Risk Management in the Energy Markets

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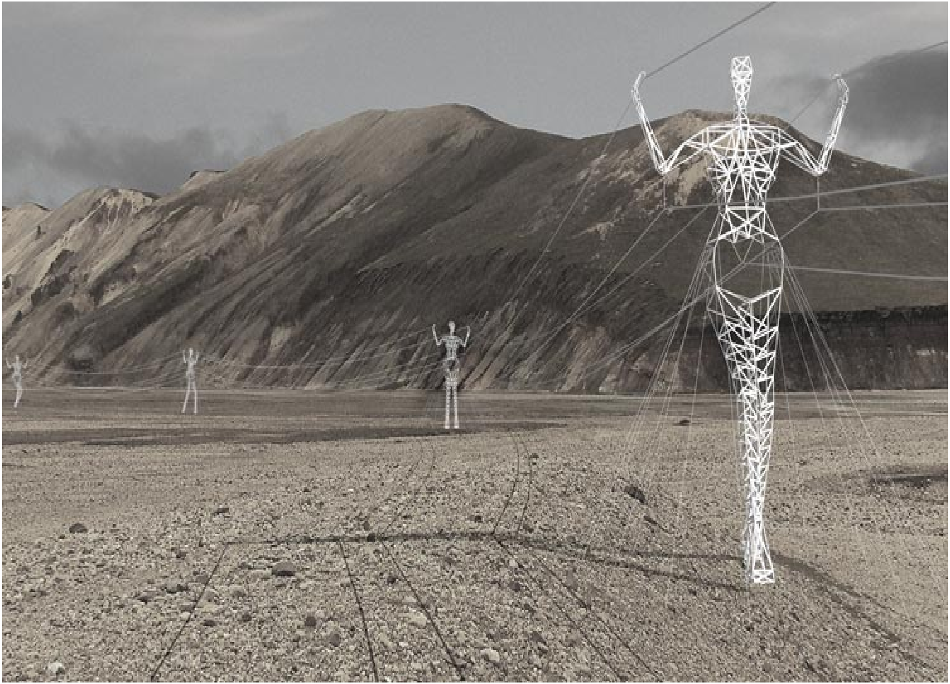
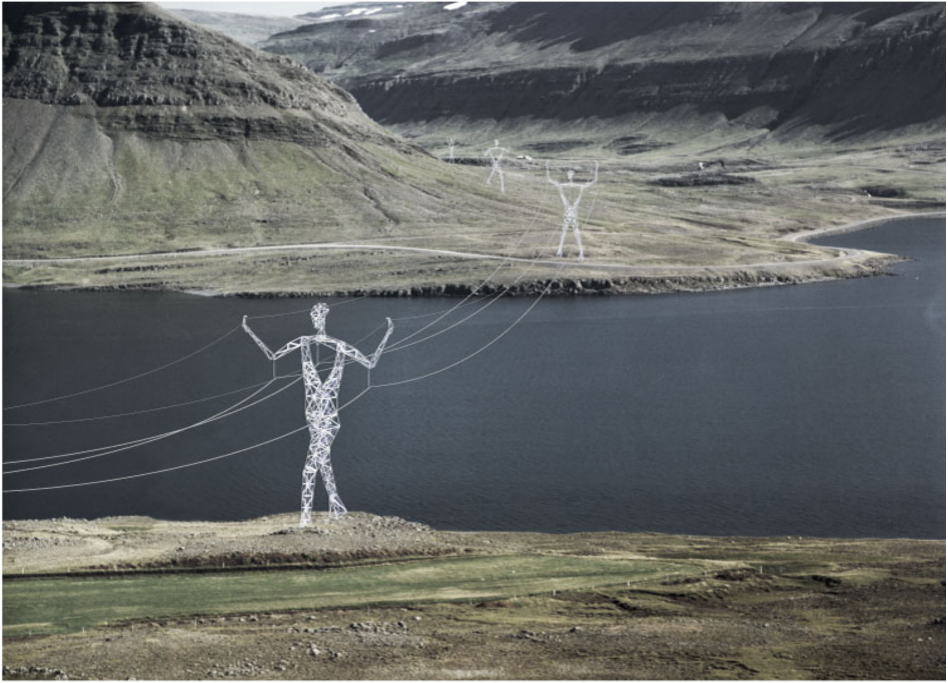
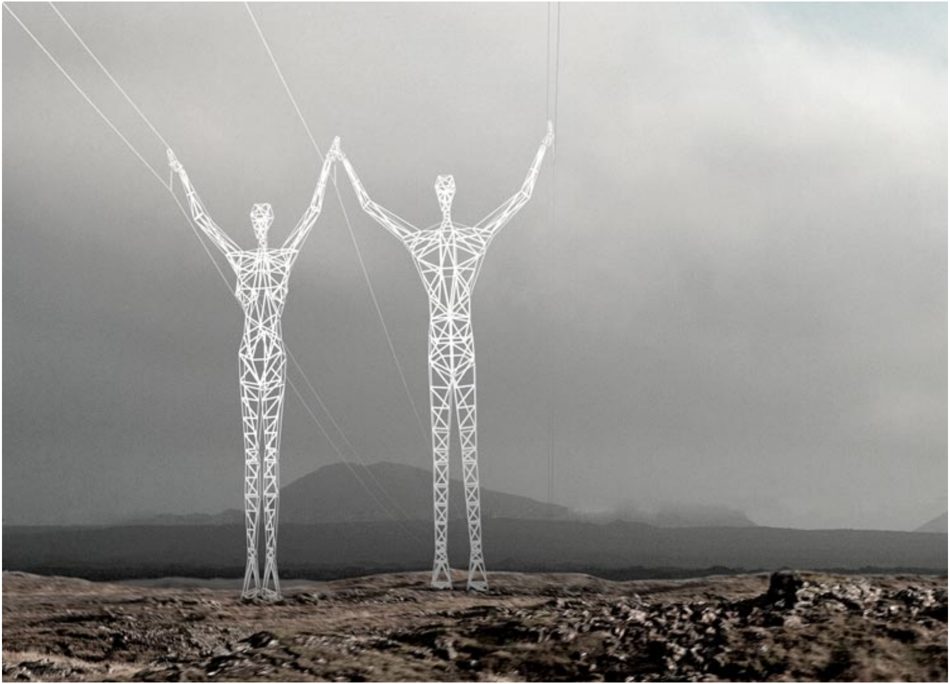






