Attilio Meucci

Fully Flexible Views:
Entropy Pooling in Theory and Practice

Swissquote Conference on Asset Management

Ecole Polytechnique Fédérale de Lausanne (EPFL), Switzerland

October 21, 2011

$\boldsymbol{m} \equiv \mathrm{E}\left\{\boldsymbol{R}_{T+\tau}\right\}$ **: expected returns**

$\boldsymbol{S} \equiv \mathrm{Cov}\left\{\boldsymbol{R}_{T+\tau}\right\}$ **: covariance**

$$\boldsymbol{w}_v \equiv \underset{\boldsymbol{w} \in \mathbf{C}}{\operatorname{argmax}}\left\{\boldsymbol{w}'\boldsymbol{m}\right\}$$

subject to $\boldsymbol{w}'\boldsymbol{S}\boldsymbol{w} \leq v$

$\boldsymbol{w}$ **: portfolio weights**

v **: grid of target variances**

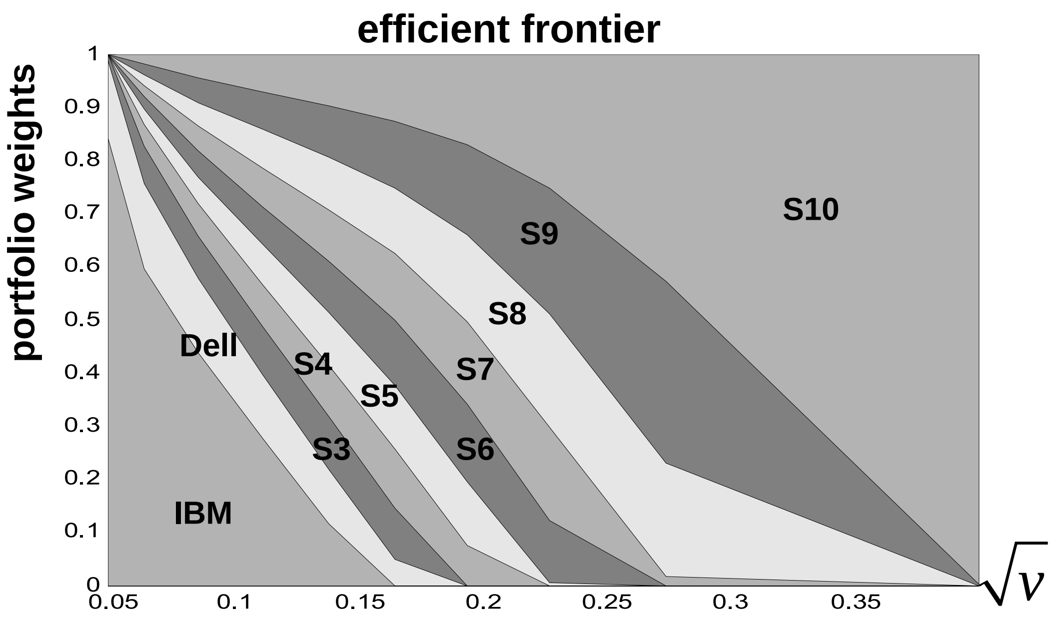
$\mathbf{C}$ **: investment constraints, e.g.** $\boldsymbol{w}'\boldsymbol{1} = 1, \ \boldsymbol{w} \geq \mathbf{0}$

$$\boldsymbol{m} \equiv \mathrm{E}\left\{\boldsymbol{R}_{T+\tau}\right\}$$: **expected returns**

$$\boldsymbol{S} \equiv \mathrm{Cov}\left\{\boldsymbol{R}_{T+\tau}\right\}$$: **covariance**

$$\boldsymbol{w}_v \equiv \underset{\boldsymbol{w}\in\mathbf{C}}{\operatorname{argmax}}\left\{\boldsymbol{w}'\boldsymbol{m}\right\}$$

subject to $\boldsymbol{w}'\boldsymbol{S}\boldsymbol{w} \leq v$



$$r_t \sim \mathrm{N}(\boldsymbol{m}, \boldsymbol{S}), \quad t = 1, \ldots, T$$

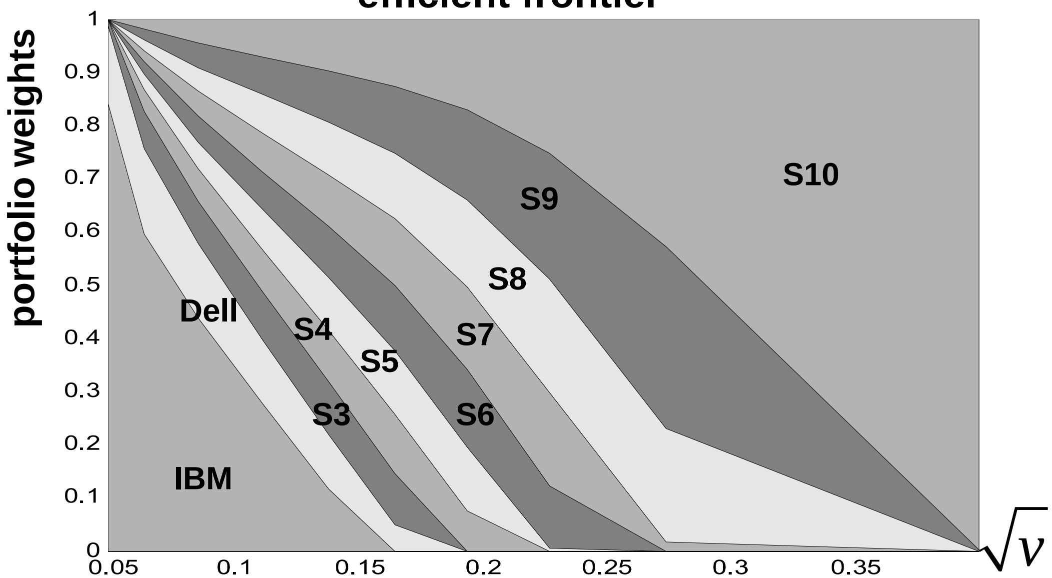
? $\boldsymbol{m} \equiv \mathrm{E}\{\boldsymbol{R}_{T+\tau}\}$: **expected returns**

? $\boldsymbol{S} \equiv \mathrm{Cov}\{\boldsymbol{R}_{T+\tau}\}$: **covariance**

$$\boldsymbol{w}_v \equiv \underset{\boldsymbol{w} \in \mathbf{C}}{\operatorname{argmax}} \{\boldsymbol{w}'\boldsymbol{m}\}$$

subject to $\boldsymbol{w}'\boldsymbol{S}\boldsymbol{w} \leq v$

efficient frontier

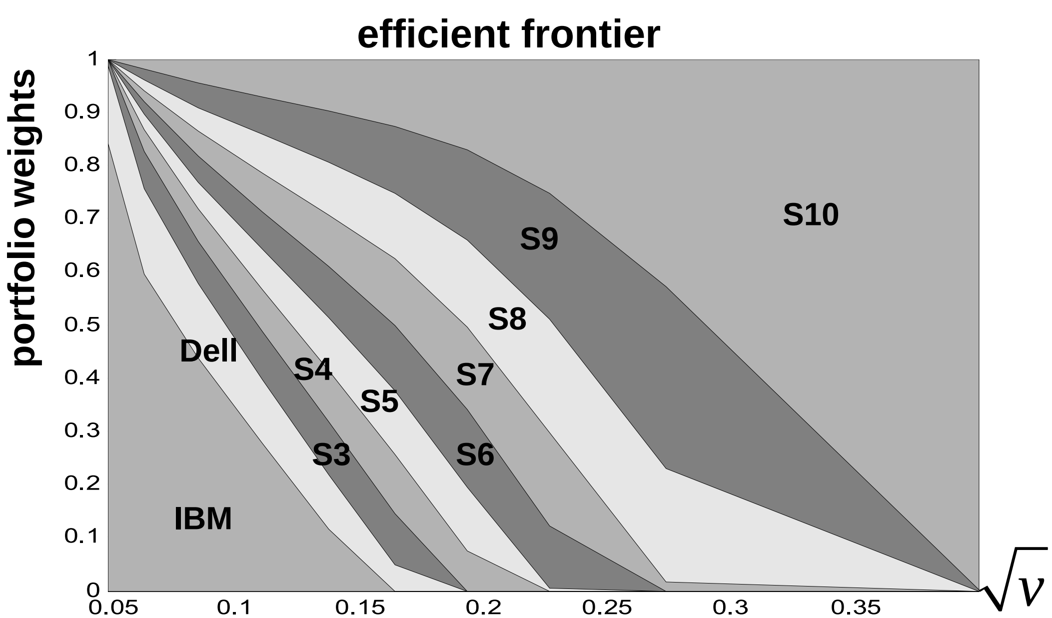


$$? \quad \boldsymbol{m} \equiv \mathrm{E}\left\{\boldsymbol{R}_{T+\tau}\right\} \text{ : expected returns}$$

$$? \quad \boldsymbol{S} \equiv \mathrm{Cov}\left\{\boldsymbol{R}_{T+\tau}\right\} \text{ : covariance}$$

$$\boldsymbol{w}_v \equiv \underset{\boldsymbol{w}\in\mathbf{C}}{\operatorname{argmax}}\left\{\boldsymbol{w}'\boldsymbol{m}\right\}$$

subject to $\boldsymbol{w}'\boldsymbol{S}\boldsymbol{w} \leq v$



$$\boldsymbol{r}_t \sim \mathrm{N}\left(\boldsymbol{m}, \boldsymbol{S}\right), \quad t = 1, \ldots, T$$

$$\tilde{\boldsymbol{m}} \equiv \frac{1}{T}\sum_{t=1}^{T}\boldsymbol{r}_t$$

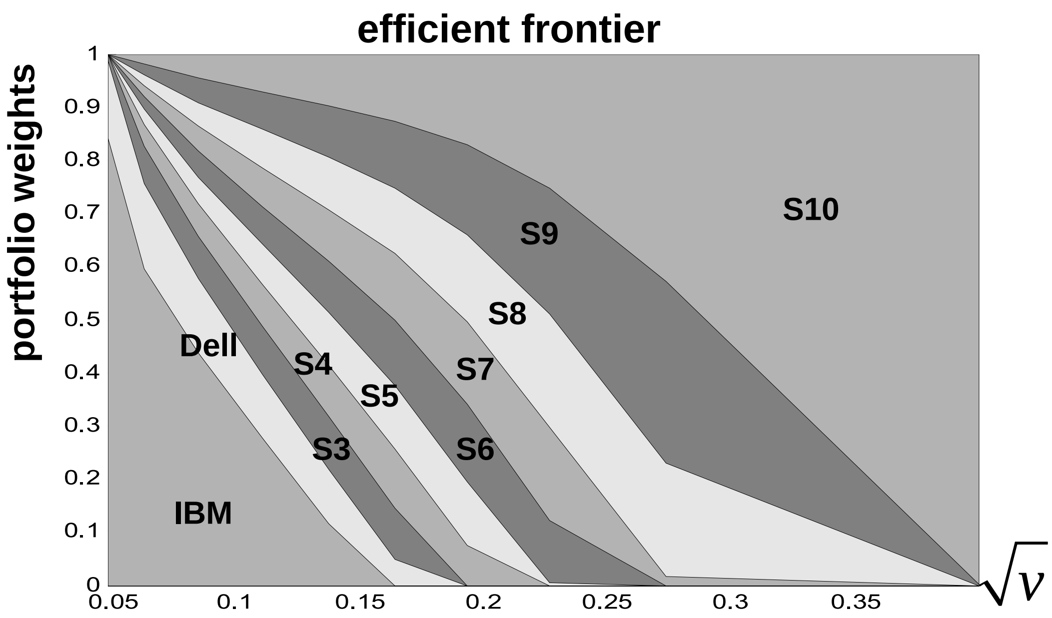
$$\tilde{\boldsymbol{S}} \equiv \frac{1}{T}\sum_{t=1}^{T}\left(\boldsymbol{r}_t - \tilde{\boldsymbol{m}}\right)\left(\boldsymbol{r}_t - \tilde{\boldsymbol{m}}\right)'$$

? $\boldsymbol{m} \equiv \mathrm{E}\left\{\boldsymbol{R}_{T+\tau}\right\}$: expected returns

? $\boldsymbol{S} \equiv \mathrm{Cov}\left\{\boldsymbol{R}_{T+\tau}\right\}$: covariance

$$\boldsymbol{w}_v \equiv \underset{\boldsymbol{w} \in \mathrm{C}}{\operatorname{argmax}} \left\{\boldsymbol{w}'\boldsymbol{m}\right\}$$

subject to $\boldsymbol{w}'\boldsymbol{S}\boldsymbol{w} \leq v$



$$\boldsymbol{r}_t \sim \mathrm{N}\left(\boldsymbol{m}, \boldsymbol{S}\right), \quad t = 1, \ldots, T$$

$$\tilde{\boldsymbol{m}} \equiv \frac{1}{T}\sum_{t=1}^{T} \boldsymbol{r}_t$$

$$\tilde{\boldsymbol{S}} \equiv \frac{1}{T}\sum_{t=1}^{T} \left(\boldsymbol{r}_t - \tilde{\boldsymbol{m}}\right)\left(\boldsymbol{r}_t - \tilde{\boldsymbol{m}}\right)'$$

$$\boldsymbol{w}_v \equiv \underset{\boldsymbol{w} \in \mathrm{C}}{\operatorname{argmax}} \left\{\boldsymbol{w}'\tilde{\boldsymbol{m}}\right\}$$

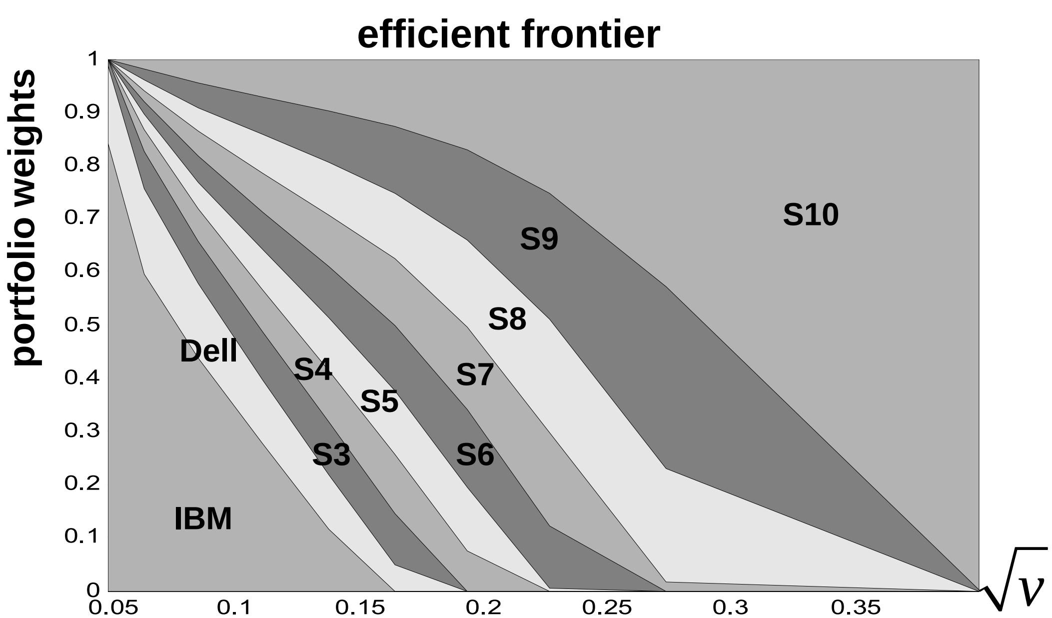
subject to $\boldsymbol{w}'\tilde{\boldsymbol{S}}\boldsymbol{w} \leq v$

? $\boldsymbol{m} \equiv \mathrm{E}\left\{\boldsymbol{R}_{T+\tau}\right\}$: **expected returns**

? $\boldsymbol{S} \equiv \mathrm{Cov}\left\{\boldsymbol{R}_{T+\tau}\right\}$: **covariance**

$$\boldsymbol{w}_v \equiv \underset{\boldsymbol{w}\in C}{\mathrm{argmax}}\left\{\boldsymbol{w}'\boldsymbol{m}\right\}$$

subject to $\boldsymbol{w}'\boldsymbol{S}\boldsymbol{w} \leq v$



$$\boldsymbol{r}_t \sim \mathrm{N}\left(\boldsymbol{m}, \boldsymbol{S}\right), \quad t = 1, \ldots, T$$

$$\tilde{\boldsymbol{m}} \equiv \frac{1}{T}\sum_{t=1}^{T} \boldsymbol{r}_t$$

$$\tilde{\boldsymbol{S}} \equiv \frac{1}{T}\sum_{t=1}^{T}\left(\boldsymbol{r}_t - \tilde{\boldsymbol{m}}\right)\left(\boldsymbol{r}_t - \tilde{\boldsymbol{m}}\right)'$$

$$\boldsymbol{w}_v \equiv \underset{\boldsymbol{w}\in C}{\mathrm{argmax}}\left\{\boldsymbol{w}'\tilde{\boldsymbol{m}}\right\}$$

subject to $\boldsymbol{w}'\tilde{\boldsymbol{S}}\boldsymbol{w} \leq v$

LIVE

1

? $\boldsymbol{m} \equiv \mathrm{E}\left\{\boldsymbol{R}_{T+\tau}\right\}$ **: expected returns**

? $\boldsymbol{S} \equiv \mathrm{Cov}\left\{\boldsymbol{R}_{T+\tau}\right\}$ **: covariance**

$$\boldsymbol{w}_v \equiv \underset{\boldsymbol{w}\in\mathbf{C}}{\operatorname{argmax}}\left\{\boldsymbol{w}'\boldsymbol{m}\right\}$$

subject to $\boldsymbol{w}'\boldsymbol{S}\boldsymbol{w} \leq v$

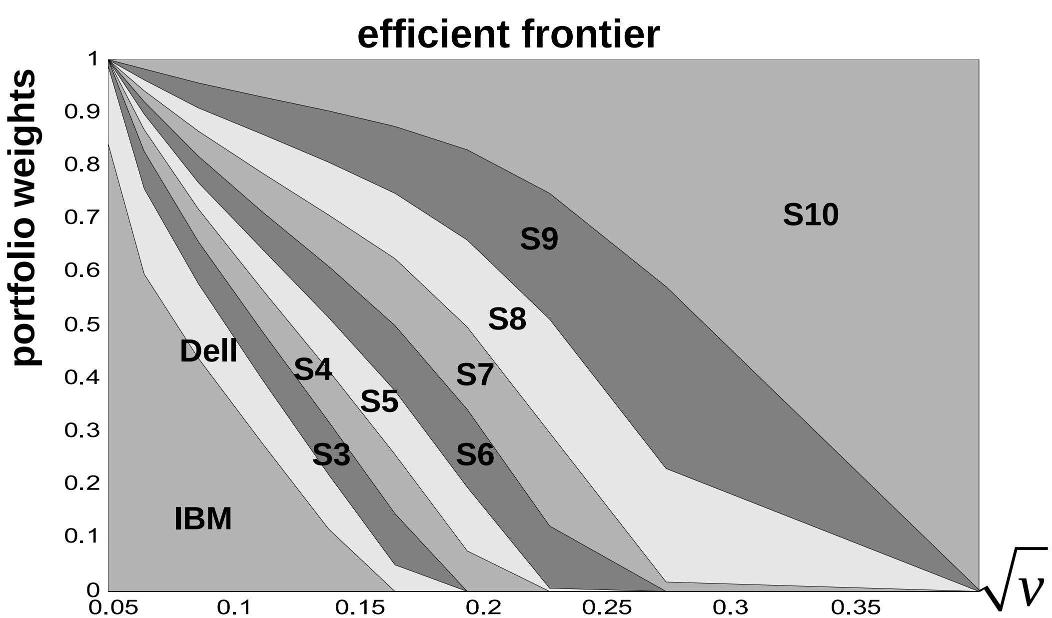
$$\boldsymbol{r}_t \sim \mathrm{N}\left(\boldsymbol{m},\boldsymbol{S}\right), \quad t = 1,\ldots,T$$

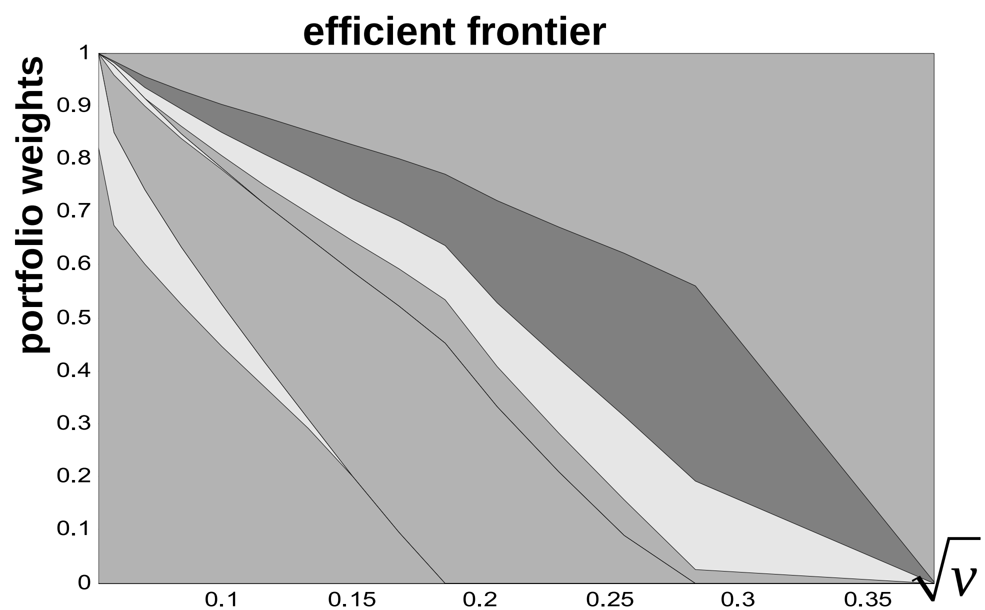
$$\tilde{\boldsymbol{m}} \equiv \frac{1}{T}\sum_{t=1}^{T}\boldsymbol{r}_t$$

$$\tilde{\boldsymbol{S}} \equiv \frac{1}{T}\sum_{t=1}^{T}\left(\boldsymbol{r}_t - \tilde{\boldsymbol{m}}\right)\left(\boldsymbol{r}_t - \tilde{\boldsymbol{m}}\right)'$$

$$\boldsymbol{w}_v \equiv \underset{\boldsymbol{w}\in\mathbf{C}}{\operatorname{argmax}}\left\{\boldsymbol{w}'\tilde{\boldsymbol{m}}\right\}$$

subject to $\boldsymbol{w}'\tilde{\boldsymbol{S}}\boldsymbol{w} \leq v$





$$\boldsymbol{r}_t \sim \mathrm{N}\left(\boldsymbol{m}, \boldsymbol{S}\right), \quad t = 1, \ldots, T$$

$$? \quad \boldsymbol{m} \equiv \mathrm{E}\left\{\boldsymbol{R}_{T+\tau}\right\}$$: **expected returns**

$$? \quad \boldsymbol{S} \equiv \mathrm{Cov}\left\{\boldsymbol{R}_{T+\tau}\right\}$$: **covariance**

$$\tilde{\boldsymbol{m}} \equiv \frac{1}{T}\sum_{t=1}^{T} \boldsymbol{r}_t$$

$$\tilde{\boldsymbol{S}} \equiv \frac{1}{T}\sum_{t=1}^{T} \left(\boldsymbol{r}_t - \tilde{\boldsymbol{m}}\right)\left(\boldsymbol{r}_t - \tilde{\boldsymbol{m}}\right)'$$

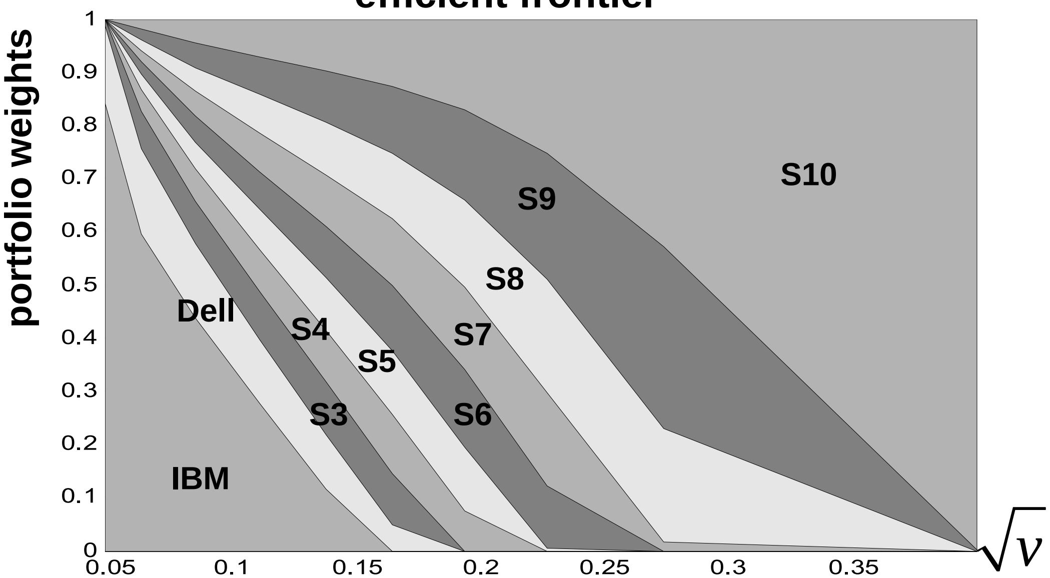
$$\boldsymbol{w}_v \equiv \underset{\boldsymbol{w} \in \mathbf{C}}{\mathrm{argmax}} \left\{\boldsymbol{w}'\boldsymbol{m}\right\}$$

subject to $\boldsymbol{w}'\boldsymbol{S}\boldsymbol{w} \leq v$

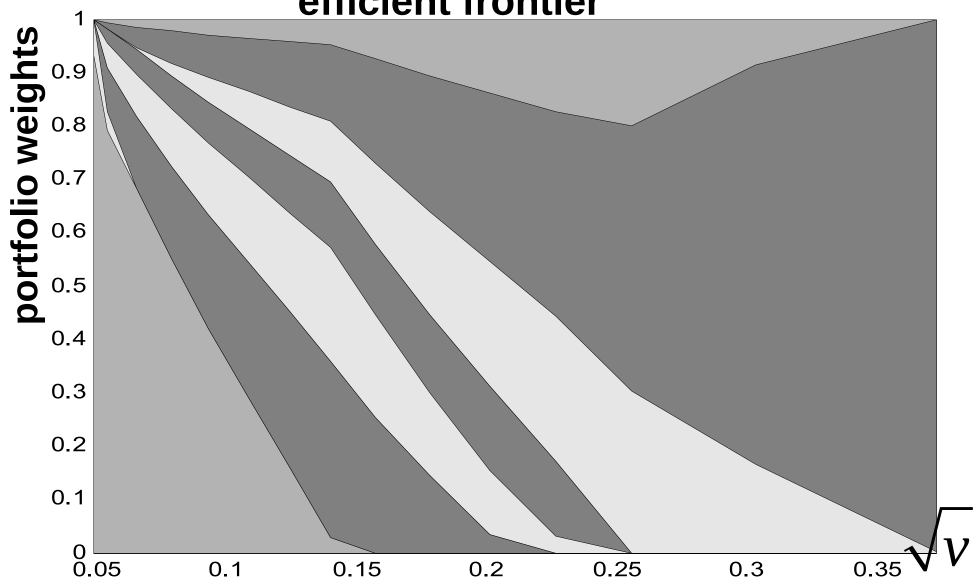
$$\boldsymbol{w}_v \equiv \underset{\boldsymbol{w} \in \mathbf{C}}{\mathrm{argmax}} \left\{\boldsymbol{w}'\tilde{\boldsymbol{m}}\right\}$$

subject to $\boldsymbol{w}'\tilde{\boldsymbol{S}}\boldsymbol{w} \leq v$

efficient frontier



efficient frontier

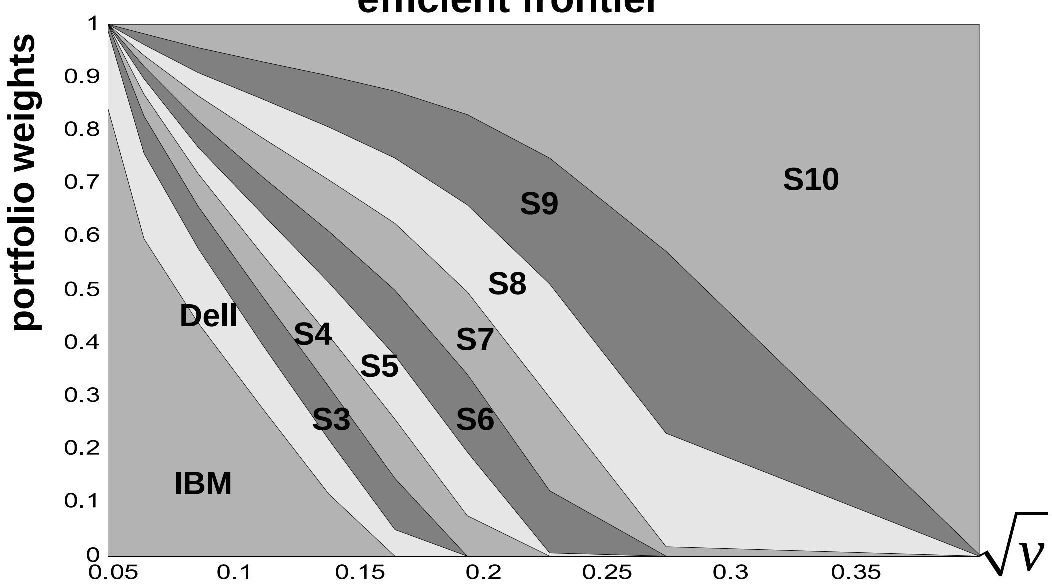


? $\boldsymbol{m} \equiv \mathrm{E}\left\{\boldsymbol{R}_{T+\tau}\right\}$: **expected returns**

? $\boldsymbol{S} \equiv \mathrm{Cov}\left\{\boldsymbol{R}_{T+\tau}\right\}$: **covariance**

$$\boldsymbol{w}_v \equiv \underset{\boldsymbol{w}\in\mathrm{C}}{\mathrm{argmax}}\left\{\boldsymbol{w}'\boldsymbol{m}\right\}$$

subject to $\boldsymbol{w}'\boldsymbol{S}\boldsymbol{w} \leq v$



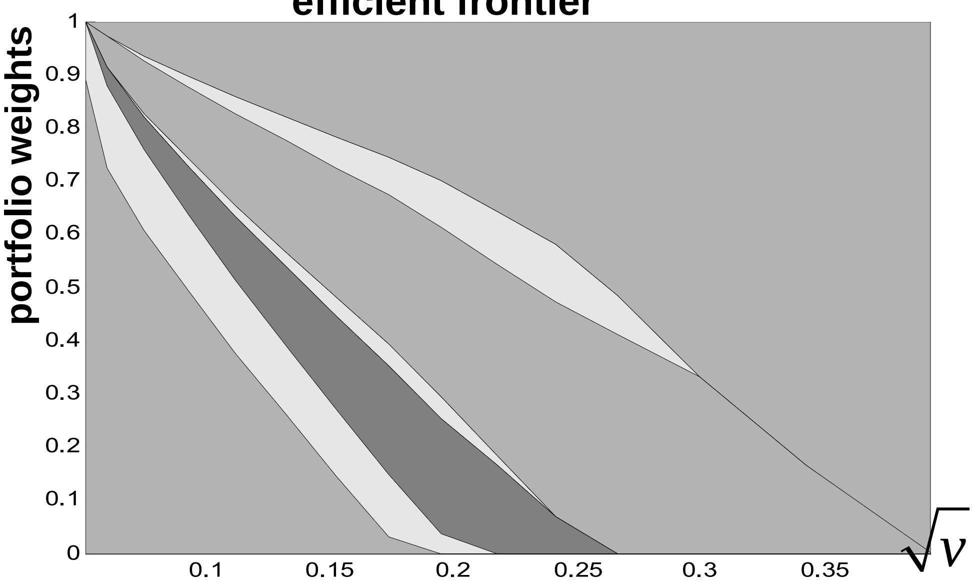
$$\boldsymbol{r}_t \sim \mathrm{N}\left(\boldsymbol{m}, \boldsymbol{S}\right), \quad t = 1, \ldots, T$$

$$\tilde{\boldsymbol{m}} \equiv \frac{1}{T}\sum_{t=1}^{T} \boldsymbol{r}_t$$

$$\tilde{\boldsymbol{S}} \equiv \frac{1}{T}\sum_{t=1}^{T}\left(\boldsymbol{r}_t - \tilde{\boldsymbol{m}}\right)\left(\boldsymbol{r}_t - \tilde{\boldsymbol{m}}\right)'$$

$$\boldsymbol{w}_v \equiv \underset{\boldsymbol{w}\in\mathrm{C}}{\mathrm{argmax}}\left\{\boldsymbol{w}'\tilde{\boldsymbol{m}}\right\}$$

subject to $\boldsymbol{w}'\tilde{\boldsymbol{S}}\boldsymbol{w} \leq v$



ESTIMATION RISK

SCENARIO ANALYSIS

THE BLACK-LITTERMAN APPROACH

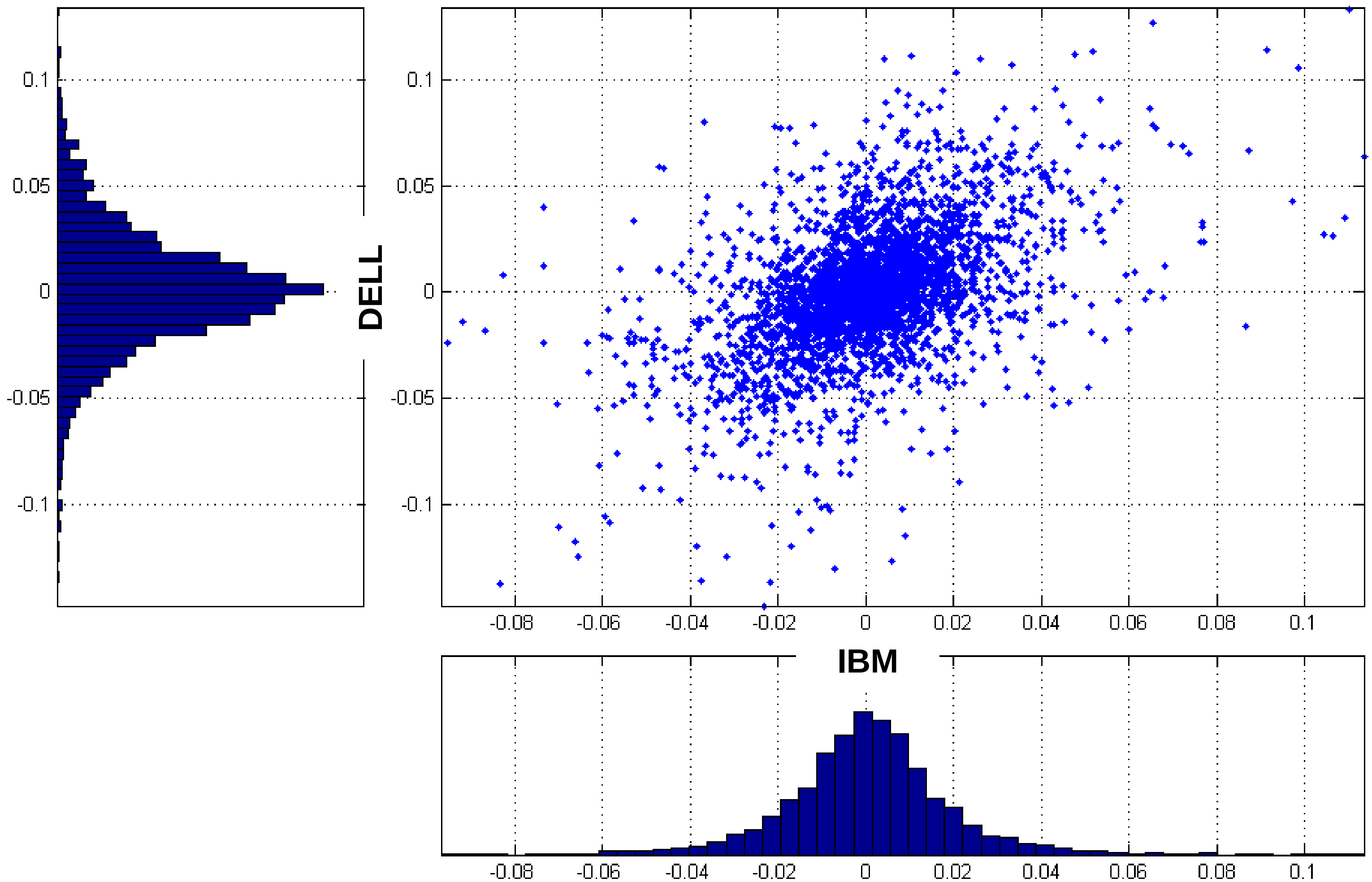
ENTROPY POOLING

CASE STUDIES

CONCLUSIONS AND REFERENCES

Market distribution

$\mathbf{X} \sim \mathrm{N}(\mu, \Sigma)$ **returns on asset classes/funds**



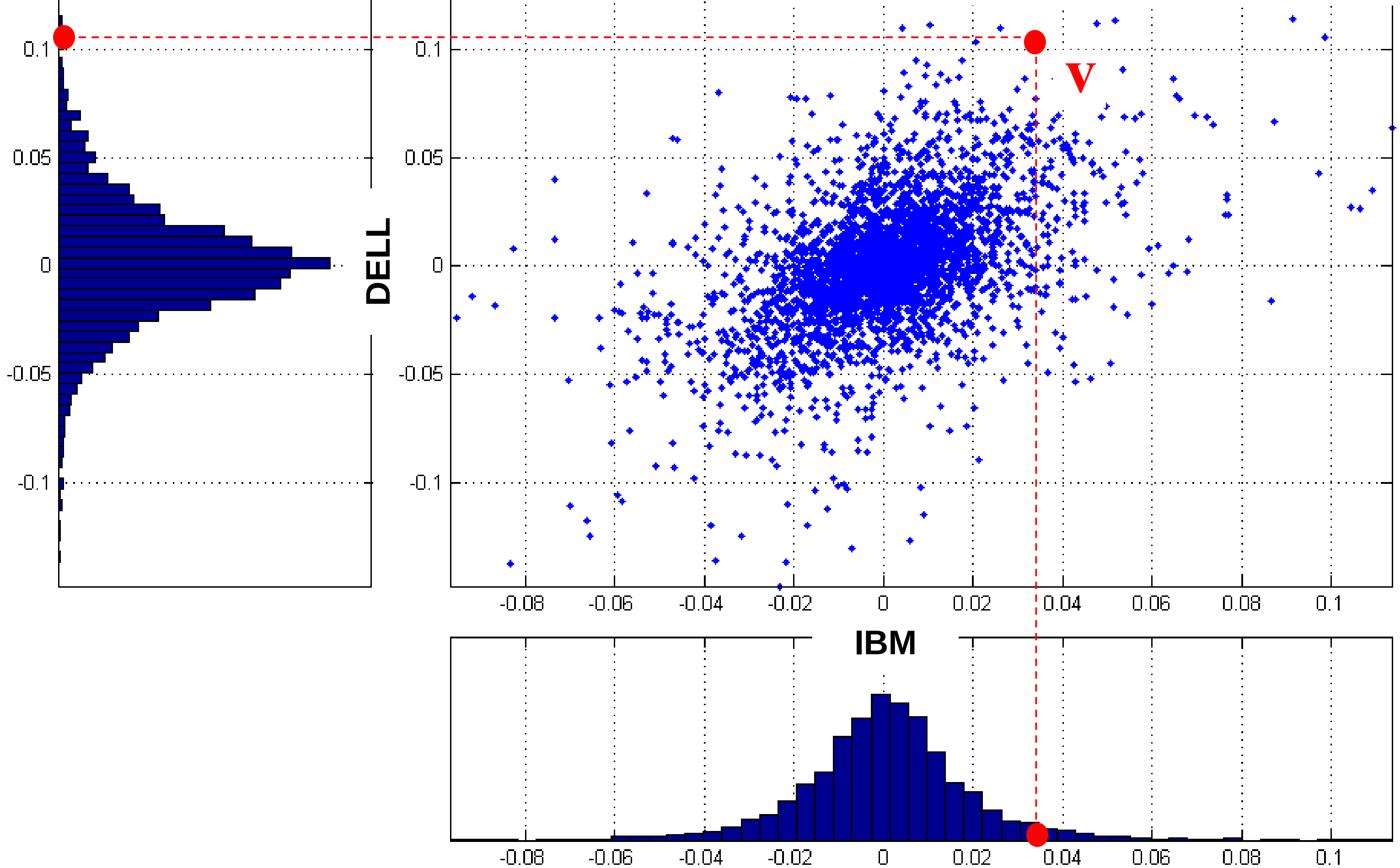
Market distribution

Scenario analysis

$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **returns on asset classes/funds**

