case of default
  - Expected Positive Exposure (EPE), the expected value under the risk neutral measure
  - It is now a considerable part of the PnL of any financial institution
  - Needs to be hedged
  - Enters in VaR

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Underlying Models

Section 3

Set-Up

- Computation of counterparty credit exposure and of CVA for portfolio of OTC transactions, including both vanillas and exotics
  - Interest Rate Swaps and Cross Currency Swaps
  - Exotic interest rate products, CMS, steepener
  - Exotic options on equity, FX, commodities
  - Credit Default Swaps, CDO

- Models need to be
  - Scenario consistent across products
  - Powerful enough to deal with exotic transactions
  - Powerful enough to be used for pricing and hedging: CVA computation

- The framework needs to be
  - Flexible enough to deal with different types of products, booked and priced in different system

- Models and framework need to be able to
  - Take into account collateral and cost of collateral
  - Possibly be extended to consider other aspects e.g. cost of funding

Choice of Models

Underlying simulations

- Risk Models
  - Physical measure
  - Simulations are **not** (necessarily) used for pricing
  - Calibration with historical values
  - Conservative measures
  - Portfolio view
  - Scenario consistency across asset classes
  - Future price distributions
  - Very large book of transactions

- Pricing Models: TV
  - Pricing measure (risk neutral)
  - Simulations are used for pricing (Monte Carlo pricing)
  - Calibration with market instruments
  - Focus on accuracy
  - Each product can be priced in isolation
  - Hedging

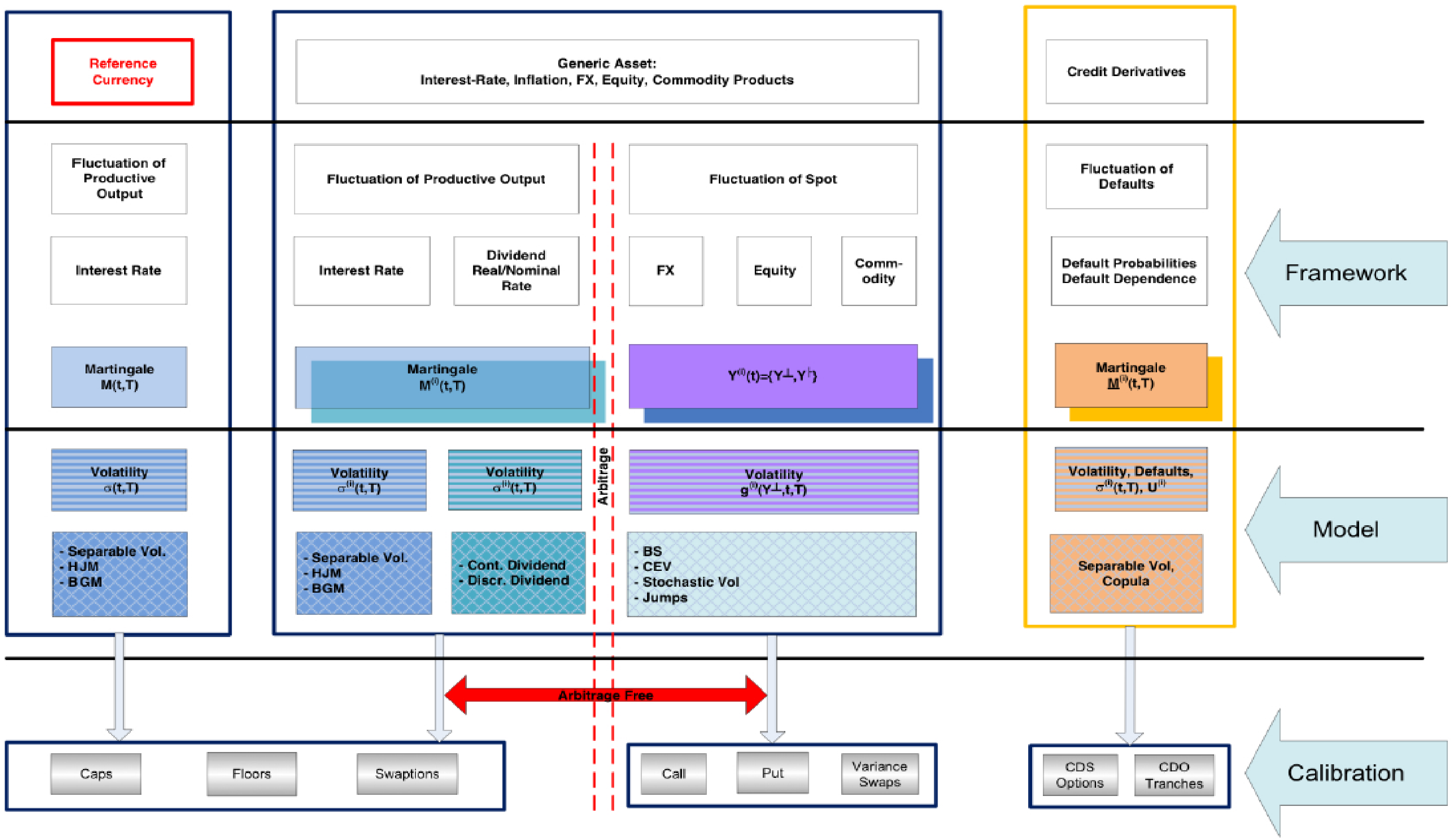
CVA Models

- Scenario consistency
- Future price distributions
- Portfolio view
- Very large book of transactions

- Pricing measure
- Simulations are used for pricing
- Calibration with market instruments
- Focus on accuracy
- Hedging

