Discussion of Good Deal Bound Pricing, with Applications to Credit Risk

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Swissquote Conference on Interest Rate and Credit Risk
EPFL, 29 October 2010

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World OTC Options Outstanding Notionals

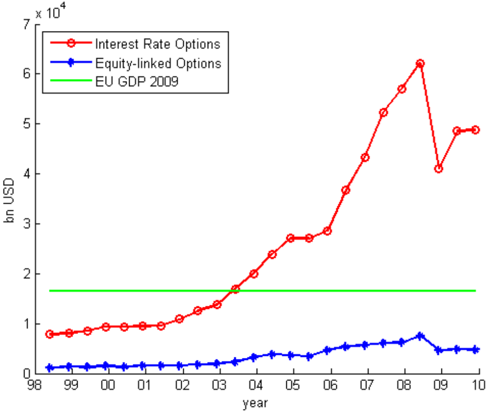

Figure: Notional amounts outstanding of OTC Options in billions of USD. Source: BIS.

Counterparty Risk

- Notional amount outstanding of OTC options Dec 2009:
  - Interest rate options: 48,808 bn USD
  - Equity-linked options: 4,762 bn USD
- Counterparty risk is substantial
- Counterparty risk implies market incompleteness (e.g. vulnerable options)

Pricing in Incomplete Markets

- Goal: value risky payoff by reference to prices of traded assets.
- Difficult if there is no perfect replicating portfolio (incompleteness).
- Challenge: find bounds on prices in this situation.
- No-arbitrage bounds (super-hedging): too large
- Ad-hoc choice: e.g. minimal martingale measure
- Cochrane and Saà Requejo (2000) impose bound on Sharpe ratios: good deal bounds

Hansen–Jagannathan (1991) Bounds

- Financial market

$$dS_0 = S_0 r\, dt$$
$$dS_i = S_i \left(\alpha_i\, dt + (\sigma\, dW)_i\right)$$

- No-arbitrage: there exists a (non-unique) market price of risk λ

$$\sigma\, \lambda = \alpha - r\, \mathbf{1}$$

- Minimal market price of risk (pseudo-inverse)

$$\hat{\lambda} = \sigma^\top (\sigma\sigma^\top)^{-1}(\alpha - r\, \mathbf{1})$$

satisfies

$$\|\hat{\lambda}\| \leq \|\lambda\|$$

  - Corresponds to minimal martingale measure

Hansen–Jagannathan (1991) Bounds

- Portfolio π, self-financing ($\pi^\top \mathbf{1} = 1$)

$$dS^\pi = S^\pi \left(\pi^\top \alpha \, dt + \pi^\top \sigma \, dW\right)$$

- Sharpe ratio $SR(\pi) = \frac{\pi^\top \alpha - r}{\|\pi^\top \sigma\|}$ satisfies

$$|SR(\pi)| = \left| \frac{\pi^\top \sigma \lambda}{\|\pi^\top \sigma\|} \right| \leq \|\lambda\|$$

- Hansen–Jagannathan Bounds:

$$|SR(\pi)| \leq \|\hat{\lambda}\| \leq \|\lambda\|$$

Björk and Slinko (2006)

- Extend Hansen–Jagannathan bounds to jump-diffusion market model
$$|SR(\pi)|^2 \leq \|\lambda\|^2 + \int_{\mathcal{X}} \varphi(x)^2 \nu(x)\, dx$$
- Extend Cochrane and Saá Requejo (2000) notion of upper/lower good-deal price by bounding the HJ bounds
$$\|\lambda\|^2 + \int_{\mathcal{X}} \varphi(x)^2 \nu(x)\, dx \leq B^2$$
- Good deal bound pricing becomes mathematically tractable.

Good Deal Bound Pricing

- Contingent claim $\Phi(S_T, Z_T)$ has upper good deal bound price

$$U_t = \sup_{\lambda,\varphi} \mathbb{E}_{\mathbb{Q}} \left[\mathrm{e}^{-\int_t^T r_s\, ds} \Phi(S_T, Z_T) \mid \mathcal{F}_t \right]$$

  subject to
  - NA and good deal bound $\|\lambda\|^2 + \int_{\mathcal{X}} \varphi(x)^2 \nu(x)\, dx \leq B^2$
- Similar definition for lower good deal bound price

$$L_t = \inf_{\lambda,\varphi} \mathbb{E}_{\mathbb{Q}} \left[\mathrm{e}^{-\int_t^T r_s\, ds} \Phi(S_T, Z_T) \mid \mathcal{F}_t \right]$$

- Both price processes, L_t and U_t, have a Sharpe ratio bounded by B, and so does (indeed?) any arbitrage-free price process

$$L_t \leq P_t \leq U_t$$

Risk Measurement

- Lower good deal bound price L_t is coherent risk-adjusted value of $\Phi(S_T, Z_T)$.
- In other words, $-L_t$ is coherent risk measure,
- Complies with axioms of coherent risk measures.
- GDB price seems robust with respect to $\mathbb{P}$-drift and GDB B specifications.
  - Good choices of B seem to be around 2–4
- Can reduce model risk!
  - Has this been exploited?

Computability

- Good deal bound pricing leads to “beautiful” mathematical problems (non-standard Hamilton–Jacobi–Bellman equations, etc.).
- Challenge: can we compute the prices efficiently and accurately?
- Systematic numerical studies needed, including Markovian credit migration models à la Donnelly

Summary

- The GDB approach provides a potential alternative risk-adjusted pricing tool.
- Further studies are needed to understand the implementation and effects.
- In particular, it should also be applied to vulnerable interest rate options (see first slide)!

References

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