$\mathbf{QX} \equiv \mathbf{v}$

FULL specification: $\mathbf{Q}$ = identity

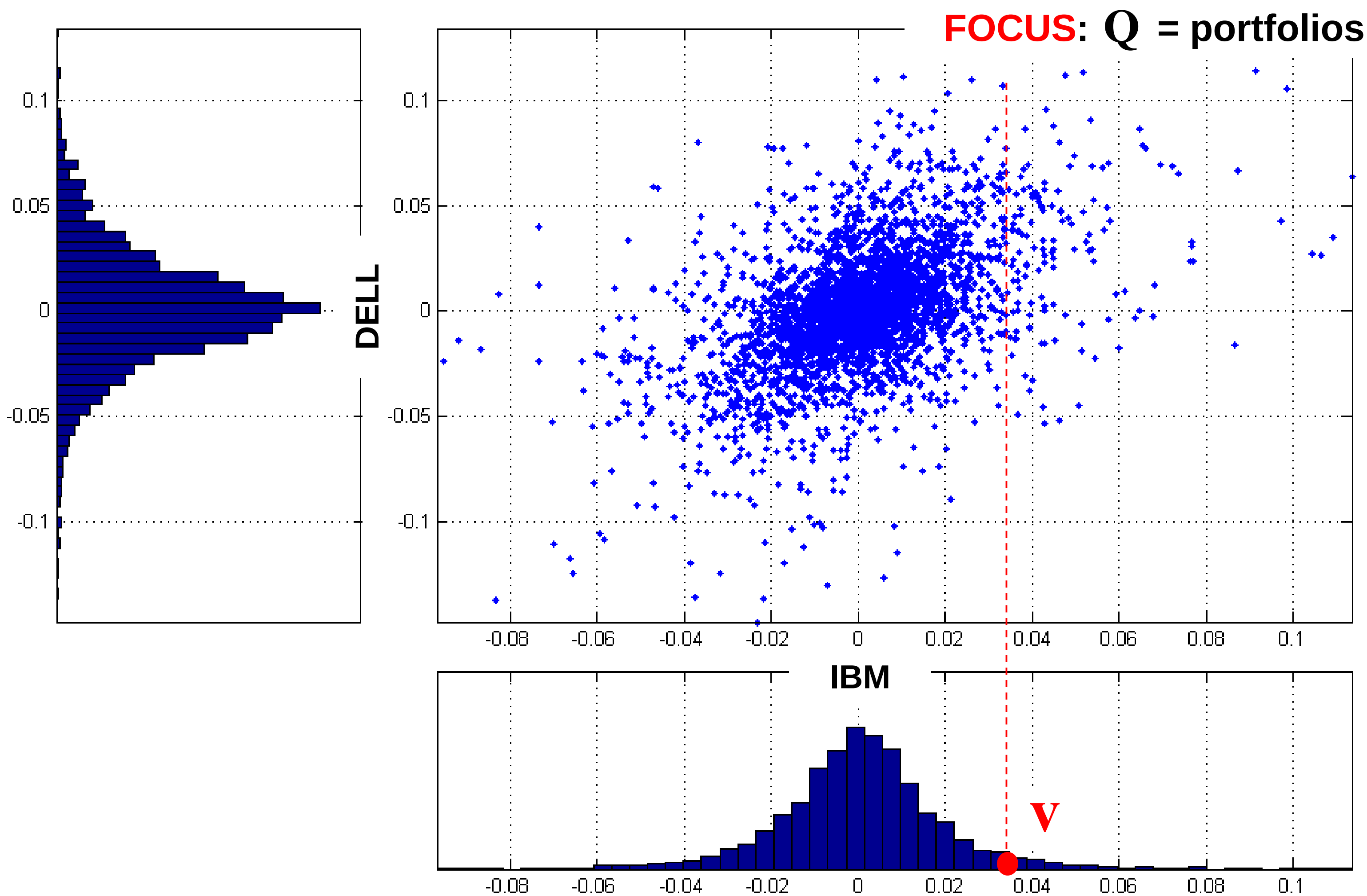


Market distribution

Scenario analysis

$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **returns on asset classes/funds**

$\mathbf{Q}\mathbf{X} \equiv \mathbf{v}$



Market distribution

$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **returns on asset classes/funds**

Scenario analysis

$\mathbf{Q}\mathbf{X} \equiv \mathbf{v}$

Conditional formula

$$\mathbf{X}|\mathbf{v} \sim \mathrm{N}\left(\boldsymbol{\mu}_{\mathbf{x}|\mathbf{v}}, \boldsymbol{\Sigma}_{\mathbf{x}|\mathbf{v}}\right)$$

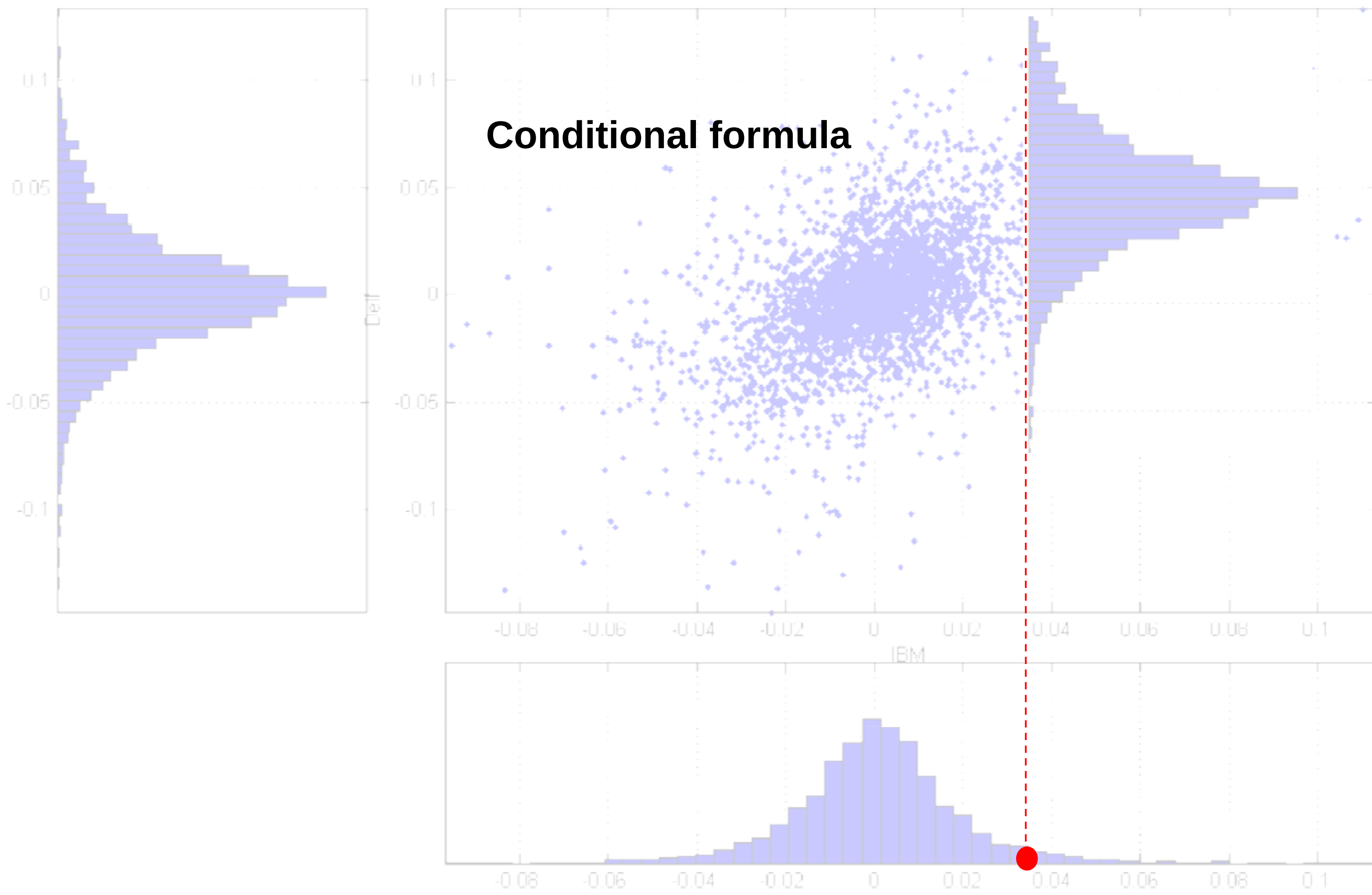
$$\begin{aligned}
\boldsymbol{\mu}_{\mathbf{x}|\mathbf{v}} &\equiv \boldsymbol{\mu} + \boldsymbol{\Sigma}\mathbf{Q}'\left(\mathbf{Q}\boldsymbol{\Sigma}\mathbf{Q}'\right)^{-1}(\mathbf{v} - \mathbf{Q}\boldsymbol{\mu}) \\
\boldsymbol{\Sigma}_{\mathbf{x}|\mathbf{v}} &\equiv \boldsymbol{\Sigma} - \boldsymbol{\Sigma}\mathbf{Q}'\left(\mathbf{Q}\boldsymbol{\Sigma}\mathbf{Q}'\right)^{-1}\mathbf{Q}\boldsymbol{\Sigma}.
\end{aligned}$$

Market distribution

$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **returns on asset classes/funds**

Scenario analysis

$\mathbf{Q}\mathbf{X} \equiv \mathbf{v}$



Market distribution

$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **returns on asset classes/funds**

Scenario analysis

$$\mathbf{Q}\mathbf{X} \equiv \mathbf{v}$$

Conditional formula

$$\mathbf{X}|\mathbf{v} \sim \mathrm{N}\left(\boldsymbol{\mu}_{\mathbf{x}|\mathbf{v}}, \boldsymbol{\Sigma}_{\mathbf{x}|\mathbf{v}}\right)$$

$$\begin{aligned}
\boldsymbol{\mu}_{\mathbf{x}|\mathbf{v}} &\equiv \boldsymbol{\mu} + \boldsymbol{\Sigma}\mathbf{Q}' \left(\mathbf{Q}\boldsymbol{\Sigma}\mathbf{Q}'\right)^{-1} (\mathbf{v} - \mathbf{Q}\boldsymbol{\mu}) \\
\boldsymbol{\Sigma}_{\mathbf{x}|\mathbf{v}} &\equiv \boldsymbol{\Sigma} - \boldsymbol{\Sigma}\mathbf{Q}' \left(\mathbf{Q}\boldsymbol{\Sigma}\mathbf{Q}'\right)^{-1} \mathbf{Q}\boldsymbol{\Sigma}.
\end{aligned}$$

Optimization

$$\mathbf{w}_{\lambda} \equiv \underset{\mathbf{w}\in\mathcal{C}}{\operatorname{argmax}} \left\{\mathbf{w}'\boldsymbol{\mu}_{\mathbf{x}|\mathbf{v}} - \lambda\mathbf{w}'\boldsymbol{\Sigma}_{\mathbf{x}|\mathbf{v}}\mathbf{w}\right\}$$

ESTIMATION RISK

SCENARIO ANALYSIS

THE BLACK-LITTERMAN APPROACH

- **Estimation risk**
- **Views**
- **Discussion**

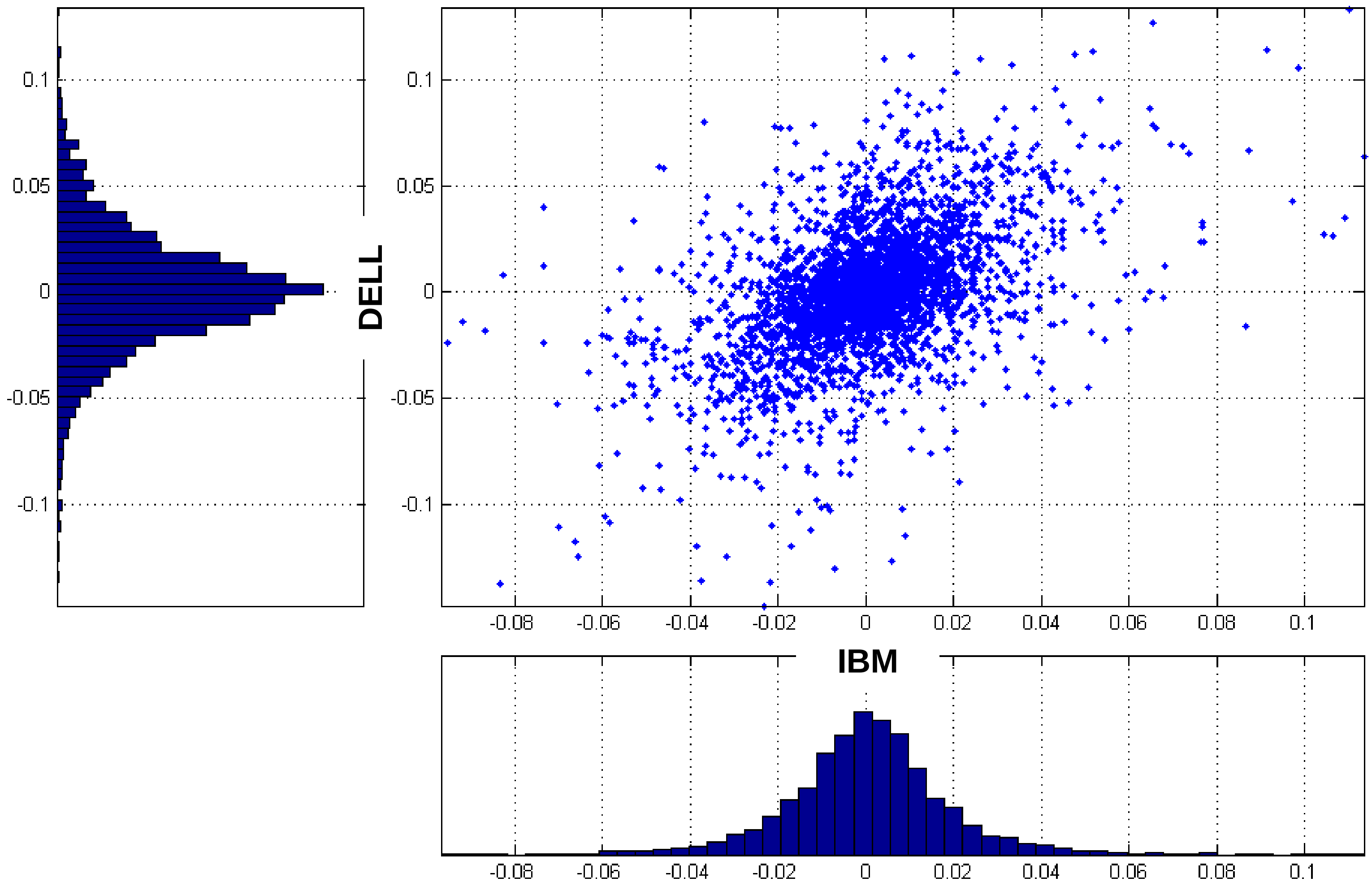
ENTROPY POOLING

CASE STUDIES

REFERENCES AND CONCLUSIONS

Market distribution

$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **returns on asset classes/funds**



Market distribution

$\mathbf{X} \sim \mathrm{N}(\boxed{\mu}, \boxed{\Sigma})$ **returns on asset classes/funds**

?

estimation risk

Market distribution

$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **returns on asset classes/funds**

$\boldsymbol{\Sigma}$ **estimated by exponential smoothing**

Market distribution

$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **returns on asset classes/funds**

$\boldsymbol{\Sigma}$ **estimated by exponential smoothing**

$\boldsymbol{\mu} \sim \mathrm{N}(\boldsymbol{\pi}, \tau\boldsymbol{\Sigma})$

Market distribution

$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **returns on asset classes/funds**

$\boldsymbol{\Sigma}$ **estimated by exponential smoothing**

$$\boldsymbol{\mu} \sim \mathrm{N}(\boldsymbol{\pi}, \boxed{\tau} \boldsymbol{\Sigma})$$

?

$$\widehat{\boldsymbol{\mu}} \equiv \frac{1}{T} \sum_{t=1}^{T} \mathbf{X}_t \sim \mathrm{N}\left(\boldsymbol{\pi}, \frac{\boldsymbol{\Sigma}}{T}\right)$$

Market distribution

$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **returns on asset classes/funds**

$\boldsymbol{\Sigma}$ **estimated by exponential smoothing**

$$\boldsymbol{\mu} \sim \mathrm{N}(\boldsymbol{\pi}, \tau\boldsymbol{\Sigma})$$

$$\tau \approx \frac{1}{T}. \qquad \Leftarrow \qquad \widehat{\boldsymbol{\mu}} \equiv \frac{1}{T}\sum_{t=1}^{T}\mathbf{X}_t \sim \mathrm{N}\left(\boldsymbol{\pi}, \frac{\boldsymbol{\Sigma}}{T}\right)$$

Market distribution

$\mathbf{X} \sim \mathrm{N}\left(\boldsymbol{\mu}, \boldsymbol{\Sigma}\right)$ **returns on asset classes/funds**

$\boldsymbol{\Sigma}$ **estimated by exponential smoothing**

$\boldsymbol{\mu} \sim \mathrm{N}\left(\boxed{\boldsymbol{\pi}}, \tau\boldsymbol{\Sigma}\right)$

?

$$\tau \approx \frac{1}{T}.$$

Market distribution

$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **returns on asset classes/funds**

$\boldsymbol{\Sigma}$ **estimated by exponential smoothing**

$\boldsymbol{\mu} \sim \mathrm{N}(\boldsymbol{\pi}, \tau\boldsymbol{\Sigma})$

$$\tau \approx \frac{1}{T}.$$

mean-variance $\mathbf{w}_\lambda \equiv \underset{\mathbf{w}}{\operatorname{argmax}} \left\{ \mathbf{w}'\boldsymbol{\pi} - \lambda \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} \right\}$

Market distribution

$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **returns on asset classes/funds**

$\boldsymbol{\Sigma}$ **estimated by exponential smoothing**

$$\boldsymbol{\mu} \sim \mathrm{N}(\boldsymbol{\pi}, \tau\boldsymbol{\Sigma})$$

$$\tau \approx \frac{1}{T}.$$

$$\mathbf{w}_\lambda = \frac{1}{2\lambda}\boldsymbol{\Sigma}^{-1}\boldsymbol{\pi}$$

mean-variance $\mathbf{w}_\lambda \equiv \underset{\mathbf{w}}{\operatorname{argmax}}\left\{\mathbf{w}'\boldsymbol{\pi} - \lambda\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}\right\}$

Market distribution

$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **returns on asset classes/funds**

$\boldsymbol{\Sigma}$ **estimated by exponential smoothing**

$\boldsymbol{\mu} \sim \mathrm{N}(\boldsymbol{\pi}, \tau\boldsymbol{\Sigma})$

$$\tau \approx \frac{1}{T}.$$

$$\boldsymbol{\pi} \equiv 2\overline{\lambda}\boldsymbol{\Sigma}\widetilde{\mathbf{w}}.$$

$$\mathbf{w}_{\lambda} = \frac{1}{2\lambda}\boldsymbol{\Sigma}^{-1}\boldsymbol{\pi}$$

mean-variance $\mathbf{w}_{\lambda} \equiv \underset{\mathbf{w}}{\operatorname{argmax}} \left\{ \mathbf{w}'\boldsymbol{\pi} - \lambda \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} \right\}$

Market distribution

$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **returns on asset classes/funds**

$\boldsymbol{\Sigma}$ **estimated by exponential smoothing**

$$\boldsymbol{\mu} \sim \mathrm{N}(\boldsymbol{\pi}, \tau\boldsymbol{\Sigma})$$

$$\tau \approx \frac{1}{T}.$$

$$\boxed{\boldsymbol{\pi}} \equiv 2\overline{\lambda}\boldsymbol{\Sigma}\widetilde{\mathbf{w}}.$$

equilibrium portfolio

equilibrium returns

Market distribution

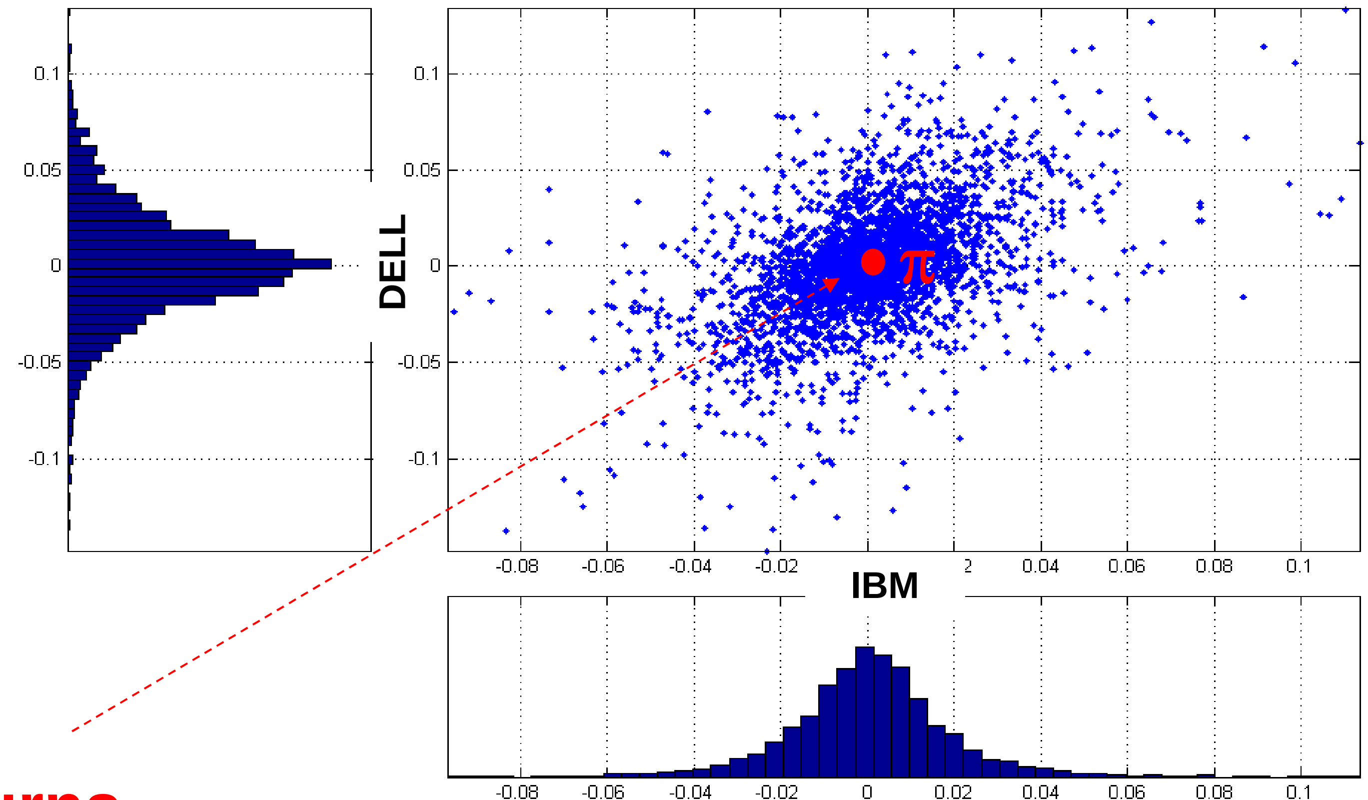
$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **returns on asset classes/funds**

$\boldsymbol{\Sigma}$ **estimated by exponential smoothing**

$$\boldsymbol{\mu} \sim \mathrm{N}(\boldsymbol{\pi}, \tau\boldsymbol{\Sigma})$$

$$\tau \approx \frac{1}{T}.$$

$$\boxed{\boldsymbol{\pi}} \equiv 2\overline{\lambda}\boldsymbol{\Sigma}\widetilde{\mathbf{w}}.$$

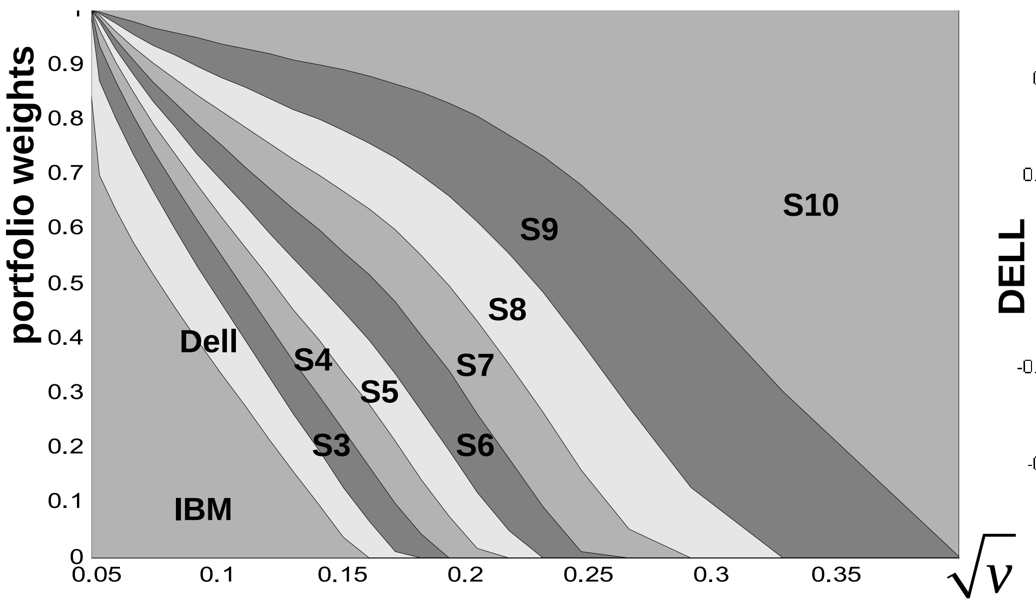


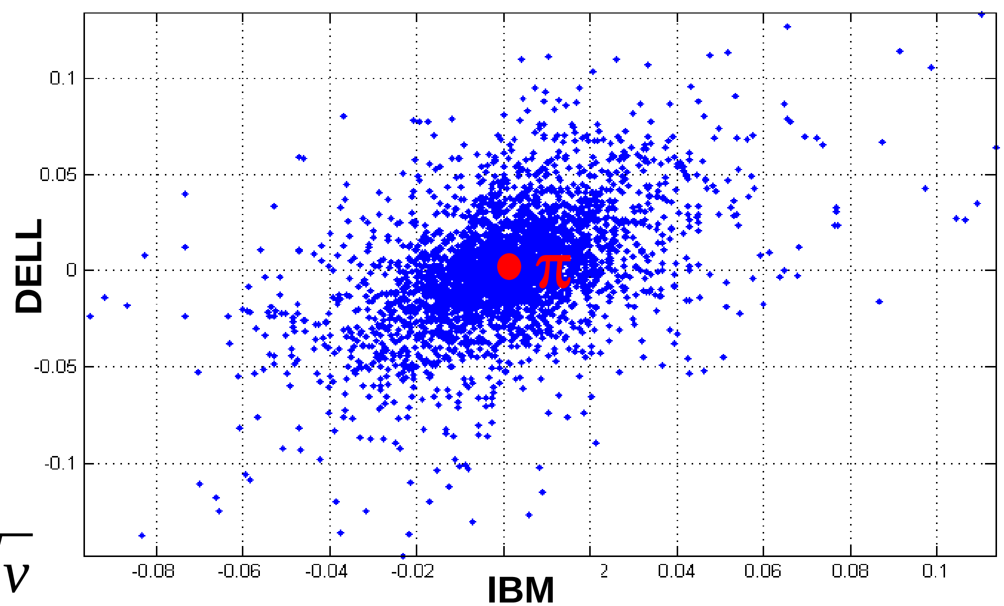
equilibrium returns

Market distribution

$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **returns on asset classes/funds**

efficient frontier





Optimization

$$\mathbf{w}_{\lambda} \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \left\{ \mathbf{w}' \boldsymbol{\mu} - \lambda \mathbf{w}' \boldsymbol{\Sigma} \mathbf{w} \right\}$$

ESTIMATION RISK

SCENARIO ANALYSIS

THE BLACK-LITTERMAN APPROACH

- **Estimation risk**
- **Views**
- **Discussion**

ENTROPY POOLING

CASE STUDIES

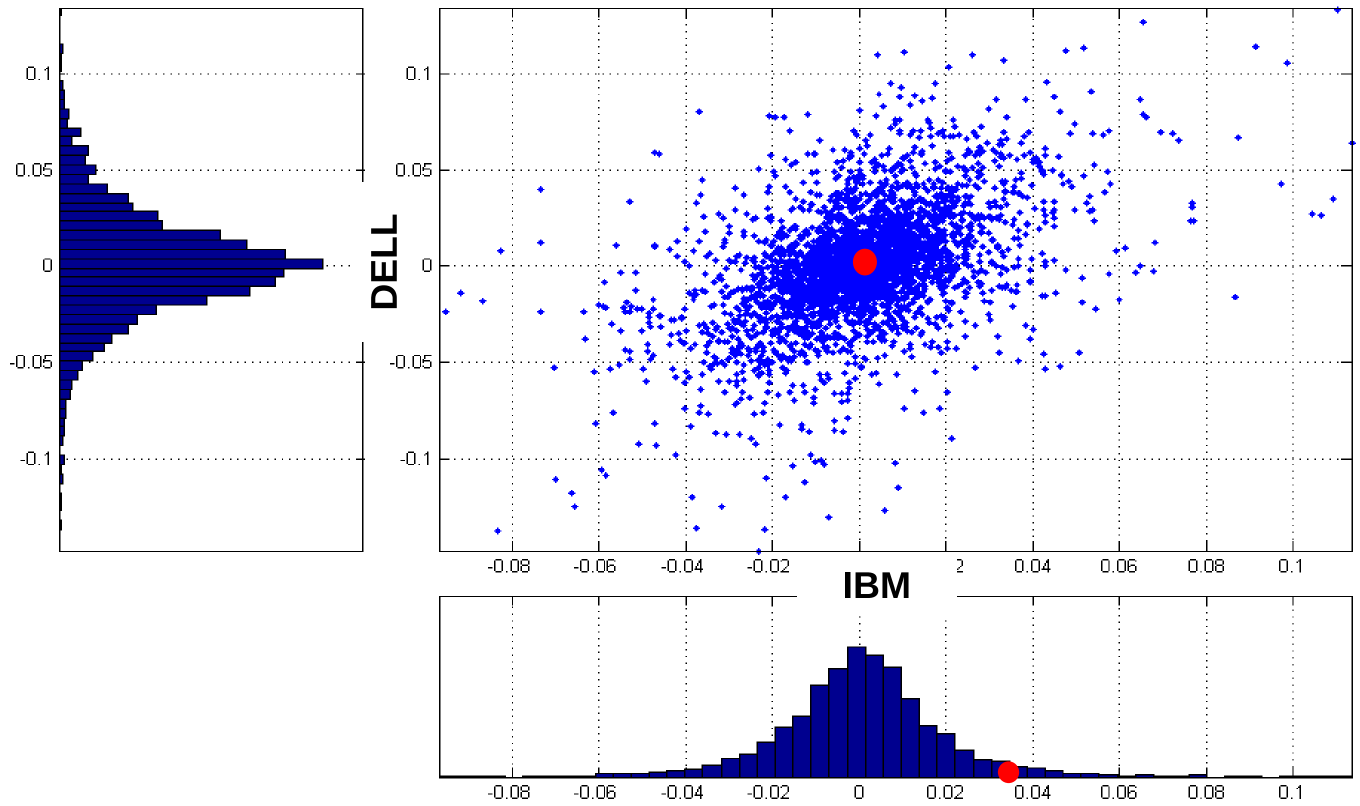
REFERENCES AND CONCLUSIONS

Market distribution

$\mathbf{X} \sim \mathrm{N}(\mu, \Sigma)$ **returns on asset classes/funds**

Views

scenario analysis with uncertainty



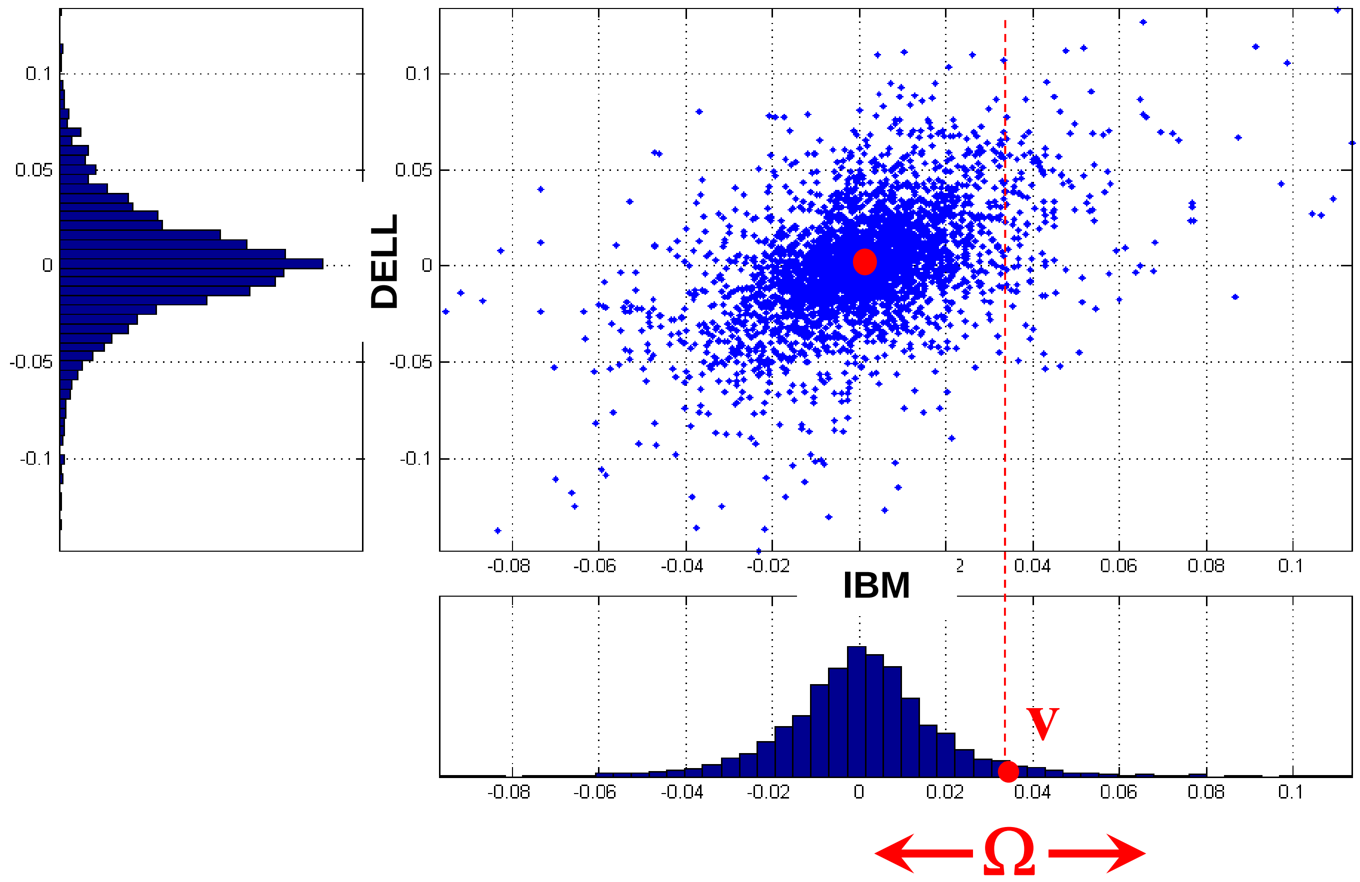
Market distribution

$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **returns on asset classes/funds**

Views

$\mathbf{Q}\boldsymbol{\mu} \sim \mathrm{N}(\mathbf{v}, \boldsymbol{\Omega})$

FOCUS: $\mathbf{Q}$ = portfolio



Market distribution

$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **returns on asset classes/funds**

Views

$\mathbf{Q}\boldsymbol{\mu} \sim \mathrm{N}(\mathbf{v}, \boldsymbol{\Omega})$

Bayes' formula

$$\mathbf{X}|\mathbf{v}; \boldsymbol{\Omega} \sim \mathrm{N}(\boldsymbol{\mu}_{BL}, \boldsymbol{\Sigma}_{BL})$$

$$\begin{aligned} \boldsymbol{\mu}_{BL} &= \boldsymbol{\pi} + \tau\boldsymbol{\Sigma}\mathbf{Q}'\left(\tau\mathbf{Q}\boldsymbol{\Sigma}\mathbf{Q}' + \boldsymbol{\Omega}\right)^{-1}(\mathbf{v} - \mathbf{Q}\boldsymbol{\pi}) \\ \boldsymbol{\Sigma}_{BL} &= (1+\tau)\boldsymbol{\Sigma} - \tau^2\boldsymbol{\Sigma}\mathbf{Q}'\left(\tau\mathbf{Q}\boldsymbol{\Sigma}\mathbf{Q}' + \boldsymbol{\Omega}\right)^{-1}\mathbf{Q}\boldsymbol{\Sigma}. \end{aligned}$$

Market distribution

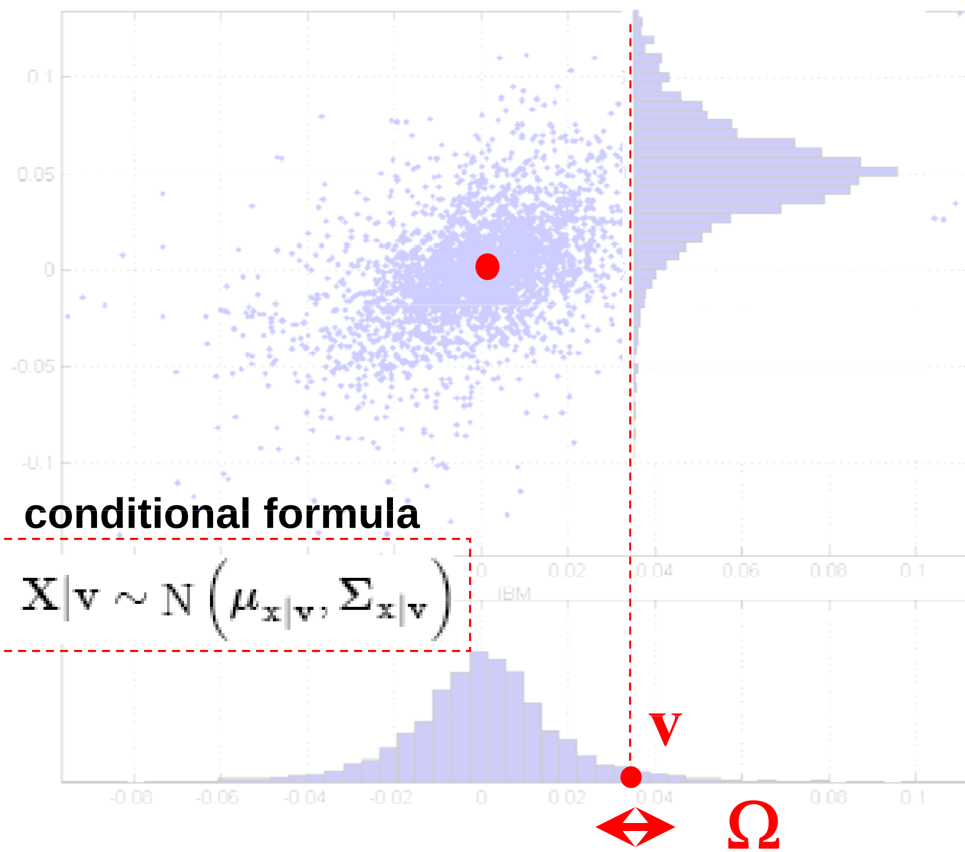
$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **returns on asset classes/funds**