Talk Based on Three Papers

1. **R.C., and Y. Sun**: Implied and Local Correlations from Spread Options (July 2011)
2. **R.C., M. Coulon, and D. Schwarz**: A Structural Model for Electricity Prices (Oct. 2011)
3. **R.C., and J. Hinz**: Least Squares Monte Carlo for Control Problems with Convex Value Functions (Oct. 2011)

Commodities Forward Markets

- Forward curve is the **Basic Data**
- **Backwardation / Contango** $\Longrightarrow$ Theory of **Convenience Yield**

In the Case of Power several obstructions

- Cannot store the physical commodity
- Delivery **over** a preiod $[T_1, T_2]$ (**Benth**)
- Which spot price? Real time? Day-ahead? Balance-of-the-week? month? on-peak? off-peak? etc
- Does the forward price converge as the time to maturity goes to 0?

Mathematical spot?

$$S_t = \lim_{T \downarrow t} F(t, T)$$

Sparse Forward Data

- Lack of **transparency** (manipulated indexes)
- Poor (or lack of) **reporting** by fear of law suits (**CCRO** white paper)

Models for Electricity Spot Price

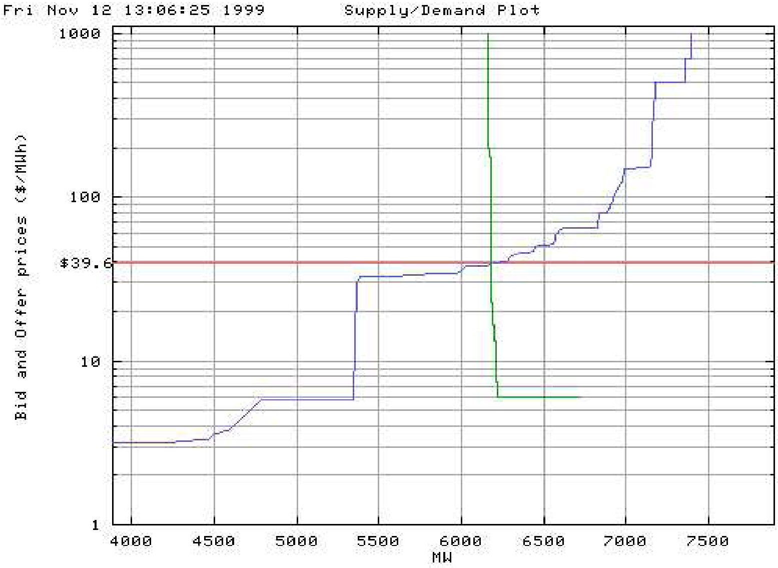
- **Mean Reversion** toward the cost of production
- **Reduced Form Models**
  - Nonlinear effects (exponential OU^2)
  - Jumps (**Geman-Roncoroni**, **Benth**, **Cartea**, **Meyer-Brandis**, ...)
- **Structural Models**
  - Inelastic Demand
  - The Supply Stack

Barlow (based on **merit** order graph)