Model Roadmap

Model Roadmap



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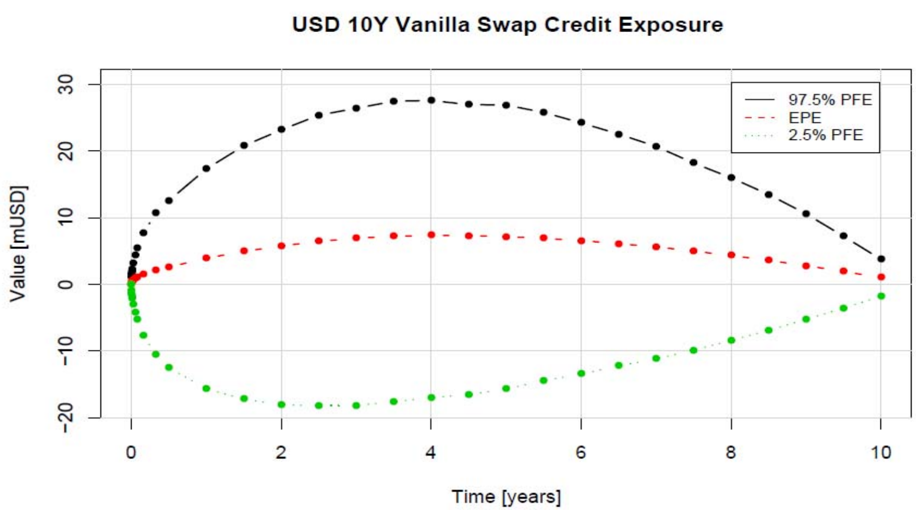
- CVA: C-CDS approach
- Other Applications

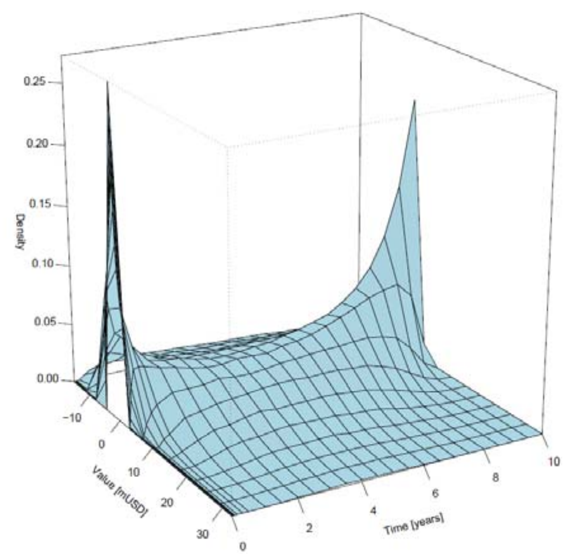
Modelling Framework: AMC

Section 4

Typical Counterparty Exposure Profile

Vanilla Interest Rate Swap



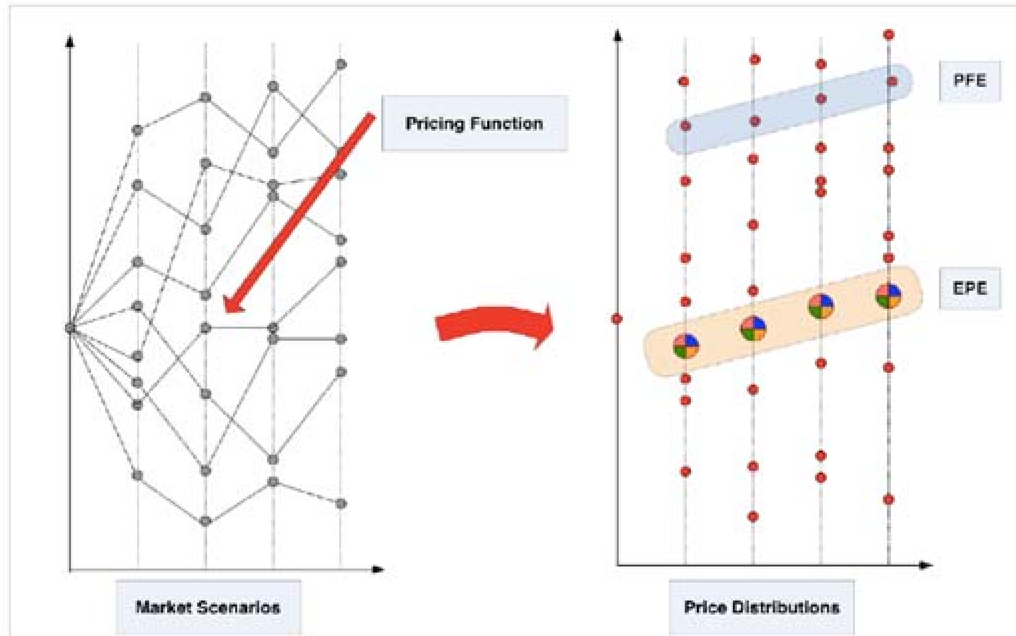


- Consider an interest rate swap
  - We receive the 6 month Libor rate on a notional of $100 million
  - We pay a fixed rate equal to the par 10 year swap rate
- The swap contract has zero value at inception
- As time passes and market condition changes accordingly
  - If the swap rate decreases, the transaction will be out of the money
  - If the swap rate increases, the transaction will be in the money to us and if the counterparty defaults, this is a mark-to-market credit loss to us
- As time passes, the amount of payments decreases and hence we have less exposure

Recipe for Computing Credit Exposure

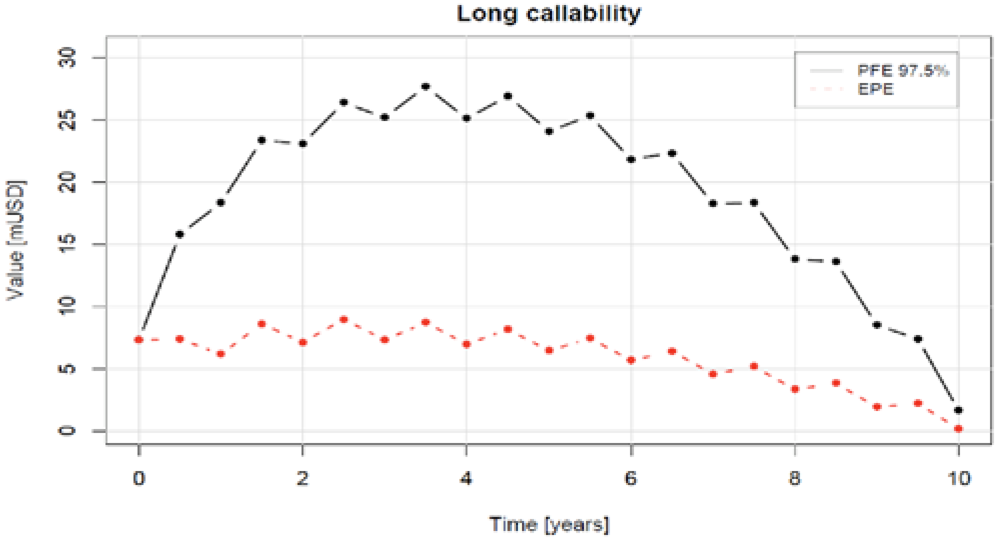
At the highest level, all credit exposure systems