Views

$\mathbf{Q}\boldsymbol{\mu} \sim \mathrm{N}(\mathbf{v}, \boldsymbol{\Omega})$

$\boldsymbol{\Omega} \rightarrow \mathbf{0}$

small uncertainty



Bayes' formula

$$\mathbf{X}|\mathbf{v};\boldsymbol{\Omega} \sim \mathrm{N}(\boldsymbol{\mu}_{BL}, \boldsymbol{\Sigma}_{BL})$$

$$\boldsymbol{\mu}_{BL} = \boldsymbol{\pi} + \tau\boldsymbol{\Sigma}\mathbf{Q}'\left(\tau\mathbf{Q}\boldsymbol{\Sigma}\mathbf{Q}' + \boldsymbol{\Omega}\right)^{-1}(\mathbf{v} - \mathbf{Q}\boldsymbol{\pi})$$

$$\boldsymbol{\Sigma}_{BL} = (1+\tau)\boldsymbol{\Sigma} - \tau^2\boldsymbol{\Sigma}\mathbf{Q}'\left(\tau\mathbf{Q}\boldsymbol{\Sigma}\mathbf{Q}' + \boldsymbol{\Omega}\right)^{-1}\mathbf{Q}\boldsymbol{\Sigma}.$$

conditional formula

$$\mathbf{X}|\mathbf{v} \sim \mathrm{N}\left(\boldsymbol{\mu}_{\mathbf{x}|\mathbf{v}}, \boldsymbol{\Sigma}_{\mathbf{x}|\mathbf{v}}\right)$$

Market distribution

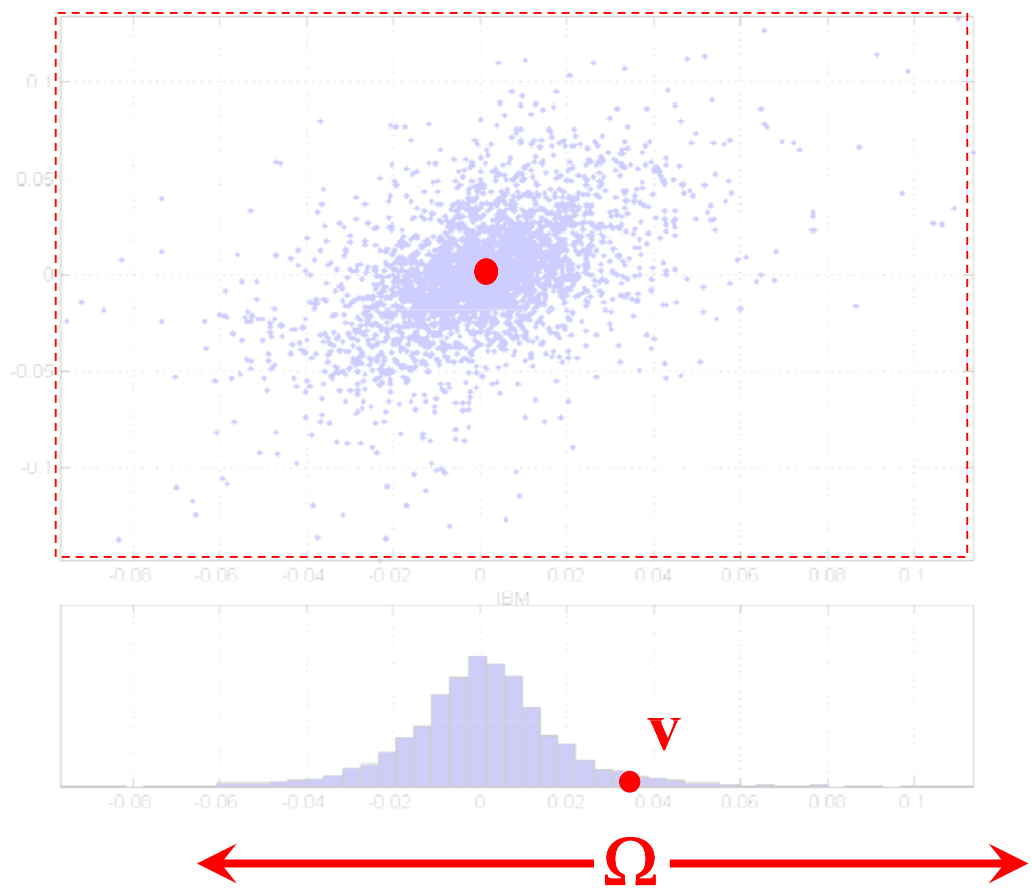
$$\boxed{\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})}$$ **returns on asset classes/funds**

Views

$$\mathbf{Q}\boldsymbol{\mu} \sim \mathrm{N}(\mathbf{v}, \boldsymbol{\Omega})$$

$\boldsymbol{\Omega} \to \infty$

large uncertainty



Bayes' formula

$$\mathbf{X}|\mathbf{v};\boldsymbol{\Omega} \sim \mathrm{N}(\boldsymbol{\mu}_{BL}, \boldsymbol{\Sigma}_{BL})$$

$$\begin{aligned} \boldsymbol{\mu}_{BL} &= \boldsymbol{\pi} + \tau\boldsymbol{\Sigma}\mathbf{Q}'\left(\tau\mathbf{Q}\boldsymbol{\Sigma}\mathbf{Q}' + \boldsymbol{\Omega}\right)^{-1}(\mathbf{v} - \mathbf{Q}\boldsymbol{\pi}) \\ \boldsymbol{\Sigma}_{BL} &= (1+\tau)\boldsymbol{\Sigma} - \tau^2\boldsymbol{\Sigma}\mathbf{Q}'\left(\tau\mathbf{Q}\boldsymbol{\Sigma}\mathbf{Q}' + \boldsymbol{\Omega}\right)^{-1}\mathbf{Q}\boldsymbol{\Sigma}. \end{aligned}$$

Market distribution

$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **returns on asset classes/funds**

Views

$\mathbf{Q}\boldsymbol{\mu} \sim \mathrm{N}(\mathbf{v}, \boldsymbol{\Omega})$

Bayes' formula

$\mathbf{X}|\mathbf{v}; \boldsymbol{\Omega} \sim \mathrm{N}(\boldsymbol{\mu}_{BL}, \boldsymbol{\Sigma}_{BL})$

Market distribution

$$\mathbf{X} \sim \mathrm{N}\left(\boldsymbol{\mu}, \boldsymbol{\Sigma}\right)$$

returns on asset classes/funds

Views

$$\mathbf{Q}\boldsymbol{\mu} \sim \mathrm{N}\left(\mathbf{v}, \boldsymbol{\Omega}\right)$$

Bayes' formula

$$\mathbf{X}|\mathbf{v}; \boldsymbol{\Omega} \sim \mathrm{N}\left(\boldsymbol{\mu}_{BL}, \boldsymbol{\Sigma}_{BL}\right)$$

Optimization

$$\mathbf{w}_{\lambda} \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \left\{\mathbf{w}'\boldsymbol{\mu} - \lambda \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}\right\}$$

Optimization

$$\mathbf{w}_{\lambda} \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \left\{\mathbf{w}'\boldsymbol{\mu}_{BL} - \lambda \mathbf{w}'\boldsymbol{\Sigma}_{BL}\mathbf{w}\right\}$$

Market distribution

$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **returns on asset classes/funds**

Views

$\mathbf{Q}\boldsymbol{\mu} \sim \mathrm{N}(\mathbf{v}, \boldsymbol{\Omega})$

LIVE

Optimization

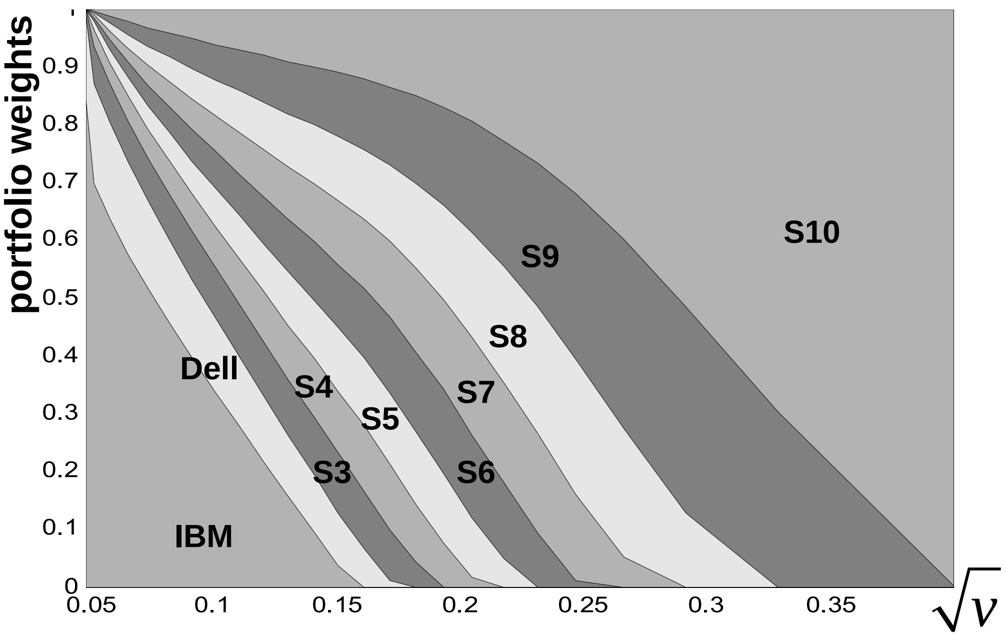
$$\mathbf{w}_\lambda \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \left\{ \mathbf{w}'\boldsymbol{\mu} - \lambda \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} \right\}$$

Optimization

$$\mathbf{w}_\lambda \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \left\{ \mathbf{w}'\boldsymbol{\mu}_{BL} - \lambda \mathbf{w}'\boldsymbol{\Sigma}_{BL}\mathbf{w} \right\}$$

Market distribution

$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **returns on asset classes/funds**

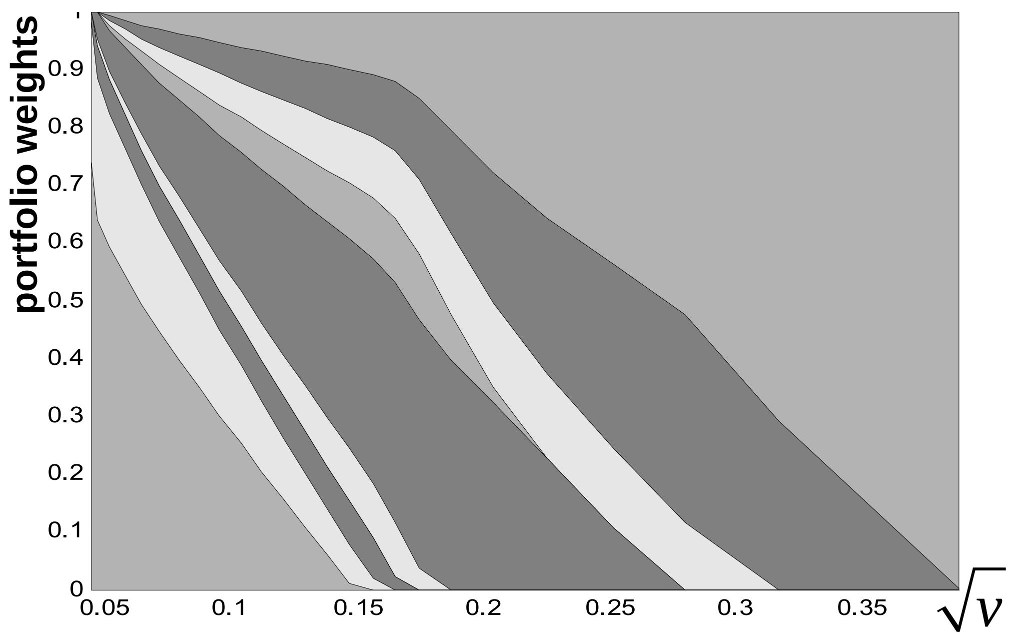


Optimization

$$\mathbf{w}_\lambda \equiv \underset{\mathbf{w}\in\mathcal{C}}{\operatorname{argmax}} \left\{ \mathbf{w}'\boldsymbol{\mu} - \lambda \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} \right\}$$

Views

$$\mathbf{Q}\boldsymbol{\mu} \sim \mathrm{N}(\mathbf{v}, \boldsymbol{\Omega})$$



Optimization

$$\mathbf{w}_\lambda \equiv \underset{\mathbf{w}\in\mathcal{C}}{\operatorname{argmax}} \left\{ \mathbf{w}'\boldsymbol{\mu}_{BL} - \lambda \mathbf{w}'\boldsymbol{\Sigma}_{BL}\mathbf{w} \right\}$$

Market distribution

$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **returns on asset classes/funds**

Views/Scenarios

$\mathbf{Q}$ $\boldsymbol{\mu}$ $\equiv \mathbf{v}$ **+ uncertainty**

Optimization

$$\mathbf{w}_{\lambda} \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \left\{ \mathbf{w}' \boldsymbol{\mu} - \lambda \mathbf{w}' \boldsymbol{\Sigma} \mathbf{w} \right\}$$

Market distribution

$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **returns on asset classes/funds**

Views/Scenarios

$\mathbf{Q} \quad \boldsymbol{\mu} \quad \equiv \mathbf{v}$ **+ uncertainty**

Market is not only returns: implied volatilities (derivatives)
rates paths (mortgages)
implied correlations (CDO's)
….

Optimization

$$\mathbf{w}_{\lambda} \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \left\{ \mathbf{w}' \boldsymbol{\mu} - \lambda \mathbf{w}' \boldsymbol{\Sigma} \mathbf{w} \right\}$$

Black-Litterman – con's

Market distribution

$\mathbf{X} \sim \cancel{\mathrm{N}}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **~~returns~~ on asset classes/funds**

Views/Scenarios

$\mathbf{Q} \quad \boldsymbol{\mu} \quad \equiv \mathbf{v}$ **+ uncertainty**

Market is not only returns

Market is not only normal: fat tails, skewness, tail-risk codependence,…

Optimization

$$\mathbf{w}_\lambda \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \left\{ \mathbf{w}' \boldsymbol{\mu} - \lambda \mathbf{w}' \boldsymbol{\Sigma} \mathbf{w} \right\}$$

Market distribution

$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **returns on asset classes/funds**

Views/Scenarios

$\mathbf{Q} \quad \boldsymbol{\mu} \quad \equiv \mathbf{v}$ **+ uncertainty**

Market is not only returns

Market is not only normal

Market is not only equilibrium: historical estimates, implied values, …

Optimization

$$\mathbf{w}_{\lambda} \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \left\{ \mathbf{w}' \boldsymbol{\mu} - \lambda \mathbf{w}' \boldsymbol{\Sigma} \mathbf{w} \right\}$$

Market distribution

$\cancel{\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})}$ ~~returns on asset classes/funds~~

Views/Scenarios

$\cancel{\mathbf{Q}} \; \boldsymbol{\mu} \equiv \mathbf{v}$ **+ uncertainty**

Market is not only returns

Market is not only normal

Market is not only equilibrium

Views are not only portfolios: generic non-linear functions

Optimization

$$\mathbf{w}_\lambda \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \left\{ \mathbf{w}' \boldsymbol{\mu} - \lambda \mathbf{w}' \boldsymbol{\Sigma} \mathbf{w} \right\}$$

Market distribution

$\cancel{\mathbf{X} \sim \mathrm{N}(\mu, \Sigma)}$ ~~**returns on asset classes/funds**~~

Views/Scenarios

$\cancel{\mathbf{Q}}\ \cancel{\mu} \equiv \mathbf{v}$ **+ uncertainty**

Market is not only returns

Market is not only normal

Market is not only equilibrium

Views are not only portfolios

Views are not only on expectations: correlations,
volatilities,
tail behavior,
copulas,
…

Optimization

$$\mathbf{w}_\lambda \equiv \underset{\mathbf{w}\in\mathcal{C}}{\operatorname{argmax}} \{\mathbf{w}'\mu - \lambda\mathbf{w}'\Sigma\mathbf{w}\}$$

Market distribution

$\cancel{\mathbf{X} \sim \mathrm{N}(\mu, \Sigma)}$ **~~returns~~ on asset classes/funds**

Views/Scenarios

$\cancel{\mathbf{Q}}$ $\cancel{\mu}$ $\cancel{\equiv \mathbf{v}}$ **+ uncertainty**

Market is not only returns

Market is not only normal

Market is not only equilibrium

Views are not only portfolios

Views are not only on expectations

Views are not only equalities: stock ranking, qualitative views

Optimization

$$\mathbf{w}_\lambda \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \left\{ \mathbf{w}' \mu - \lambda \mathbf{w}' \Sigma \mathbf{w} \right\}$$

Market distribution

$\mathbf{X} \cancel{\sim} \cancel{\mathrm{N}(\mu, \Sigma)}$ ~~returns~~ on asset classes/funds

Views/Scenarios

$\cancel{\mathbf{Q}} \; \cancel{\mu} \; \cancel{\equiv} \, \mathbf{v}$ + uncertainty

Market is not only returns

Market is not only normal

Market is not only equilibrium

Views are not only portfolios

Views are not only on expectations

Views are not only equalities

Optimization is not only mean variance: mean-CVaR, mean-VaR, …

Optimization

$$\mathbf{w}_{\lambda} \equiv \cancel{\underset{\mathbf{w}\in\mathcal{C}}{\operatorname{argmax}}} \left\{ \mathbf{w}'\mu - \lambda \mathbf{w}'\Sigma\mathbf{w} \right\}$$

ESTIMATION RISK

SCENARIO ANALYSIS

THE BLACK-LITTERMAN APPROACH

ENTROPY POOLING

- **Theory**
- **Implementation**

CASE STUDIES

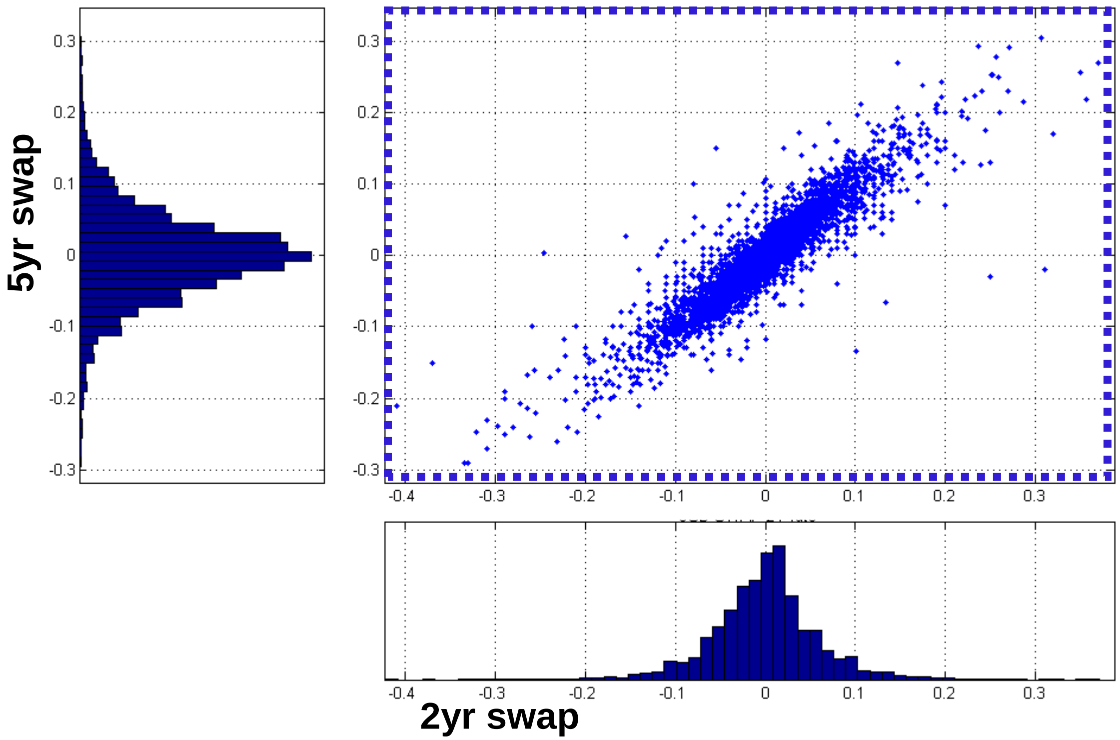
REFERENCES AND CONCLUSIONS

e.g.

X_1 **2-yr swap rate**

X_2 **5-yr swap rate**

Market distr. $\mathbf{X} \sim \boxed{f_{\mathbf{X}}}$ **not returns, not normal, not equilibrium**

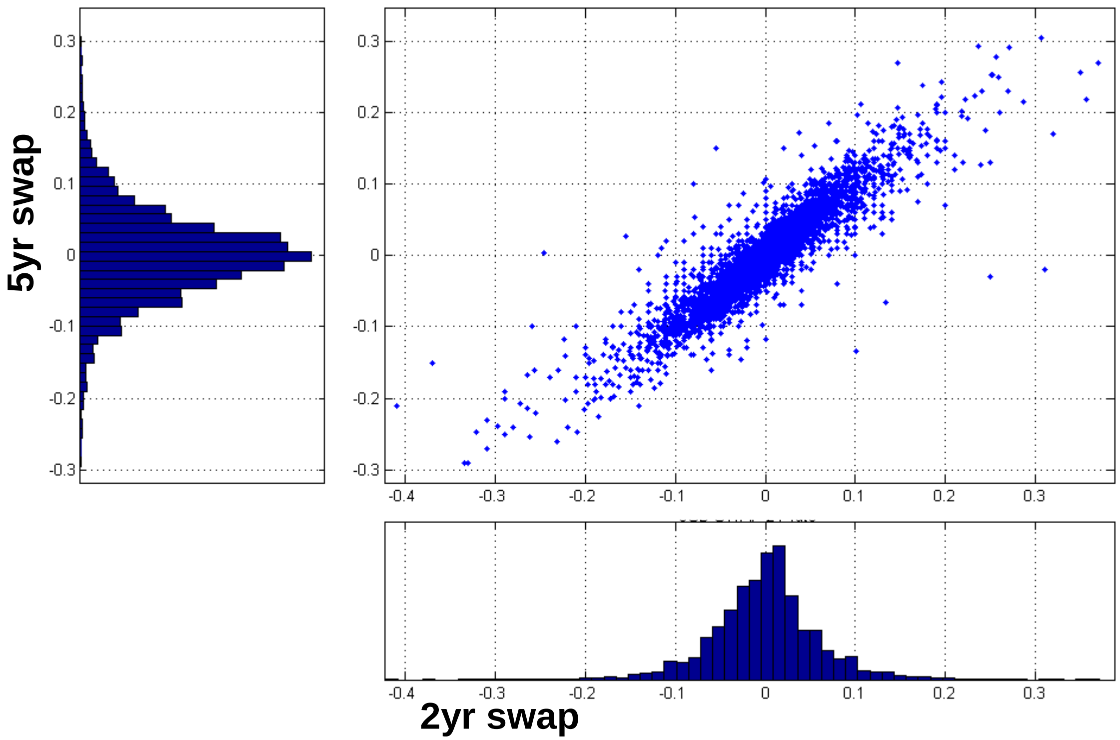


Market distr. $\mathbf{X} \sim f_{\mathbf{X}}$. **not returns, not normal, not equilibrium**

e.g.

X_1 **2-yr swap rate**

X_2 **5-yr swap rate**



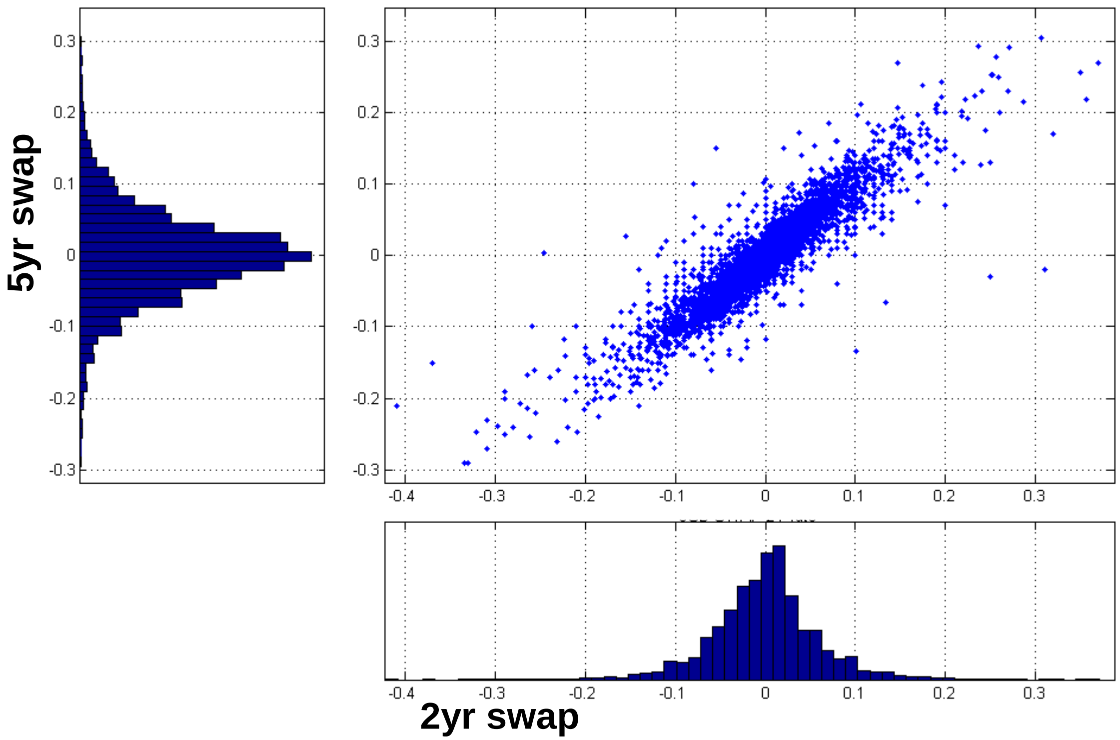
Pricing $P_{t+\tau} \equiv P\left(\mathbf{X}, \mathcal{I}_t\right)$ **delta/gamma/vega, full pricing, …**

e.g.

duration + convexity

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}$. **not returns, not normal, not equilibrium**

e.g.
X_1 **2-yr swap rate**
X_2 **5-yr swap rate**



Pricing $P_{t+\tau} \equiv P(\mathbf{X}, \mathcal{I}_t)$ **delta/gamma/vega, full pricing, …**

Optimization $\mathbf{w}^* \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \{\mathcal{S}(\mathbf{w}; f_{\mathbf{X}})\}$ **utility, mean-CVaR, …**

e.g.
duration + convexity
mean-variance

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}.$ **not returns, not normal, not equilibrium**

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$ **non-linear functions and external factors**

e.g.

X_1 **2-yr swap rate**

X_2 **5-yr swap rate**

$V \equiv X_1^{\,2} + X_2^{\,2}$

convexity factor

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}$. **not returns, not normal, not equilibrium**

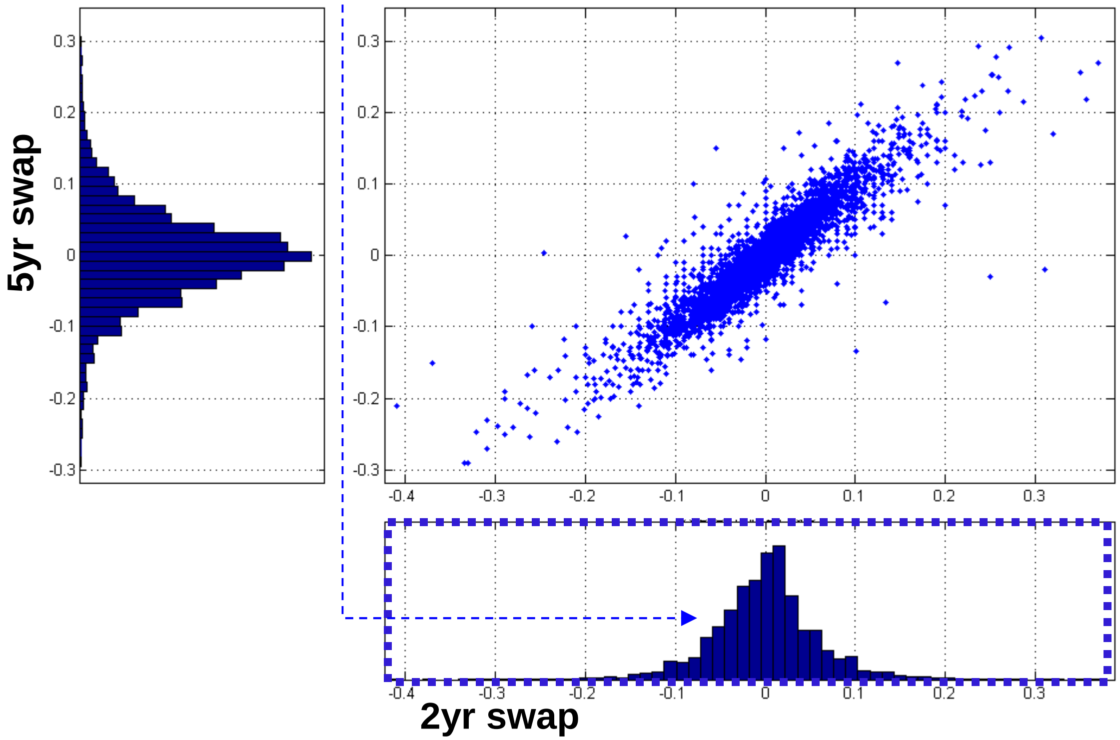
Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim \boxed{f_{\mathbf{V}}}$ **non-linear functions and external factors**

e.g.

X_1 **2-yr swap rate**

X_2 **5-yr swap rate**

$V \equiv X_1$



A. Meucci - *Fully Flexible Views (Entropy Pooling)*

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}$. **not returns, not normal, not equilibrium**

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim \boxed{f_{\mathbf{V}}}$ **non-linear functions and external factors**

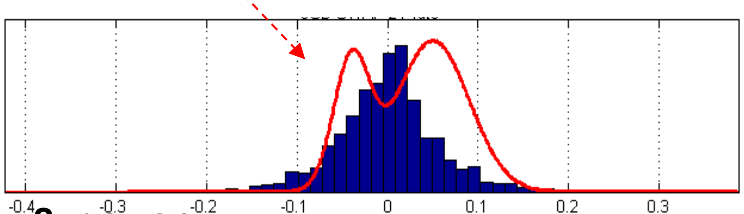
Views ✓ $\mathbf{V} \sim \boxed{\tilde{f}_{\mathbf{V}}} \neq f_{\mathbf{V}}$. **full distribution specification**

e.g.

X_1 **2-yr swap rate**

X_2 **5-yr swap rate**

$V \equiv X_1$



2yr swap

e.g.

X_1 **2-yr swap rate**

X_2 **5-yr swap rate**

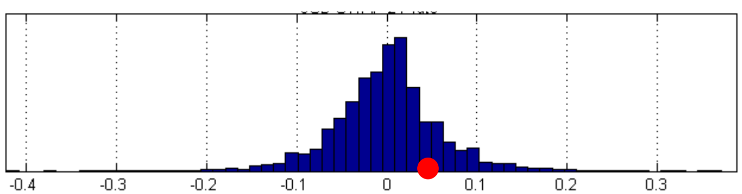
$V \equiv X_1$

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}.$ **not returns, not normal, not equilibrium**

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$ **non-linear functions and external factors**

Views $\mathbf{V} \sim \tilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}.$ **full distribution specification**

✓ $\tilde{m}\{V_k\} \gtreqless \tilde{\mu}_{\mathbf{v},k}:$ **view on expectations (BL), medians**



2yr swap

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}.$ **not returns, not normal, not equilibrium**

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$ **non-linear functions and external factors**

Views $\mathbf{V} \sim \widetilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}.$ **full distribution specification**

$\widetilde{m}\{V_k\} \gtreqless \widetilde{\mu}_{\mathbf{v},k}:$ **view on expectations (BL), medians**

$\widetilde{m}\{V_1\} \geq \widetilde{m}\{V_2\} \geq \cdots \geq \widetilde{m}\{V_K\}$ **ranking**

e.g.

X_1 **2-yr swap rate**

X_2 **5-yr swap rate**

$V \equiv X_1$

e.g.

X_1 **2-yr swap rate**

X_2 **5-yr swap rate**

$V \equiv X_1$

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}.$ **not returns, not normal, not equilibrium**

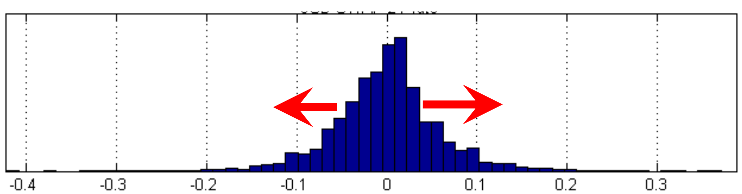
Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$ **non-linear functions and external factors**

Views $\mathbf{V} \sim \widetilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}.$ **full distribution specification**

$\widetilde{m}\{V_k\} \gtreqless \widetilde{\mu}_{\mathbf{v},k}:$ **view on expectations (BL), medians**

$\widetilde{m}\{V_1\} \geq \widetilde{m}\{V_2\} \geq \cdots \geq \widetilde{m}\{V_K\}$ **ranking**

✓ $\widetilde{\sigma}\{V_k\} \gtreqless \varkappa\sigma\{V_k\}$ **views on volatilities**



2yr swap

e.g.

X_1 **2-yr swap rate**

X_2 **5-yr swap rate**

$V \equiv X_1$

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}$. **not returns, not normal, not equilibrium**

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$ **non-linear functions and external factors**

Views $\mathbf{V} \sim \tilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}$. **full distribution specification**

$\widetilde{m}\{V_k\} \gtreqless \widetilde{\mu}_{\mathbf{v},k}$: **view on expectations (BL), medians**

$\widetilde{m}\{V_1\} \geq \widetilde{m}\{V_2\} \geq \cdots \geq \widetilde{m}\{V_K\}$ **ranking**

$\widetilde{\sigma}\{V_k\} \gtreqless \varkappa\sigma\{V_k\}$ **views on volatilities**

$\widetilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1 \mathbf{I} + \rho_2 \mathbb{C}\{\mathbf{V}\} + \rho_3 \mathbf{1}\mathbf{1}'$, **correlation stress-testing**

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}.$ **not returns, not normal, not equilibrium**

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$ **non-linear functions and external factors**

Views $\mathbf{V} \sim \widetilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}.$ **full distribution specification**

$\widetilde{m}\{V_k\} \gtreqless \widetilde{\mu}_{\mathbf{v},k}:$ **view on expectations (BL), medians**

$\widetilde{m}\{V_1\} \geq \widetilde{m}\{V_2\} \geq \cdots \geq \widetilde{m}\{V_K\}$ **ranking**

$\widetilde{\sigma}\{V_k\} \gtreqless \varkappa\sigma\{V_k\}$ **views on volatilities**

$\widetilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1\mathbf{I} + \rho_2\mathbb{C}\{\mathbf{V}\} + \rho_3\mathbf{1}\mathbf{1}',$ **correlation stress-testing**

$\widetilde{Q}_V(u) \gtreqless Q_V(u)$ **view on tail behavior**

e.g.

X_1 **2-yr swap rate**

X_2 **5-yr swap rate**

$V \equiv X_1$

A. Meucci - Fully Flexible Views (Entropy Pooling)

e.g.

X_1 **2-yr swap rate**

X_2 **5-yr swap rate**

$V \equiv X_1$

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}.$ **not returns, not normal, not equilibrium**

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$ **non-linear functions and external factors**

Views $\mathbf{V} \sim \widetilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}.$ **full distribution specification**

$$\left.\begin{array}{l}
\widetilde{m}\{V_k\} \gtreqless \widetilde{\mu}_{\mathbf{v},k}; \\
\widetilde{m}\{V_1\} \geq \widetilde{m}\{V_2\} \geq \cdots \geq \widetilde{m}\{V_K\} \\
\widetilde{\sigma}\{V_k\} \gtreqless \varkappa\sigma\{V_k\} \\
\widetilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1 \mathbf{I} + \rho_2 \mathbb{C}\{\mathbf{V}\} + \rho_3 \mathbf{1}\mathbf{1}', \\
\widetilde{Q}_V(u) \gtreqless Q_V(u)
\end{array}\right\}$$

partial distribution specification

A. Meucci - Fully Flexible Views (Entropy Pooling)

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}$. **not returns, not normal, not equilibrium**

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$ **non-linear functions and external factors**

Views $\mathbf{V} \sim \widetilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}$. **full distribution specification**

$$
\left.
\begin{array}{l}
\widetilde{m}\{V_k\} \gtreqqless \widetilde{\mu}_{\mathbf{v},k}; \\
\widetilde{m}\{V_1\} \geq \widetilde{m}\{V_2\} \geq \cdots \geq \widetilde{m}\{V_K\} \\
\widetilde{\sigma}\{V_k\} \gtreqqless \varkappa\sigma\{V_k\} \\
\widetilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1\mathbf{I} + \rho_2\mathbb{C}\{\mathbf{V}\} + \rho_3\mathbf{1}\mathbf{1}', \\
\widetilde{Q}_V(u) \gtreqqless Q_V(u)
\end{array}
\right\}
$$

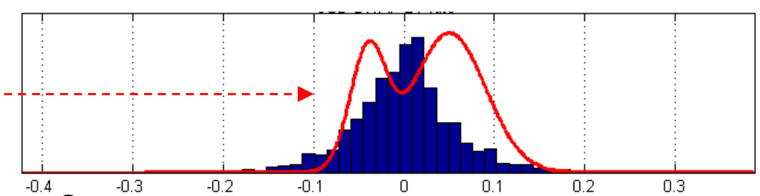
partial distribution specification

e.g.

X_1 **2-yr swap rate**

X_2 **5-yr swap rate**

$V \equiv X_1$



2yr swap

e.g.

X_1 **2-yr swap rate**

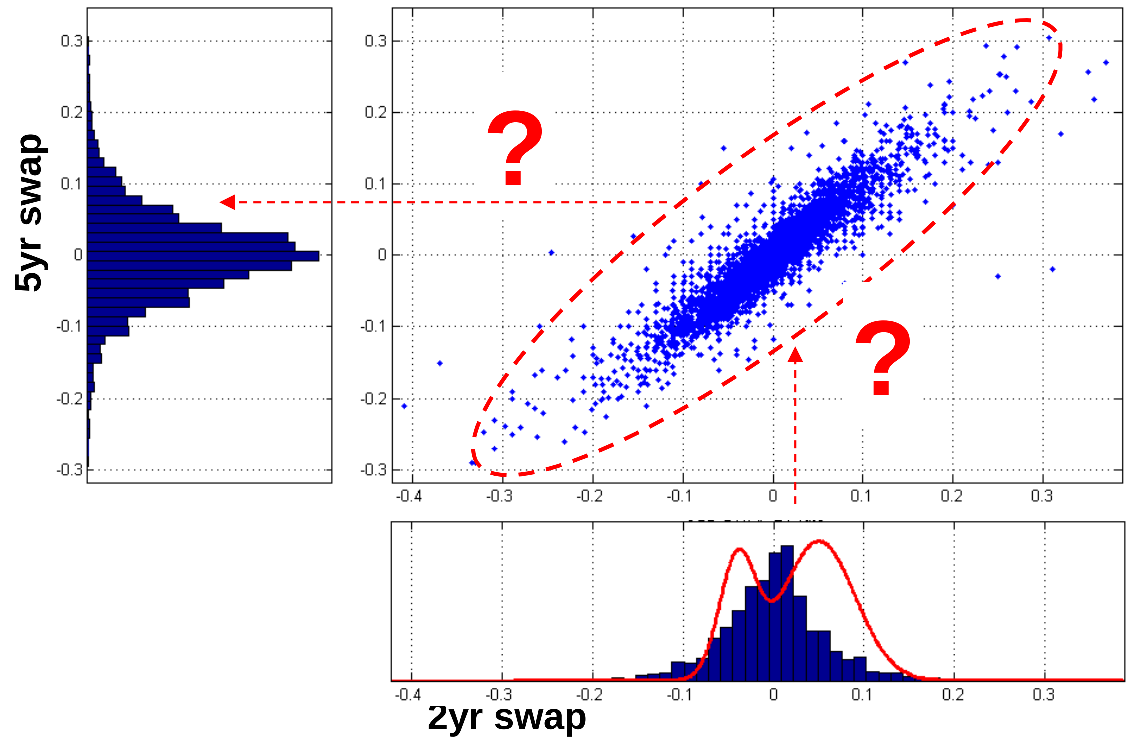
X_2 **5-yr swap rate**

$V \equiv X_1$

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}$ **not returns, not normal, not equilibrium**

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$ **non-linear functions and external factors**

Views



e.g.