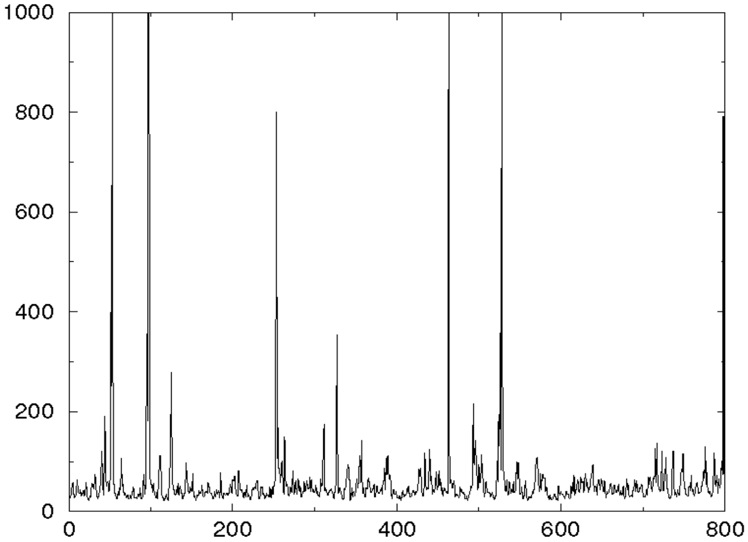
- $s_t(x)$ supply at time t when power price is x
- $d_t(x)$ demand at time t when power price is x

Power price at time t is number P_t such that

$$s_t(P_t) = d_t(P_t)$$



Example of a merit graph (Alberta Power Pool, courtesy M. Barlow)



Monte Carlo Sample from Barlow's Spot Model (courtesy M. Barlow)

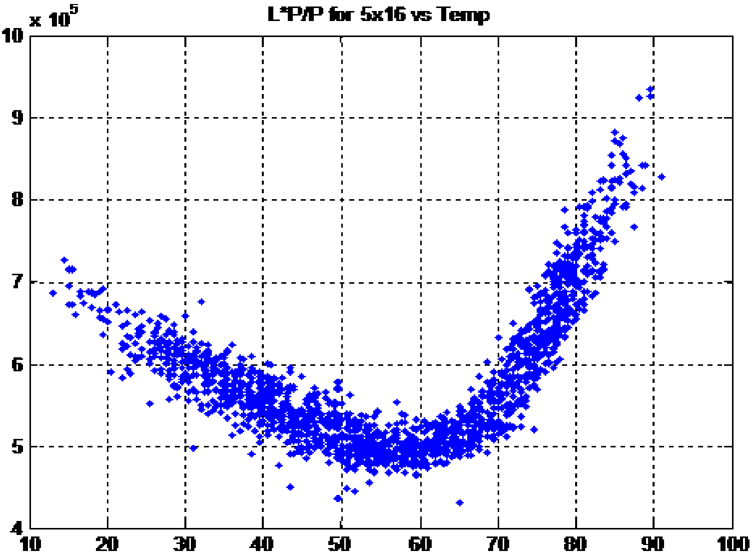
Still Another Issue: Negative Prices

Consider the case of **PJM**

(Pennsylvania - New Jersey - Maryland)

- Over $3,000$ nodes in the transmission network
- Each day, and for each node
  - Real time prices
  - Day-ahead prices
  - Hour by hour load prediction for the following day
- **Historical prices**
- In 2003 over $100,000$ instances of **NEGATIVE PRICES**
  - Geographic clusters
  - Time of the year (**shoulder months**)
  - Time of the day (**night**)
- **Possible Explanations**
  - Load miss-predicted
  - High temperature volatility

Modeling the Demand: Load / Temperature



Daily Load versus Daily Temperature (PJM)

More Structural Models for Power

Alternatives to **reduced-form** and **equilibrium models**

- Choice of **Factors**: Demand, Fuel Prices, Outages, etc.
- Choice of **Function**: $P_t = B(t, D_t, G_t, \ldots)$ to map to spot power.
- **Calibration**

Examples:

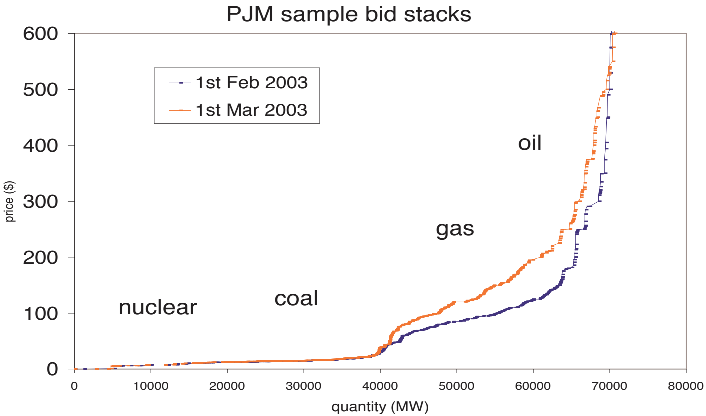
- **Barlow** (2002): $P_t = B(D_t) = (1 + \alpha_c D_t)^{1/\alpha_c}$
- **Burger et al.** (2004), **Cartea et al.** (2007): $P_t = B(t, D_t, \xi_t)$
- **Pirrong- Jermakyan** (2005): $P_t = B(D_t, G_t) = G_t f(D_t)$

Others include: **Eydeland & Wolyniec** (2003), **Davison et. al.** (2002), **Cartea-Figueroa-Geman** (2009), **Aid et. al.** (2009, 2011)

Challenge: Overlapping fuels in many markets!