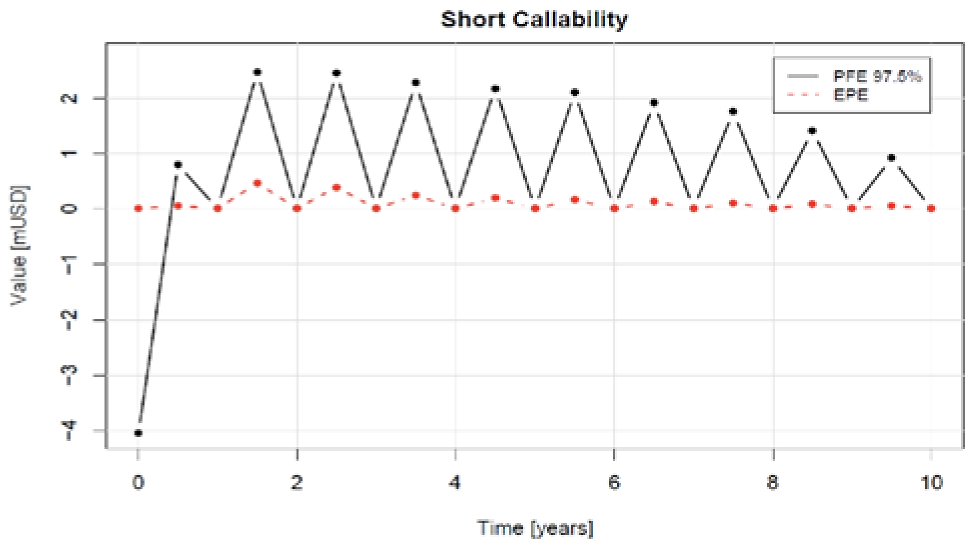
- Scenario Generation
  - Generate the scenario from a model, calibrated using the latest market data
- Pricing
  - Price the instruments on each scenario in the future
- Aggregation
  - Add up all the prices of each product at each scenario and each time point



Challenge to the Monte Carlo Approach

Products with embedded optionality





- Now suppose that we have the option to cancel a trade at no cost
  - We are *long callability*
  - Conversely, we are *short callability* if the other side can cancel a trade at no cost
- We would walk away from the trade if the mark-to-market value of the swap plus the option is negative
  - The profile is similar to a normal swap, except the starting point is the value of the option
- From a computational point of view, there is a fundamental difference between vanilla swap and this embedded optionality
  - Vanilla swaps can be priced off the yield curve, while the Bermudan swap requires a model to value

Other Challenges…

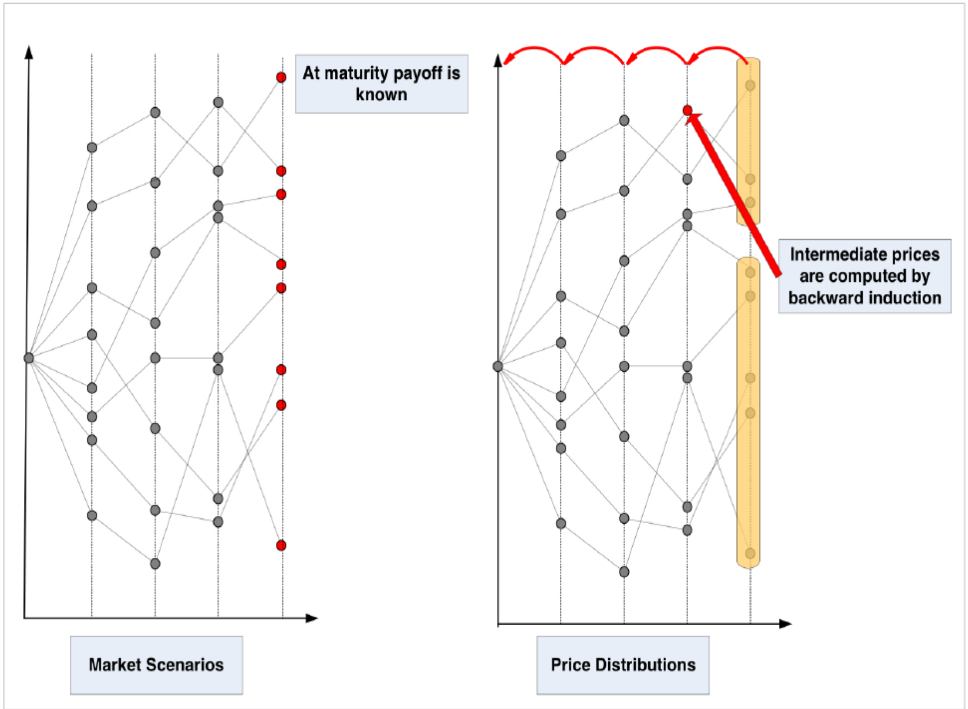
- The Monte Carlo framework seems to give a good implementation recipe. In practice, there are issues that needs to be addressed
- The generation of correlated scenarios is not trivial, potentially thousands of different risk factors driving the dynamics of different and often complex products
- The scenarios have to be consistent across all systems to build a counterparty view
  - This is the key issue with the current generation of front office systems, it is not designed with this in mind
  - Need the same family of underlying models for all product types, same numeraire
- Pricing functions developed in various libraries are not necessary designed to be integrated in a counterparty exposure framework.
  - This has implications from both a software and architecture prospective
- Not all products can be computed in analytic form. Most exotics are priced either using PDE or Monte Carlo approaches

Need of an alternative approach!