X_1 **2-yr swap rate**

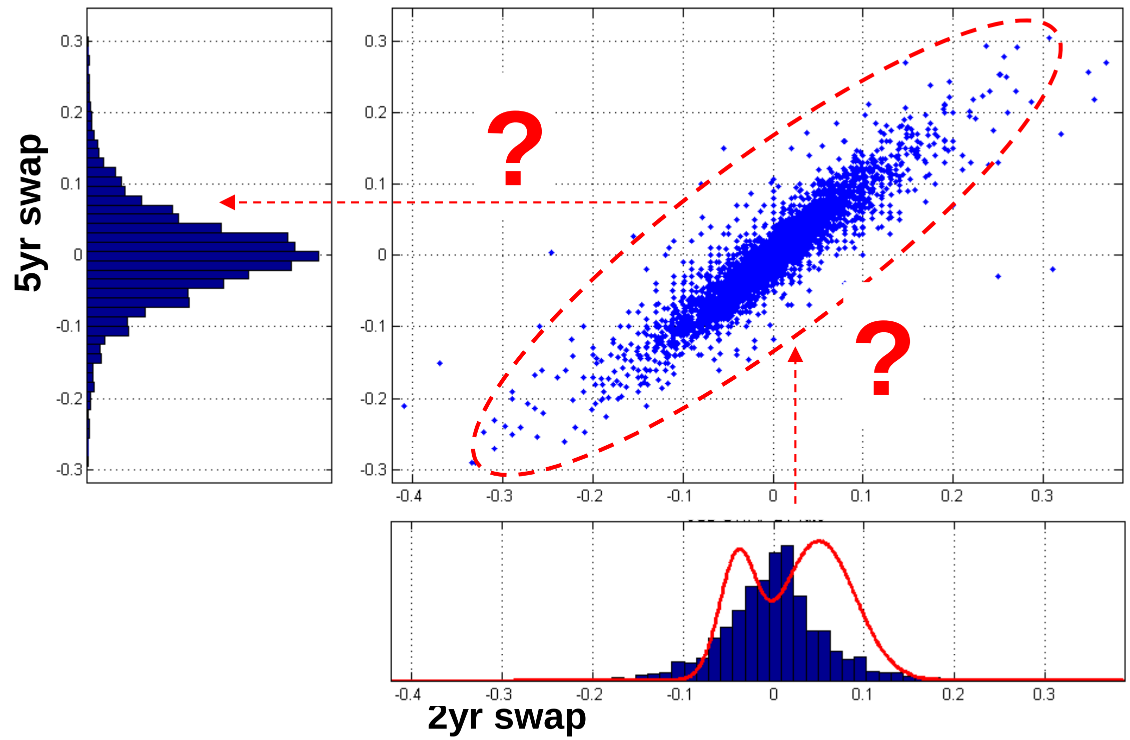
X_2 **5-yr swap rate**

$V \equiv X_1$

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}$ **not returns, not normal, not equilibrium**

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$ **non-linear functions and external factors**

Views



Posterior $\tilde{f}_{\mathbf{X}}$ **?**

e.g.

X_1 **2-yr swap rate**

X_2 **5-yr swap rate**

$V \equiv X_1$

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}.$ **not returns, not normal, not equilibrium**

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$ **non-linear functions and external factors**

Views $\mathbf{V} \sim \tilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}.$ **full distribution specification**

$$\left.\begin{array}{l}
\widetilde{m}\{V_k\} \gtreqless \widetilde{\mu}_{\mathbf{V},k}; \\
\widetilde{m}\{V_1\} \geq \widetilde{m}\{V_2\} \geq \cdots \geq \widetilde{m}\{V_K\} \\
\widetilde{\sigma}\{V_k\} \gtreqless \varkappa\sigma\{V_k\} \\
\widetilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1\mathbf{I} + \rho_2\mathbb{C}\{\mathbf{V}\} + \rho_3\mathbf{1}\mathbf{1}', \\
\widetilde{Q}_V(u) \gtreqless Q_V(u)
\end{array}\right\}$$

partial distribution specification

Posterior $\tilde{f}_{\mathbf{X}}$ **?**

relative entropy
"distance" btw. distributions $\mathcal{E}\left(\tilde{f}_{\mathbf{X}}, f_{\mathbf{X}}\right) \equiv \int \tilde{f}_{\mathbf{X}}(\mathbf{x}) \left[\ln \tilde{f}_{\mathbf{X}}(\mathbf{x}) - \ln f_{\mathbf{X}}(\mathbf{x})\right] d\mathbf{x}.$

A. Meucci - Fully Flexible Views (Entropy Pooling)

e.g.

X_1 **2-yr swap rate**

X_2 **5-yr swap rate**

$V \equiv X_1$

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}.$ **not returns, not normal, not equilibrium**

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$ **non-linear functions and external factors**

Views $\mathbf{V} \sim \tilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}.$ **full distribution specification**

$$\tilde{m}\{V_k\} \gtreqqless \tilde{\mu}_{\mathbf{v},k}:$$

$$\tilde{m}\{V_1\} \geq \tilde{m}\{V_2\} \geq \cdots \geq \tilde{m}\{V_K\}$$

$$\tilde{\sigma}\{V_k\} \gtreqqless \varkappa\sigma\{V_k\}$$

$$\tilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1\mathbf{I} + \rho_2\mathbb{C}\{\mathbf{V}\} + \rho_3\mathbf{1}\mathbf{1}',$$

$$\tilde{Q}_V(u) \gtreqqless Q_V(u)$$

partial distribution specification

Posterior $\tilde{f}_{\mathbf{X}} \equiv \operatorname{argmin}\{\mathcal{E}(f, f_{\mathbf{X}})\}$ **least distance from prior**

relative entropy
"distance" btw. distributions $\mathcal{E}\left(\tilde{f}_{\mathbf{X}}, f_{\mathbf{X}}\right) \equiv \int \tilde{f}_{\mathbf{X}}(\mathbf{x})\left[\ln \tilde{f}_{\mathbf{X}}(\mathbf{x}) - \ln f_{\mathbf{X}}(\mathbf{x})\right] d\mathbf{x}.$

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}.$ **not returns, not normal, not equilibrium**

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$ **non-linear functions and external factors**

Views $\mathbf{V} \sim \widetilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}.$ **full distribution specification**

$$\left.\begin{array}{l}
\widetilde{m}\{V_k\} \gtreqless \widetilde{\mu}_{\mathbf{V},k}; \\
\widetilde{m}\{V_1\} \geq \widetilde{m}\{V_2\} \geq \cdots \geq \widetilde{m}\{V_K\} \\
\widetilde{\sigma}\{V_k\} \gtreqless \varkappa\sigma\{V_k\} \\
\widetilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1 \mathbf{I} + \rho_2 \mathbb{C}\{\mathbf{V}\} + \rho_3 \mathbf{1}\mathbf{1}', \\
\widetilde{Q}_V(u) \gtreqless Q_V(u)
\end{array}\right\}$$

partial distribution specification

Posterior $\widetilde{f}_{\mathbf{X}} \equiv \underset{\boxed{f \in \mathbb{V}}}{\operatorname{argmin}} \{\mathcal{E}(f, f_{\mathbf{X}})\}$ **least distance from prior, views satisfied**

relative entropy
“distance” btw. distributions $\mathcal{E}\left(\widetilde{f}_{\mathbf{X}}, f_{\mathbf{X}}\right) \equiv \int \widetilde{f}_{\mathbf{X}}(\mathrm{x}) \left[\ln \widetilde{f}_{\mathbf{X}}(\mathrm{x}) - \ln f_{\mathbf{X}}(\mathrm{x})\right] d\mathrm{x}.$

A. Meucci - Fully Flexible Views (Entropy Pooling)

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}.$ **not returns, not normal, not equilibrium**

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$ **non-linear functions and external factors**

Views $\mathbf{V} \sim \tilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}.$ **full distribution specification**

$$\left.\begin{array}{l}
\tilde{m}\{V_k\} \gtreqqless \tilde{\mu}_{\mathbf{v},k}; \\
\tilde{m}\{V_1\} \geq \tilde{m}\{V_2\} \geq \cdots \geq \tilde{m}\{V_K\} \\
\tilde{\sigma}\{V_k\} \gtreqqless \varkappa\sigma\{V_k\} \\
\tilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1 \mathbf{I} + \rho_2 \mathbb{C}\{\mathbf{V}\} + \rho_3 \mathbf{1}\mathbf{1}', \\
\tilde{Q}_V(u) \gtreqqless Q_V(u)
\end{array}\right\} \text{partial distribution specification}$$

Posterior $\boxed{\tilde{f}_{\mathbf{X}} \equiv \underset{f \in \mathbb{V}}{\operatorname{argmin}} \{\mathcal{E}(f, f_{\mathbf{X}})\}}$ **least distance from prior, views satisfied**

A. Meucci - Fully Flexible Views (Entropy Pooling)

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}.$ **not returns, not normal, not equilibrium**

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$ **non-linear functions and external factors**

Views $\mathbf{V} \sim \tilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}.$ **full distribution specification**

$$\left.\begin{aligned}
&\tilde{m}\{V_k\} \gtreqless \tilde{\mu}_{\mathbf{v},k}: \\
&\tilde{m}\{V_1\} \geq \tilde{m}\{V_2\} \geq \cdots \geq \tilde{m}\{V_K\} \\
&\tilde{\sigma}\{V_k\} \gtreqless \varkappa\sigma\{V_k\} \\
&\tilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1\mathbf{I} + \rho_2\mathbb{C}\{\mathbf{V}\} + \rho_3\mathbf{1}\mathbf{1}', \\
&\tilde{Q}_V(u) \gtreqless Q_V(u)
\end{aligned}\right\}$$

partial distribution specification

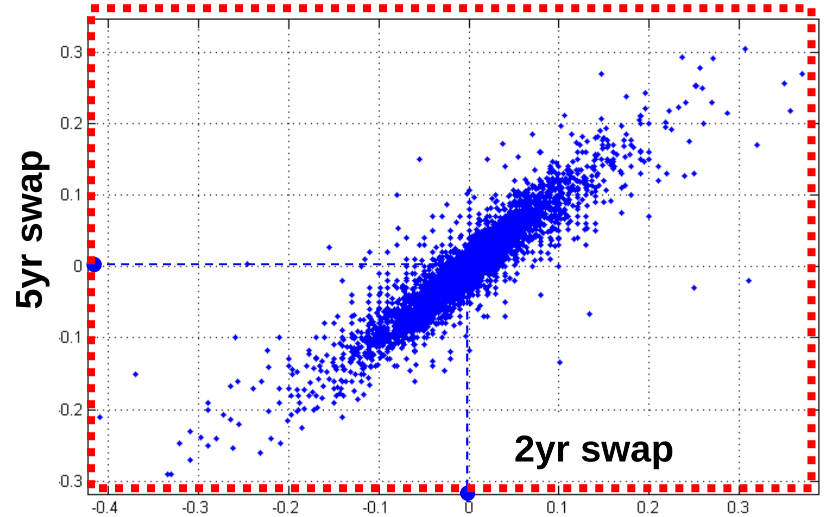
Posterior $\tilde{f}_{\mathbf{X}} \equiv \underset{f \in \mathbb{V}}{\operatorname{argmin}}\{\mathcal{E}(f, f_{\mathbf{X}})\}$ **least distance from prior, views satisfied**

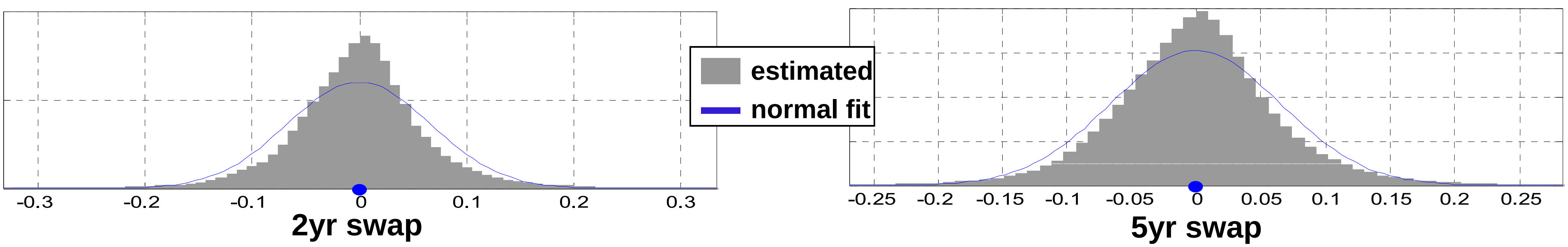
LIVE

Market distr. $\mathbf{X} \sim \boxed{f_{\mathbf{X}}}$

X_1 **2-yr swap rate**

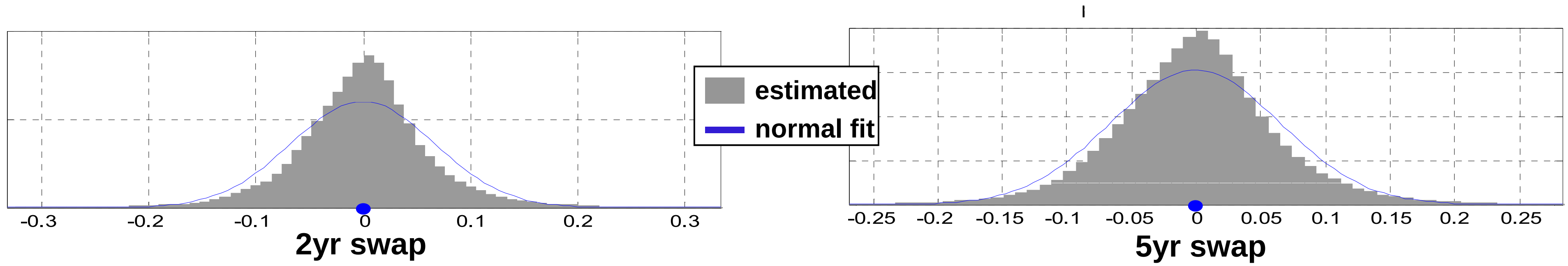
X_2 **5-yr swap rate**





Market distr. $\mathbf{X} \sim \boxed{f_{\mathbf{X}}}$.

X_1 **2-yr swap rate**
X_2 **5-yr swap rate**



A. Meucci - Fully Flexible Views (Entropy Pooling)

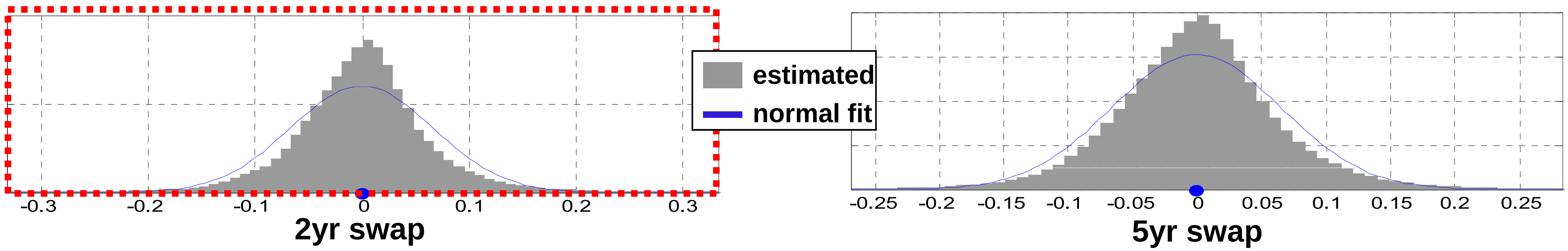
Market distr. $\mathbf{X} \sim f_{\mathbf{X}}.$

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim \boxed{f_{\mathbf{V}}}$

X_1 **2-yr swap rate**

X_2 **5-yr swap rate**

$$\boxed{V \equiv X_1}$$



Market distr. $\mathbf{X} \sim f_{\mathbf{X}}.$

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$

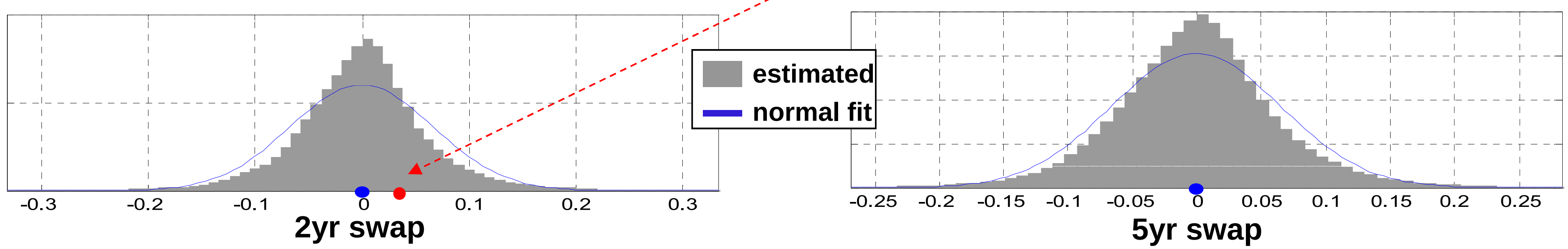
Views

X_1 **2-yr swap rate**

X_2 **5-yr swap rate**

$$V \equiv X_1$$

$$\tilde{m}\{V\} \equiv m\{V\} + \frac{\sigma\{V\}}{2} \approx 3.26\ bp$$



X_1 **2-yr swap rate**

X_2 **5-yr swap rate**

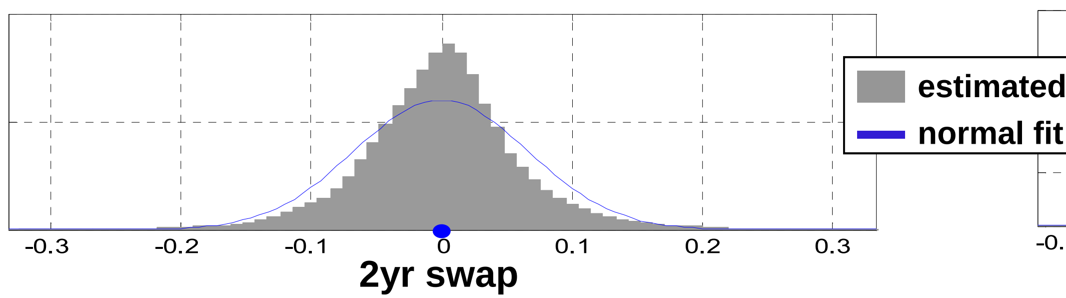
Market distr. $\mathbf{X} \sim f_{\mathbf{X}}$.

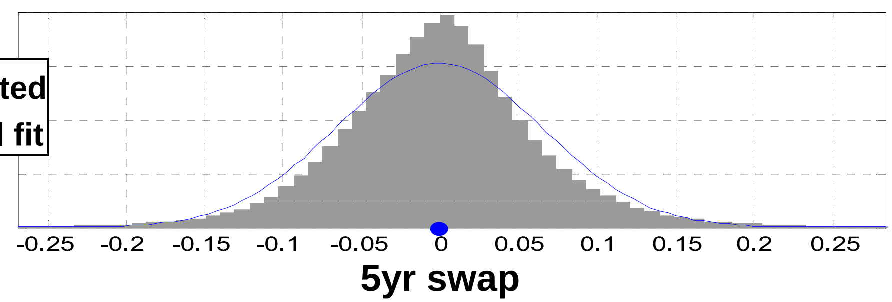
Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$

$$V \equiv X_1$$

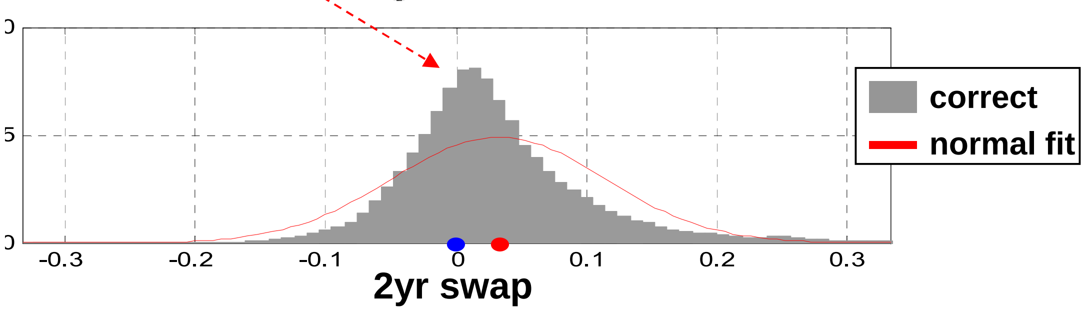
Views

$$\tilde{m}\{V\} \equiv m\{V\} + \frac{\sigma\{V\}}{2} \approx 3.26\ bp$$





Posterior $\tilde{f}_{\mathbf{X}} \equiv \underset{f \in \mathbb{V}}{\operatorname{argmin}} \{\mathcal{E}(f, f_{\mathbf{X}})\}$



Market distr. $\mathbf{X} \sim f_{\mathbf{X}}$.

X_1 **2-yr swap rate**

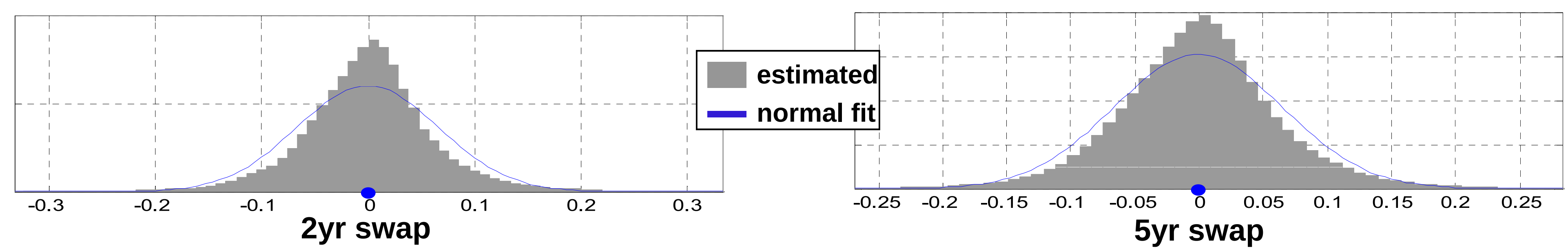
X_2 **5-yr swap rate**

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$

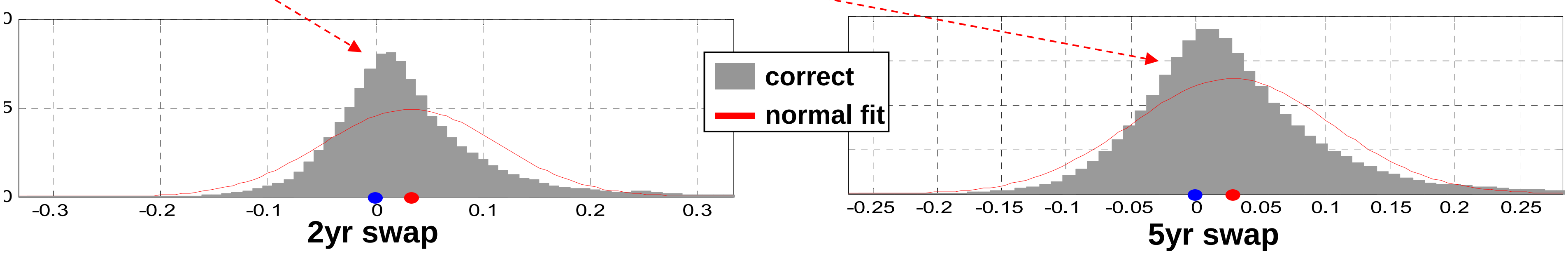
$$V \equiv X_1$$

Views

$$\tilde{m}\{V\} \equiv m\{V\} + \frac{\sigma\{V\}}{2} \approx 3.26\ bp$$



Posterior $\tilde{f}_{\mathbf{X}} \equiv \underset{f \in \mathbb{V}}{\operatorname{argmin}} \{\mathcal{E}(f, f_{\mathbf{X}})\}$



A. Meucci - Fully Flexible Views (Entropy Pooling)

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}.$ **not returns, not normal, not equilibrium**

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$ **non-linear functions and external factors**

Views $\mathbf{V} \sim \widetilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}.$ **full distribution specification**

$$\left.\begin{aligned}
&\widetilde{m}\{V_k\} \gtreqqless \widetilde{\mu}_{\mathbf{v},k}; \\
&\widetilde{m}\{V_1\} \geq \widetilde{m}\{V_2\} \geq \cdots \geq \widetilde{m}\{V_K\} \\
&\widetilde{\sigma}\{V_k\} \gtreqqless \varkappa\sigma\{V_k\} \\
&\widetilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1\mathbf{I} + \rho_2\mathbb{C}\{\mathbf{V}\} + \rho_3\mathbf{1}\mathbf{1}', \\
&\widetilde{Q}_V(u) \gtreqqless Q_V(u)
\end{aligned}\right\}$$

partial distribution specification

Posterior $\widetilde{f}_{\mathbf{X}} \equiv \underset{f \in \mathbb{V}}{\operatorname{argmin}}\{\mathcal{E}(f, f_{\mathbf{X}})\}$ **least distance from prior, views satisfied**

Market distr.	$\mathbf{X} \sim f_{\mathbf{X}}.$	**not returns, not normal, not equilibrium**
Focus	$\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$	**non-linear functions and external factors**
Views	$\mathbf{V} \sim \widetilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}.$	**full distribution specification**
	$\widetilde{m}\{V_k\} \gtreqless \widetilde{\mu}_{\mathbf{v},k}$; $\widetilde{m}\{V_1\} \geq \widetilde{m}\{V_2\} \geq \cdots \geq \widetilde{m}\{V_K\}$; $\widetilde{\sigma}\{V_k\} \gtreqless \varkappa\sigma\{V_k\}$; $\widetilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1\mathbf{I} + \rho_2\mathbb{C}\{\mathbf{V}\} + \rho_3\mathbf{1}\mathbf{1}'$; $\widetilde{Q}_V(u) \gtreqless Q_V(u)$	**partial distribution specification**
Posterior	$\widetilde{f}_{\mathbf{X}} \equiv \underset{f \in \mathbb{V}}{\operatorname{argmin}} \{\mathcal{E}(f, f_{\mathbf{X}})\}$	**least distance from prior, views satisfied**
Confidence	$\widetilde{f}_{\mathbf{X}}^{c} \equiv (1-c) f_{\mathbf{X}} + c\widetilde{f}_{\mathbf{X}}.$	**multi-user, multi-confidence**

A. Meucci - Fully Flexible Views (Entropy Pooling)

Market distr. $\boxed{\mathbf{X} \sim f_{\mathbf{X}}}$ **not returns, not normal, not equilibrium**

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$ **non-linear functions and external factors**

Views $\mathbf{V} \sim \widetilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}.$ **full distribution specification**

$$\left.\begin{array}{l}
\widetilde{m}\{V_k\} \gtreqless \widetilde{\mu}_{\mathbf{v},k}; \\
\widetilde{m}\{V_1\} \geq \widetilde{m}\{V_2\} \geq \cdots \geq \widetilde{m}\{V_K\} \\
\widetilde{\sigma}\{V_k\} \gtreqless \varkappa\sigma\{V_k\} \\
\widetilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1\mathbf{I} + \rho_2\mathbb{C}\{\mathbf{V}\} + \rho_3\mathbf{1}\mathbf{1}', \\
\widetilde{Q}_V(u) \gtreqless Q_V(u)
\end{array}\right\}$$

partial distribution specification

Posterior $\widetilde{f}_{\mathbf{X}} \equiv \underset{f \in \mathbb{V}}{\operatorname{argmin}} \{\mathcal{E}(f, f_{\mathbf{X}})\}$ **least distance from prior, views satisfied**

Confidence $\widetilde{f}^{c}_{\mathbf{X}} \equiv (1-c)\boxed{f_{\mathbf{X}}} + c\widetilde{f}_{\mathbf{X}}.$ **multi-user, multi-confidence**

100(1-c) % of times:
PRIOR

A. Meucci - Fully Flexible Views (Entropy Pooling)

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}$. **not returns, not normal, not equilibrium**

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$ **non-linear functions and external factors**

Views $\mathbf{V} \sim \widetilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}$. **full distribution specification**

$$
\left.
\begin{aligned}
&\widetilde{m}\{V_k\} \gtreqless \widetilde{\mu}_{\mathbf{v},k}:\\
&\widetilde{m}\{V_1\} \geq \widetilde{m}\{V_2\} \geq \cdots \geq \widetilde{m}\{V_K\}\\
&\widetilde{\sigma}\{V_k\} \gtreqless \varkappa\sigma\{V_k\}\\
&\widetilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1\mathbf{I} + \rho_2\mathbb{C}\{\mathbf{V}\} + \rho_3\mathbf{1}\mathbf{1}',\\
&\widetilde{Q}_V(u) \gtreqless Q_V(u)
\end{aligned}
\right\}
$$

partial distribution specification

Posterior $\boxed{\widetilde{f}_{\mathbf{X}} \equiv \underset{f \in \mathbb{V}}{\operatorname{argmin}} \{\mathcal{E}(f, f_{\mathbf{X}})\}}$ **least distance from prior, views satisfied**

Confidence $\widetilde{f}^c_{\mathbf{X}} \equiv (1-c) f_{\mathbf{X}} + c\boxed{\widetilde{f}_{\mathbf{X}}}$. **multi-user, multi-confidence**

100c % of times:
POSTERIOR

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}.$ **not returns, not normal, not equilibrium**

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$ **non-linear functions and external factors**

Views $\mathbf{V} \sim \widetilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}.$ **full distribution specification**

$$\left.\begin{array}{l}
\widetilde{m}\{V_k\} \gtreqless \widetilde{\mu}_{\mathbf{v},k}; \\
\widetilde{m}\{V_1\} \geq \widetilde{m}\{V_2\} \geq \cdots \geq \widetilde{m}\{V_K\} \\
\widetilde{\sigma}\{V_k\} \gtreqless \varkappa\sigma\{V_k\} \\
\widetilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1 \mathbf{I} + \rho_2 \mathbb{C}\{\mathbf{V}\} + \rho_3 \mathbf{1}\mathbf{1}', \\
\widetilde{Q}_V(u) \gtreqless Q_V(u)
\end{array}\right\}$$

partial distribution specification

Posterior $\widetilde{f}_{\mathbf{X}} \equiv \underset{f \in \mathbb{V}}{\operatorname{argmin}} \{\mathcal{E}(f, f_{\mathbf{X}})\}$ **least distance from prior, views satisfied**

Confidence $\widetilde{f}^c_{\mathbf{X}} \equiv (1-c) f_{\mathbf{X}} + c\widetilde{f}_{\mathbf{X}}.$ **multi-user, multi-confidence**

Pricing $P_{t+\tau} \equiv P(\widetilde{\mathbf{X}}, \mathcal{I}_t)$ **delta/gamma/vega, full pricing, …**

A. Meucci - Fully Flexible Views (Entropy Pooling)

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}$. **not returns, not normal, not equilibrium**

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$ **non-linear functions and external factors**

Views $\mathbf{V} \sim \tilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}$. **full distribution specification**

$$\left.\begin{array}{l}
\tilde{m}\{V_k\} \gtreqless \tilde{\mu}_{\mathbf{v},k}; \\
\tilde{m}\{V_1\} \geq \tilde{m}\{V_2\} \geq \cdots \geq \tilde{m}\{V_K\} \\
\tilde{\sigma}\{V_k\} \gtreqless \varkappa\sigma\{V_k\} \\
\tilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1\mathbf{I} + \rho_2\mathbb{C}\{\mathbf{V}\} + \rho_3\mathbf{1}\mathbf{1}', \\
\tilde{Q}_V(u) \gtreqless Q_V(u)
\end{array}\right\} \text{ partial distribution specification}$$

Posterior $\tilde{f}_{\mathbf{X}} \equiv \underset{f\in\mathbb{V}}{\operatorname{argmin}}\{\mathcal{E}(f, f_{\mathbf{X}})\}$ **least distance from prior, views satisfied**

Confidence $\tilde{f}^{c}_{\mathbf{X}} \equiv (1-c)\, f_{\mathbf{X}} + c\tilde{f}_{\mathbf{X}}$. **multi-user, multi-confidence**

Pricing $P_{t+\tau} \equiv P(\tilde{\mathbf{X}}, \mathcal{I}_t)$ **delta/gamma/vega, full pricing, …**

Optimization $\mathbf{w}^* \equiv \underset{\mathbf{w}\in\mathcal{C}}{\operatorname{argmax}}\{\mathcal{S}(\mathbf{w}; \tilde{f}^{c}_{\mathbf{X}})\}$ **mean-variance, mean-CVaR, …**

ESTIMATION RISK

SCENARIO ANALYSIS

THE BLACK-LITTERMAN APPROACH

ENTROPY POOLING

- **Theory**
- **Implementation**

CASE STUDIES

REFERENCES AND CONCLUSIONS

Entropy Pooling

$$\widetilde{f}_{\mathbf{X}} = \underset{f \in \mathcal{V}}{\operatorname{argmin}} \, \mathcal{E}\left(f, f_{\mathbf{X}}\right)$$

Parametric Distributions

$$f \Leftrightarrow \{f_{\boldsymbol{\vartheta}}\}_{\boldsymbol{\vartheta} \in \boldsymbol{\Theta}}$$

Fully Flexible Probabilities

$$f \Leftrightarrow \{\mathbf{x}_j, p_j\}_{j=1,\ldots,J}$$

Entropy Pooling

$$\widetilde{f}_{\mathbf{X}} = \underset{f \in \mathcal{V}}{\operatorname{argmin}} \, \mathcal{E}\left(f, f_{\mathbf{X}}\right)$$

Parametric Distributions

$$f \Leftrightarrow \{f_{\boldsymbol{\vartheta}}\}_{\boldsymbol{\vartheta} \in \boldsymbol{\Theta}}$$

Parametric Entropy Pooling

$$f_{\widetilde{\boldsymbol{\theta}}} = \underset{\substack{\boldsymbol{\vartheta} \in \boldsymbol{\Theta} \\ f_{\boldsymbol{\vartheta}} \in \mathcal{V}}}{\operatorname{argmin}} \, \mathcal{E}\left(f_{\boldsymbol{\vartheta}}, f_{\boldsymbol{\theta}}\right)$$

Entropy Pooling

$$\widetilde{f}_{\mathbf{X}} = \underset{f \in \mathcal{V}}{\operatorname{argmin}} \, \mathcal{E}\left(f, f_{\mathbf{X}}\right)$$

Parametric Distributions

$$f \Leftrightarrow \{f_{\boldsymbol{\vartheta}}\}_{\boldsymbol{\vartheta} \in \boldsymbol{\Theta}}$$

Parametric Entropy Pooling

$$f_{\widetilde{\boldsymbol{\theta}}} = \underset{\substack{\boldsymbol{\vartheta} \in \boldsymbol{\Theta} \\ f_{\boldsymbol{\vartheta}} \in \mathcal{V}}}{\operatorname{argmin}} \, \mathcal{E}\left(f_{\boldsymbol{\vartheta}}, f_{\boldsymbol{\theta}}\right)$$

$f_{\boldsymbol{\vartheta}}$ **Normal (+dimension reduction)**

Entropy Pooling

$$\widetilde{f}_{\mathbf{X}} = \underset{f \in \mathcal{V}}{\operatorname{argmin}}\, \mathcal{E}\left(f, f_{\mathbf{X}}\right)$$

Parametric Distributions

$$f \Leftrightarrow \{f_{\boldsymbol{\vartheta}}\}_{\boldsymbol{\vartheta} \in \boldsymbol{\Theta}}$$

Parametric Entropy Pooling

$$f_{\widetilde{\boldsymbol{\theta}}} = \underset{\substack{\boldsymbol{\vartheta} \in \boldsymbol{\Theta} \\ f_{\boldsymbol{\vartheta}} \in \mathcal{V}}}{\operatorname{argmin}}\, \mathcal{E}\left(f_{\boldsymbol{\vartheta}}, f_{\boldsymbol{\theta}}\right)$$

$f_{\boldsymbol{\vartheta}}$ **Normal (+dimension reduction)**

$f_{\boldsymbol{\vartheta}}$ **Normal**

$\mathcal{V}$ **Linear-Quadratic**

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}.$ $\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$

Views $\mathbf{V} \sim \widetilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}.$

$$\widetilde{m}\{V_k\} \gtreqqless \widetilde{\mu}_{\mathbf{v},k}:$$

$$\widetilde{m}\{V_1\} \geq \widetilde{m}\{V_2\} \geq \cdots \geq \widetilde{m}\{V_K\}$$

$$\widetilde{\sigma}\{V_k\} \gtreqqless \varkappa\sigma\{V_k\}$$

$$\widetilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1\mathbf{I} + \rho_2\mathbb{C}\{\mathbf{V}\} + \rho_3\mathbf{1}\mathbf{1}',$$

$$\widetilde{Q}_V(u) \gtreqqless Q_V(u)$$

Posterior $\widetilde{f}_{\mathbf{X}} \equiv \underset{f \in \mathbb{V}}{\operatorname{argmin}}\{\mathcal{E}(f, f_{\mathbf{X}})\}$

Confidence $\widetilde{f}^{c}_{\mathbf{X}} \equiv (1-c) f_{\mathbf{X}} + c\widetilde{f}_{\mathbf{X}}.$

Pricing $P_{t+\tau} \equiv P(\widetilde{\mathbf{X}}, \mathcal{I}_t)$

Optimization $\mathbf{w}^* \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}}\{\mathcal{S}(\mathbf{w}; \widetilde{f}^{c}_{\mathbf{X}})\}$

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}$. $\quad \mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$ $\quad \mathbf{QX} \quad \mathbf{GX}$

Views $\mathbf{V} \sim \widetilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}$.

$$\widetilde{m}\{V_k\} \gtreqless \widetilde{\mu}_{\mathbf{v},k}:$$

$$\widetilde{m}\{V_1\} \geq \widetilde{m}\{V_2\} \geq \cdots \geq \widetilde{m}\{V_K\}$$

$$\widetilde{\sigma}\{V_k\} \gtreqless \varkappa\sigma\{V_k\}$$

$$\widetilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1 \mathbf{I} + \rho_2 \mathbb{C}\{\mathbf{V}\} + \rho_3 \mathbf{1}\mathbf{1}',$$

$$\widetilde{Q}_V(u) \gtreqless Q_V(u)$$

Posterior $\widetilde{f}_{\mathbf{X}} \equiv \underset{f \in \mathbb{V}}{\operatorname{argmin}} \{\mathcal{E}(f, f_{\mathbf{X}})\}$

Confidence $\widetilde{f}^{c}_{\mathbf{X}} \equiv (1-c) f_{\mathbf{X}} + c\widetilde{f}_{\mathbf{X}}$.