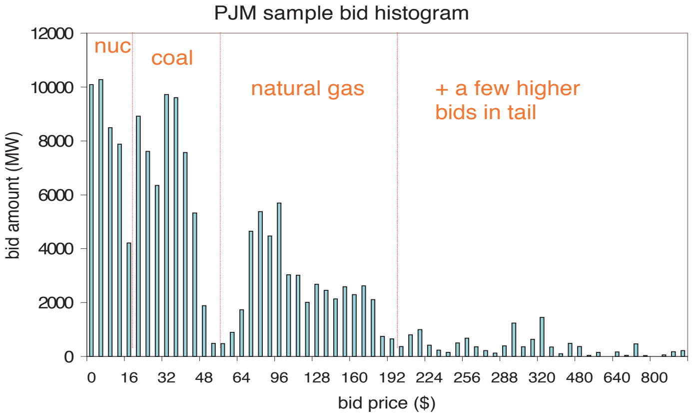
THE BID STACK FUNCTION

- Day-ahead generator bids arranged by price to form the **bid stack**
- Spot price P_t is highest bid needed to match demand D_t



An alternative perspective

- Can look at bid stack as a histogram of bids
- Merit order is often visible through clusters of bids



Distribution-based Bid Stack Model

Coulon-Howison (2009)

- Fuel types $i = 1, \ldots, N$
- $F_1(x), \ldots, F_N(x)$ proportions of bids below x
- Weights $w_1, \ldots, w_N$ (observable percentage of total capacity $\bar{\xi}$ in the market).
- Assume $0 < D_t < \bar{\xi}$. (demand cannot exceed max capacity)
- Then the spot power price P_t solves:

$$\sum_{i=1}^{N} w_i F_i(P_t) = D_t / \bar{\xi}$$

- The bid stack function is the quantile function of the distribution of bids.
- Extensions
  - $\bar{\xi}$ replaced by a process ξ_t for capacity available, or alternatively $\xi_t = D_t + M_t$ where M_t is reserve margin available.
  - Two-parameter distributions for bids (location m_i, scale s_i) such as Gaussian, Logistic, Cauchy, Weibull.

Challenge: Closed-form!

R.C - M.Coulon - D. Schwarz (2011)

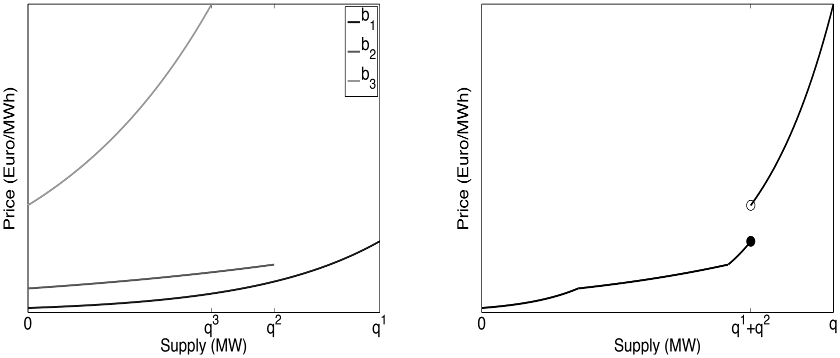
- **Fact**: Multi-fuel case: no explicit expressions even for spot or forward.
- **Alternative**: Exchange *flexibility* in the stack for *closed-form* expressions for forwards, options, etc.
- **Key assumption**: within each fuel type, heat rate differences lead to *exponential* bid stacks. (multiplicative in fuel price)
  - Example: Two Fuel Case (coal and natural gas)
    - Capacities $\bar{\xi}^c$ and $\bar{\xi}^g$.
    - Aggregation of bids to get **'sub bid stacks'**:

$$b_c(D) = C_t e^{k_c + m_c D}, \text{ for } 0 \leq D \leq \bar{\xi}^c$$

and:

$$b_g(D) = G_t e^{k_g + m_g D}, \text{ for } 0 \leq D \leq \bar{\xi}^g$$

EXPONENTIAL 'SUB BID STACKS'



A schematic of individual fuel bid curves and the resulting market bid stack for $I := \{1, 2, 3\}$, $q := \bar{\xi}$. Fuel bid curves b_i (left), Market bid stack b (right)

Exponential 'sub bid stacks'

- The total market bid stack (as a function of demand) is given by:

$$B(D) = (b_c^{-1} + b_g^{-1})^{-1}(D), \qquad \text{for } 0 \leq D \leq \bar{\xi} = \bar{\xi}^c + \bar{\xi}^g$$