American Monte Carlo

AMC neatly resolves the problem of pricing and exposure calculation in one step

- The basic idea is to approach the counterparty exposure as a pricing problem and thus use pricing algorithms



- American Monte Carlo algorithm
  - Instead of building a price moving *forward* in time
  - Starts from maturity, where the value of the product is known and goes *backward*
- AMC is used in general for products with Callability
  - Products whose value depends a strategy which can only be determined by only knowing future states of the world
  - The benefit of this approach is that a price distribution is also provided
- The algorithm is generic an hence only the payoff is required

The Credit Exposure Problem

Defining a product with early exercise features

- Suppose that we have a generic product with early-exercise features, which we denote by P. The holder is entitled to cash flows X
  - Apart from X, P also gives the holder the replace, at specific points in time, to a post-exercise portfolio Q. Write the set of possible exercise time as

$$\mathscr{T} := \{\tau_1, \tau_2, \ldots, \tau_{n_E}\} \cup \{\infty\}$$

  - If exercise happens at maturity, then the value of the trade is provided by P and is embodied in

$$\Pi_t^{no} = N_t \mathbb{E}\left[\int_t^{\tau_E \wedge T_X} \frac{X_u}{N_u} \, du \,\middle|\, \mathscr{F}_t\right]$$

Numeraire

Expectation in the N-measure

  - The optimality criterion by which the holder chooses the optimal time to exercise the option will be described later

The Credit Exposure Problem

Assuming optimal exercise time, the valuation can be given in two parts

- The price distribution of product P can be given as

$$V_t = \begin{cases} V_t^P, & t < \tau_E^* \\ V_t^Q, & t \geq \tau_E^*, \end{cases}$$

Optimal Exercise Time

- The value prior to exercise is given by

$$V_t^P = N_t \sup_{\tau_E \in \mathscr{T}_t} \left\{ \mathbb{E}\left[\int_t^{\tau_E \wedge T_X} \frac{X_u}{N_u} du \,\middle|\, \mathscr{F}_t \right] + \mathbb{E}\left[\frac{V_{\tau_E}^Q}{N_{\tau_E}} \,\middle|\, \mathscr{F}_t \right] \right\}$$

Numeraire

$\mathscr{T}_t = \{\tau \in \mathscr{T} \mid \tau \geq t\}$

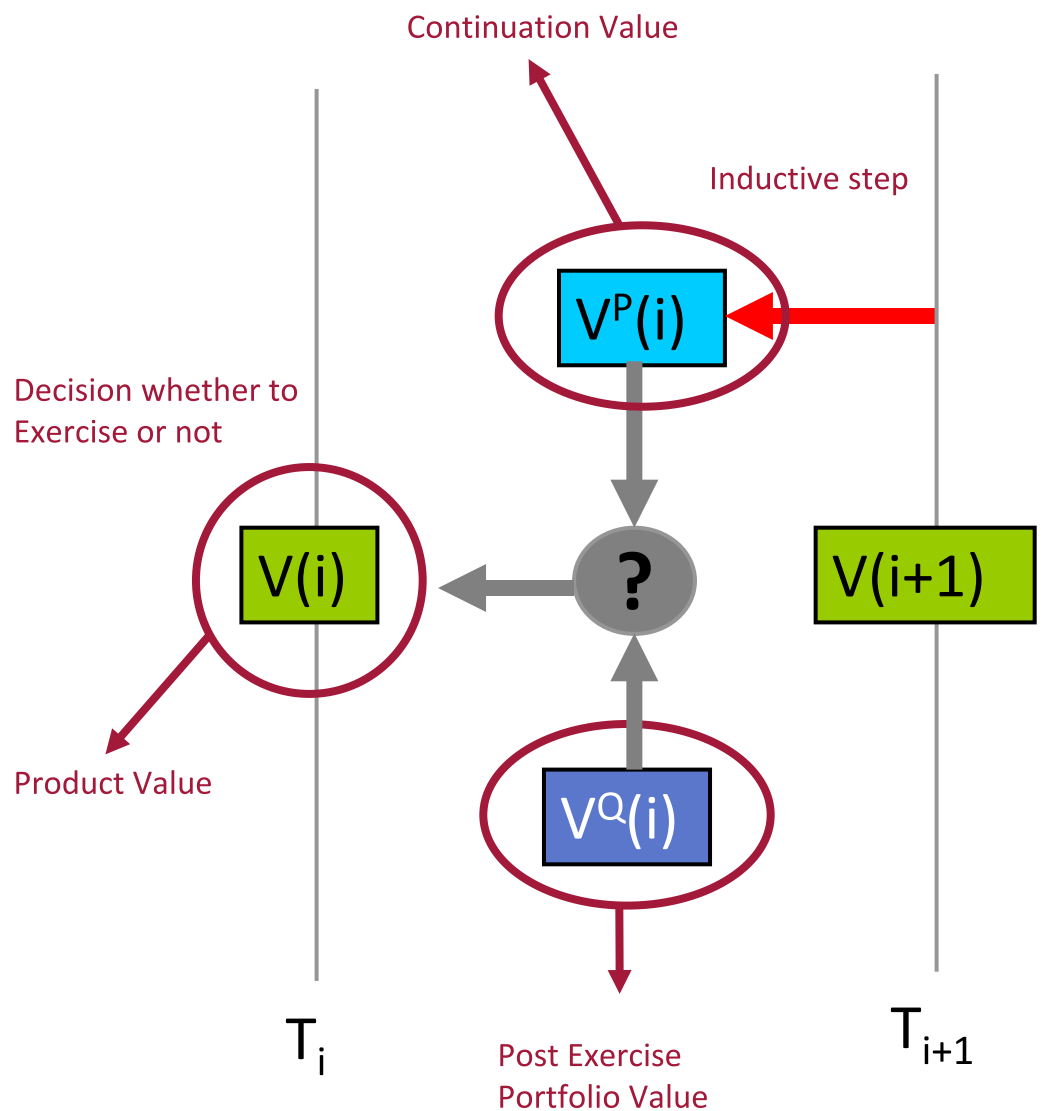
Pre-Exercise Cash Flow Values

Post-Exercise Cash Flow Values

American Monte Carlo

The valuation is done via a recursive procedure

- There are several approaches that may be employed to compute the optimal exercise decision rule
- This involves estimating at each time step at the expected value of *not* exercising, conditional on the current value and the value of the observables
- The key is to estimate the conditional expectations of the product and the post exercise portfolio