Pricing $P_{t+\tau} \equiv P(\widetilde{\mathbf{X}}, \mathcal{I}_t)$

Optimization $\mathbf{w}^* \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \{\mathcal{S}(\mathbf{w}; \widetilde{f}^{c}_{\mathbf{X}})\}$

Market distr.	$\mathbf{X} \sim f_{\mathbf{X}}$.	$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$
Focus	$\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$	$\mathbf{QX}$. $\mathbf{GX}$.
Views	$\mathbf{V} \sim \widetilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}$.	
	$\widetilde{m}\{V_k\} \gtreqless \widetilde{\mu}_{\mathbf{v},k}$:	$\mathbb{E}\{\mathbf{QX}\} \equiv \widetilde{\boldsymbol{\mu}}_{\mathbf{Q}}$
	$\widetilde{m}\{V_1\} \geq \widetilde{m}\{V_2\} \geq \cdots \geq \widetilde{m}\{V_K\}$	$\mathbb{C}ov\{\mathbf{GX}\} \equiv \widetilde{\boldsymbol{\Sigma}}_{\mathbf{G}}$
	$\widetilde{\sigma}\{V_k\} \gtreqless \varkappa\sigma\{V_k\}$	
	$\widetilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1\mathbf{I} + \rho_2\mathbb{C}\{\mathbf{V}\} + \rho_3\mathbf{1}\mathbf{1}'$,	
	$\widetilde{Q}_V(u) \gtreqless Q_V(u)$	
Posterior	$\widetilde{f}_{\mathbf{X}} \equiv \underset{f\in\mathbb{V}}{\operatorname{argmin}}\{\mathcal{E}(f, f_{\mathbf{X}})\}$	
Confidence	$\widetilde{f}_{\mathbf{X}}^{c} \equiv (1-c) f_{\mathbf{X}} + c\widetilde{f}_{\mathbf{X}}$.	
Pricing	$P_{t+\tau} \equiv P(\widetilde{\mathbf{X}}, \mathcal{I}_t)$	
Optimization	$\mathbf{w}^* \equiv \underset{\mathbf{w}\in\mathcal{C}}{\operatorname{argmax}}\{\mathcal{S}(\mathbf{w}; \widetilde{f}_{\mathbf{X}}^{c})\}$	

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}.$ $\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$ $\mathbf{QX}$ $\mathbf{GX}$

Views $\mathbf{V} \sim \widetilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}.$

$$\widetilde{m}\{V_k\} \gtreqless \widetilde{\mu}_{\mathbf{v},k};$$

$$\widetilde{m}\{V_1\} \geq \widetilde{m}\{V_2\} \geq \cdots \geq \widetilde{m}\{V_K\}$$

$$\widetilde{\sigma}\{V_k\} \gtreqless \varkappa\sigma\{V_k\}$$

$$\widetilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1\mathbf{I} + \rho_2\mathbb{C}\{\mathbf{V}\} + \rho_3\mathbf{1}\mathbf{1}',$$

$$\widetilde{Q}_V(u) \gtreqless Q_V(u)$$

$$\mathbb{E}\{\mathbf{QX}\} \equiv \widetilde{\boldsymbol{\mu}}_{\mathbf{Q}}$$

$$\mathbb{C}ov\{\mathbf{GX}\} \equiv \widetilde{\boldsymbol{\Sigma}}_{\mathbf{G}}$$

Posterior $\widetilde{f}_{\mathbf{X}} \equiv \underset{f \in \mathbb{V}}{\operatorname{argmin}} \{\mathcal{E}(f, f_{\mathbf{X}})\}$ $\mathbf{X} \sim \mathrm{N}\left(\widetilde{\boldsymbol{\mu}}, \widetilde{\boldsymbol{\Sigma}}\right)$

$$\begin{cases} \widetilde{\boldsymbol{\mu}} & \equiv \boldsymbol{\mu} + \boldsymbol{\Sigma}\mathbf{Q}'\left(\mathbf{Q}\boldsymbol{\Sigma}\mathbf{Q}'\right)^{-1}\left(\widetilde{\boldsymbol{\mu}}_{\mathbf{Q}} - \mathbf{Q}\boldsymbol{\mu}\right) \\ \widetilde{\boldsymbol{\Sigma}} & \equiv \boldsymbol{\Sigma} + \boldsymbol{\Sigma}\mathbf{G}'\Big(\left(\mathbf{G}\boldsymbol{\Sigma}\mathbf{G}'\right)^{-1}\widetilde{\boldsymbol{\Sigma}}_{\mathbf{G}}\left(\mathbf{G}\boldsymbol{\Sigma}\mathbf{G}'\right)^{-1} \\ & \qquad - \left(\mathbf{G}\boldsymbol{\Sigma}\mathbf{G}'\right)^{-1}\Big)\mathbf{G}\boldsymbol{\Sigma}. \end{cases}$$

Confidence $\widetilde{f}^{c}_{\mathbf{X}} \equiv (1-c)\, f_{\mathbf{X}} + c\widetilde{f}_{\mathbf{X}}.$

Pricing $P_{t+\tau} \equiv P\left(\widetilde{\mathbf{X}}, \mathcal{I}_t\right)$

Optimization $\mathbf{w}^{*} \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \left\{\mathcal{S}\left(\mathbf{w}; \widetilde{f}^{\mathbf{c}}_{\mathbf{X}}\right)\right\}$

Market distr.	$\mathbf{X} \sim f_{\mathbf{X}}.$	$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$
Focus	$\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$	$\mathbf{QX}$ $\mathbf{GX}$
Views	$\mathbf{V} \sim \widetilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}.$ $\widetilde{m}\{V_k\} \gtreqless \widetilde{\mu}_{\mathbf{v},k}:$ $\widetilde{m}\{V_1\} \geq \widetilde{m}\{V_2\} \geq \cdots \geq \widetilde{m}\{V_K\}$ $\widetilde{\sigma}\{V_k\} \gtreqless \varkappa\sigma\{V_k\}$ $\widetilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1\mathbf{I} + \rho_2\mathbb{C}\{\mathbf{V}\} + \rho_3\mathbf{1}\mathbf{1}',$ $\widetilde{Q}_V(u) \gtreqless Q_V(u)$	$\mathbb{E}\{\mathbf{QX}\} \equiv \widetilde{\boldsymbol{\mu}}_{\mathbf{Q}}$ $\mathbb{C}ov\{\mathbf{GX}\} \equiv \widetilde{\boldsymbol{\Sigma}}_{\mathbf{G}}$
Posterior	$\widetilde{f}_{\mathbf{X}} \equiv \underset{f \in \mathbb{V}}{\operatorname{argmin}} \{\mathcal{E}(f, f_{\mathbf{X}})\}$	$\mathbf{X} \sim \mathrm{N}\left(\widetilde{\boldsymbol{\mu}}, \widetilde{\boldsymbol{\Sigma}}\right)$ $\left\{\begin{array}{lcl} \widetilde{\boldsymbol{\mu}} & \equiv & \boldsymbol{\mu} + \boldsymbol{\Sigma}\mathbf{Q}'(\mathbf{Q}\boldsymbol{\Sigma}\mathbf{Q}')^{-1}(\widetilde{\boldsymbol{\mu}}_{\mathbf{Q}} - \mathbf{Q}\boldsymbol{\mu}) \\ \widetilde{\boldsymbol{\Sigma}} & \equiv & \boldsymbol{\Sigma} + \boldsymbol{\Sigma}\mathbf{G}'\left((\mathbf{G}\boldsymbol{\Sigma}\mathbf{G}')^{-1}\widetilde{\boldsymbol{\Sigma}}_{\mathbf{G}}(\mathbf{G}\boldsymbol{\Sigma}\mathbf{G}')^{-1} - (\mathbf{G}\boldsymbol{\Sigma}\mathbf{G}')^{-1}\right)\mathbf{G}\boldsymbol{\Sigma}. \end{array}\right.$
Confidence	$\widetilde{f}^{c}_{\mathbf{X}} \equiv (1-c) f_{\mathbf{X}} + c\widetilde{f}_{\mathbf{X}}.$	$\mathbf{X} \sim \begin{cases} \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) & (\text{probability: } 1-c) \\ \mathrm{N}\left(\widetilde{\boldsymbol{\mu}}, \widetilde{\boldsymbol{\Sigma}}\right) & (\text{probability: } c) \end{cases}$
Pricing	$P_{t+\tau} \equiv P\left(\widetilde{\mathbf{X}}, \mathcal{I}_t\right)$	
Optimization	$\mathbf{w}^* \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \{\mathcal{S}(\mathbf{w}; \widetilde{f}^{c}_{\mathbf{X}})\}$	

Market distr.	$\mathbf{X} \sim f_{\mathbf{X}}.$	$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$
Focus	$\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$	$\mathbf{QX}$ $\mathbf{GX}$
Views	$\mathbf{V} \sim \tilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}.$ $\tilde{m}\{V_k\} \gtreqless \tilde{\mu}_{\mathbf{v},k}$ $\tilde{m}\{V_1\} \geq \tilde{m}\{V_2\} \geq \cdots \geq \tilde{m}\{V_K\}$ $\tilde{\sigma}\{V_k\} \gtreqless \varkappa\sigma\{V_k\}$ $\tilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1 \mathbf{I} + \rho_2 \mathbb{C}\{\mathbf{V}\} + \rho_3 \mathbf{1}\mathbf{1}',$ $\tilde{Q}_V(u) \gtreqless Q_V(u)$	$\mathbb{E}\{\mathbf{QX}\} \equiv \tilde{\boldsymbol{\mu}}_{\mathbf{Q}}$ $\mathbb{C}ov\{\mathbf{GX}\} \equiv \tilde{\boldsymbol{\Sigma}}_{\mathbf{G}}$
Posterior	$\tilde{f}_{\mathbf{X}} \equiv \underset{f \in \mathbb{V}}{\operatorname{argmin}} \{\mathcal{E}(f, f_{\mathbf{X}})\}$	$\mathbf{X} \sim \mathrm{N}\left(\tilde{\boldsymbol{\mu}}, \tilde{\boldsymbol{\Sigma}}\right)$ $\left\{\begin{array}{lcl} \tilde{\boldsymbol{\mu}} & \equiv & \boldsymbol{\mu} + \boldsymbol{\Sigma}\mathbf{Q}'(\mathbf{Q}\boldsymbol{\Sigma}\mathbf{Q}')^{-1}(\tilde{\boldsymbol{\mu}}_{\mathbf{Q}} - \mathbf{Q}\boldsymbol{\mu}) \\ \tilde{\boldsymbol{\Sigma}} & \equiv & \boldsymbol{\Sigma} + \boldsymbol{\Sigma}\mathbf{G}'\left((\mathbf{G}\boldsymbol{\Sigma}\mathbf{G}')^{-1}\tilde{\boldsymbol{\Sigma}}_{\mathbf{G}}(\mathbf{G}\boldsymbol{\Sigma}\mathbf{G}')^{-1} - (\mathbf{G}\boldsymbol{\Sigma}\mathbf{G}')^{-1}\right)\mathbf{G}\boldsymbol{\Sigma}. \end{array}\right.$
Confidence	$\tilde{f}^{c}_{\mathbf{X}} \equiv (1-c) f_{\mathbf{X}} + c\tilde{f}_{\mathbf{X}}.$	$\mathbf{X} \sim$ $\mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ (probability: $1-c$) $\mathrm{N}\left(\tilde{\boldsymbol{\mu}}, \tilde{\boldsymbol{\Sigma}}\right)$ (probability: c)
Pricing	$P_{t+\tau} \equiv P\left(\tilde{\mathbf{X}}, \mathcal{I}_t\right)$	...
Optimization	$\mathbf{w}^* \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \{\mathcal{S}(\mathbf{w}; \tilde{f}^{c}_{\mathbf{X}})\}$	...

Entropy Pooling

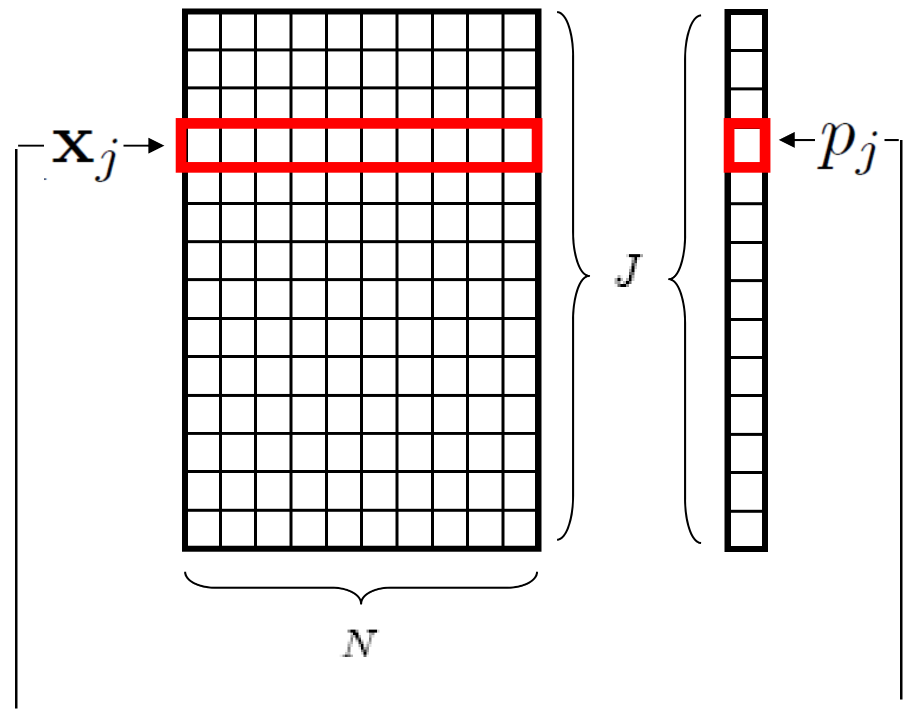
$$\widetilde{f}_{\mathbf{X}} = \underset{f \in \mathcal{V}}{\operatorname{argmin}}\, \mathcal{E}\left(f, f_{\mathbf{X}}\right)$$

Parametric Distributions

$$f \Leftrightarrow \{f_{\boldsymbol{\vartheta}}\}_{\boldsymbol{\vartheta} \in \boldsymbol{\Theta}}$$

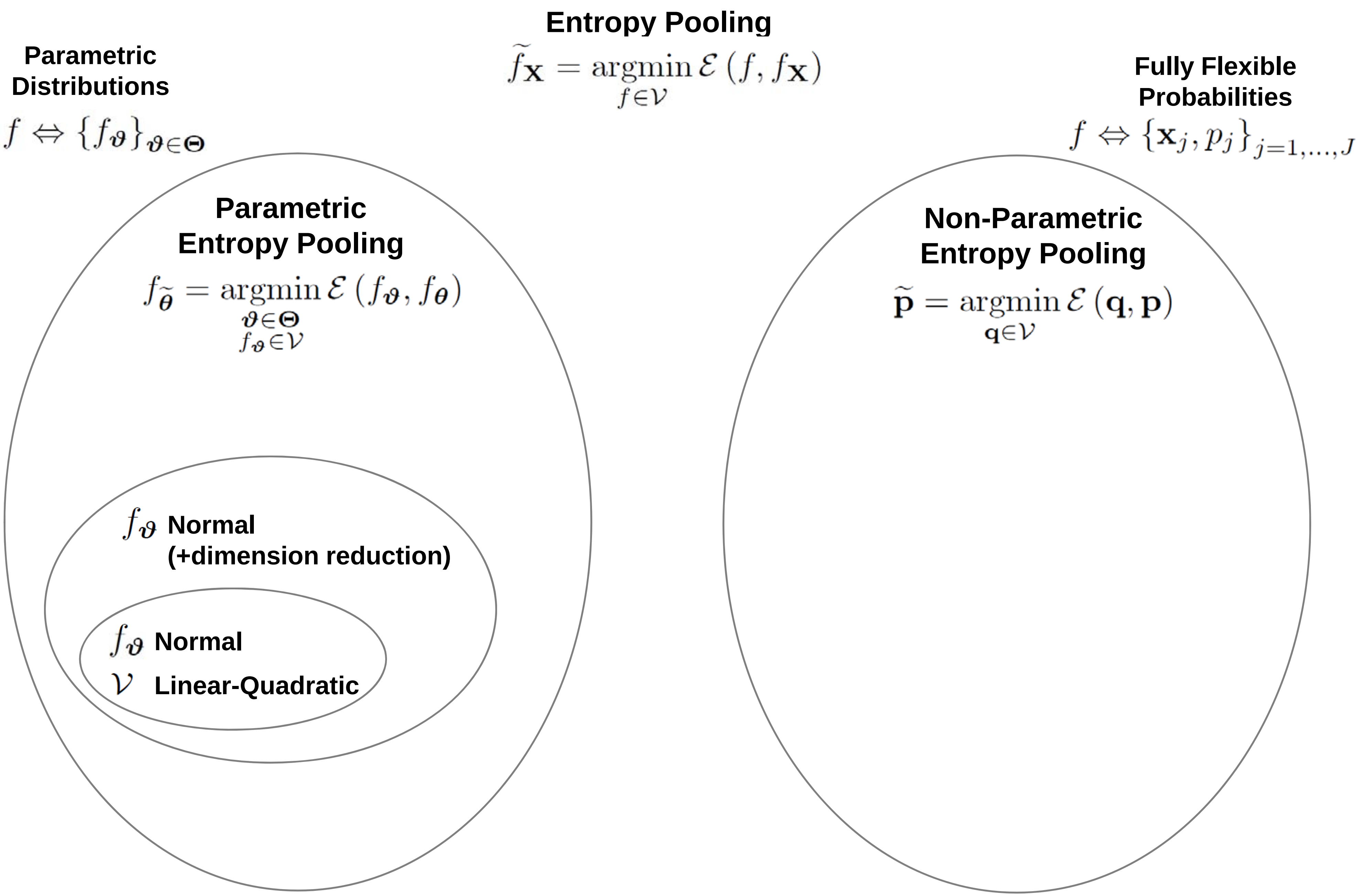
Fully Flexible Probabilities

$$f \Leftrightarrow \{\mathbf{x}_j, p_j\}_{j=1,\ldots,J}$$

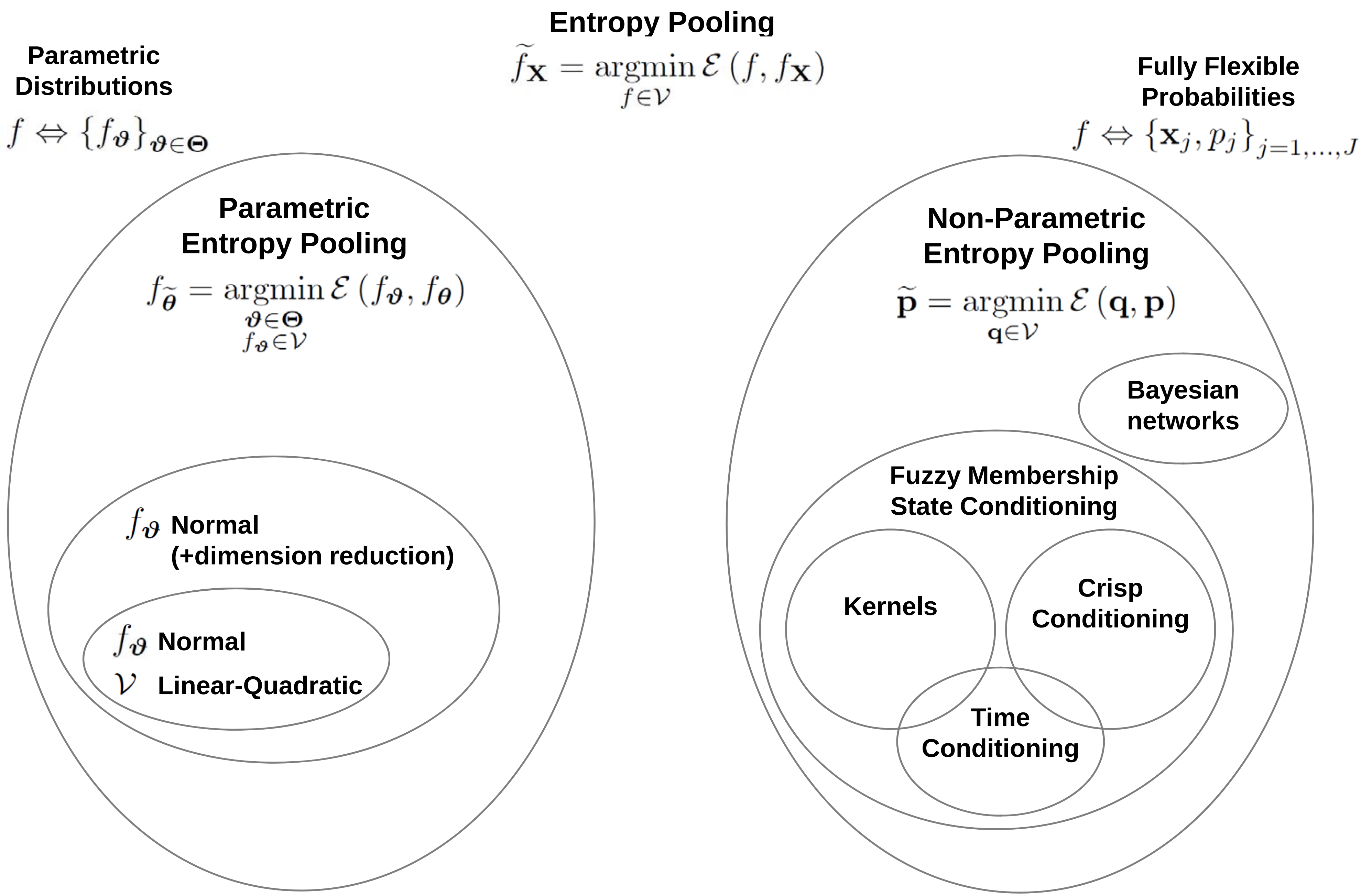


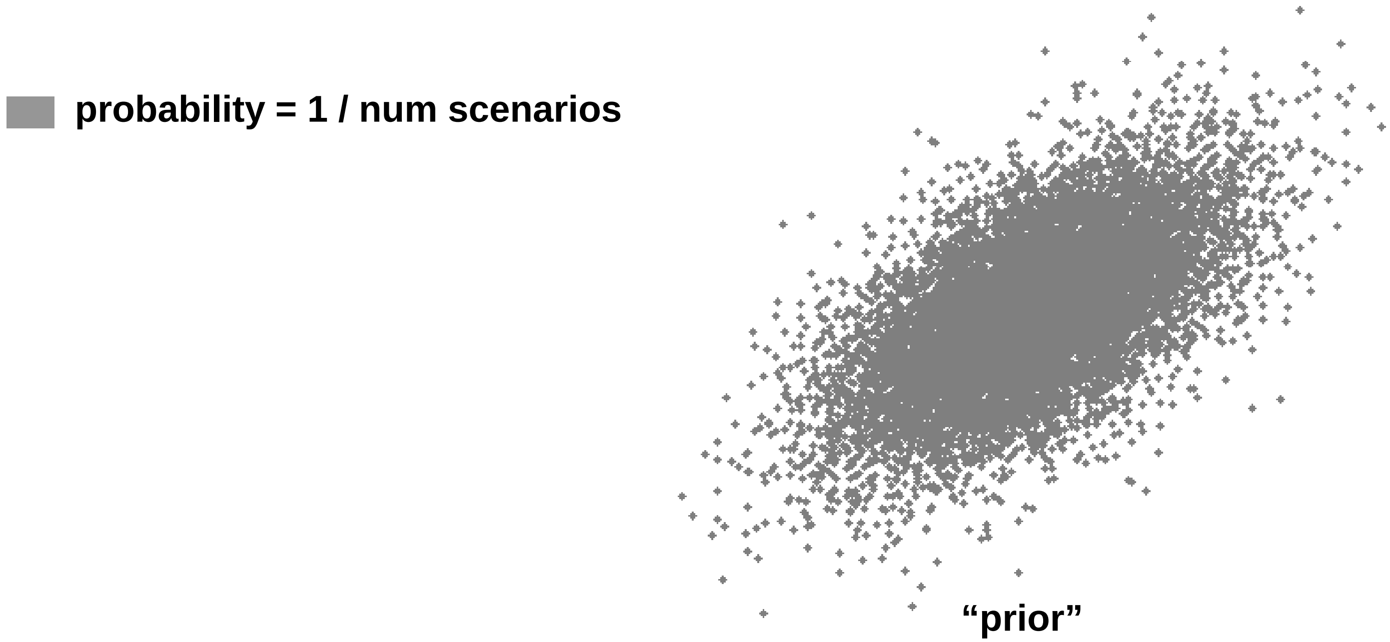
joint scenario of N risk drivers

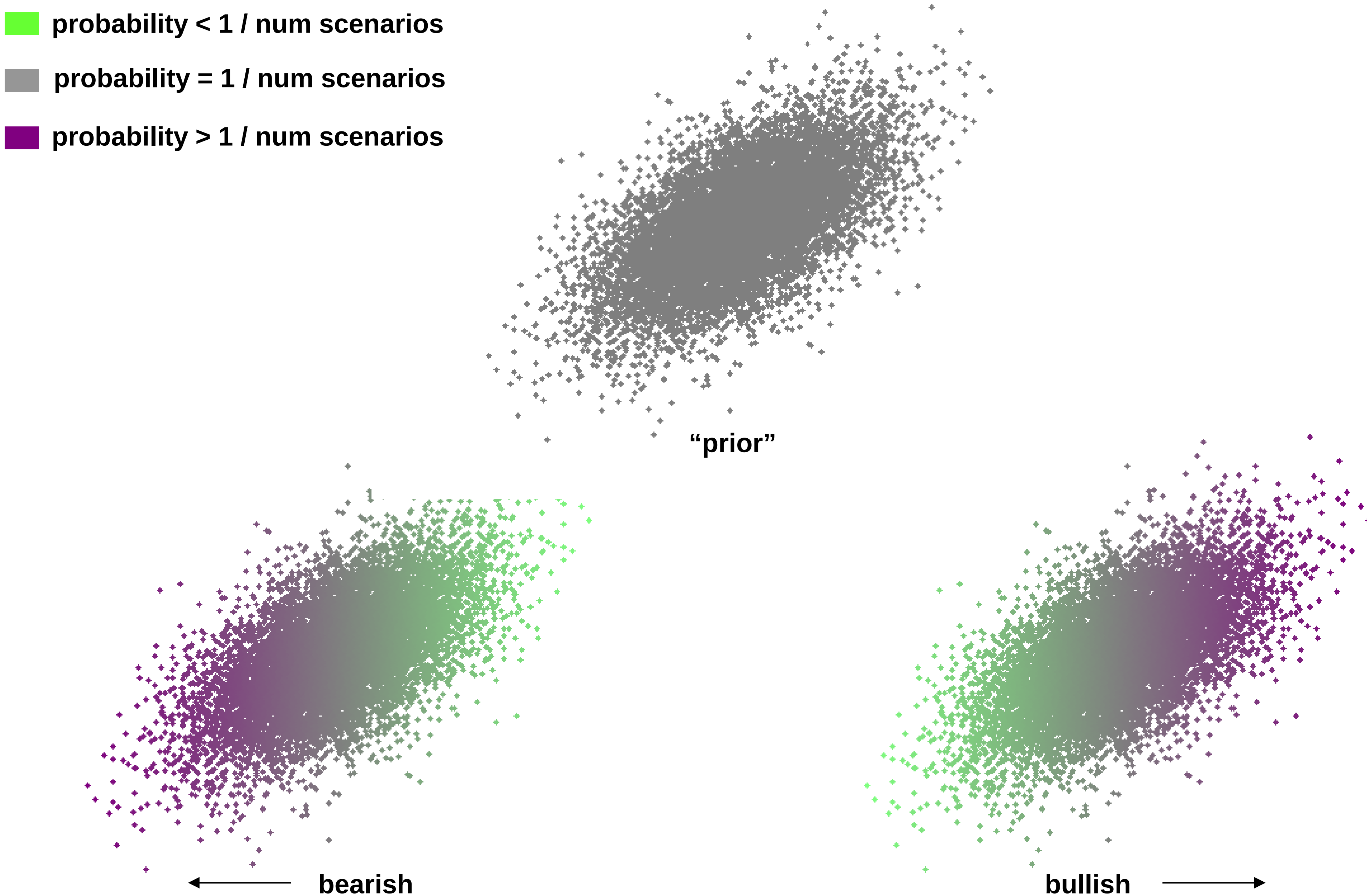
probability of joint scenario

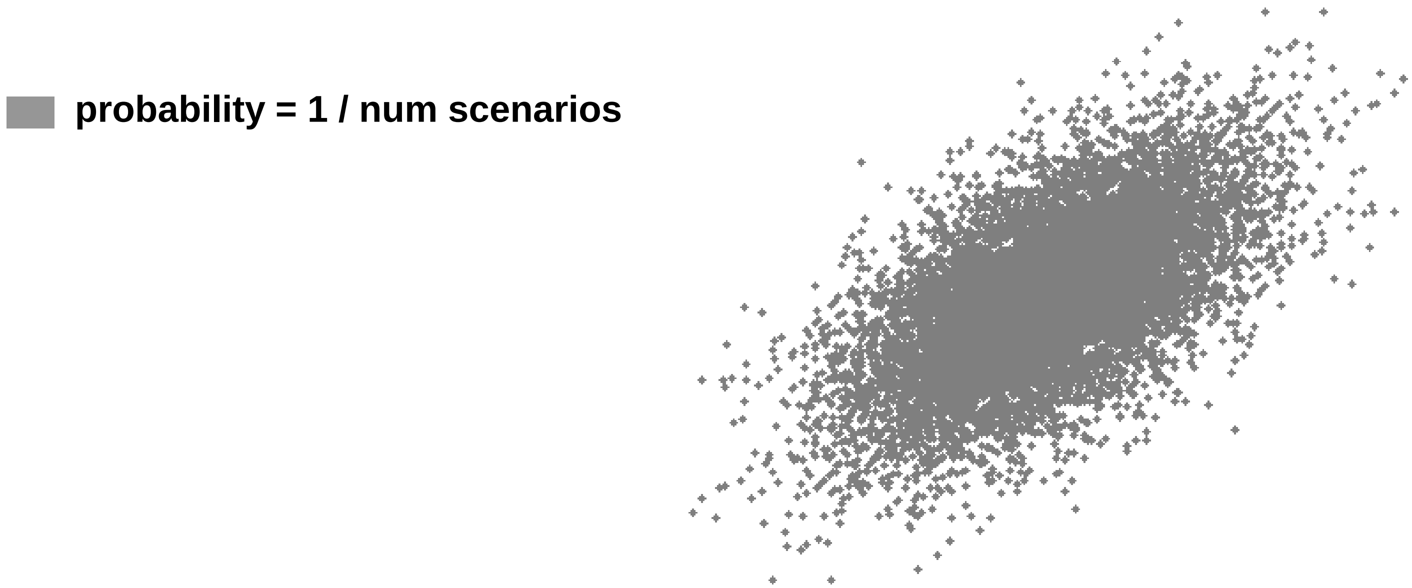


Entropy Pooling
$\widetilde{f}_{\mathbf{X}} = \underset{f \in \mathcal{V}}{\operatorname{argmin}} \, \mathcal{E}\left(f, f_{\mathbf{X}}\right)$
Parametric Distributions
$f \Leftrightarrow \{f_{\boldsymbol{\vartheta}}\}_{\boldsymbol{\vartheta} \in \boldsymbol{\Theta}}$
Fully Flexible Probabilities
$f \Leftrightarrow \{\mathbf{x}_j, p_j\}_{j=1,\ldots,J}$
Parametric Entropy Pooling
$f_{\widetilde{\boldsymbol{\theta}}} = \underset{\substack{\boldsymbol{\vartheta} \in \boldsymbol{\Theta} \\ f_{\boldsymbol{\vartheta}} \in \mathcal{V}}}{\operatorname{argmin}} \, \mathcal{E}\left(f_{\boldsymbol{\vartheta}}, f_{\boldsymbol{\theta}}\right)$
$f_{\boldsymbol{\vartheta}}$ Normal (+dimension reduction)
$f_{\boldsymbol{\vartheta}}$ Normal
$\mathcal{V}$ Linear-Quadratic
Non-Parametric Entropy Pooling
$\widetilde{\mathbf{p}} = \underset{\mathbf{q} \in \mathcal{V}}{\operatorname{argmin}} \, \mathcal{E}\left(\mathbf{q}, \mathbf{p}\right)$


Entropy Pooling
$\widetilde{f}_{\mathbf{X}} = \underset{f \in \mathcal{V}}{\operatorname{argmin}} \, \mathcal{E}\left(f, f_{\mathbf{X}}\right)$
Parametric Distributions
$f \Leftrightarrow \{f_{\boldsymbol{\vartheta}}\}_{\boldsymbol{\vartheta} \in \boldsymbol{\Theta}}$
Fully Flexible Probabilities
$f \Leftrightarrow \{\mathbf{x}_j, p_j\}_{j=1,\ldots,J}$
Parametric Entropy Pooling
$f_{\widetilde{\boldsymbol{\theta}}} = \underset{\substack{\boldsymbol{\vartheta} \in \boldsymbol{\Theta} \\ f_{\boldsymbol{\vartheta}} \in \mathcal{V}}}{\operatorname{argmin}} \, \mathcal{E}\left(f_{\boldsymbol{\vartheta}}, f_{\boldsymbol{\theta}}\right)$
$f_{\boldsymbol{\vartheta}}$ **Normal (+dimension reduction)**
$f_{\boldsymbol{\vartheta}}$ **Normal**
$\mathcal{V}$ **Linear-Quadratic**
Non-Parametric Entropy Pooling
$\widetilde{\mathbf{p}} = \underset{\mathbf{q} \in \mathcal{V}}{\operatorname{argmin}} \, \mathcal{E}\left(\mathbf{q}, \mathbf{p}\right)$
Bayesian networks
Fuzzy Membership State Conditioning
Kernels
Crisp Conditioning
Time Conditioning


probability = 1 / num scenarios
"prior"


probability < 1 / num scenarios
probability = 1 / num scenarios
probability > 1 / num scenarios
“prior”
bearish
bullish



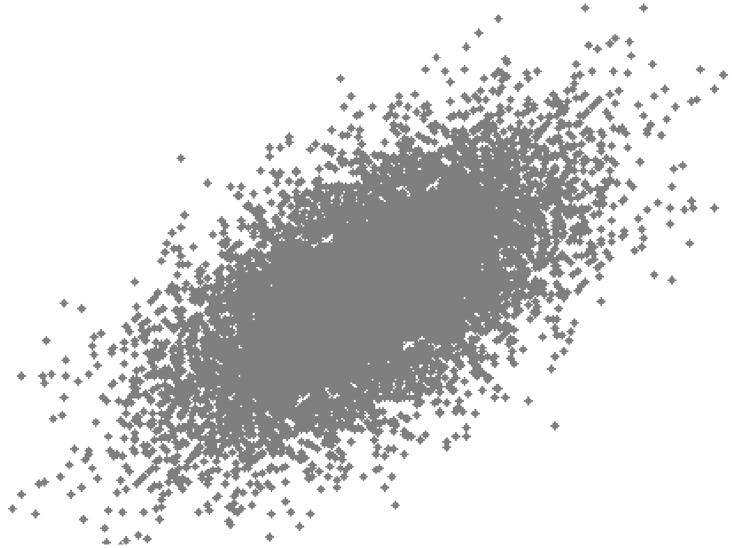
regular market

Non-parametric implementation

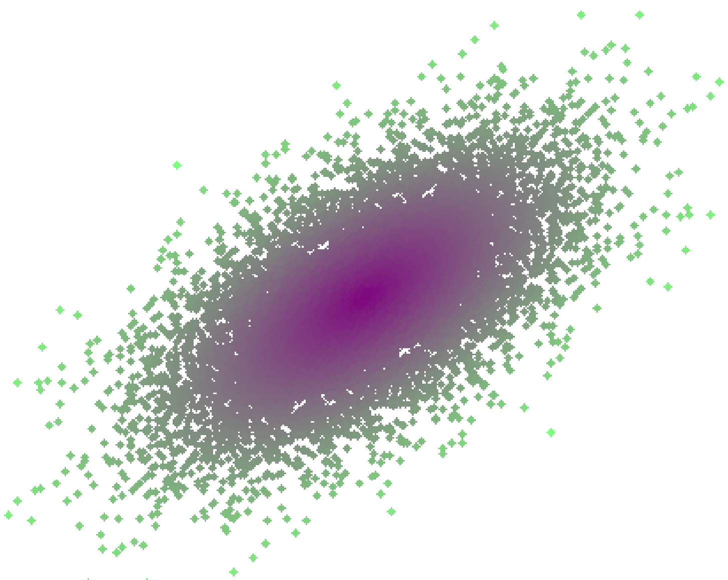
probability < 1 / num scenarios

probability = 1 / num scenarios

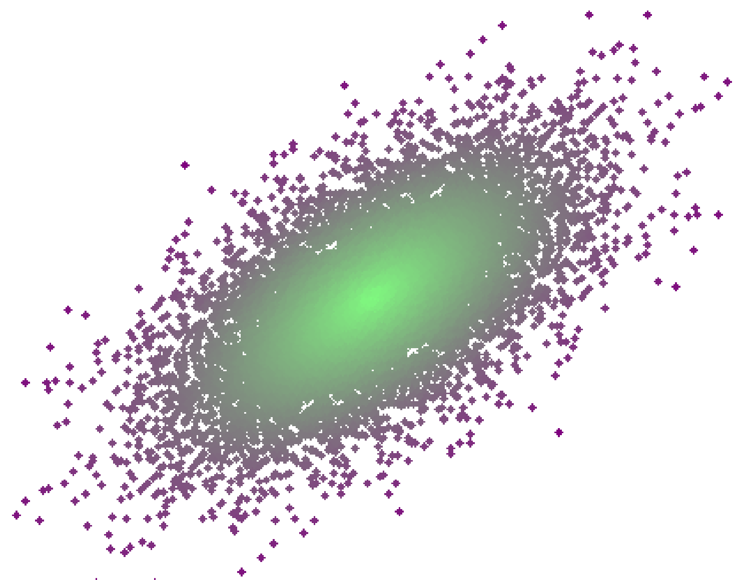
probability > 1 / num scenarios



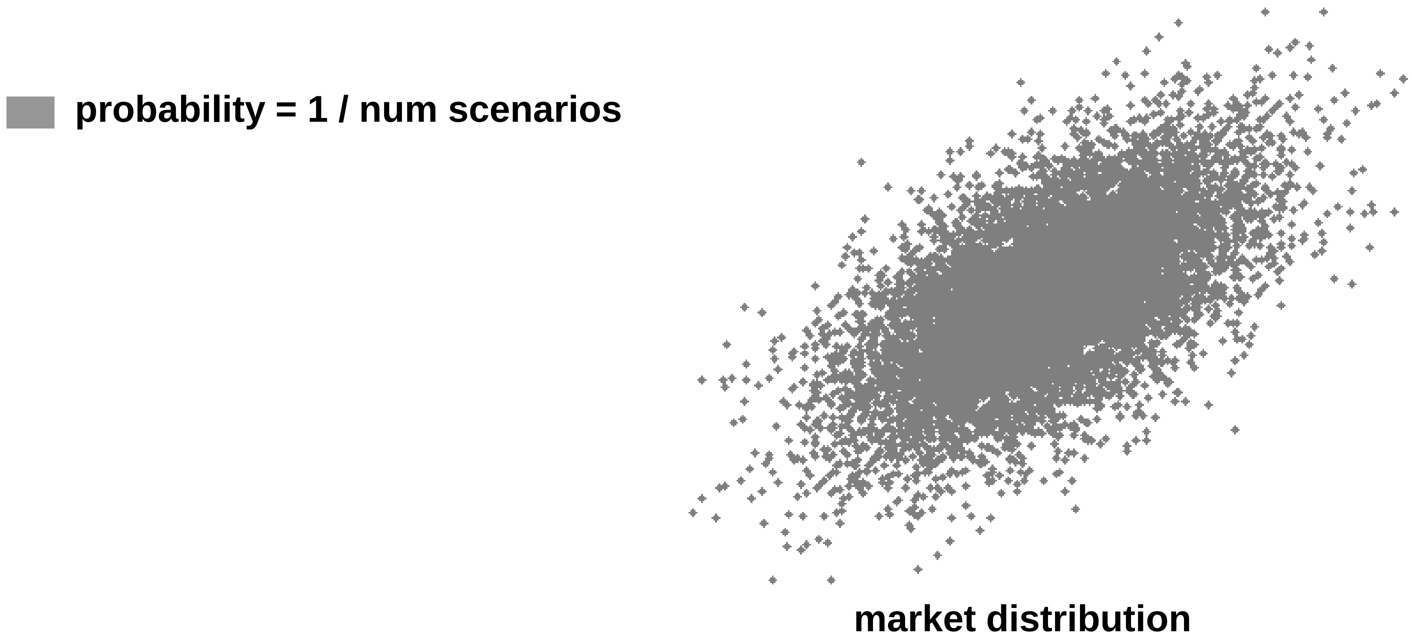
regular market



low volatility



high volatility

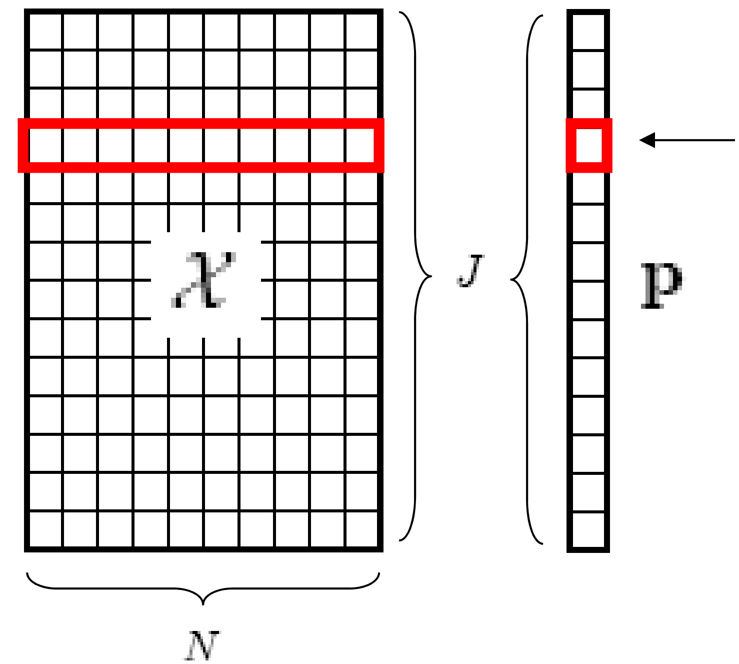

probability = 1 / num scenarios
market distribution


probability = 0
probability = 1
scenario analysis
market distribution

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}$ $\Longleftrightarrow$ $\mathcal{X}$ $J \times N$ panel $\mathbf{p}$ probabilities $1/J$

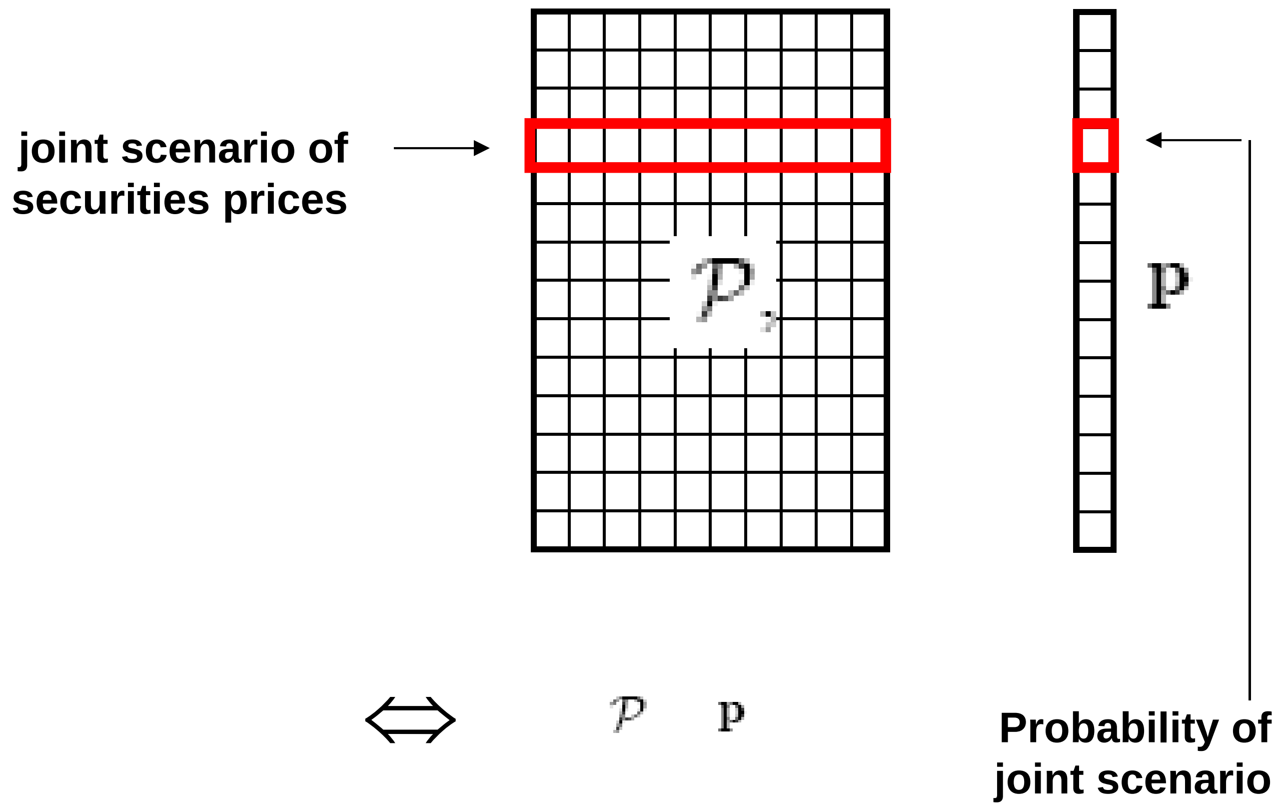
Market distr. $\mathbf{X} \sim f_{\mathbf{X}}$. $\Longleftrightarrow$ $\mathcal{X}$ $J \times N$ panel $\mathbf{p}$ probabilities $1/J$;

joint scenario of N risk drivers $\mathbf{X}$



Probability of joint scenario

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}$. $\Leftrightarrow$ $\mathcal{X}$ $J \times N$ panel $\mathbf{p}$ probabilities $1/J$.