- Hence, the result is **piecewise exponential**, although the precise form depends on ordering of start and endpoints of coal and gas stacks.
- For example, if $C_t e^{k_c} < G_t e^{k_g} < C_t e^{k_c + m_c \bar{\xi}_c} < G_t e^{k_g + m_g \bar{\xi}_g}$ (coal below gas but some **overlap**), then spot price P_t has three regions:

$$P_t(D, C_t, G_t) = \begin{cases} b_c(D) = C_t e^{k_c + m_c D} & \text{for } 0 \leq D \leq D_1 \\ C_t^{\alpha_c} G_t^{\alpha_g} e^{\beta + \gamma D} & \text{for } D_1 \leq D \leq D_2 \\ b_g(D - \bar{\xi}^c) = G_t e^{k_g + m_g (D - \bar{\xi}^c)} & \text{for } D_2 \leq D \leq \bar{\xi} \end{cases}$$

$$\alpha_c = \frac{m_g}{m_c + m_g}, \quad \alpha_g = 1 - \alpha_c, \quad \beta = \frac{k_c m_g + k_g m_c}{m_c + m_g}, \quad \gamma = \frac{m_c m_g}{m_c + m_g},$$

and with $D_1 = \frac{1}{m_c}(\log(G_t / C_t) + k_g - k_c)$, and D_2 similar.

EXPONENTIAL 'SUB BID STACKS' (CONT)

- Power spot price P_t must be given by one of the following five expressions:

P_t	Criteria	Description
$b_c(D) = C_t e^{k_c + m_c D}$	$b_c(D) < b_g(0)$	Coal sets price; No gas used
$b_g(D) = G_t e^{k_g + m_g D}$	$b_g(D) < b_c(0)$	Gas sets price; No coal used
$b_c(D - \bar{\xi}^g) = C_t e^{k_c + m_c (D - \bar{\xi}^g)}$	$b_g(\bar{\xi}^g) < b_c(D - \bar{\xi}^g)$	Coal sets price; All gas used
$b_g(D - \bar{\xi}^c) = G_t e^{k_g + m_g (D - \bar{\xi}^c)}$	$b_c(\bar{\xi}^c) < b_g(D - \bar{\xi}^c)$	Gas sets price; All coal used
$b_{cg}(D) = C_t^{\alpha_c} G_t^{\alpha_g} e^{\beta + \gamma D}$	otherwise	Both set price (overlap region)

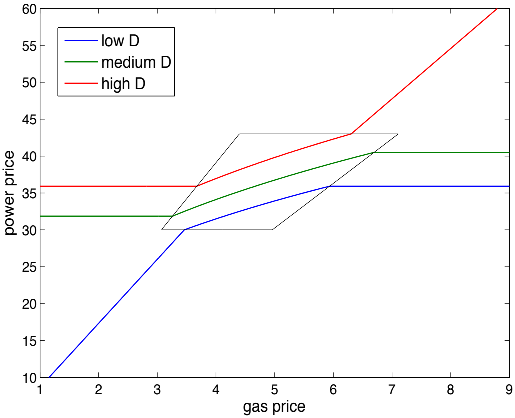
- Note that this can be extended easily to more than two fuels (and still piecewise exponential in D). However, for n fuels, number of cases is

$$\sum_{i=1}^{n} \binom{n}{i} \left[\sum_{j=0}^{n-i} \binom{n-i}{j} \right].$$

(For $n = 3$, we have 19 cases, for $n = 6$, we have 665 cases!)

Exponential 'sub bid stacks'

Alternatively, depicting power price P_t as a function of G_t (or similarly C_t) leads to three different demand 'regimes' (Case of $\bar{\xi}_c > \bar{\xi}_g$ plotted below):



High Demand: $D > \bar{\xi}_c$
(i.e., $D > \max(\bar{\xi}_c, \bar{\xi}_g)$)

Medium Demand:
$\bar{\xi}_g < D < \bar{\xi}_c$

Low Demand: $D < \bar{\xi}_g$
(i.e., $D < \min(\bar{\xi}_c, \bar{\xi}_g)$)

Quadrilateral in middle of plot represents region of coal and gas price overlap (ie, both generators at margin, setting price).

Exponential Stacks - Spot Prices

Summary

- **three** regimes for demand (low, medium, high)
  - **three** cases (fuel price dependent) for each regime.
- For each regime, spot prices have a convenient form, e.g. for low D,

$$P_t^{low} = C_t e^{\lambda^c(D)} \mathbb{I}_{\{G_t > C_t e^{\lambda^c(D) - \lambda^g(0)}\}} + G_t e^{\lambda^g(D)} \mathbb{I}_{\{G_t < C_t e^{\lambda^c(0) - \lambda^g(D)}\}} + (C_t)^{\alpha_c} (G_t)^{\alpha_g} e^{\beta + \gamma D} \mathbb{I}_{\{C_t e^{\lambda^c(0) - \lambda^g(D)} < G_t < C_t e^{\lambda^c(D) - \lambda^g(0)}\}},$$

  where $\lambda^c(x) = k_i + m_i x$ for $i \in \{c, g\}$ and $x \in [0, \bar{\xi}^i]$.
- **Log-normal** case (at fixed maturity) T,

$$\begin{pmatrix} \log C_T \\ \log G_T \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_c \\ \mu_g \end{pmatrix}, \begin{pmatrix} \sigma_c^2 & \rho\sigma_c\sigma_g \\ \rho\sigma_c\sigma_g & \sigma_g^2 \end{pmatrix} \right)$$

