Fully Flexible Views: Theory and Practice

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Summary

- Non-linear views in non-linear markets

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Important Contributions

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- No repricing in the numerical approach
- High dimensional problems

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Relative entropy

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- KL belongs to the Ali-Silvey class of information theoretic distance measures

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- $f(x) = x$, $c(x) = x \log x \quad \Rightarrow$ KL

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- Appropriate for opinion pooling ?

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Consistency Requirement

It would be reasonable to observe

$$f(x) \xrightarrow{\quad G \quad} \widetilde{f}(x)$$

$$\widetilde{f}(x) \xrightarrow{\quad G^{-1} \quad} f(x)$$

G is a view on the distribution: increase the variance by 1 %

G^{-1} is a view on the distribution: decrease the variance by 1 %

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A simple example: non linear transformation

- Start with an initial distribution P_0 of the random variable X

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$$E^{P\in\mathbb{V}}\left[(X - E^{P\in\mathbb{V}}(X))^2\right] = E^{P_0}\left[(X - E^{P_0}(X))^2\right] + 1\%$$

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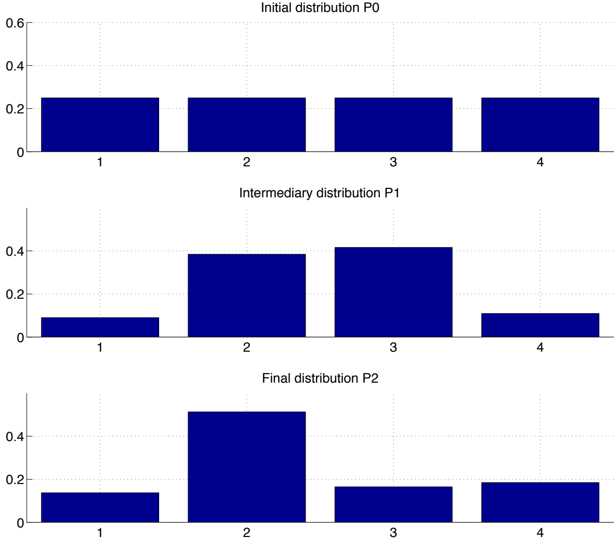
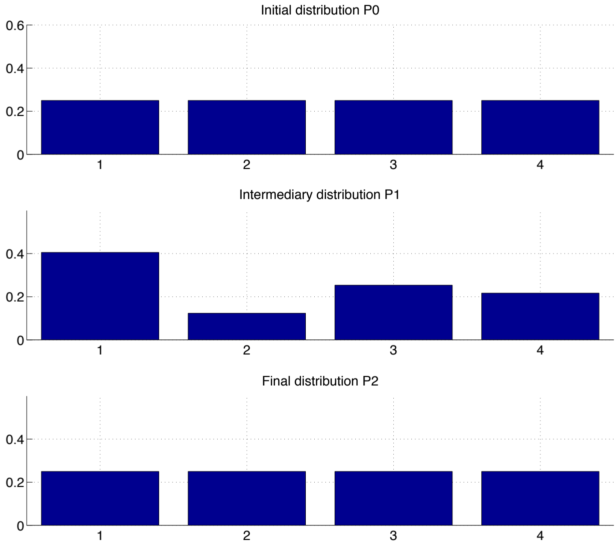
A simple example: non linear transformation



A simple(r) example: linear transformation



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