Pricing $P_{t+\tau} \equiv P\left(\mathbf{X}, \mathcal{I}_t\right)$ $\Leftrightarrow$ $\mathcal{P}$ $\mathbf{p}$

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}$. $\Leftrightarrow$ $\mathcal{X}$ $J \times N$ panel $\mathbf{p}$ probabilities $1/J$;

Pricing $P_{t+\tau} \equiv P\left(\mathbf{X}, \mathcal{I}_t\right)$ $\Leftrightarrow$ $\mathcal{P}$ $\mathbf{p}$

Optimization $\mathbf{w}^* \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \left\{\mathcal{S}\left(\mathbf{w}; f_{\mathbf{X}}\right)\right\}$ $\Leftrightarrow$ ...

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}.$ $\Longleftrightarrow$ $\mathcal{X}$ $J \times N$ panel $\mathbf{p}$ probabilities $1/J;$

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$

Views $\mathbf{V} \sim \widetilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}.$

$$\widetilde{m}\{V_k\} \gtreqqless \widetilde{\mu}_{\mathbf{V},k};$$

$$\widetilde{m}\{V_1\} \geq \widetilde{m}\{V_2\} \geq \cdots \geq \widetilde{m}\{V_K\}$$

$$\widetilde{\sigma}\{V_k\} \gtreqqless \varkappa\sigma\{V_k\}$$

$$\widetilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1\mathbf{I} + \rho_2\mathbb{C}\{\mathbf{V}\} + \rho_3\mathbf{1}\mathbf{1}',$$

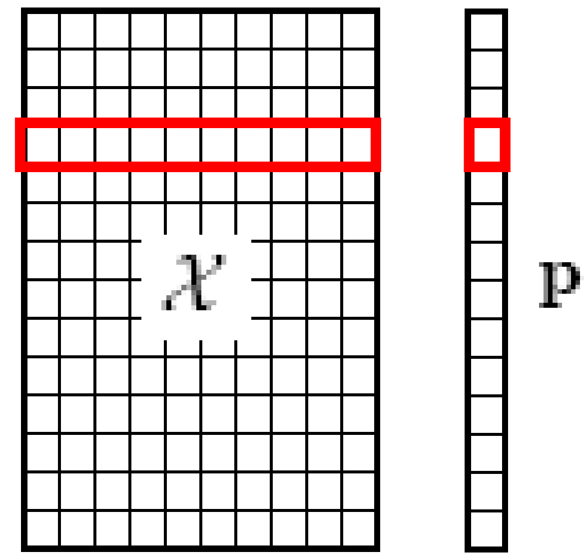
$$\widetilde{Q}_V(u) \gtreqqless Q_V(u)$$

Posterior $\widetilde{f}_{\mathbf{X}} \equiv \underset{f \in \mathbb{V}}{\operatorname{argmin}}\{\mathcal{E}(f, f_{\mathbf{X}})\}$

Confidence $\widetilde{f}^{c}_{\mathbf{X}} \equiv (1-c) f_{\mathbf{X}} + c\widetilde{f}_{\mathbf{X}}.$

Pricing $P_{t+\tau} \equiv P(\widetilde{\mathbf{X}}, \mathcal{I}_t)$

Optimization $\mathbf{w}^* \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}}\{\mathcal{S}(\mathbf{w}; \widetilde{f}^{c}_{\mathbf{X}})\}$



Market distr. $\mathbf{X} \sim f_{\mathbf{X}}$. $\Longleftrightarrow$ $\mathcal{X}$ $J \times N$ panel $\mathbf{p}$ probabilities $1/J$.

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$ $\Longleftrightarrow$ $\mathcal{V}_{j,k} \equiv g_k(\mathcal{X}_{j,1}, \ldots, \mathcal{X}_{j,N})$

scenario index

Views $\mathbf{V} \sim \tilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}$.

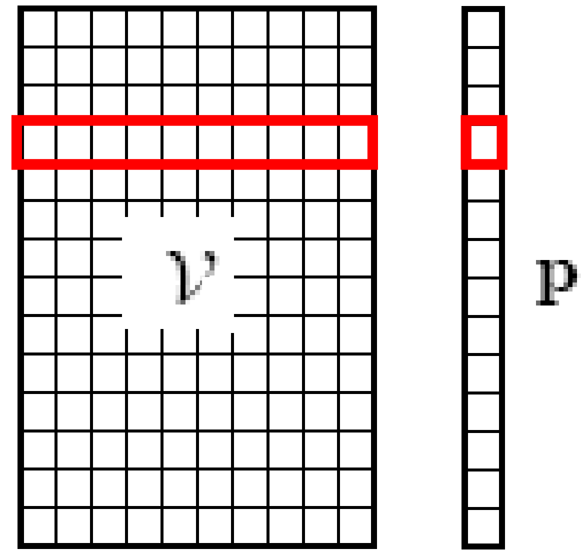
$$\tilde{m}\{V_k\} \gtreqless \tilde{\mu}_{\mathbf{v},k};$$

$$\tilde{m}\{V_1\} \geq \tilde{m}\{V_2\} \geq \cdots \geq \tilde{m}\{V_K\}$$

$$\tilde{\sigma}\{V_k\} \gtreqless \varkappa\sigma\{V_k\}$$

$$\tilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1 \mathbf{I} + \rho_2 \mathbb{C}\{\mathbf{V}\} + \rho_3 \mathbf{1}\mathbf{1}',$$

$$\tilde{Q}_V(u) \gtreqless Q_V(u)$$



Posterior $\tilde{f}_{\mathbf{X}} \equiv \underset{f \in \mathbb{V}}{\operatorname{argmin}} \{\mathcal{E}(f, f_{\mathbf{X}})\}$

Confidence $\tilde{f}^c_{\mathbf{X}} \equiv (1-c) f_{\mathbf{X}} + c\tilde{f}_{\mathbf{X}}$.

Pricing $P_{t+\tau} \equiv P(\tilde{\mathbf{X}}, \mathcal{I}_t)$

Optimization $\mathbf{w}^* \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \{\mathcal{S}(\mathbf{w}; \tilde{f}^c_{\mathbf{X}})\}$

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}.$ $\Longleftrightarrow$ $\mathcal{X}$ $J \times N$ panel $\mathbf{p}$ probabilities $1/J$.

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$ $\Longleftrightarrow$ $\mathcal{V}_{j,k} \equiv g_k(\mathcal{X}_{j,1}, \ldots, \mathcal{X}_{j,N})$

Views $\mathbf{V} \sim \widetilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}.$

$$\left.\begin{array}{l}
\widetilde{m}\{V_k\} \gtreqless \widetilde{\mu}_{\mathbf{v},k}; \\
\widetilde{m}\{V_1\} \geq \widetilde{m}\{V_2\} \geq \cdots \geq \widetilde{m}\{V_K\} \\
\widetilde{\sigma}\{V_k\} \gtreqless \varkappa\sigma\{V_k\} \\
\widetilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1\mathbf{I} + \rho_2\mathbb{C}\{\mathbf{V}\} + \rho_3\mathbf{1}\mathbf{1}', \\
\widetilde{Q}_V(u) \gtreqless Q_V(u)
\end{array}\right\} \Longleftrightarrow \underline{\mathbf{a}} \leq \mathbf{A}\widetilde{\mathbf{p}} \leq \overline{\mathbf{a}}.$$

e.g.

X_1 **2-yr swap rate**

X_2 **5-yr swap rate**

$V \equiv X_1$

$$\tilde{m}\{V\} \equiv \tilde{\mu}$$

$$\Updownarrow$$

$$\tilde{\mu} \leq \sum_{j=1}^{J} \mathrm{V}_j \tilde{p}_j \leq \tilde{\mu}$$

Posterior $\widetilde{f}_{\mathbf{X}} \equiv \underset{f \in \mathbb{V}}{\operatorname{argmin}} \{\mathcal{E}(f, f_{\mathbf{X}})\}$

Confidence $\widetilde{f}_{\mathbf{X}}^{c} \equiv (1-c) f_{\mathbf{X}} + c\widetilde{f}_{\mathbf{X}}.$

Pricing $P_{t+\tau} \equiv P(\widetilde{\mathbf{X}}, \mathcal{I}_t)$

Optimization $\mathbf{w}^* \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \{\mathcal{S}(\mathbf{w}; \widetilde{f}_{\mathbf{X}}^{c})\}$

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}.$ $\Longleftrightarrow$ $\mathcal{X}$ $J \times N$ panel $\mathbf{p}$ probabilities $1/J$.

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$ $\Longleftrightarrow$ $\mathcal{V}_{j,k} \equiv g_k\left(\mathcal{X}_{j,1}, \ldots, \mathcal{X}_{j,N}\right)$

Views $\mathbf{V} \sim \widetilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}.$

$$\left.\begin{array}{l}
\widetilde{m}\{V_k\} \gtreqless \widetilde{\mu}_{\mathbf{v},k}; \\
\widetilde{m}\{V_1\} \geq \widetilde{m}\{V_2\} \geq \cdots \geq \widetilde{m}\{V_K\} \\
\widetilde{\sigma}\{V_k\} \gtreqless \varkappa\sigma\{V_k\} \\
\widetilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1\mathbf{I} + \rho_2\mathbb{C}\{\mathbf{V}\} + \rho_3\mathbf{1}\mathbf{1}', \\
\widetilde{Q}_V(u) \gtreqless Q_V(u)
\end{array}\right\} \Longleftrightarrow \quad \underline{\mathbf{a}} \leq \mathbf{A}\widetilde{\mathbf{p}} \leq \overline{\mathbf{a}}.$$

Posterior $\widetilde{f}_{\mathbf{X}} \equiv \underset{f \in \mathbb{V}}{\operatorname{argmin}}\left\{\mathcal{E}\left(f, f_{\mathbf{X}}\right)\right\}$

$$\mathcal{E}\left(\widetilde{f}_{\mathbf{X}}, f_{\mathbf{X}}\right) \equiv \int \widetilde{f}_{\mathbf{X}}(\mathrm{x})\left[\ln \widetilde{f}_{\mathbf{X}}(\mathrm{x}) - \ln f_{\mathbf{X}}(\mathrm{x})\right] d\mathrm{x}.$$

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}.$ $\Leftrightarrow$ $\mathcal{X}$ $J \times N$ panel $\mathbf{p}$ probabilities $1/J.$

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$ $\Leftrightarrow$ $\mathcal{V}_{j,k} \equiv g_k(\mathcal{X}_{j,1}, \ldots, \mathcal{X}_{j,N})$

Views $\mathbf{V} \sim \widetilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}.$

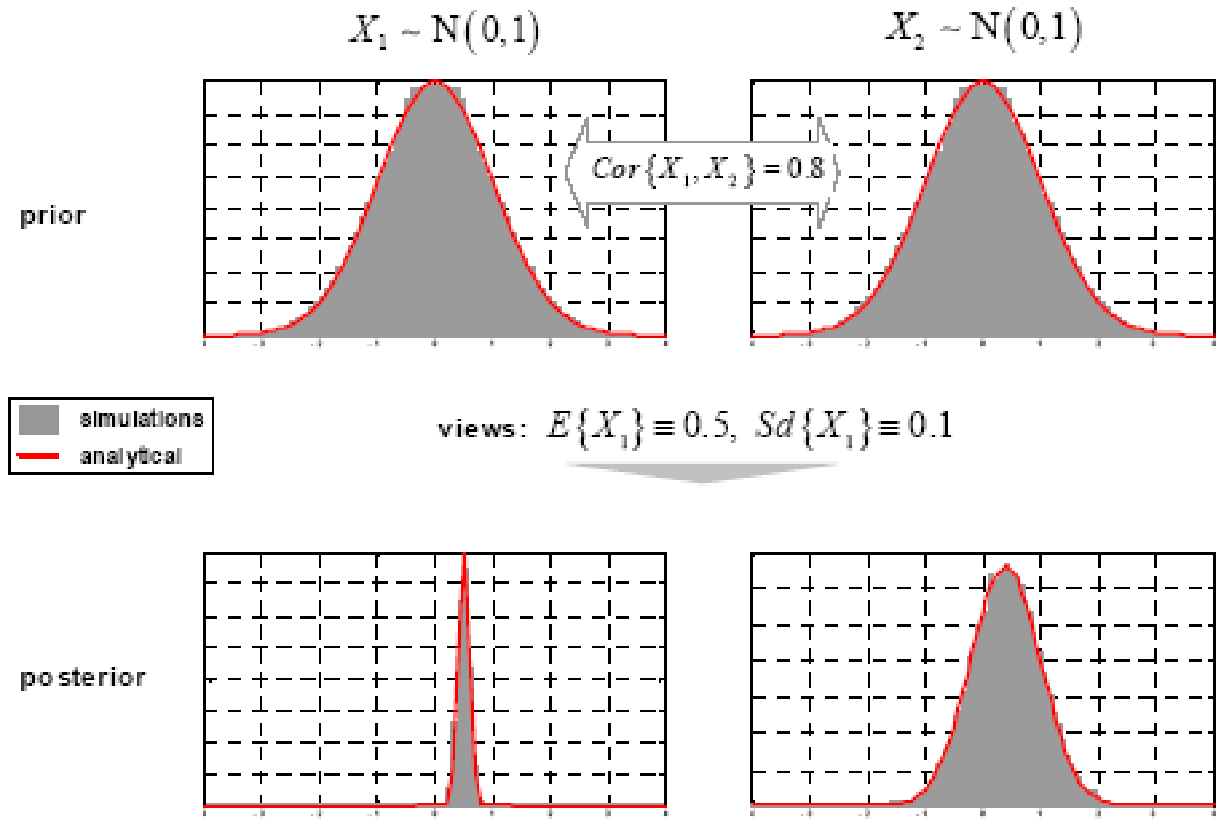
$$\left.\begin{array}{l}
\widetilde{m}\{V_k\} \gtreqless \widetilde{\mu}_{\mathbf{v},k}; \\
\widetilde{m}\{V_1\} \geq \widetilde{m}\{V_2\} \geq \cdots \geq \widetilde{m}\{V_K\} \\
\widetilde{\sigma}\{V_k\} \gtreqless \varkappa\sigma\{V_k\} \\
\widetilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1\mathbf{I} + \rho_2\mathbb{C}\{\mathbf{V}\} + \rho_3\mathbf{11}', \\
\widetilde{Q}_V(u) \gtreqless Q_V(u)
\end{array}\right\} \Leftrightarrow \quad \underline{\mathbf{a}} \leq \mathbf{A}\widetilde{\mathbf{p}} \leq \overline{\mathbf{a}}.$$

Posterior $\widetilde{f}_{\mathbf{X}} \equiv \underset{f \in \mathbb{V}}{\operatorname{argmin}}\{\mathcal{E}(f, f_{\mathbf{X}})\}$

$$\mathcal{E}\left(\widetilde{f}_{\mathbf{X}}, f_{\mathbf{X}}\right) \equiv \int \widetilde{f}_{\mathbf{X}}(\mathrm{x})\left[\ln \widetilde{f}_{\mathbf{X}}(\mathrm{x}) - \ln f_{\mathbf{X}}(\mathrm{x})\right] d\mathrm{x}. \quad \Leftrightarrow \quad \mathcal{E}(\widetilde{\mathbf{p}}, \mathbf{p}) \equiv \sum_{j=1}^{J} \widetilde{p}_j \left[\ln(\widetilde{p}_j) - \ln(p_j)\right]$$

Market distr.	$\mathbf{X} \sim f_{\mathbf{X}}.$	$\Leftrightarrow$	$\mathcal{X}$ $J \times N$ panel $\quad \mathbf{p}$ probabilities $1/J$;
Focus	$\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$	$\Leftrightarrow$	$\mathcal{V}_{j,k} \equiv g_k(\mathcal{X}_{j,1}, \ldots, \mathcal{X}_{j,N})$
Views	$\mathbf{V} \sim \widetilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}.$ $\widetilde{m}\{V_k\} \gtreqless \widetilde{\mu}_{\mathbf{v},k};$ $\widetilde{m}\{V_1\} \geq \widetilde{m}\{V_2\} \geq \cdots \geq \widetilde{m}\{V_K\}$ $\widetilde{\sigma}\{V_k\} \gtreqless \varkappa\sigma\{V_k\}$ $\widetilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1 \mathbf{I} + \rho_2 \mathbb{C}\{\mathbf{V}\} + \rho_3 \mathbf{1}\mathbf{1}',$ $\widetilde{Q}_V(u) \gtreqless Q_V(u)$	$\Leftrightarrow$	$\underline{\mathbf{a}} \leq \mathbf{A}\widetilde{\mathbf{p}} \leq \overline{\mathbf{a}}$
Posterior	$\widetilde{f}_{\mathbf{X}} \equiv \underset{f \in \mathbb{V}}{\operatorname{argmin}} \{\mathcal{E}(f, f_{\mathbf{X}})\}$	$\Leftrightarrow$	$\mathcal{X} \quad \widetilde{\mathbf{p}} \equiv \underset{\underline{\mathbf{a}} \leq \mathbf{A}\mathbf{f} \leq \overline{\mathbf{a}}}{\operatorname{argmin}} \{\mathcal{E}(\mathbf{f}, \mathbf{p})\}$

$$\mathcal{E}\left(\widetilde{f}_{\mathbf{X}}, f_{\mathbf{X}}\right) \equiv \int \widetilde{f}_{\mathbf{X}}(\mathrm{x}) \left[\ln \widetilde{f}_{\mathbf{X}}(\mathrm{x}) - \ln f_{\mathbf{X}}(\mathrm{x})\right] d\mathrm{x}. \quad \Leftrightarrow \quad \mathcal{E}(\widetilde{\mathbf{p}}, \mathbf{p}) \equiv \sum_{j=1}^{J} \widetilde{p}_j \left[\ln(\widetilde{p}_j) - \ln(p_j)\right]$$



Posterior $\tilde{f}_{\mathbf{X}} \equiv \underset{f \in \mathbb{V}}{\operatorname{argmin}} \{\mathcal{E}(f, f_{\mathbf{X}})\}$ $\Leftrightarrow$ $\mathcal{X}$ $\boxed{\tilde{\mathbf{p}} \equiv \underset{\underline{\mathbf{a}} \leq \mathbf{A}\mathbf{f} \leq \overline{\mathbf{a}}}{\operatorname{argmin}} \{\mathcal{E}(\mathbf{f}, \mathbf{p})\}}$

Dual formulation: linearly constrained convex optimization in # variables = # views

A. Meucci - Fully Flexible Views (Entropy Pooling) *Non-parametric implementation*

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}$. $\Leftrightarrow$ $\mathcal{X}$ $J \times N$ panel $\mathbf{p}$ probabilities $1/J$.

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$ $\Leftrightarrow$ $\mathcal{V}_{j,k} \equiv g_k(\mathcal{X}_{j,1}, \ldots, \mathcal{X}_{j,N})$

Views $\mathbf{V} \sim \widetilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}$.

$$\left.\begin{array}{l} \widetilde{m}\{V_k\} \gtreqless \widetilde{\mu}_{\mathbf{v},k}; \\ \widetilde{m}\{V_1\} \geq \widetilde{m}\{V_2\} \geq \cdots \geq \widetilde{m}\{V_K\} \\ \widetilde{\sigma}\{V_k\} \gtreqless \varkappa\sigma\{V_k\} \\ \widetilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1 \mathbf{I} + \rho_2 \mathbb{C}\{\mathbf{V}\} + \rho_3 \mathbf{1}\mathbf{1}', \\ \widetilde{Q}_V(u) \gtreqless Q_V(u) \end{array}\right\} \Leftrightarrow \quad \underline{\mathbf{a}} \leq \mathbf{A}\widetilde{\mathbf{p}} \leq \overline{\mathbf{a}}$$

Posterior $\widetilde{f}_{\mathbf{X}} \equiv \underset{f \in \mathcal{V}}{\operatorname{argmin}} \{\mathcal{E}(f, f_{\mathbf{X}})\}$ $\Leftrightarrow$ $\mathcal{X}$ $\widetilde{\mathbf{p}} \equiv \underset{\underline{\mathbf{a}} \leq \mathbf{A}\mathbf{f} \leq \overline{\mathbf{a}}}{\operatorname{argmin}} \{\mathcal{E}(\mathbf{f}, \mathbf{p})\}$

Confidence $\widetilde{f}^{c}_{\mathbf{X}} \equiv (1-c) f_{\mathbf{X}} + c\widetilde{f}_{\mathbf{X}}$.

Pricing $P_{t+\tau} \equiv P(\widetilde{\mathbf{X}}, \mathcal{I}_t)$

Optimization $\mathbf{w}^* \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \{\mathcal{S}(\mathbf{w}; \widetilde{f}^{c}_{\mathbf{X}})\}$

Market distr.	$\mathbf{X} \sim f_{\mathbf{X}}$.	$\Leftrightarrow$	$\mathcal{X}$ $J \times N$ panel $\mathbf{p}$ probabilities $1/J$.
Focus	$\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$	$\Leftrightarrow$	$\mathcal{V}_{j,k} \equiv g_k(\mathcal{X}_{j,1}, \ldots, \mathcal{X}_{j,N})$
Views	$\mathbf{V} \sim \widetilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}$. $\widetilde{m}\{V_k\} \gtreqless \widetilde{\mu}_{\mathbf{v},k}$: $\widetilde{m}\{V_1\} \geq \widetilde{m}\{V_2\} \geq \cdots \geq \widetilde{m}\{V_K\}$ $\widetilde{\sigma}\{V_k\} \gtreqless \varkappa\sigma\{V_k\}$ $\widetilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1\mathbf{I} + \rho_2\mathbb{C}\{\mathbf{V}\} + \rho_3\mathbf{1}\mathbf{1}'$, $\widetilde{Q}_V(u) \gtreqless Q_V(u)$	$\Leftrightarrow$	$\underline{\mathbf{a}} \leq \mathbf{A}\widetilde{\mathbf{p}} \leq \overline{\mathbf{a}}$
Posterior	$\widetilde{f}_{\mathbf{X}} \equiv \underset{f \in \mathbb{V}}{\operatorname{argmin}} \{\mathcal{E}(f, f_{\mathbf{X}})\}$	$\Leftrightarrow$	$\mathcal{X}$ $\widetilde{\mathbf{p}} \equiv \underset{\underline{\mathbf{a}} \leq \mathbf{A}\mathbf{f} \leq \overline{\mathbf{a}}}{\operatorname{argmin}} \{\mathcal{E}(\mathbf{f}, \mathbf{p})\}$
Confidence	$\widetilde{f}_{\mathbf{X}}^{c} \equiv (1-c) f_{\mathbf{X}} + c\widetilde{f}_{\mathbf{X}}$.	$\Leftrightarrow$	$\mathcal{X}$ $\mathbf{p}_c \equiv (1-c)\mathbf{p} + c\widetilde{\mathbf{p}}$.
Pricing	$P_{t+\tau} \equiv P(\widetilde{\mathbf{X}}, \mathcal{I}_t)$		
Optimization	$\mathbf{w}^* \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \{\mathcal{S}(\mathbf{w}; \widetilde{f}_{\mathbf{X}}^{\mathbf{c}})\}$		

A. Meucci - Fully Flexible Views (Entropy Pooling) — Non-parametric implementation

Market distr.	$\mathbf{X} \sim f_{\mathbf{X}}.$	$\Leftrightarrow$	$\mathcal{X}$ $J \times N$ panel $\mathbf{p}$ probabilities $1/J$.
Focus	$\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$	$\Leftrightarrow$	$\mathcal{V}_{j,k} \equiv g_k(\mathcal{X}_{j,1}, \ldots, \mathcal{X}_{j,N})$
Views	$\left.\begin{array}{l} \mathbf{V} \sim \tilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}. \\ \tilde{m}\{V_k\} \gtreqless \tilde{\mu}_{\mathbf{v},k}; \\ \tilde{m}\{V_1\} \geq \tilde{m}\{V_2\} \geq \cdots \geq \tilde{m}\{V_K\} \\ \tilde{\sigma}\{V_k\} \gtreqless \varkappa\sigma\{V_k\} \\ \tilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1 \mathbf{I} + \rho_2 \mathbb{C}\{\mathbf{V}\} + \rho_3 \mathbf{11}', \\ \tilde{Q}_V(u) \gtreqless Q_V(u) \end{array}\right\}$	$\Leftrightarrow$	$\underline{\mathbf{a}} \leq \mathbf{A}\tilde{\mathbf{p}} \leq \overline{\mathbf{a}}$
Posterior	$\tilde{f}_{\mathbf{X}} \equiv \operatorname*{argmin}_{f \in \mathbb{V}} \{\mathcal{E}(f, f_{\mathbf{X}})\}$	$\Leftrightarrow$	$\mathcal{X}$ $\tilde{\mathbf{p}} \equiv \operatorname*{argmin}_{\underline{\mathbf{a}} \leq \mathbf{Af} \leq \overline{\mathbf{a}}} \{\mathcal{E}(\mathbf{f}, \mathbf{p})\}$
Confidence	$\tilde{f}^c_{\mathbf{X}} \equiv (1-c) f_{\mathbf{X}} + c\tilde{f}_{\mathbf{X}}.$	$\Leftrightarrow$	$\mathcal{X}$ $\mathbf{p}_c \equiv (1-c)\mathbf{p} + c\tilde{\mathbf{p}}.$
~~**Pricing**~~	$P_{t+\tau} \equiv P(\tilde{\mathbf{X}}, \mathcal{I}_t)$	$\Leftrightarrow$	$\boxed{\mathcal{P}}$ $\mathbf{p}_c$
Optimization	$\mathbf{w}^* \equiv \operatorname*{argmax}_{\mathbf{w} \in \mathcal{C}} \{\mathcal{S}(\mathbf{w}; \tilde{f}^c_{\mathbf{X}})\}$		

A. Meucci - Fully Flexible Views (Entropy Pooling) — *Non-parametric implementation*

Market distr.	$\mathbf{X} \sim f_{\mathbf{X}}$.	$\Leftrightarrow$	$\mathcal{X}$ $J \times N$ panel $\mathbf{p}$ probabilities $1/J$.
Focus	$\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$	$\Leftrightarrow$	$\mathcal{V}_{j,k} \equiv g_k(\mathcal{X}_{j,1}, \ldots, \mathcal{X}_{j,N})$
Views	$\mathbf{V} \sim \tilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}$. $\tilde{m}\{V_k\} \gtreqless \tilde{\mu}_{\mathbf{v},k}$; $\tilde{m}\{V_1\} \geq \tilde{m}\{V_2\} \geq \cdots \geq \tilde{m}\{V_K\}$ $\tilde{\sigma}\{V_k\} \gtreqless \varkappa\sigma\{V_k\}$ $\tilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1 \mathbf{I} + \rho_2 \mathbb{C}\{\mathbf{V}\} + \rho_3 \mathbf{1}\mathbf{1}'$, $\tilde{Q}_V(u) \gtreqless Q_V(u)$	$\Leftrightarrow$	$\underline{\mathbf{a}} \leq \mathbf{A}\tilde{\mathbf{p}} \leq \overline{\mathbf{a}}$.
Posterior	$\tilde{f}_{\mathbf{X}} \equiv \underset{f \in \mathbb{V}}{\operatorname{argmin}} \{\mathcal{E}(f, f_{\mathbf{X}})\}$	$\Leftrightarrow$	$\mathcal{X}$ $\tilde{\mathbf{p}} \equiv \underset{\underline{\mathbf{a}} \leq \mathbf{A}\mathbf{f} \leq \overline{\mathbf{a}}}{\operatorname{argmin}} \{\mathcal{E}(\mathbf{f}, \mathbf{p})\}$
Confidence	$\tilde{f}^c_{\mathbf{X}} \equiv (1-c) f_{\mathbf{X}} + c \tilde{f}_{\mathbf{X}}$.	$\Leftrightarrow$	$\mathcal{X}$ $\mathbf{p}_c \equiv (1-c)\mathbf{p} + c\tilde{\mathbf{p}}$.
Pricing	$P_{t+\tau} \equiv P(\tilde{\mathbf{X}}, \mathcal{I}_t)$	$\Leftrightarrow$	$\mathcal{P}$ $\mathbf{p}_c$
Optimization	$\mathbf{w}^* \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \{\mathcal{S}(\mathbf{w}; \tilde{f}^c_{\mathbf{X}})\}$	$\Leftrightarrow$	...

ESTIMATION RISK

SCENARIO ANALYSIS

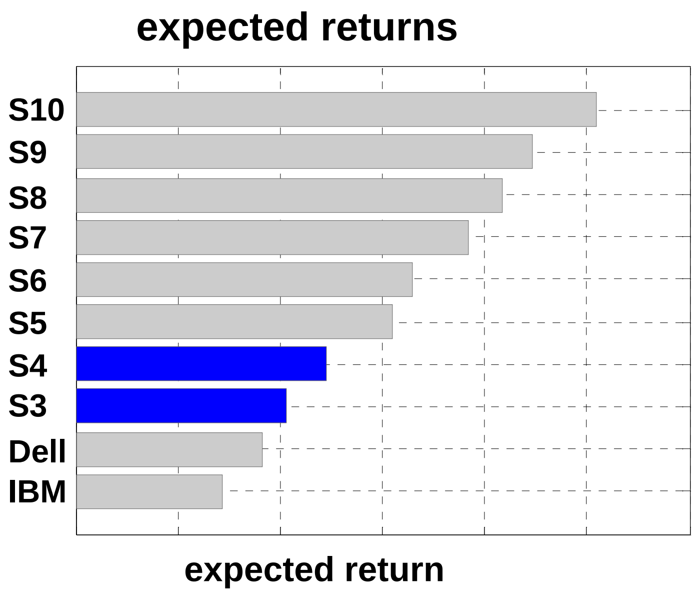
THE BLACK-LITTERMAN APPROACH

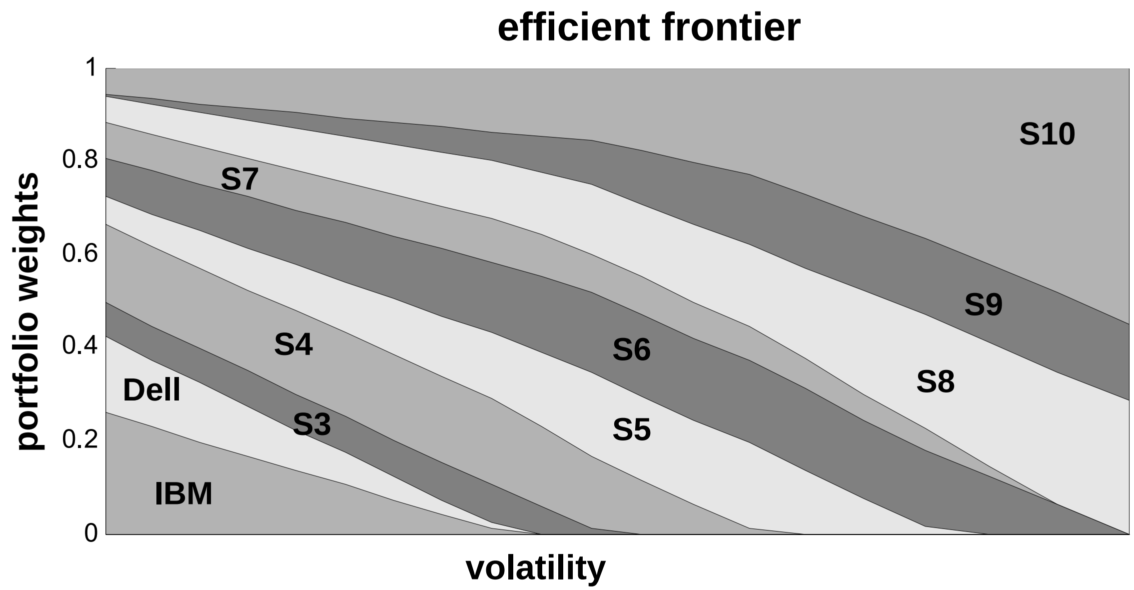
ENTROPY POOLING

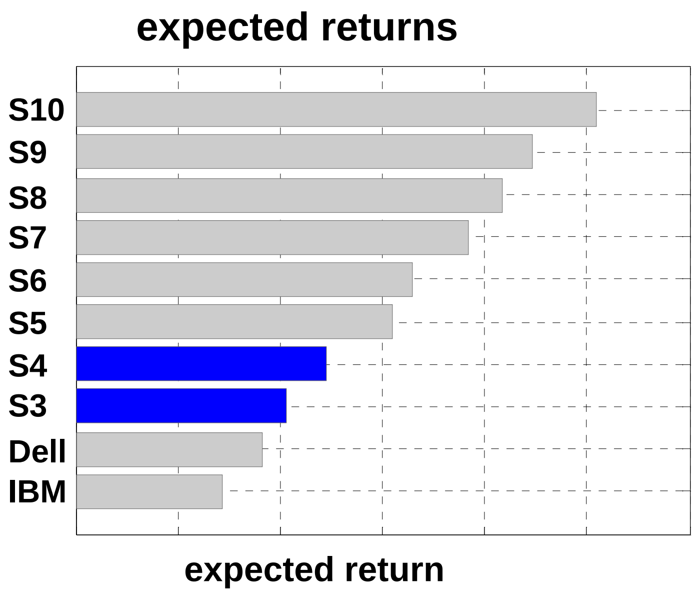
CASE STUDIES

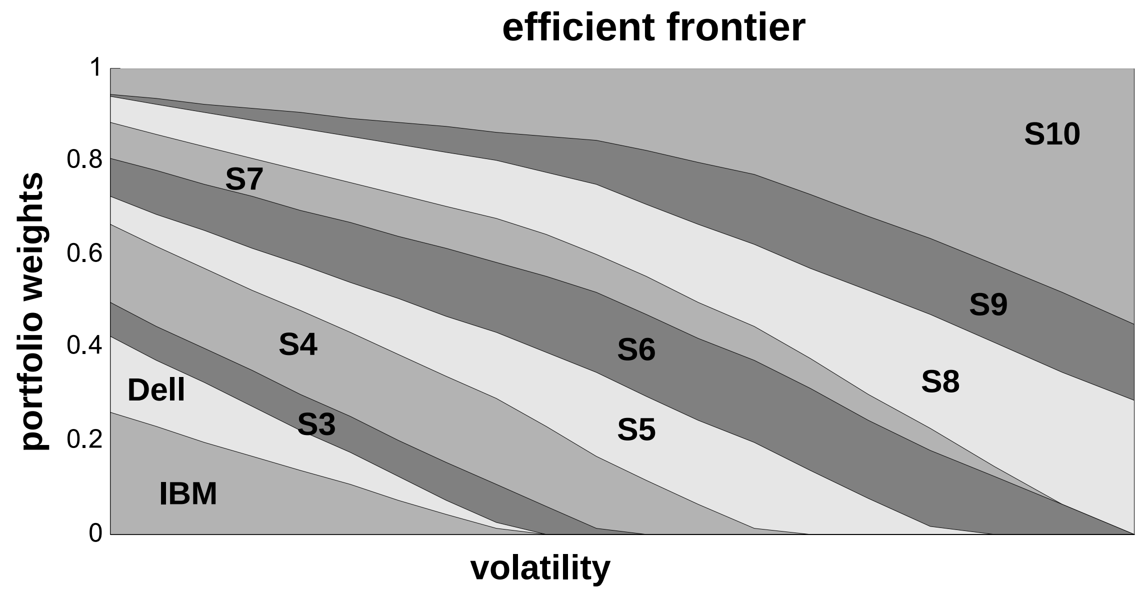
- **Portfolio from sorts**
- **Implied expected returns**
- **Distributional stress-testing**
- **Option trading**