Exponential Stacks - Forward Prices

For **FIXED** demand D, need formula for:

$$\mathbb{E}^{\mathbb{Q}}\left[\tilde{a}_0 C_t^{\tilde{a}_1} G_t^{\tilde{a}_2} \mathbb{I}_{\{\tilde{b}_0 C_t^{\tilde{b}_1} G_t^{\tilde{b}_2} < 1\}}\right] = \mathbb{E}\left[\left(e^{a_0 + a_1 X + a_2 Y}\right) \mathbb{I}_{\{b_0 + b_1 X + b_2 Y < 0\}}\right]$$

for (correlated) jointly Gaussians X and Y.

- We use:

$$\int_{-\infty}^{\infty} e^{cx} \Phi\left(\frac{a + bx}{d}\right) \frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} dx = e^{\frac{1}{2}c^2} \Phi\left(\frac{a + bc}{\sqrt{b^2 + d^2}}\right)$$

- and

$$\int_{-\infty}^{h} e^{cx} \Phi\left(\frac{a + bx}{d}\right) \frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} dx = e^{\frac{1}{2}c^2} \Phi_2\left(h - c, \frac{a + bc}{\sqrt{b^2 + d^2}}; \frac{-b}{\sqrt{b^2 + d^2}}\right)$$

where $\Phi(z)$ and $\Phi_2(z_1, z_2, \rho_{12})$ are the univariate and bivariate standard Gaussian cdf.

Exponential Stacks - Forward Prices

- Example: **low D regime**

$$
\begin{aligned}
F_t^{low} \quad = \quad & b_c\left(D, F_t^c\right)\Phi\left(\frac{R_c(D,0)}{\sigma}\right) + b_g\left(D, F_t^g\right)\Phi\left(\frac{R_g(D,0)}{\sigma}\right) + \\
& b_{cg}(D, F_t^c, F_t^g)e^{-\frac{1}{2}\alpha_c\alpha_g\sigma^2}\left[1 - \sum_{i\in I}\Phi\left(\frac{R_i(D,0)+\alpha_j\sigma^2}{\sigma}\right)\right],
\end{aligned}
$$

where $I = \{c, g\}$, $j = I \setminus \{i\}$ and

$$\sigma^2 = \sigma_c^2 - 2\rho\sigma_c\sigma_g + \sigma_g^2,$$

$$R_i\left(\xi_i, \xi_j\right) = k_j + m_j\xi_j - k_i - m_i\xi_i + \log(F_t^j) - \log(F_t^i) - \frac{1}{2}\sigma^2.$$

- Similar expressions exist for F_t^{mid} and F_t^{high}, the other regions.
- D enters **linearly** inside cdf's $\Phi(\cdot)$ and in **exponential** outside.

Exponential Stacks - Random Demand

- Demand D is random but **independent** (of fuels). Then integrate (or sum) over demand distribution $f(x)$:

$$F_t^T = \int_0^{\bar{\xi}^g} F_t^{low}(x) f(x) dx + \int_{\bar{\xi}^g}^{\bar{\xi}^c} F_t^{med}(x) f(x) dx + \int_{\bar{\xi}^c}^{\bar{\xi}} F_t^{high}(x) f(x) dx$$

- Example: **Capped Gaussian** demand:

$$D_t = \max\left(0, \min(\bar{\xi}, \tilde{D}_t)\right) \quad \text{where } \tilde{D} \sim N(\mu_z, \sigma_z^2)$$

- Again, closed form formulae for F_t^T (though rather involved)
- Using notation:

$$\Phi_2^{2\times 1}\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, y; \rho\right) = \Phi_2(x_1, y; \rho) - \Phi_2(x_2, y; \rho)$$