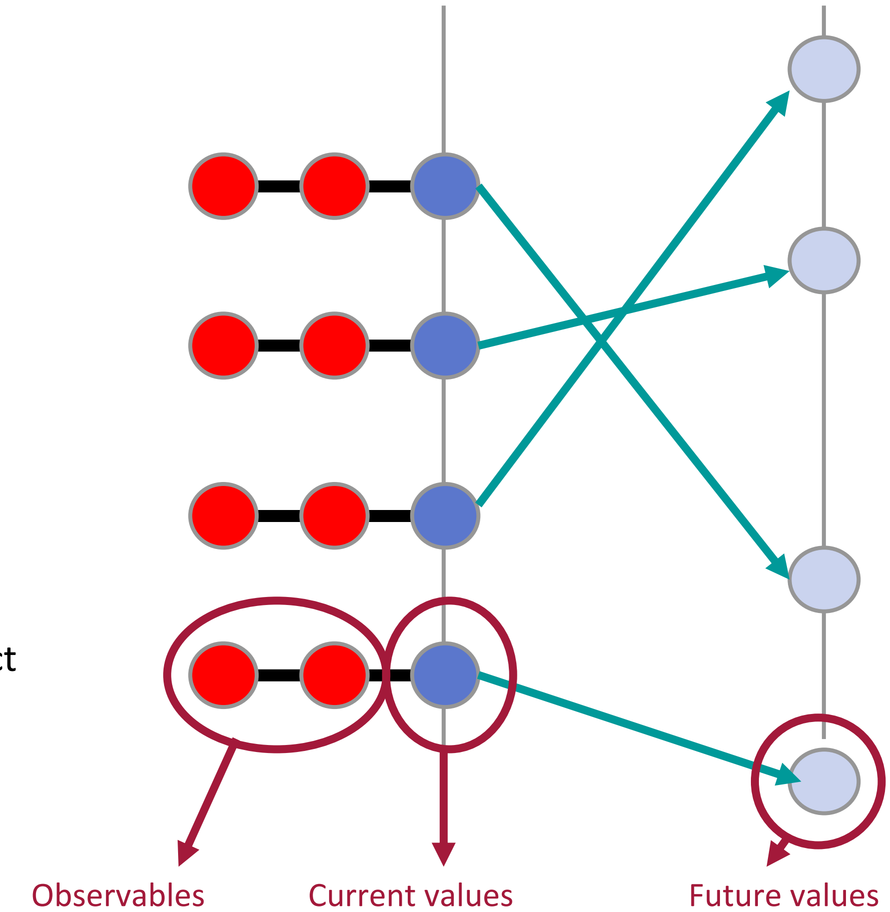
American Monte Carlo

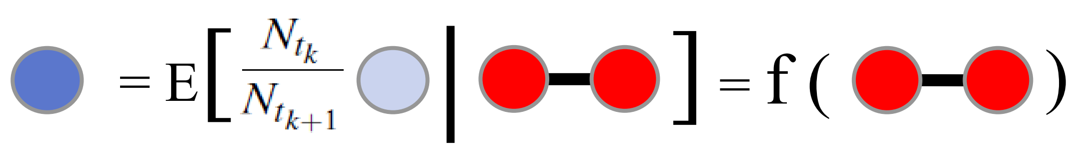
The conditional expectation is estimated using a regression

- The only remaining question is on how to estimate the conditional expectation

- We construct an estimator using a regression on polynomial functions on the observables
  - Regressing the discounted future values against the current observables

- There are many possible basis functions to choose from, our implementation uses polynomials
  - The choice of basis function have very limited impact on the quality of result
  - The choice of the observable itself is important