REFERENCES AND CONCLUSIONS

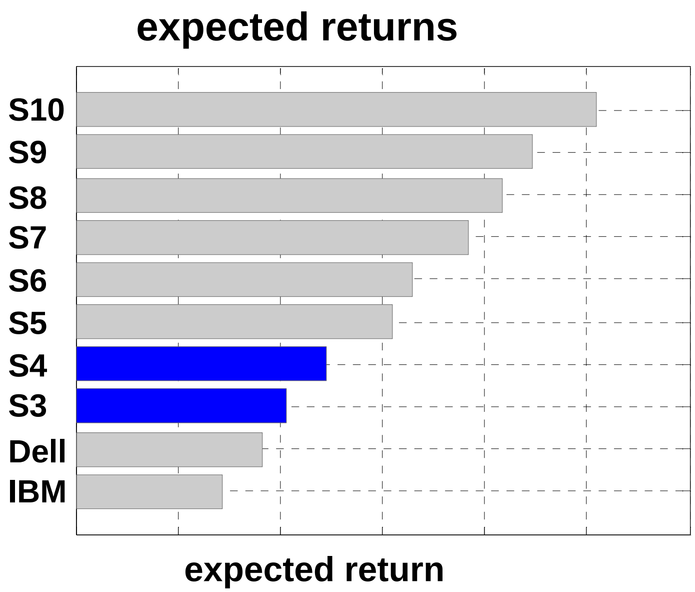

expected returns
S10
S9
S8
S7
S6
S5
S4
S3
Dell
IBM
expected return

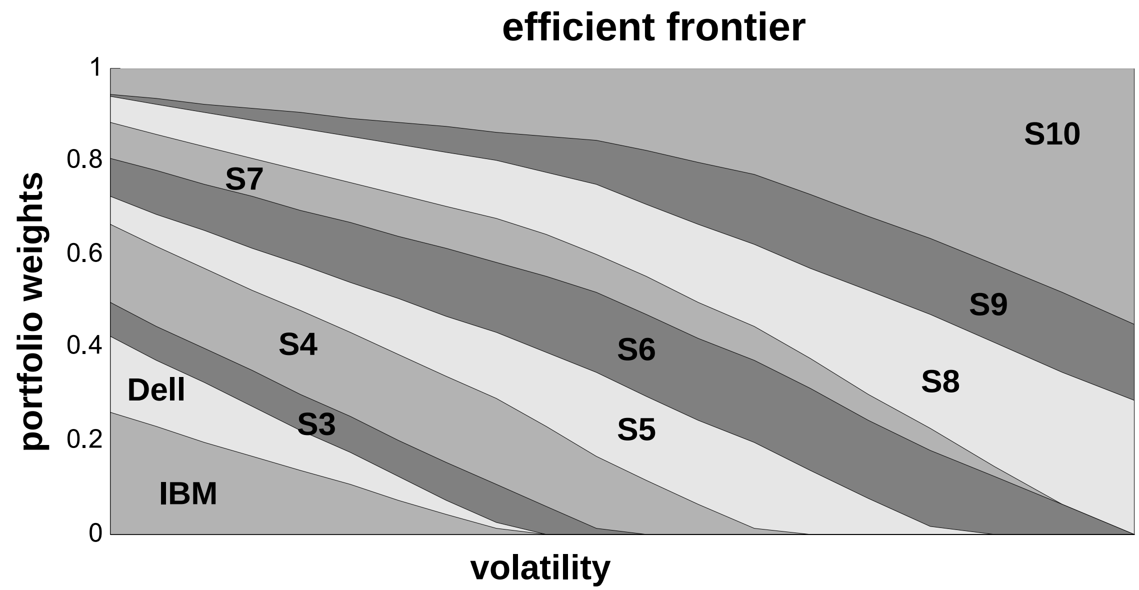

efficient frontier
portfolio weights
1
0.8
0.6
0.4
0.2
0
S10
S9
S8
S7
S6
S5
S4
S3
Dell
IBM
volatility



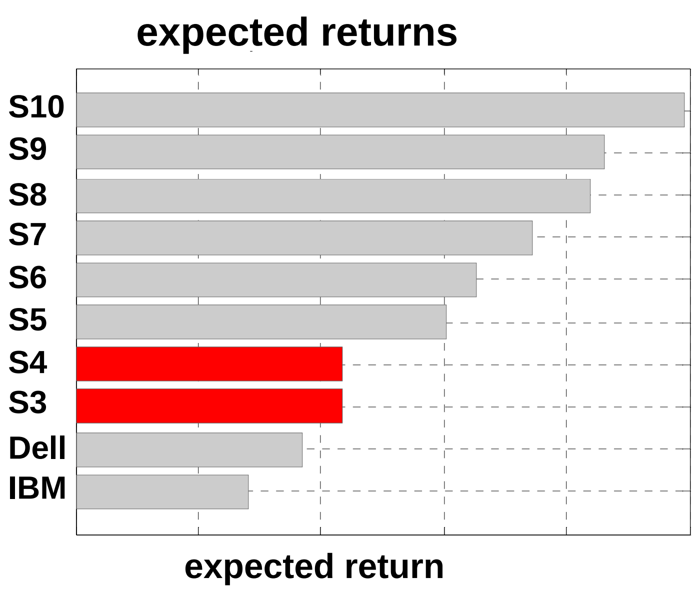


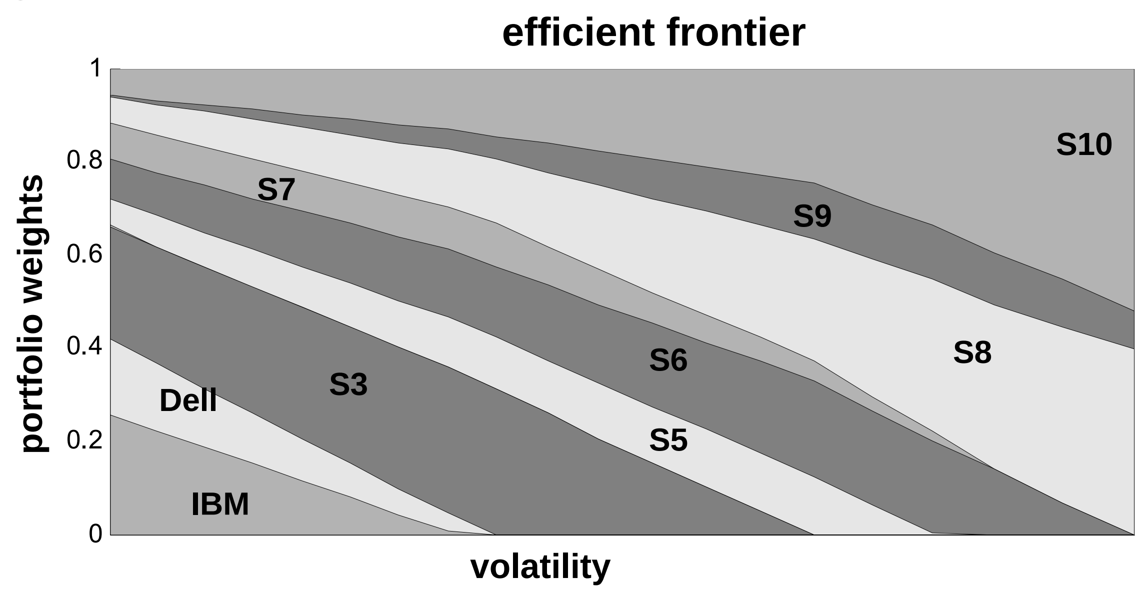
view: $E\left\{R_3\right\} \geq E\left\{R_4\right\}$



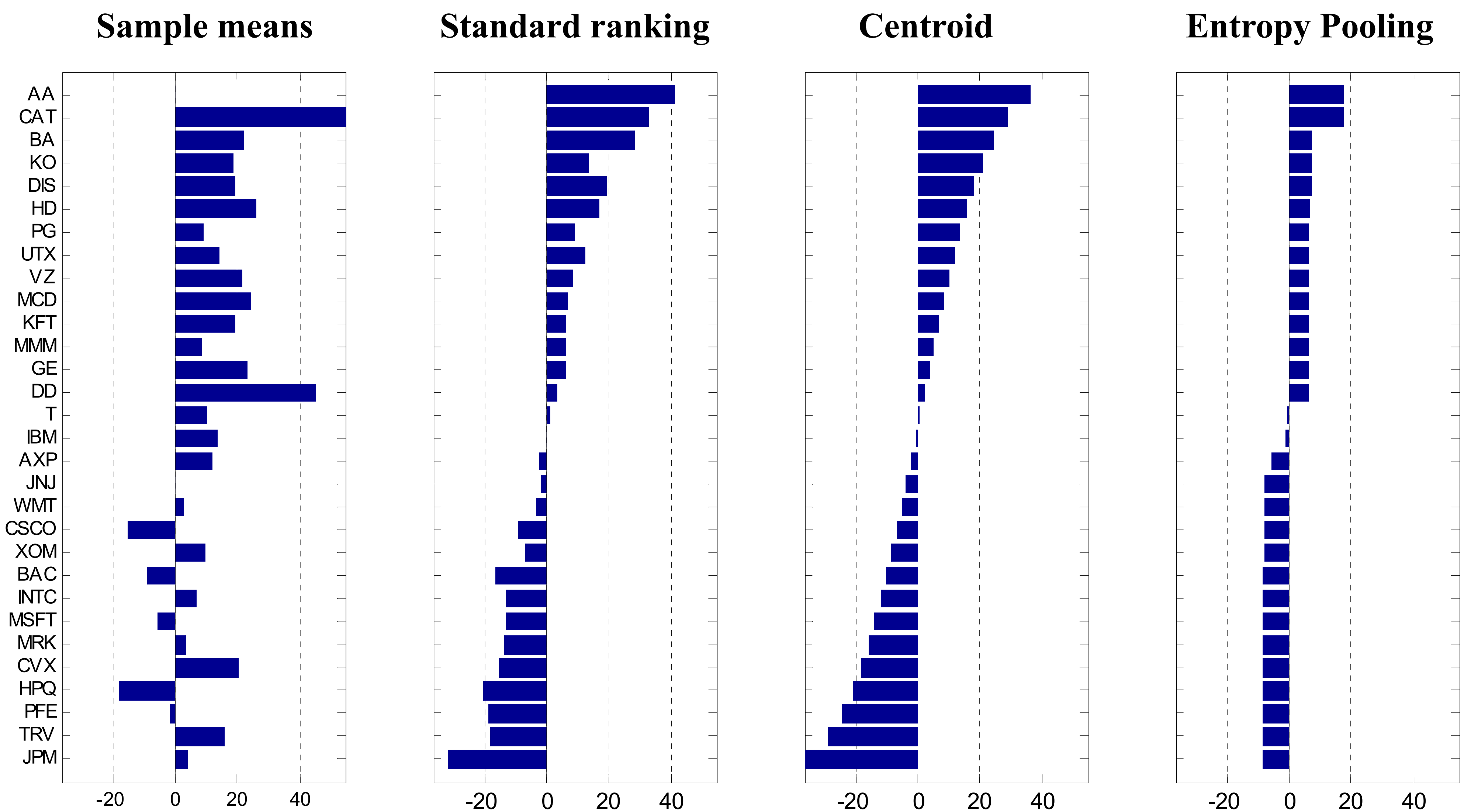


view: $E\left\{R_3\right\} \geq E\left\{R_4\right\}$





Views: $\mathcal{V} : \mathrm{E}\{R_1\} \geq \mathrm{E}\{R_2\} \geq \cdots \geq \mathrm{E}\{R_N\}$



ESTIMATION RISK

SCENARIO ANALYSIS

THE BLACK-LITTERMAN APPROACH

ENTROPY POOLING

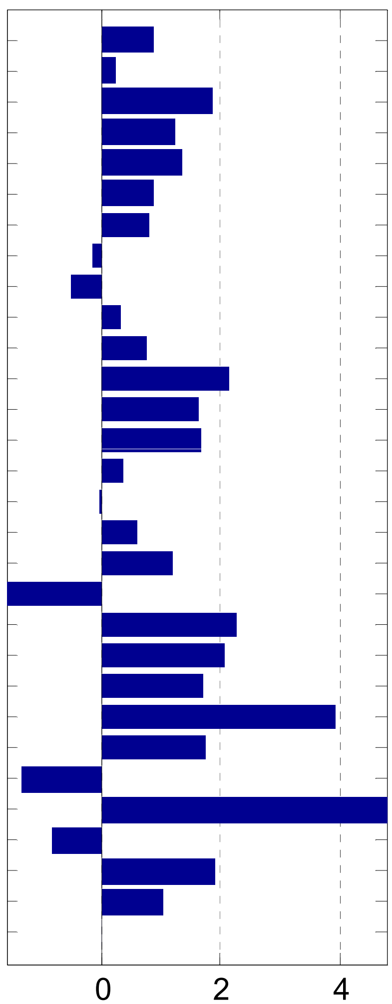
CASE STUDIES

- **Portfolio from sorts**
- **Implied expected returns**
- **Distributional stress-testing**
- **Option trading**

REFERENCES AND CONCLUSIONS

$$\mathrm{E}\{\mathbf{R}\}$$

sample means

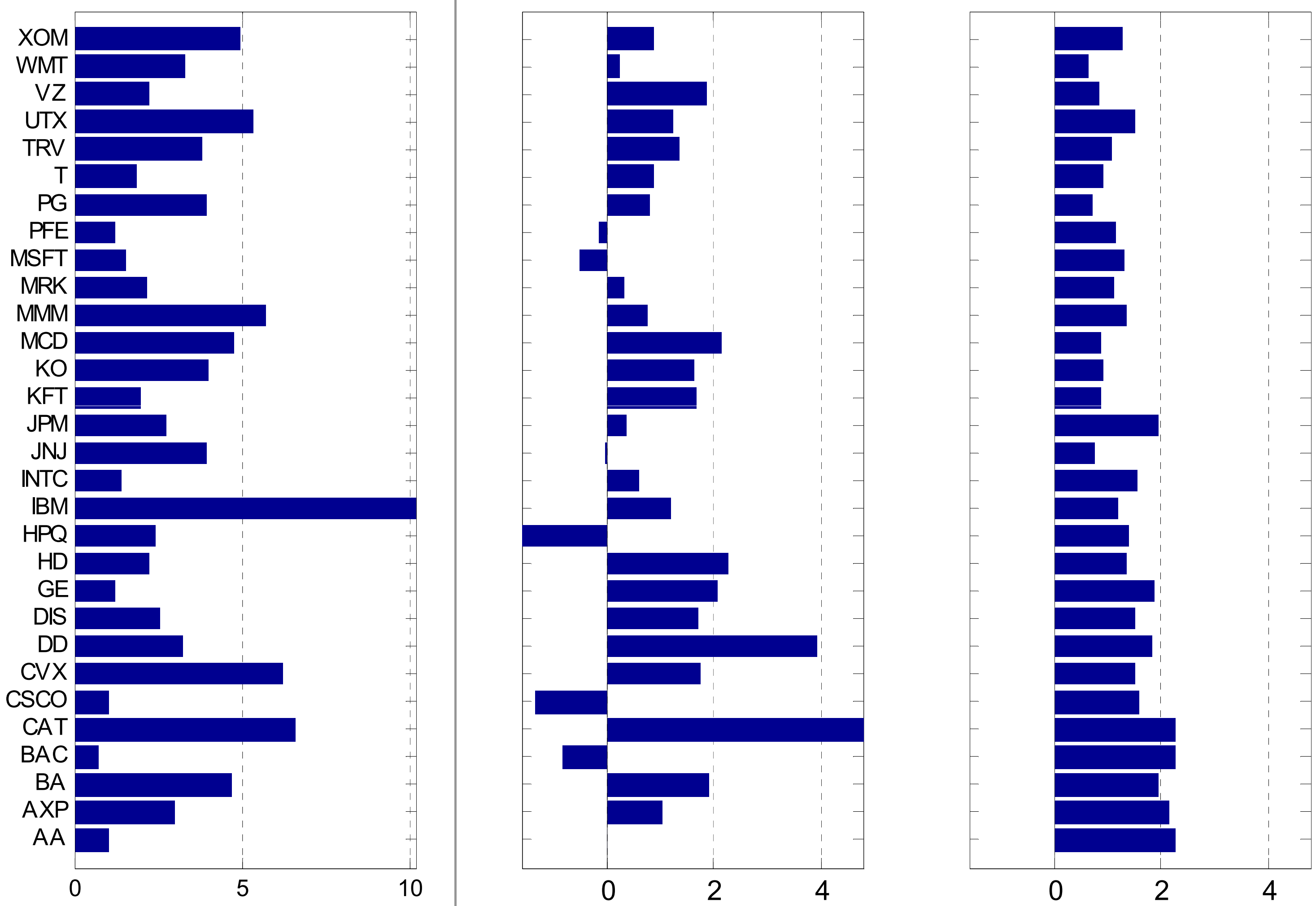


$$\mathrm{E}\left\{\mathbf{R}\right\} \equiv \gamma\,\mathrm{Cov}\left\{\mathbf{R}\right\}\mathbf{w}_{eq}$$

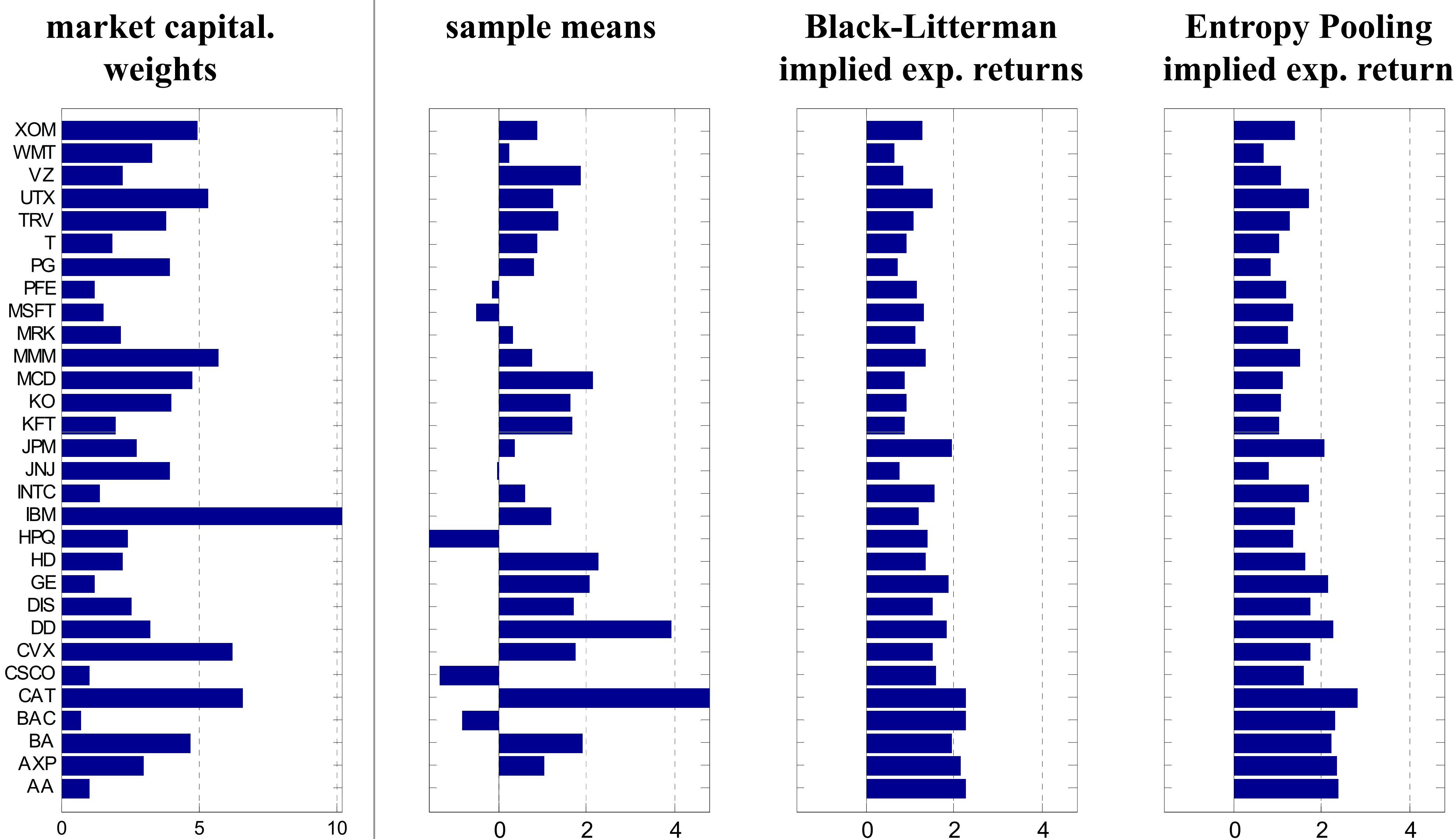
market capital. weights

sample means

Black-Litterman implied exp. returns

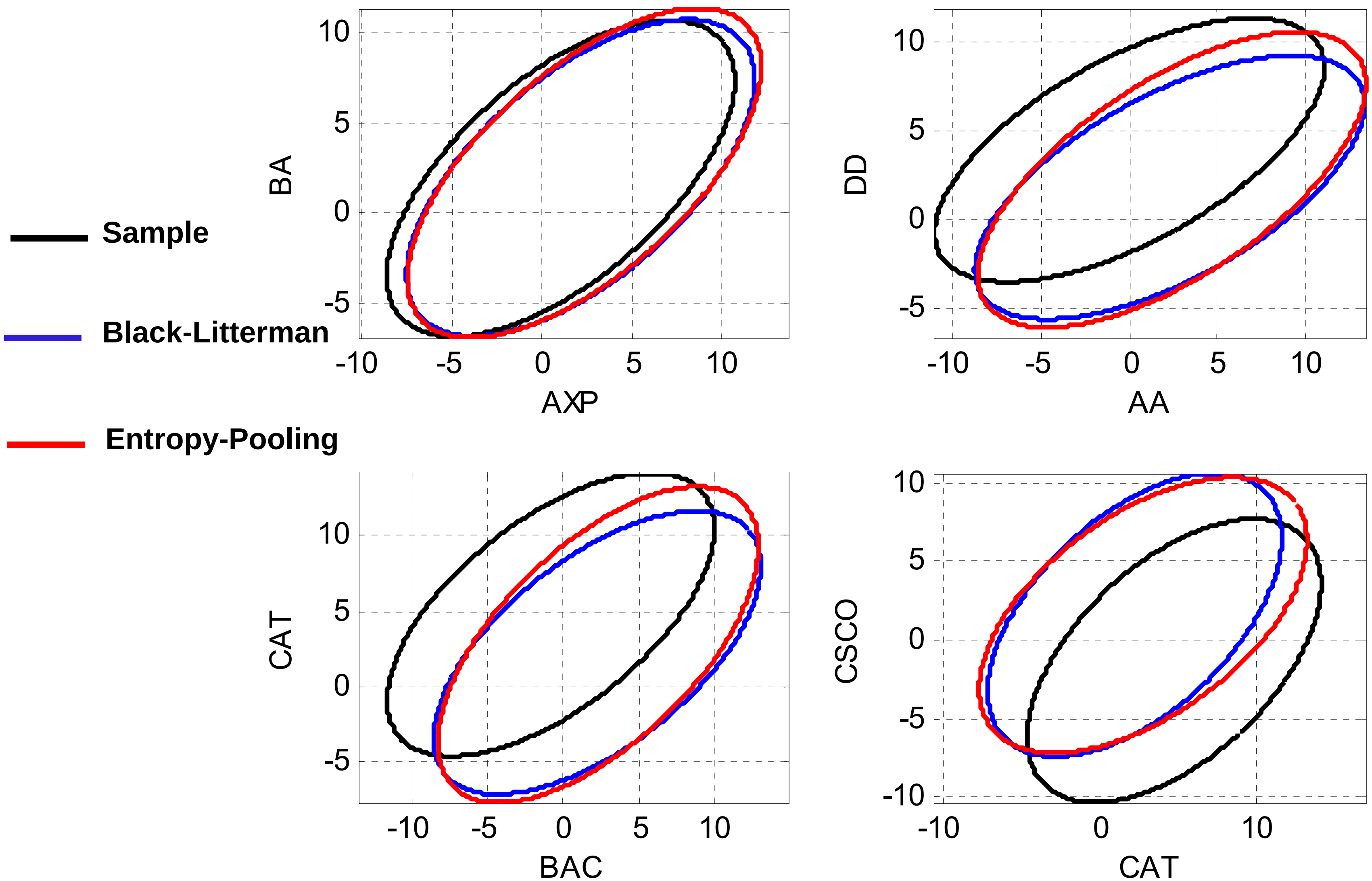


$$\mathcal{V} : \mathrm{E}\left\{\mathbf{R}\right\} \equiv \gamma\, \mathrm{Cov}\left\{\mathbf{R}\right\} \mathbf{w}_{eq}$$



$$\mathcal{V} : \mathrm{E}\{\mathbf{R}\} \equiv \gamma\, \mathrm{Cov}\{\mathbf{R}\}\, \mathbf{w}_{eq}$$

Expectation -covariance ellipsoids



ESTIMATION RISK

SCENARIO ANALYSIS

THE BLACK-LITTERMAN APPROACH

ENTROPY POOLING

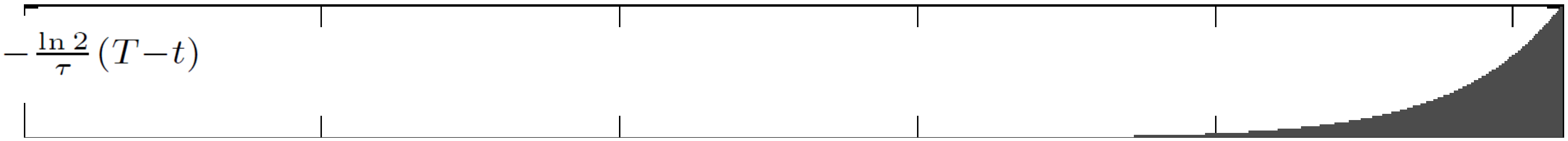
CASE STUDIES

- **Portfolio from sorts**
- **Implied expected returns**
- **Distributional stress-testing**
- **Option trading**

REFERENCES AND CONCLUSIONS

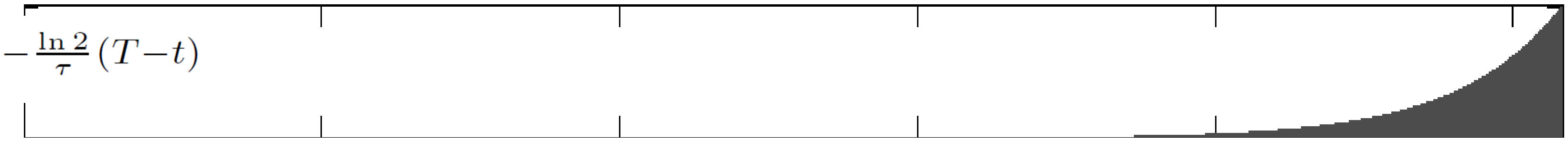
Prior (exponential time decay)

$$p_t^{(0)} \propto e^{-\frac{\ln 2}{\tau}(T-t)}$$

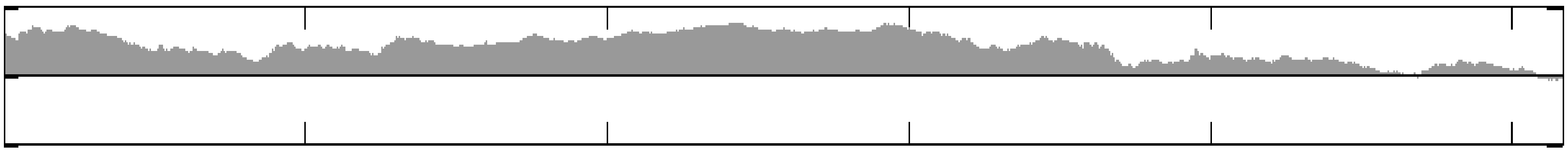


Prior (exponential time decay)

$$p_t^{(0)} \propto e^{-\frac{\ln 2}{\tau}(T-t)}$$



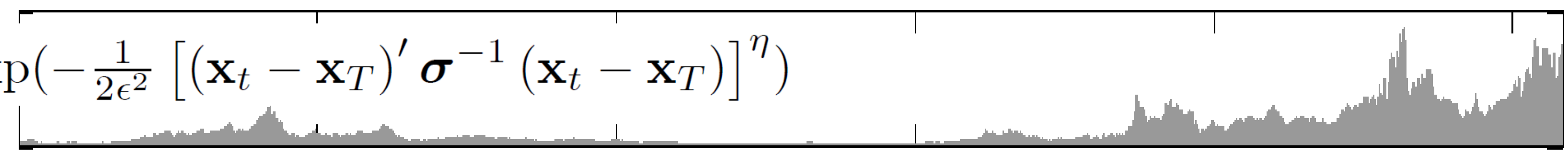
State indicator (2yr swap rate - current level)



State indicator (1->5yr ATM implied swaption vol - current level)

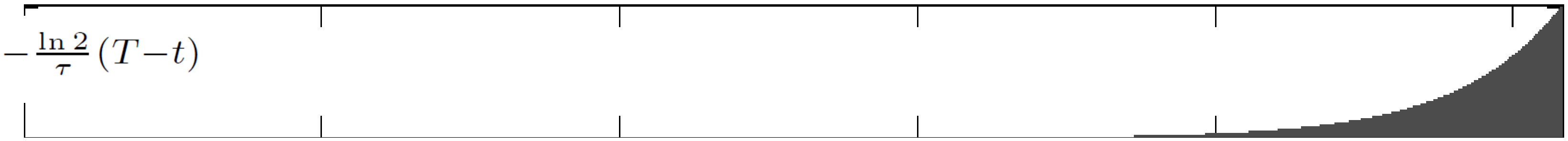
Membership (pseudo Gaussian kernel)

$$m_t \equiv \exp\left(-\frac{1}{2\epsilon^2}\left[(\mathbf{x}_t - \mathbf{x}_T)' \boldsymbol{\sigma}^{-1} (\mathbf{x}_t - \mathbf{x}_T)\right]^{\eta}\right)$$

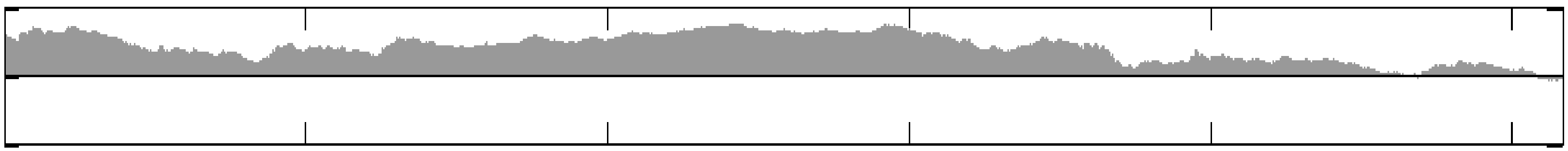


Prior (exponential time decay)

$$p_t^{(0)} \propto e^{-\frac{\ln 2}{\tau}(T-t)}$$



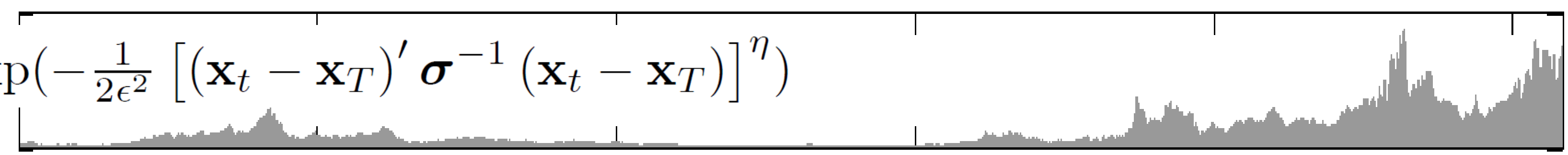
State indicator (2yr swap rate - current level)



State indicator (1->5yr ATM implied swaption vol - current level)

Membership (pseudo Gaussian kernel)

$$m_t \equiv \exp\left(-\frac{1}{2\epsilon^2}\left[(\mathbf{x}_t - \mathbf{x}_T)' \boldsymbol{\sigma}^{-1} (\mathbf{x}_t - \mathbf{x}_T)\right]^{\eta}\right)$$

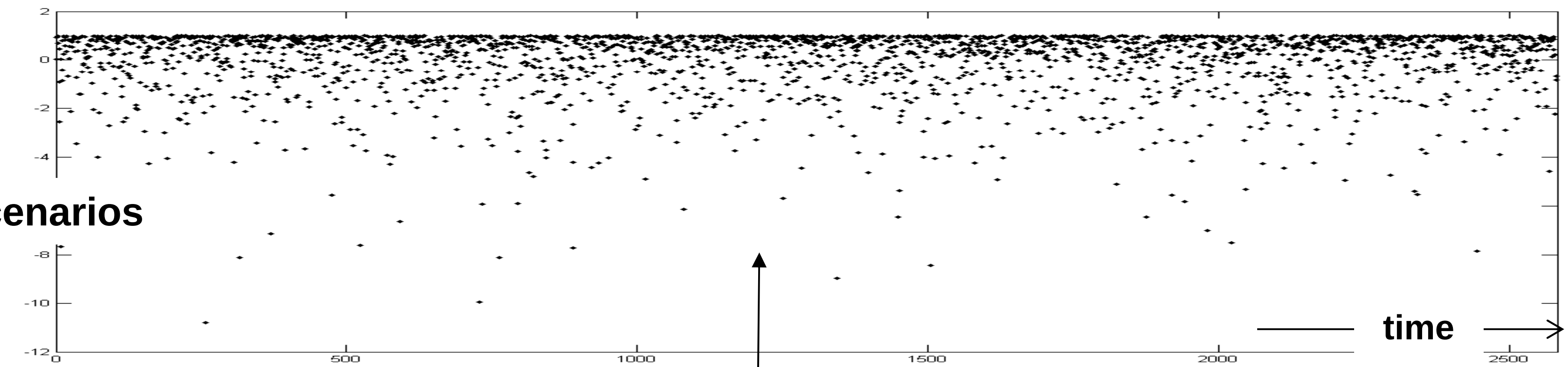


Posterior (Entropy Pooling mixture)

$$\mathbf{p} \equiv \underset{\mathbf{q}' \ln \mathbf{m} \geq v_*}{\operatorname{argmin}} \; \mathcal{E}(\mathbf{q}, \mathbf{p}^{(0)})$$

time $\longrightarrow$

P&L scenarios



P&L distribution

$$f_\Pi \iff \{\pi_t, p_t\}_{t=1,\ldots,T}$$

P&L ex-ante performance statistics

	Time decay	Kernel	Entropy Pooling
Exp. value	0.12	0.05	0.10
St. dev.	1.18	1.30	1.26
Skew.	-2.65	-2.56	-2.76
Kurt.	12.58	11.76	13.39
VaR 99%	-4.62	-5.53	-5.11
Cond. VaR 99%	-6.16	-6.70	-6.85
Effective scenarios	471	1,644	897

ESTIMATION RISK

SCENARIO ANALYSIS

THE BLACK-LITTERMAN APPROACH

ENTROPY POOLING

CASE STUDIES

- **Portfolio from sorts**
- **Implied expected returns**
- **Distributional stress-testing**
- **Option trading**

REFERENCES AND CONCLUSIONS

Black-Scholes formula:
deterministic function of risk into price

$$C_{BS}(y,\sigma;\kappa,T,r) \equiv yF(d_1) - \kappa e^{-rT}F(d_2)$$

$$d_1 \equiv \left(\ln(y/\kappa) + \left(r + \sigma^2/2\right)T\right)/\sigma\sqrt{T}, \qquad d_2 \equiv d_1 - \sigma\sqrt{T};$$

Black-Scholes formula:
deterministic function of risk into price

$$\boxed{C}_{BS}\left(\boxed{y},\boxed{\sigma};\kappa,T,r\right)\equiv yF\left(d_1\right)-\kappa e^{-rT}F\left(d_2\right)$$

$$d_1\equiv\left(\ln\left(\boxed{y}/\kappa\right)+\left(r+\boxed{\sigma}^2/2\right)T\right)/\boxed{\sigma}\sqrt{T},\qquad d_2\equiv d_1-\boxed{\sigma}\sqrt{T};$$

$$C_{BS}(y,\sigma;\kappa,T,r) \equiv yF(d_1) - \kappa e^{-rT}F(d_2)$$

$$d_1 \equiv \left(\ln(y/\kappa) + \left(r + \sigma^2/2\right)T\right)/\sigma\sqrt{T}, \qquad d_2 \equiv d_1 - \sigma\sqrt{T};$$

$$h(y,\sigma;\kappa,T) \equiv \sigma + a\frac{\ln(y/\kappa)}{\sqrt{T}} + b\left(\frac{\ln(y/\kappa)}{\sqrt{T}}\right)^2$$

empirical smirk and smile

call option price at horizon $P_{t+\tau} = C_{BS}\left(y_t e^{X_y}, h\left(y_t e^{X_y}, \sigma_t + X_\sigma, \kappa, T-\tau\right); \kappa, T-\tau, r\right)$

$$X_y \equiv \ln\left(y_{t+\tau}/y_t\right)$$

$$X_\sigma \equiv \sigma_{t+\tau} - \sigma_t$$

$$C_{BS}\left(y, \sigma; \kappa, T, r\right) \equiv yF\left(d_1\right) - \kappa e^{-rT} F\left(d_2\right)$$

$$d_1 \equiv \left(\ln\left(y/\kappa\right) + \left(r + \sigma^2/2\right) T\right) / \sigma\sqrt{T}, \qquad d_2 \equiv d_1 - \sigma\sqrt{T};$$

$$h\left(y, \sigma; \kappa, T\right) \equiv \sigma + a\frac{\ln\left(y/\kappa\right)}{\sqrt{T}} + b\left(\frac{\ln\left(y/\kappa\right)}{\sqrt{T}}\right)^2$$

call option price at horizon $P_{t+\tau} = C_{BS}\left(y_t e^{X_y}, h\left(y_t e^{X_y}, \sigma_t + X_\sigma, \kappa, T-\tau\right); \kappa, T-\tau, r\right)$

$$X_y \equiv \ln\left(y_{t+\tau}/y_t\right)$$

$$X_\sigma \equiv \sigma_{t+\tau} - \sigma_t$$

$$C_{BS}\left(y, \sigma; \kappa, T, r\right) \equiv yF\left(d_1\right) - \kappa e^{-rT} F\left(d_2\right)$$

$$d_1 \equiv \left(\ln\left(y/\kappa\right) + \left(r + \sigma^2/2\right)T\right)/\sigma\sqrt{T}, \qquad d_2 \equiv d_1 - \sigma\sqrt{T};$$

$$h\left(y, \sigma; \kappa, T\right) \equiv \sigma + a\frac{\ln\left(y/\kappa\right)}{\sqrt{T}} + b\left(\frac{\ln\left(y/\kappa\right)}{\sqrt{T}}\right)^2$$

Portfolio: Microsoft 1 month
Microsoft 2 months
Microsoft 6 months
Yahoo 1 month
Yahoo 2 months
Yahoo 6 months
Google 1 month
Google 2 months
Google 6 months

$$\mathbf{X} \equiv \left(X^M, X^M_{1m}, X^M_{2m}, X^M_{6m}, \ldots, X^G_{6m}, \underbrace{X_{2y}, X_{10y}}\right)'$$

curve change (growth/inflation)
not directly in pricing

call option price at horizon $P_{t+\tau} = C_{BS}\left(y_t e^{X_y}, h\left(y_t e^{X_y}, \sigma_t + X_\sigma, \kappa, T-\tau\right); \kappa, T-\tau, r\right)$

$$X_y \equiv \ln\left(y_{t+\tau}/y_t\right)$$

$$X_\sigma \equiv \sigma_{t+\tau} - \sigma_t$$

$$C_{BS}\left(y,\sigma;\kappa,T,r\right) \equiv yF\left(d_1\right) - \kappa e^{-rT}F\left(d_2\right)$$

$$d_1 \equiv \left(\ln\left(y/\kappa\right) + \left(r + \sigma^2/2\right)T\right)/\sigma\sqrt{T}, \qquad d_2 \equiv d_1 - \sigma\sqrt{T};$$

$$\boxed{h\left(y,\sigma;\kappa,T\right) \equiv \sigma + a\frac{\ln\left(y/\kappa\right)}{\sqrt{T}} + b\left(\frac{\ln\left(y/\kappa\right)}{\sqrt{T}}\right)^2}$$

$$\mathbf{X} \equiv \left(X^M, X^M_{1m}, X^M_{2m}, X^M_{6m}, \ldots, X^G_{6m}, X_{2y}, X_{10y}\right)' \sim \cancel{\mathrm{N}}\left(\boldsymbol{\pi}, \boldsymbol{\Sigma}\right)$$

call option price at horizon $P_{t+\tau} = C_{BS}\left(y_t e^{X_y}, h\left(y_t e^{X_y}, \sigma_t + X_\sigma, \kappa, T-\tau\right); \kappa, T-\tau, r\right)$

$$X_y \equiv \ln\left(y_{t+\tau}/y_t\right)$$

$$X_\sigma \equiv \sigma_{t+\tau} - \sigma_t$$

$$C_{BS}\left(y,\sigma;\kappa,T,r\right) \equiv yF\left(d_1\right) - \kappa e^{-rT} F\left(d_2\right)$$

$$d_1 \equiv \left(\ln\left(y/\kappa\right) + \left(r + \sigma^2/2\right)T\right)/\sigma\sqrt{T}, \qquad d_2 \equiv d_1 - \sigma\sqrt{T};$$

$$h\left(y,\sigma;\kappa,T\right) \equiv \sigma + a\frac{\ln\left(y/\kappa\right)}{\sqrt{T}} + b\left(\frac{\ln\left(y/\kappa\right)}{\sqrt{T}}\right)^2$$

$$\mathbf{X} \equiv \left(X^M, X^M_{1m}, X^M_{2m}, X^M_{6m}, \ldots, X^G_{6m}, X_{2y}, X_{10y}\right)' \nsim \mathrm{N}\left(\boldsymbol{\pi}, \boldsymbol{\Sigma}\right)$$

$$\Pi_{\mathbf{w}} \equiv \sum_{i=1}^{I} w_i \left(C_{BS,i}\left(\mathbf{X}, \mathcal{I}_t\right) - C_{i,t}\right)$$