Random Demand - Forward Prices

T-Forward power price at time $t < T$

$$
\begin{aligned}
F_t^p(T) = {} & \Phi\left(\frac{-\mu_Z}{\sigma_Z}\right)\sum_{i\in I} b_i(0, F_t^i)\Phi\left(\frac{R_i(0,0)}{\sigma}\right) + \Phi\left(\frac{\mu_Z-\bar{\xi}}{\sigma_Z}\right)\sum_{i\in I} b_i(\bar{\xi}^i, F_t^i)\Phi\left(\frac{-R_i(\bar{\xi}^i,\bar{\xi}^i)}{\sigma}\right) + \\
& \sum_{i\in I} b_i(\mu_Z, F_t^i)e^{\frac{1}{2}m_i^2\sigma_Z^2}\Phi_2^{2\times 1}\left(\begin{bmatrix} \frac{\bar{\xi}^i-\mu_Z}{\sigma_Z} - m_i\sigma_Z \\ \frac{-\mu_Z}{\sigma_Z} - m_i\sigma_Z \end{bmatrix}, \frac{R_i(\mu_Z,0) - m_i^2\sigma_Z^2}{\sigma_{i,Z}}; \frac{m_i\sigma_Z}{\sigma_{i,Z}}\right) + \\
& \sum_{i\in I} b_i(\mu_Z - \bar{\xi}^j, F_t^i)e^{\frac{1}{2}m_i^2\sigma_Z^2}\Phi_2^{2\times 1}\left(\begin{bmatrix} \frac{\bar{\xi}-\mu_Z}{\sigma_Z} - m_i\sigma_Z \\ \frac{\bar{\xi}^j-\mu_Z}{\sigma_Z} - m_i\sigma_Z \end{bmatrix}, \frac{-R_i(\mu_Z - \bar{\xi}^j, \bar{\xi}^j) + m_i^2\sigma_Z^2}{\sigma_{i,Z}}; \frac{-m_i\sigma_Z}{\sigma_{i,Z}}\right) + \\
& (F_t^c)^{\alpha_c}(F_t^g)^{\alpha_g}e^{\eta}\left\{-\sum_{i\in I}\Phi_2^{2\times 1}\left(\begin{bmatrix} \frac{\bar{\xi}^i-\mu_Z}{\sigma_Z} - \gamma\sigma_Z \\ \frac{-\mu_Z}{\sigma_Z} - \gamma\sigma_Z \end{bmatrix}, \frac{R_i(\mu_Z,0) + \alpha_j\sigma^2 - \gamma m_i\sigma_Z^2}{\sigma_{i,Z}}; \frac{m_i\sigma_Z}{\sigma_{i,Z}}\right) + \right. \\
& \left. \sum_{i\in I}\Phi_2^{2\times 1}\left(\begin{bmatrix} \frac{\bar{\xi}-\mu_Z}{\sigma_Z} - \gamma\sigma_Z \\ \frac{\bar{\xi}^j-\mu_Z}{\sigma_Z} - \gamma\sigma_Z \end{bmatrix}, \frac{R_i(\mu_Z - \bar{\xi}^j, \bar{\xi}^j) + \alpha_j\sigma^2 + \gamma m_i\sigma_Z^2}{\sigma_{i,Z}}; \frac{-m_i\sigma_Z}{\sigma_{i,Z}}\right) - \xi + \xi_x + \xi_y\right\}.
\end{aligned}
$$

where $x := \text{argmax}_{i\in I}(\bar{\xi}^i)$, $y := \text{argmin}_{i\in I}(\bar{\xi}^i)$, $\sigma_{i,Z}^2 = m_i^2\sigma_Z^2 + \sigma^2$ for $i \in I$ and $\eta = \beta + \gamma\mu_Z + \frac{1}{2}\gamma^2\sigma_Z^2 + -\frac{1}{2}\alpha_c\alpha_g\sigma^2$.

The Importance of Spread Options

European Call written on

- the **Difference** between **two** Underlying Interests
- a **Linear Combination** of **several** Underlying Interests

Calendar Spread Options

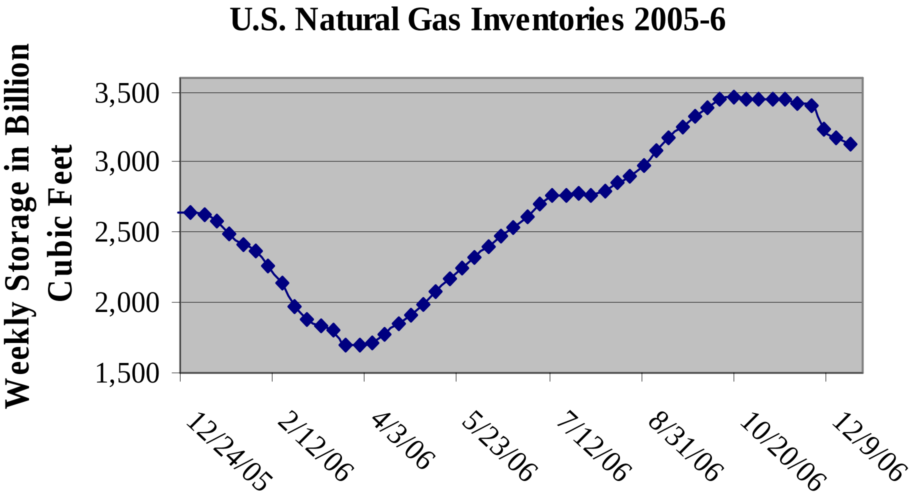
- Single Commodity at two different times