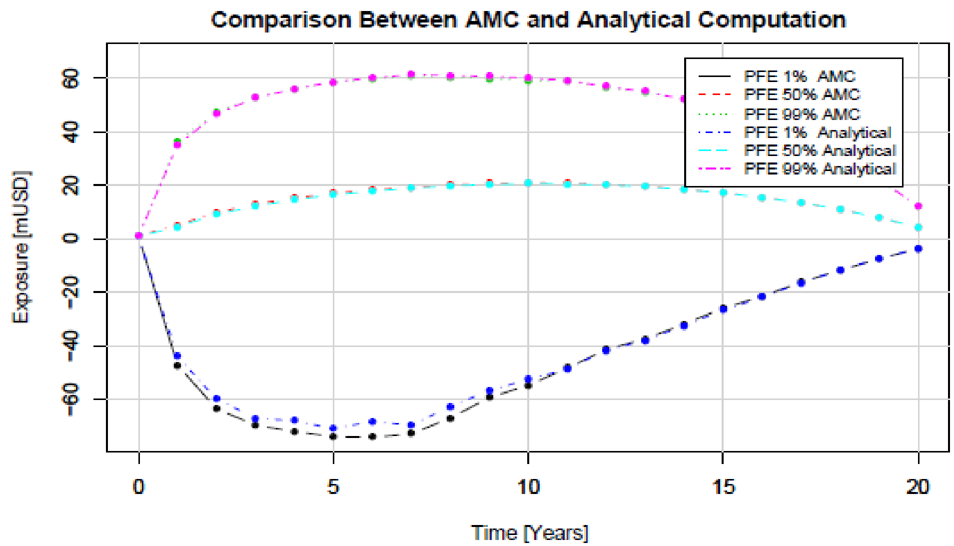




Valuation Errors

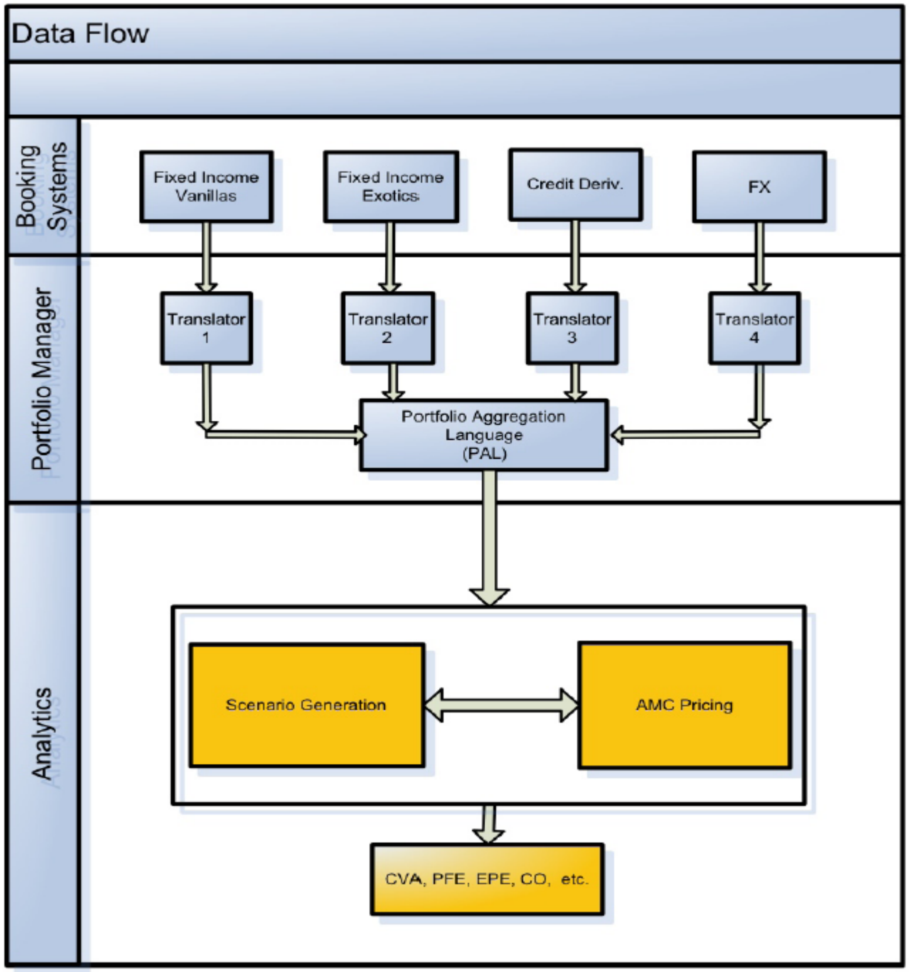
AMC is an approximation

- The price distribution computed via AMC yields an *estimate* of the true price
- Errors can come from the following
  - Choice of observables – As observables are the parameters driving prices, the wrong choice could lead to unreliable result
  - Regression error – The type of regression function and their order could impact the result
  - Bundling – The size of bundling can influence result
- The graph on the right shows the difference in profile for a vanilla interest rate swap
  - We pay floating and receive fixed
  - The EPE is near identical
  - The lower PFE is subject to more numerical noise



High Level Architecture Description

The key idea is to homogenize the booking descriptions and models for the purpose of portfolio evaluation

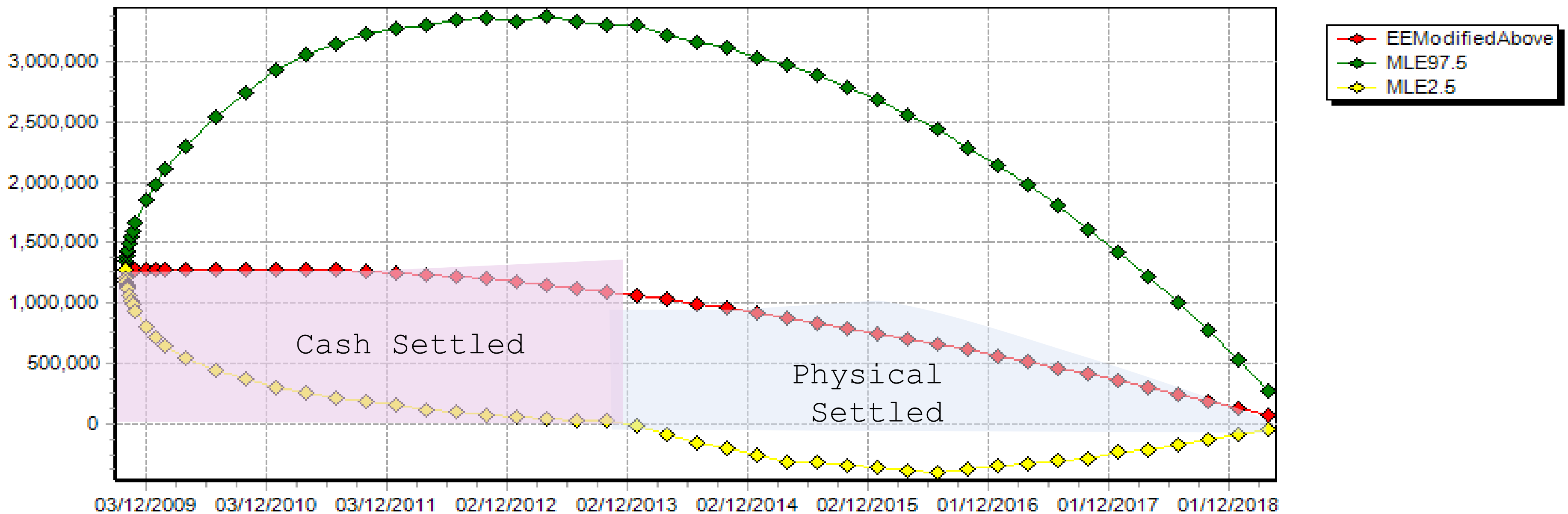


- In order to compute exposure at portfolio level, it is necessary to collect all trades that are booked on different pricing systems
  - Easily compute exposure of trades that usually are described via termsheet
  - Decouple trade description from implementation of analytics
  - Bring trades from existing booking systems into a single unified booking representation

Example 1

A Physically Settled Swaption

```
Notional = 10 mm USD;
Schedule = From 2009/03/31 to 2019/03/31 Every 3 Months;
Swap = Receive (Notional * IR:USD6M * 0.25) USD on Schedule;
Swap += Pay (Notional * 3% * 0.25) USD on Schedule;
Long callable on 2013/03/31 into swap;
```



Example 2

Steepener

```
Notional = 10 mm EUR;
Schedule = from 2009/05/09 to 2029/11/29 Every 6 Months;
Steepener = Receive Notional * (4.84% + 2*Max(0,(1.33%-(EUR 20y - EUR2y)))
            on Schedule;
Steepener += Pay (Notional * EUR 6m) on Schedule;
Long callable every 1 year from 2010/05/21 to 2029/11/21;
```