?

profit and loss is highly <u>non-linear</u>, highly <u>non-normal</u>

call option price at horizon $P_{t+\tau} = C_{BS}\left(y_t e^{X_y}, h\left(y_t e^{X_y}, \sigma_t + X_\sigma, \kappa, T-\tau\right); \kappa, T-\tau, r\right)$

$$X_y \equiv \ln\left(y_{t+\tau}/y_t\right)$$

$$X_\sigma \equiv \sigma_{t+\tau} - \sigma_t$$

$$C_{BS}\left(y,\sigma;\kappa,T,r\right) \equiv yF\left(d_1\right) - \kappa e^{-rT}F\left(d_2\right)$$

$$d_1 \equiv \left(\ln\left(y/\kappa\right) + \left(r + \sigma^2/2\right)T\right)/\sigma\sqrt{T}, \qquad d_2 \equiv d_1 - \sigma\sqrt{T};$$

$$h\left(y,\sigma;\kappa,T\right) \equiv \sigma + a\frac{\ln\left(y/\kappa\right)}{\sqrt{T}} + b\left(\frac{\ln\left(y/\kappa\right)}{\sqrt{T}}\right)^2$$

$$\mathbf{X} \equiv \left(X^M, X^M_{1m}, X^M_{2m}, X^M_{6m}, \ldots, X^G_{6m}, X_{2y}, X_{10y}\right)' \not\sim \mathrm{N}\left(\boldsymbol{\pi}, \boldsymbol{\Sigma}\right)$$

$$\Pi_{\mathbf{w}} \equiv \sum_{i=1}^{I} w_i\left(C_{BS,i}\left(\mathbf{X}, \mathcal{I}_t\right) - C_{i,t}\right)$$ **?**

Mean-CVaR optimization **?**

$$\mathbf{w}_\lambda \equiv \underset{\underline{\mathbf{b}} \leq \mathbf{B}\mathbf{w} \leq \overline{\mathbf{b}}}{\operatorname{argmax}} \left\{\mathbb{E}\left\{\Pi_{\mathbf{w}}\right\} - \lambda\,\mathrm{CVaR}_\gamma\left\{\Pi_{\mathbf{w}}\right\}\right\}$$

- long-short delta-neutral
- no cash upfront
- limit on leverage

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}.$

Pricing $P_{t+\tau} \equiv P(\mathbf{X}, \mathcal{I}_t)$ **?**

Optimization $\mathbf{w}^* \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \{\mathcal{S}(\mathbf{w}; f_{\mathbf{X}})\}$ **?**

call option price at horizon $P_{t+\tau} = C_{BS}\left(y_t e^{X_y}, h\left(y_t e^{X_y}, \sigma_t + X_\sigma, \kappa, T-\tau\right); \kappa, T-\tau, r\right)$

$$X_y \equiv \ln\left(y_{t+\tau}/y_t\right)$$

$$X_\sigma \equiv \sigma_{t+\tau} - \sigma_t$$

$$C_{BS}\left(y,\sigma;\kappa,T,r\right) \equiv yF\left(d_1\right) - \kappa e^{-rT}F\left(d_2\right)$$

$$d_1 \equiv \left(\ln\left(y/\kappa\right) + \left(r + \sigma^2/2\right)T\right)/\sigma\sqrt{T}, \qquad d_2 \equiv d_1 - \sigma\sqrt{T};$$

$$h\left(y,\sigma;\kappa,T\right) \equiv \sigma + a\frac{\ln\left(y/\kappa\right)}{\sqrt{T}} + b\left(\frac{\ln\left(y/\kappa\right)}{\sqrt{T}}\right)^2$$

$$\mathbf{X} \equiv \left(X^M, X^M_{1m}, X^M_{2m}, X^M_{6m}, \ldots, X^G_{6m}, X_{2y}, X_{10y}\right)'$$

simulations

$$\Pi_{\mathbf{w}} \equiv \sum_{i=1}^{I} w_i\left(C_{BS,i}\left(\mathbf{X}, \mathcal{I}_t\right) - C_{i,t}\right) \quad \Longleftrightarrow \quad \mathcal{P}_{j,i} \equiv C_{BS,i}\left(\mathcal{X}_{j,\cdot}, \mathcal{I}_t\right) - C_{i,t},$$

$$\mathbf{w}_\lambda \equiv \underset{\underline{\mathbf{b}} \leq \mathbf{B}\mathbf{w} \leq \overline{\mathbf{b}}}{\operatorname{argmax}} \left\{\mathbb{E}\left\{\Pi_{\mathbf{w}}\right\} - \lambda\, \mathrm{CVaR}_\gamma\left\{\Pi_{\mathbf{w}}\right\}\right\}$$

call option price at horizon $P_{t+\tau} = C_{BS}\left(y_t e^{X_y}, h\left(y_t e^{X_y}, \sigma_t + X_\sigma, \kappa, T-\tau\right); \kappa, T-\tau, r\right)$

$$X_y \equiv \ln\left(y_{t+\tau}/y_t\right)$$

$$X_\sigma \equiv \sigma_{t+\tau} - \sigma_t$$

$$C_{BS}\left(y,\sigma;\kappa,T,r\right) \equiv yF\left(d_1\right) - \kappa e^{-rT}F\left(d_2\right)$$

$$d_1 \equiv \left(\ln\left(y/\kappa\right) + \left(r + \sigma^2/2\right)T\right)/\sigma\sqrt{T}, \qquad d_2 \equiv d_1 - \sigma\sqrt{T};$$

$$h\left(y,\sigma;\kappa,T\right) \equiv \sigma + a\frac{\ln\left(y/\kappa\right)}{\sqrt{T}} + b\left(\frac{\ln\left(y/\kappa\right)}{\sqrt{T}}\right)^2$$

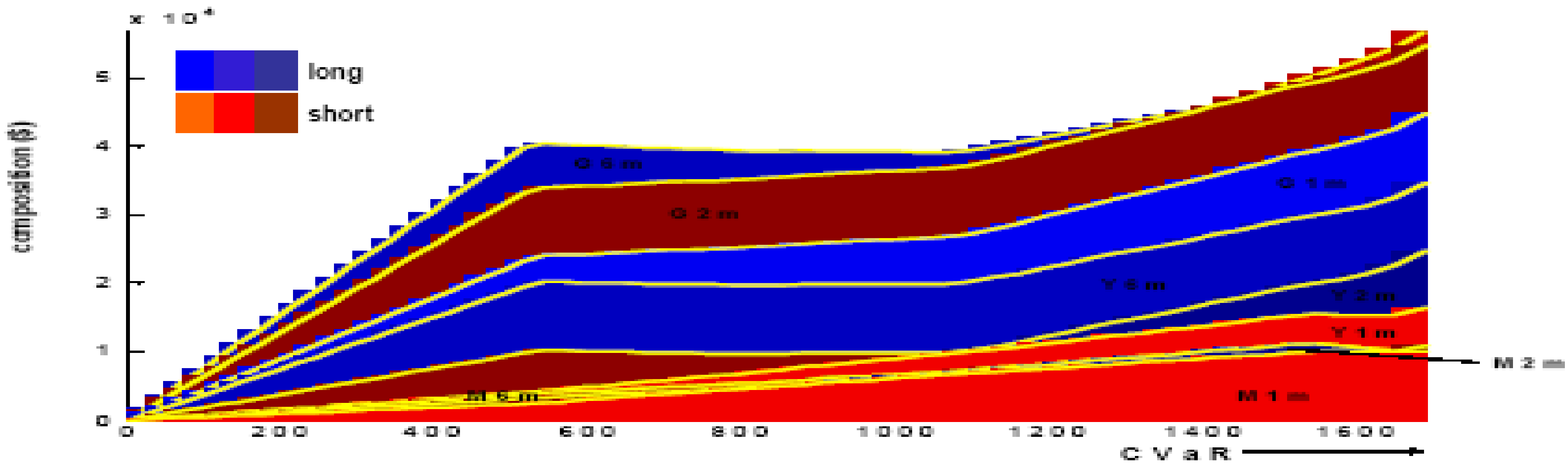
$$\mathbf{X} \equiv \left(X^M, X^M_{1m}, X^M_{2m}, X^M_{6m}, \ldots, X^G_{6m}, X_{2y}, X_{10y}\right)'$$

simulations

$$\Pi_{\mathbf{w}} \equiv \sum_{i=1}^{I} w_i\left(C_{BS,i}\left(\mathbf{X},\mathcal{I}_t\right) - C_{i,t}\right) \iff \mathcal{P}_{j,i} \equiv C_{BS,i}\left(\mathcal{X}_{j,\cdot},\mathcal{I}_t\right) - C_{i,t},$$

$$\mathbf{w}_\lambda \equiv \underset{\underline{\mathbf{b}} \leq \mathbf{B}\mathbf{w} \leq \overline{\mathbf{b}}}{\operatorname{argmax}}\left\{\mathbb{E}\left\{\Pi_{\mathbf{w}}\right\} - \lambda\,\mathrm{CVaR}_\gamma\left\{\Pi_{\mathbf{w}}\right\}\right\} \iff \textbf{linear programming}$$

Market distr.	$\mathbf{X} \sim f_{\mathbf{X}}$.	$\Longleftrightarrow$	$\mathcal{X}$ $\mathbf{p}$
Pricing	$P_{t+\tau} \equiv P\left(\mathbf{X}, \mathcal{I}_t\right)$	$\Longleftrightarrow$	$\mathcal{P}$, $\mathbf{p}$
Optimization	$\mathbf{w}^* \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \left\{\mathcal{S}\left(\mathbf{w}; f_{\mathbf{X}}\right)\right\}$	$\Longleftrightarrow$	**linear programming**



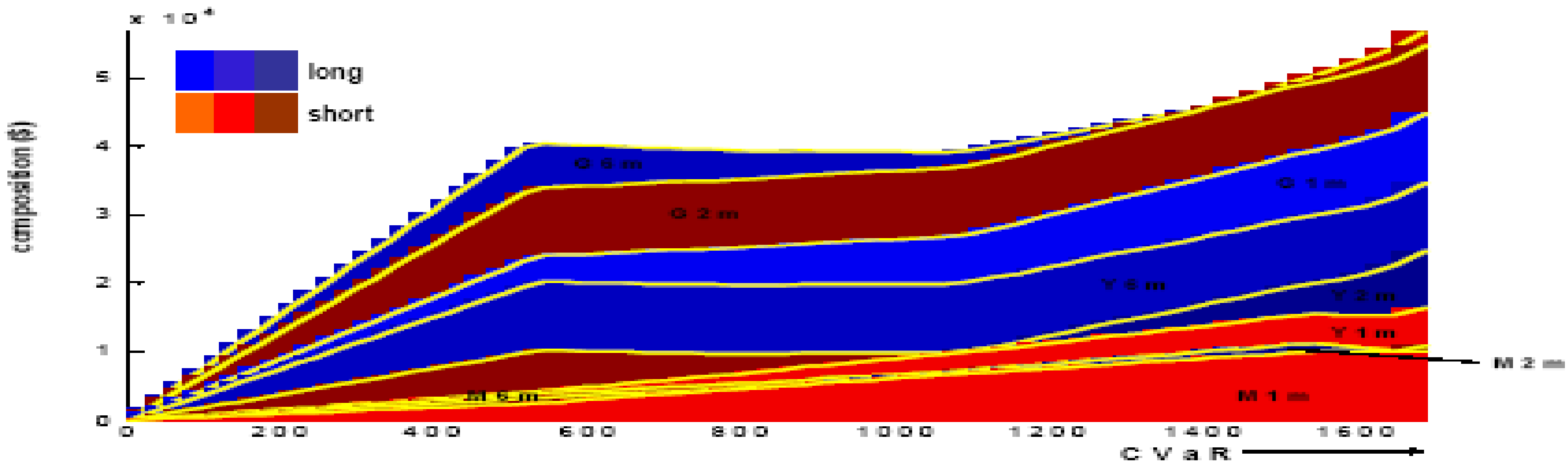
x 10^4
long
short
composition ($)
0
1
2
3
4
5
0
200
400
600
800
1000
1200
1400
1600
CVaR
G 6 m
G 2 m
G 1 m
Y 6 m
Y 2 m
Y 1 m
M 2 m
M 6 m
M 1 m

Market distr. $\mathbf{X} \sim f_{\mathbf{X}}.$ $\Longleftrightarrow$ $\mathcal{X}$ $J \times N$ panel $\quad \mathbf{p}$ probabilities $1/J$.

Focus $\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$ $\Longleftrightarrow$ $\mathcal{V}_{j,k} \equiv g_k(\mathcal{X}_{j,1}, \ldots, \mathcal{X}_{j,N})$

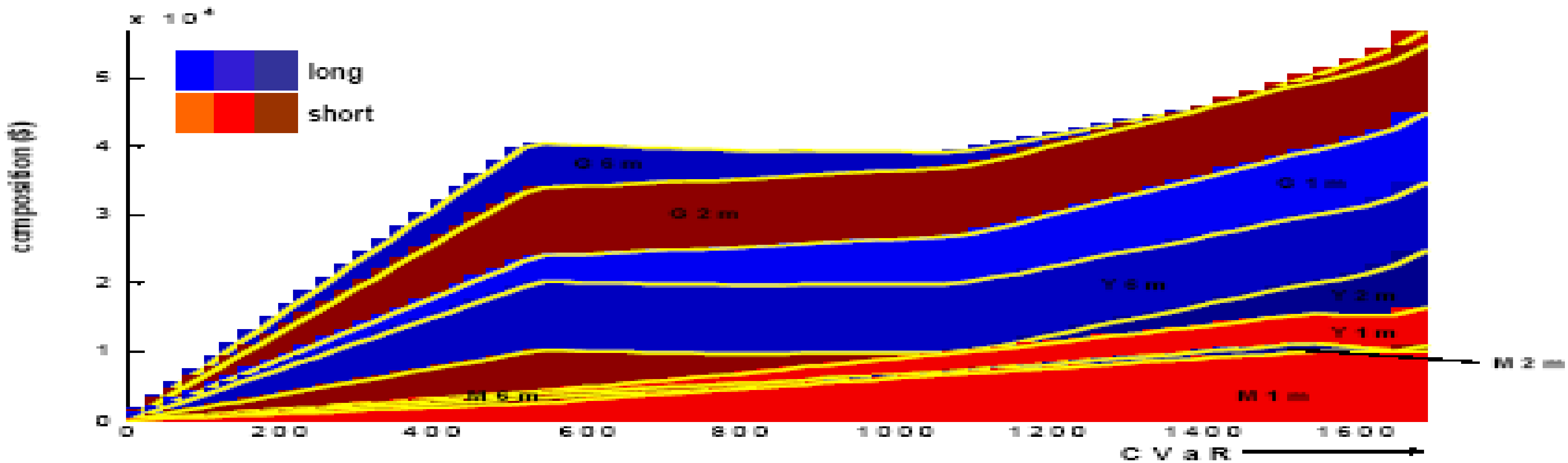
Views

$$\left.\begin{array}{l}
\mathbf{V} \sim \widetilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}. \\
\widetilde{m}\{V_k\} \gtreqqless \widetilde{\mu}_{\mathbf{v},k}; \\
\widetilde{m}\{V_1\} \geq \widetilde{m}\{V_2\} \geq \cdots \geq \widetilde{m}\{V_K\} \\
\widetilde{\sigma}\{V_k\} \gtreqqless \varkappa\sigma\{V_k\} \\
\widetilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1 \mathbf{I} + \rho_2 \mathbb{C}\{\mathbf{V}\} + \rho_3 \mathbf{1}\mathbf{1}', \\
\widetilde{Q}_V(u) \gtreqqless Q_V(u)
\end{array}\right\} \Longleftrightarrow \quad \underline{\mathbf{a}} \leq \mathbf{A}\widetilde{\mathbf{p}} \leq \overline{\mathbf{a}}.$$

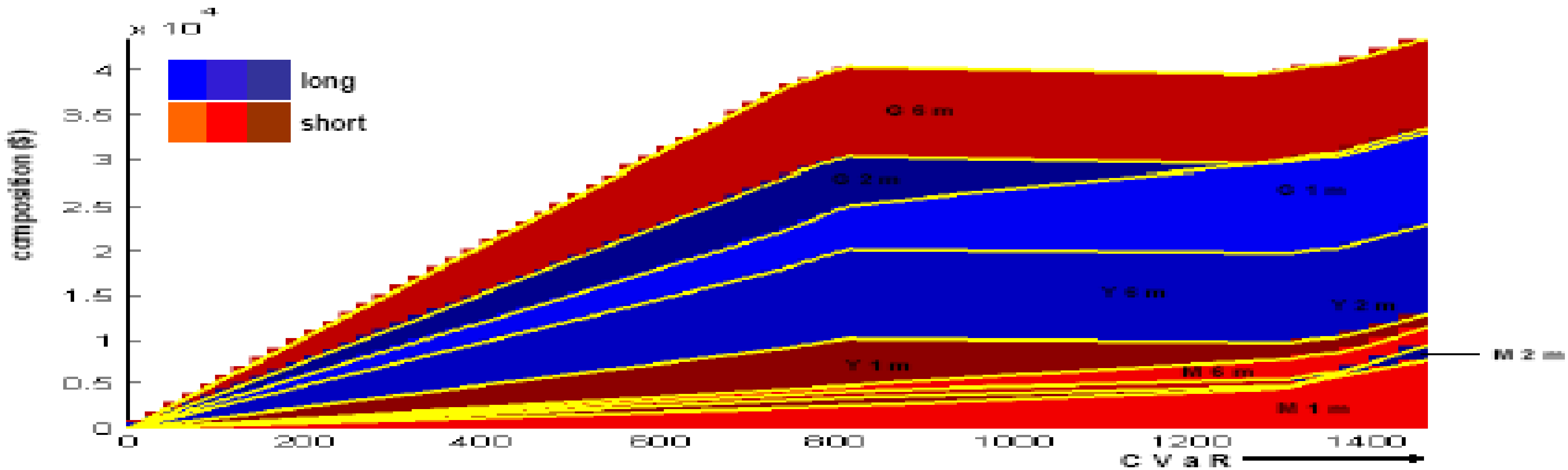


view: G 6m $\leq$ G 2m

Market distr.	$\mathbf{X} \sim f_{\mathbf{X}}.$	$\Longleftrightarrow$	$\mathcal{X}$ $J \times N$ panel $\quad \mathbf{p}$ probabilities $1/J$.
Focus	$\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim f_{\mathbf{V}}$	$\Longleftrightarrow$	$\mathcal{V}_{j,k} \equiv g_k(\mathcal{X}_{j,1}, \ldots, \mathcal{X}_{j,N})$
Views	$\mathbf{V} \sim \widetilde{f}_{\mathbf{V}} \neq f_{\mathbf{V}}.$ $\widetilde{m}\{V_k\} \gtreqless \widetilde{\mu}_{\mathbf{v},k}.$ $\widetilde{m}\{V_1\} \geq \widetilde{m}\{V_2\} \geq \cdots \geq \widetilde{m}\{V_K\}$ $\widetilde{\sigma}\{V_k\} \gtreqless \varkappa\sigma\{V_k\}$ $\widetilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1\mathbf{I} + \rho_2\mathbb{C}\{\mathbf{V}\} + \rho_3\mathbf{1}\mathbf{1}',$ $\widetilde{Q}_V(u) \gtreqless Q_V(u)$	$\Longleftrightarrow$	$\underline{\mathbf{a}} \leq \mathbf{A}\widetilde{\mathbf{p}} \leq \overline{\mathbf{a}}.$
Posterior	$\widetilde{f}_{\mathbf{X}} \equiv \underset{f \in \mathbb{V}}{\operatorname{argmin}}\{\mathcal{E}(f, f_{\mathbf{X}})\}$	$\Longleftrightarrow$	$\mathcal{X} \quad \widetilde{\mathbf{p}} \equiv \underset{\underline{\mathbf{a}} \leq \mathbf{A}\mathbf{f} \leq \overline{\mathbf{a}}}{\operatorname{argmin}}\{\mathcal{E}(\mathbf{f}, \mathbf{p})\}$
Confidence	$\widetilde{f}^{c}_{\mathbf{X}} \equiv (1-c) f_{\mathbf{X}} + c\widetilde{f}_{\mathbf{X}}.$	$\Longleftrightarrow$	$\mathcal{X} \quad \mathbf{p}_c \equiv (1-c)\mathbf{p} + c\widetilde{\mathbf{p}}.$
~~Pricing~~	$P_{t+\tau} \equiv P(\widetilde{\mathbf{X}}, \mathcal{I}_t)$	$\Longleftrightarrow$	$\boxed{\mathcal{P}} \quad \mathbf{p}_c$
Optimization	$\mathbf{w}^* \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}}\{\mathcal{S}(\mathbf{w}; \widetilde{f}^{c}_{\mathbf{X}})\}$	$\Longleftrightarrow$	...



view: G 6m $\leq$ G 2m



ESTIMATION RISK

SCENARIO ANALYSIS

THE BLACK-LITTERMAN APPROACH

ENTROPY POOLING

CASE STUDIES

REFERENCES AND CONCLUSIONS

Black, F., and R. Litterman, 1990, Asset allocation: combining investor views with market equilibrium, *Goldman Sachs Fixed Income Research*.

	BL
normal market & linear views	✓
scenario analysis	·
correlation stress-test	·
trading desk: non-linear pricing	·
external factors: macro, etc.	·
partial specifications	·
non-normal market	·
multiple users	·
non-linear views	·
trading desk: costly pricing	·
lax constraints: ranking	·

Almgren, R., and N. Chriss, 2006, Optimal portfolios from ordering information, *Journal of Risk* 9, 1–47.

Black, F., and R. Litterman, 1990, Asset allocation: combining investor views with market equilibrium, *Goldman Sachs Fixed Income Research*.

	BL	AC
normal market & linear views	✓	·
scenario analysis	·	·
correlation stress-test	·	·
trading desk: non-linear pricing	·	·
external factors: macro, etc.	·	·
partial specifications	·	·
non-normal market	·	·
multiple users	·	·
non-linear views	·	·
trading desk: costly pricing	·	·
lax constraints: ranking	·	✓

Almgren, R., and N. Chriss, 2006, Optimal portfolios from ordering information, *Journal of Risk* 9, 1–47.

Black, F., and R. Litterman, 1990, Asset allocation: combining investor views with market equilibrium, *Goldman Sachs Fixed Income Research*.

Qian, E., and S. Gorman, 2001, Conditional distribution in portfolio theory, *Financial Analyst Journal* 57, 44–51.

	BL	AC	QG
normal market & linear views	✓	·	✓
scenario analysis	·	·	✓
correlation stress-test	·	·	✓
trading desk: non-linear pricing	·	·	·
external factors: macro, etc.	·	·	·
partial specifications	·	·	·
non-normal market	·	·	·
multiple users	·	·	·
non-linear views	·	·	·
trading desk: costly pricing	·	·	·
lax constraints: ranking	·	✓	·

Almgren, R., and N. Chriss, 2006, Optimal portfolios from ordering information, *Journal of Risk* 9, 1–47.

Black, F., and R. Litterman, 1990, Asset allocation: combining investor views with market equilibrium, *Goldman Sachs Fixed Income Research*.

Pezier, J., 2007, Global portfolio optimization revisited: A least discrimination alternantive to Black-Litterman, *ICMA Centre Discussion Papers in Finance*.

Qian, E., and S. Gorman, 2001, Conditional distribution in portfolio theory, *Financial Analyst Journal* 57, 44–51.

	BL	AC	QG	P
normal market & linear views	✓	·	✓	✓
scenario analysis	·	·	✓	✓
correlation stress-test	·	·	✓	✓
trading desk: non-linear pricing	·	·	·	·
external factors: macro, etc.	·	·	·	·
partial specifications	·	·	·	✓
non-normal market	·	·	·	·
multiple users	·	·	·	·
non-linear views	·	·	·	·
trading desk: costly pricing	·	·	·	·
lax constraints: ranking	·	✓	·	·

References

Almgren, R., and N. Chriss, 2006, Optimal portfolios from ordering information, *Journal of Risk* 9, 1–47.

Black, F., and R. Litterman, 1990, Asset allocation: combining investor views with market equilibrium, *Goldman Sachs Fixed Income Research*.

Meucci, A.,8b, Enhancing the Black-Litterman and related approaches: Views and stress-test on risk factors, *Working Paper* Available at symmys.com > Reasearch > Working Papers.

Pezier, J., 2007, Global portfolio optimization revisited: A least discrimination alternantive to Black-Litterman, *ICMA Centre Discussion Papers in Finance*.

Qian, E., and S. Gorman, 2001, Conditional distribution in portfolio theory, *Financial Analyst Journal* 57, 44–51.

	BL	AC	QG	P	M
normal market & linear views	✓	·	✓	✓	✓
scenario analysis	·	·	✓	✓	✓
correlation stress-test	·	·	✓	✓	✓
trading desk: non-linear pricing	·	·	·	·	✓
external factors: macro, etc.	·	·	·	·	✓
partial specifications	·	·	·	✓	·
non-normal market	·	·	·	·	·
multiple users	·	·	·	·	·
non-linear views	·	·	·	·	·
trading desk: costly pricing	·	·	·	·	·
lax constraints: ranking	·	✓	·	·	·

References

Almgren, R., and N. Chriss, 2006, Optimal portfolios from ordering information, *Journal of Risk* 9, 1–47.

Black, F., and R. Litterman, 1990, Asset allocation: combining investor views with market equilibrium, *Goldman Sachs Fixed Income Research*.

Meucci, A., 2006, Beyond Black-Litterman in practice: A five-step recipe to input views on non-normal markets, *Risk* 19, 114–119.

——— , 2008b, Enhancing the Black-Litterman and related approaches: Views and stress-test on risk factors, *Working Paper* Available at symmys.com > Reasearch > Working Papers.

Pezier, J., 2007, Global portfolio optimization revisited: A least discrimination alternantive to Black-Litterman, *ICMA Centre Discussion Papers in Finance*.

Qian, E., and S. Gorman, 2001, Conditional distribution in portfolio theory, *Financial Analyst Journal* 57, 44–51.

	BL	AC	QG	P	M	COP
normal market & linear views	✓	·	✓	✓	✓	✓
scenario analysis	·	·	✓	✓	✓	✓
correlation stress-test	·	·	✓	✓	✓	·
trading desk: non-linear pricing	·	·	·	·	✓	✓
external factors: macro, etc.	·	·	·	·	✓	✓
partial specifications	·	·	·	✓	·	·
non-normal market	·	·	·	·	·	✓
multiple users	·	·	·	·	·	✓
non-linear views	·	·	·	·	·	·
trading desk: costly pricing	·	·	·	·	·	·
lax constraints: ranking	·	✓	·	·	·	·

Meucci (2008)
http://symmys.com/node/158

Entropy Pooling

$$\widetilde{f}_{\mathbf{X}} = \underset{f \in \mathcal{V}}{\operatorname{argmin}} \, \mathcal{E}\left(f, f_{\mathbf{X}}\right)$$

	BL	AC	QG	P	M	COP	EP
normal market & linear views	✓	·	✓	✓	✓	✓	✓
scenario analysis	·	·	✓	✓	✓	✓	✓
correlation stress-test	·	·	✓	✓	✓	·	✓
trading desk: non-linear pricing	·	·	·	·	✓	✓	✓
external factors: macro, etc.	·	·	·	·	✓	✓	✓
partial specifications	·	·	·	✓	·	·	✓
non-normal market	·	·	·	·	·	✓	✓
multiple users	·	·	·	·	·	✓	✓
non-linear views	·	·	·	·	·	·	✓
trading desk: costly pricing	·	·	·	·	·	·	✓
lax constraints: ranking	·	✓	·	·	·	·	✓

Meucci (2008)
http://symmys.com/node/158

Entropy Pooling

$$\widetilde{f}_{\mathbf{X}} = \underset{f \in \mathcal{V}}{\operatorname{argmin}} \, \mathcal{E}\left(f, f_{\mathbf{X}}\right)$$

Parametric Entropy Pooling

$$f_{\widetilde{\boldsymbol{\theta}}} = \underset{\substack{\boldsymbol{\vartheta} \in \boldsymbol{\Theta} \\ f_{\boldsymbol{\vartheta}} \in \mathcal{V}}}{\operatorname{argmin}} \, \mathcal{E}\left(f_{\boldsymbol{\vartheta}}, f_{\boldsymbol{\theta}}\right)$$

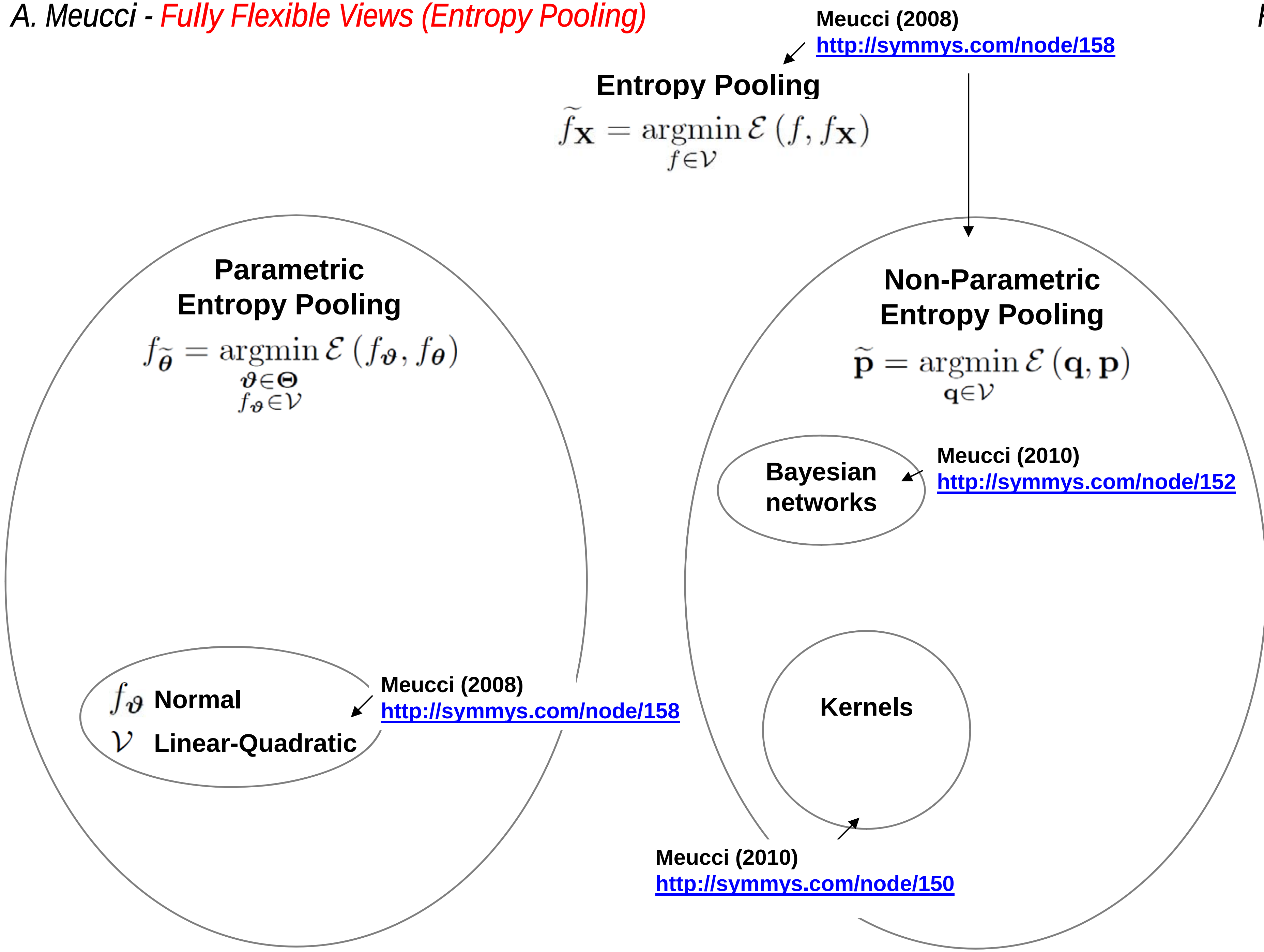
$f_{\boldsymbol{\vartheta}}$ **Normal**

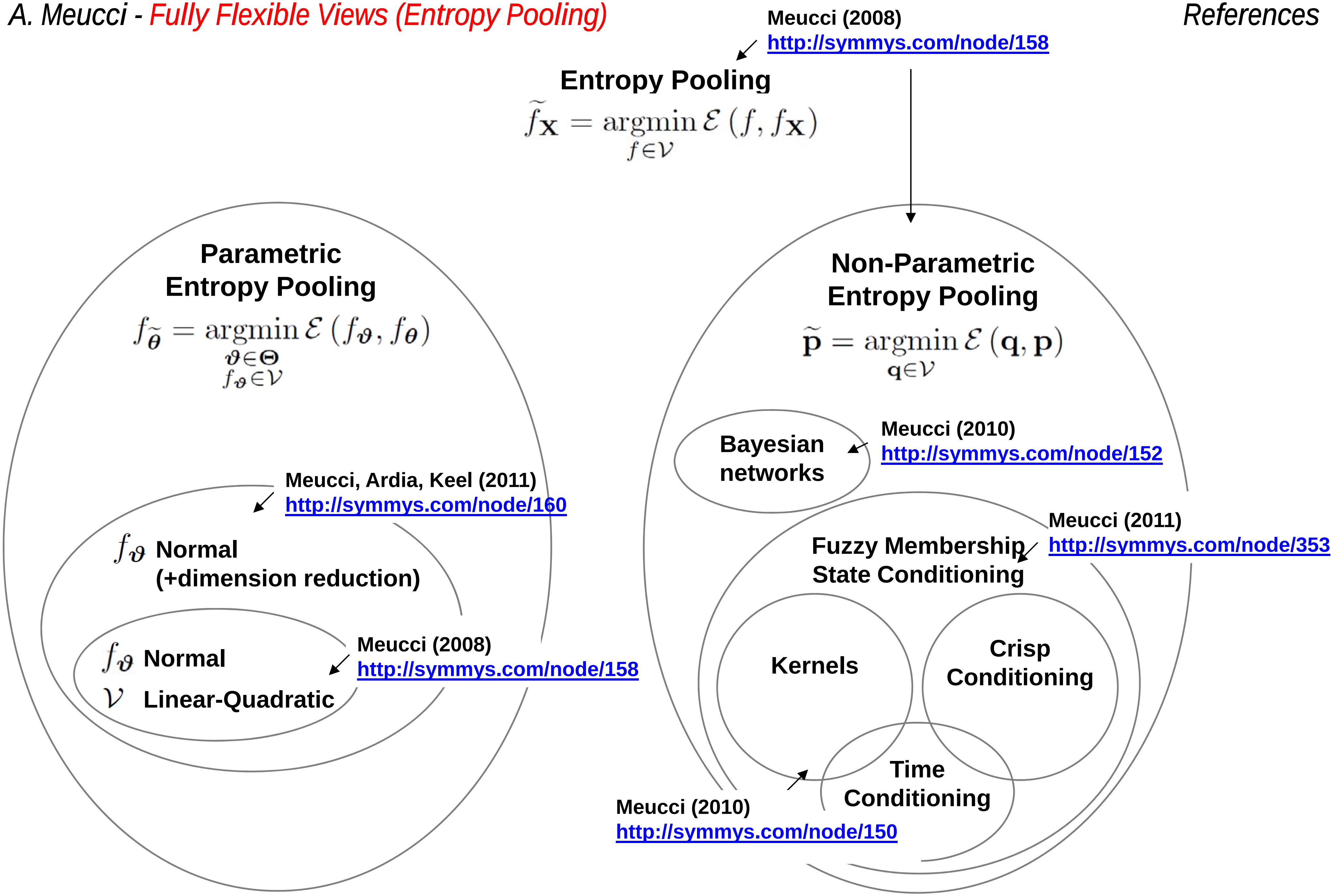
$\mathcal{V}$ **Linear-Quadratic**

Meucci (2008)
http://symmys.com/node/158

Non-Parametric Entropy Pooling

$$\widetilde{\mathbf{p}} = \underset{\mathbf{q} \in \mathcal{V}}{\operatorname{argmin}} \, \mathcal{E}\left(\mathbf{q}, \mathbf{p}\right)$$


Meucci (2008)
http://symmys.com/node/158
Entropy Pooling
$\widetilde{f}_{\mathbf{X}} = \underset{f \in \mathcal{V}}{\operatorname{argmin}}\, \mathcal{E}\left(f, f_{\mathbf{X}}\right)$
Parametric Entropy Pooling
$f_{\widetilde{\boldsymbol{\theta}}} = \underset{\substack{\boldsymbol{\vartheta} \in \boldsymbol{\Theta} \\ f_{\boldsymbol{\vartheta}} \in \mathcal{V}}}{\operatorname{argmin}}\, \mathcal{E}\left(f_{\boldsymbol{\vartheta}}, f_{\boldsymbol{\theta}}\right)$
$f_{\boldsymbol{\vartheta}}$ Normal
$\mathcal{V}$ Linear-Quadratic
Meucci (2008)
http://symmys.com/node/158
Non-Parametric Entropy Pooling
$\widetilde{\mathbf{p}} = \underset{\mathbf{q} \in \mathcal{V}}{\operatorname{argmin}}\, \mathcal{E}\left(\mathbf{q}, \mathbf{p}\right)$
Bayesian networks
Meucci (2010)
http://symmys.com/node/152
Kernels
Meucci (2010)
http://symmys.com/node/150


Meucci (2008)
http://symmys.com/node/158
Entropy Pooling
$\tilde{f}_{\mathbf{X}} = \underset{f \in \mathcal{V}}{\operatorname{argmin}} \, \mathcal{E}\left(f, f_{\mathbf{X}}\right)$
Parametric Entropy Pooling
$f_{\tilde{\boldsymbol{\theta}}} = \underset{\substack{\boldsymbol{\vartheta} \in \boldsymbol{\Theta} \\ f_{\boldsymbol{\vartheta}} \in \mathcal{V}}}{\operatorname{argmin}} \, \mathcal{E}\left(f_{\boldsymbol{\vartheta}}, f_{\boldsymbol{\theta}}\right)$
Meucci, Ardia, Keel (2011)
http://symmys.com/node/160
$f_{\boldsymbol{\vartheta}}$ Normal
(+dimension reduction)
$f_{\boldsymbol{\vartheta}}$ Normal
$\mathcal{V}$ Linear-Quadratic
Meucci (2008)
http://symmys.com/node/158
Non-Parametric Entropy Pooling
$\tilde{\mathbf{p}} = \underset{\mathbf{q} \in \mathcal{V}}{\operatorname{argmin}} \, \mathcal{E}\left(\mathbf{q}, \mathbf{p}\right)$
Bayesian networks
Meucci (2010)
http://symmys.com/node/152
Meucci (2011)
http://symmys.com/node/353
Fuzzy Membership State Conditioning
Kernels
Crisp Conditioning
Time Conditioning
Meucci (2010)
http://symmys.com/node/150

Entropy Pooling:

✓ **Market represented by generic non-linear risk factors, <u>not only returns</u>**

✓ **Market distribution fully general, <u>not only normal</u>**

✓ **Views/stress-testing on any function of the market, <u>not only linear portfolios</u>**

✓ **Views on <u>any feature</u>, not only on expectations: median, volatility, correlations, tails**

✓ **Views are equalities and <u>inequalities</u>: ranking is possible**

✓**Applications span several areas of <u>portfolio management</u> and <u>risk management</u>**