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CVA: C-CDS Approach

Section 5

CVA Computation

Dynamic EPE - the C-CDS approach

- CVA can be computed as “EPE x Spread”
- In reality, EPE is itself risky: underlying portfolio may have interest rate, FX, credit, equity, inflation risk
  - Portfolio effects might further complicate this: correlation risk
  - EPE is always positive part of portfolio: embedded optionality → volatility risk
- It can be useful to have a view on how CVA can could change during the life of the trade
- Right-Way / Wrong –Way effects might alter CVA pricing and risk / hedging

All these effects are difficult to capture through the traditional “EPE x Spread” approach

CVA Computation

Dynamic EPE - The C-CDS approach

- Rather than seeing CVA as a reserve, see it as the value of a derivative
- We call this derivative a C-CDS - Contingent Credit Default Swap
- Contingent, because value paid upon default of the counterparty is dependent on the value of an underlying transaction/portfolio
- CVA = C-CDS value
- Valuation of CVA through a C-CDS approach requires Monte Carlo valuation techniques
- This allows to directly control Right/Wrong-Way effects linking underlying risk drivers to default of the counterparty

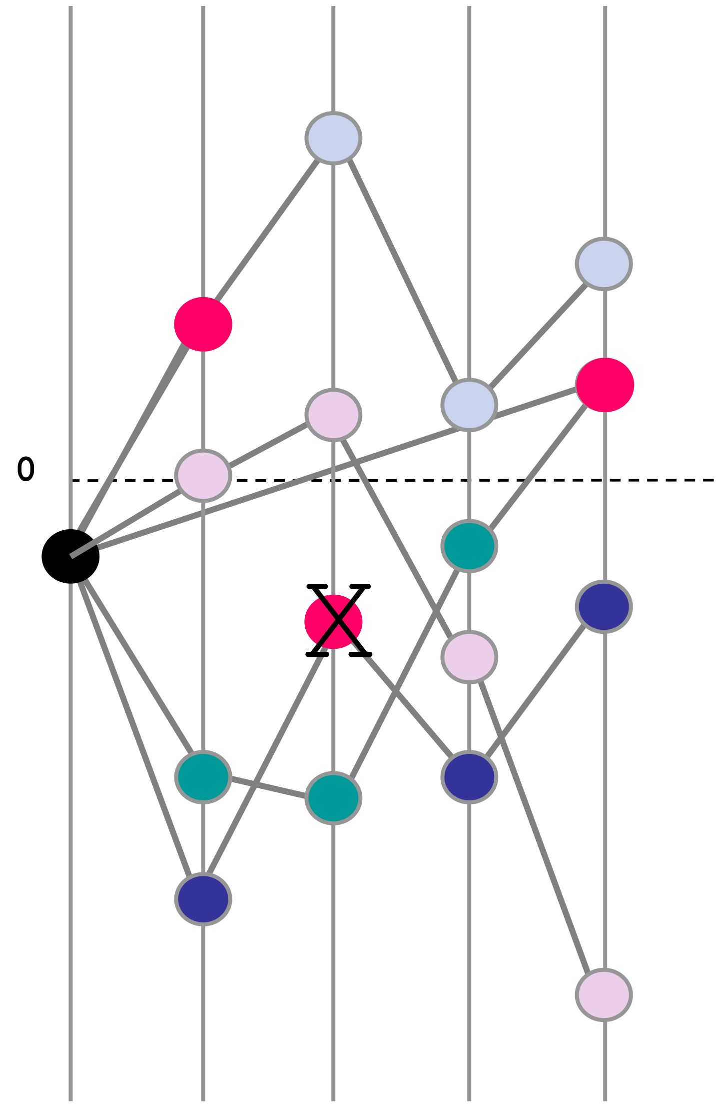
CVA Computation

Dynamic EPE - The C-CDS approach

- The valuation can then be performed by Monte Carlo technique using the following payoff

$$\Pi_{CCDS}(T_i) = V_{T_i}^{+} \mathbf{1}_{\tau \in]T_{i-1}, T_i]}$$

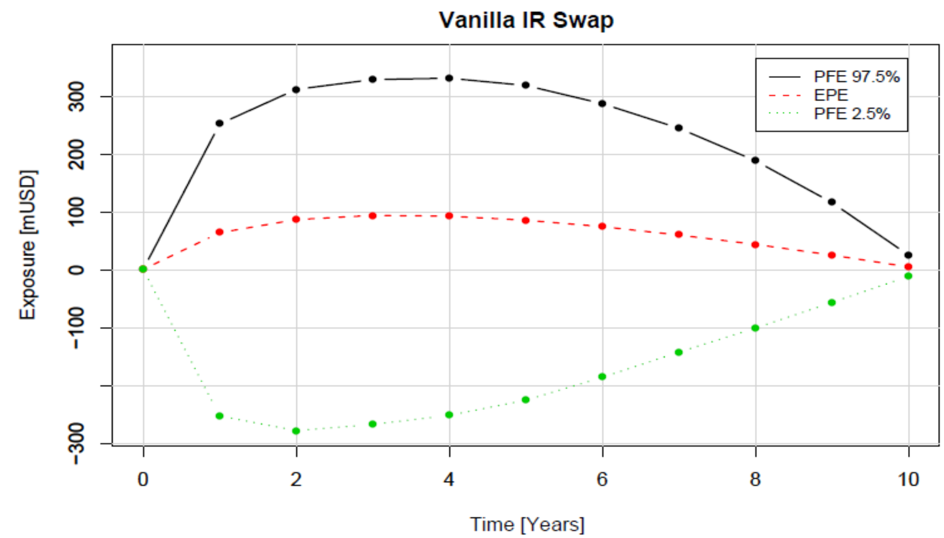
- Suppose we have the full simulation of the underlying portfolio value
- Simulate the default time of the counterparty at each path and then take the value of the portfolio at that time
  - It is possible for the counterparty not to default during the life of the trade
- Take expectation across all paths to compute the C-CDS price from the payoff
- The price of the C-CDS is the CVA

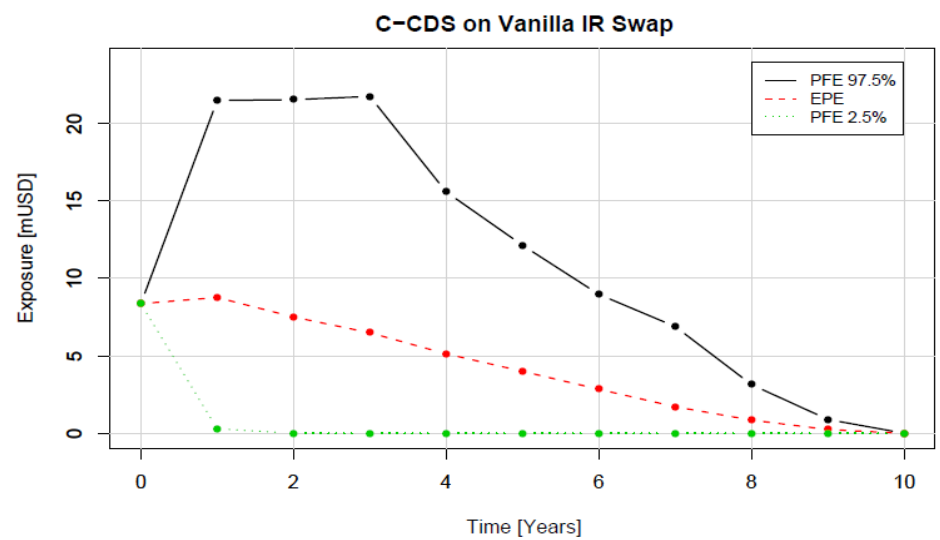


C-CDS

Existence of the price distribution means that we can have a long term view of the risk due to CVA

- As an illustration, consider a 10 year USD swap on a notional of 1000m USD
  - Receive 3 month USD Libor fixed in advance
  - Pay a fixed coupon equal to today's par
- Assume the counterparty's CDS curve is flat 130 bps
- The initial point is equal to today's CVA at around 8.4m USD,
- The underlying interest rate and spread risk means that the CVA could reach up to 22m USD at 97.5% confidence level





Wrong Way – Right Way Risk

Advantages of using a C-CDS approach

- Using a C-CDS approach it is possible to include in the simulation of counterparty defaults correlation with other risk factors
- In the case of credit derivatives (e.g. CDS, or CDO) it is straightforward to include correlation between defaults of the underlying and of the counterparty
- Correlation with other risk factors can be more challenging

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Next Steps

Section 6

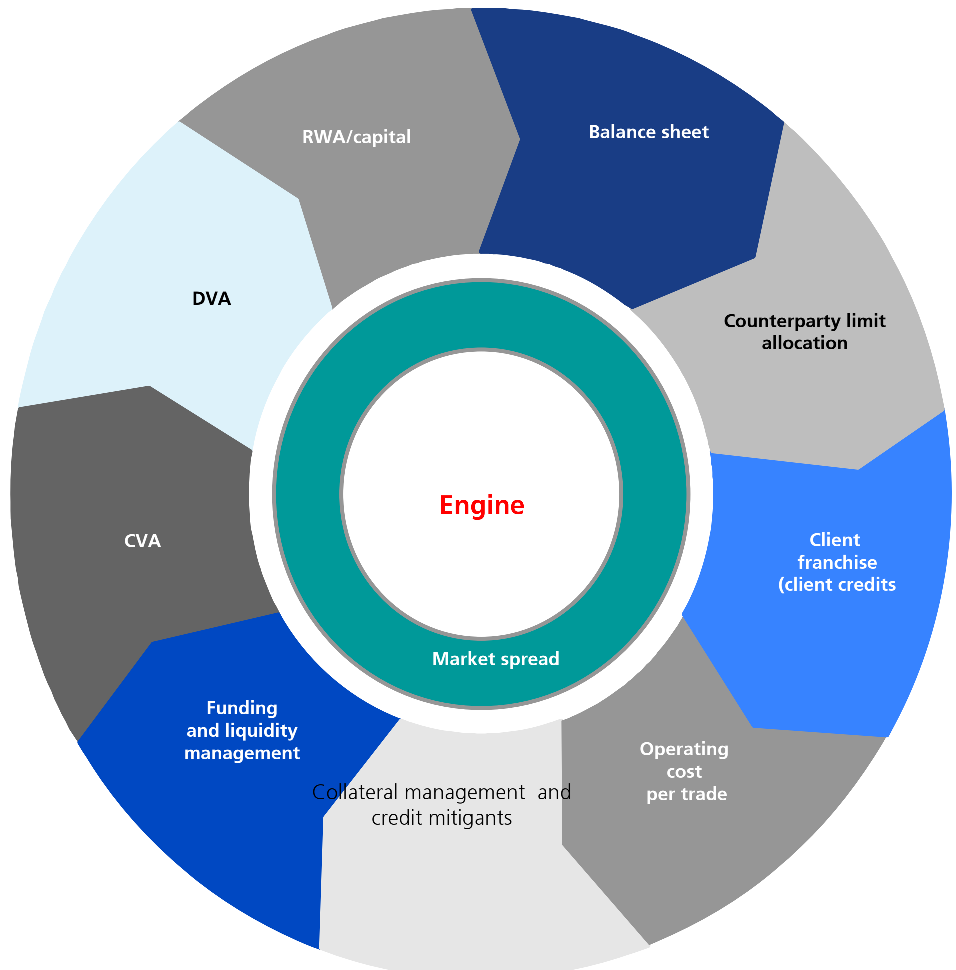
Open Questions and Challenges (From a Quant Perspective)

- CVA vs. counterparty exposure
  - Do we want different models for CVA (pricing) and counterparty exposure (control)?
  - Physical vs risk neutral measure
- Models
  - What is the level of accuracy required (e.g. interest rate exotics)?
  - What is the required level of consistency with other pricing systems (e.g. CDO)?
  - Can we use the AMC approach for all products?
- Hedging
  - Which sensitivities are needed, how often should they be computed?
- Collateral, Close-out and CVA
  - Should we take into account close-out risk?
  - How should we model collateral – which curve should be used?
- Cost of collateral cost of funding and DVA
  - Should we recognize DVA?
  - How do we include cost of funding?

Need of having accurate models across portfolios

Managing Banks Scarce Resources

- Resource allocation has to be performed on a portfolio basis
  - Models need to be flexible and powerful enough to price accurately transactions in future scenarios
- A time-zero pricing view is not enough
- A "risk" view is not accurate enough
- We have all the ingredients to be able to compute different risk measures across all asset classes and portfolios



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