$$\mathbb{E}\{(I(T_2) - I(T_1) - K)^+\}$$

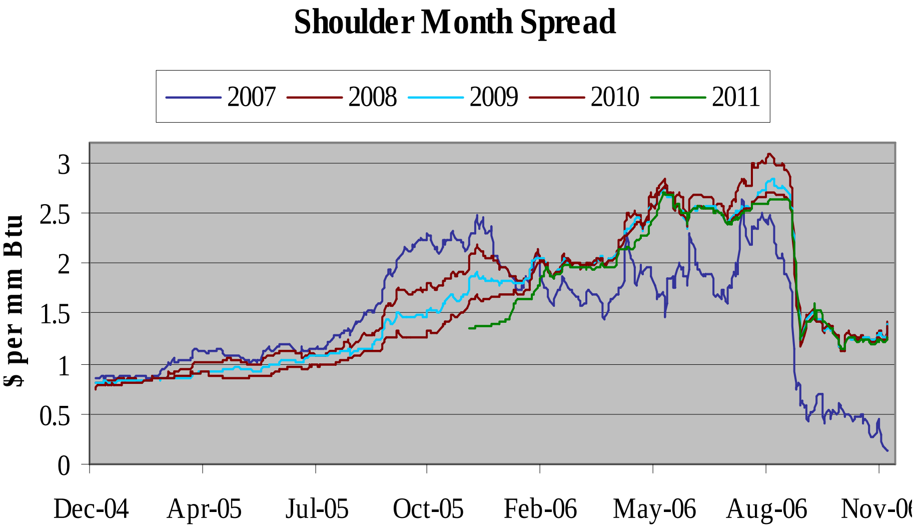
- Mathematically easier (only one underlier)

- **Amaranth** largest (and **fatal**) positions
  - Shoulder Natural Gas Spread (play on inventories)
  - **Long** March Gas / **Short** April Gas
    - Depletion stops in March / injection starts in April
    - Can be fatal: ***widow maker spread***

SEASONALITY OF GAS INVENTORY



What Went Wrong with Amaranth?



Cross Commodity Spread Options

- **Crush Spread**
  - between Soybean and soybean products (meal & oil)
- **Crack Spread**
  - gasoline crack spread between Crude and Unleaded
  - heating oil crack spread between Crude and HO
- **(Dirty) Spark Spread**
  - between price of 1 MWhe of Electric Power and cost of Natural Gas needed to produce it

$$S_t = F_E(t) - H_{eff} F_G(t)$$

- **(Dirty) Dark Spread**
  - with Coal instead of Natural Gas

$$S_t = F_E(t) - H_{eff} F_C(t)$$

- **(Clean) Spark Spread**
  - including the cost of CO_2 Emissions

$$S_t = F_E(t) - H_{eff} F_G(t) - e_G A_t$$

H_{eff} **Heat Rate** of the plant

Real Option NG Power Plant Valuation

Real Option Approach

- Lifetime of the plant $[T_1, T_2]$
- C **capacity** of the plant (in MWh)
- H **heat rate** of the plant (in MMBtu/MWh)
- P_t price of **power** on day t
- G_t price of **fuel** (gas) on day t
- K fixed **Operating Costs**
- **Value of the Plant (ORACLE)**

$$C \sum_{t=T_1}^{T_2} e^{-rt} \mathbb{E}\{(P_t - HG_t - K)^+\}$$

String of Spark Spread Options

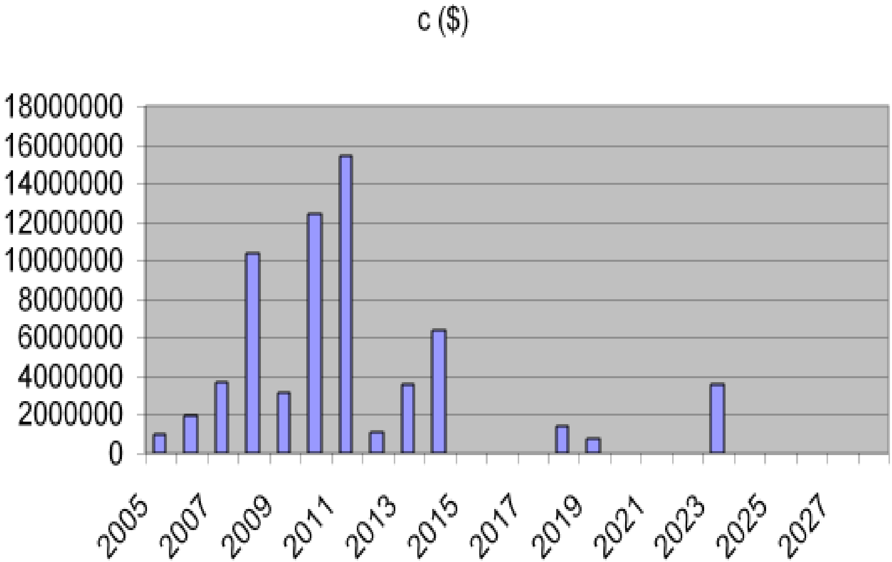
Beyond Plant Valuation: Credit Enhancement

(Flash Back)

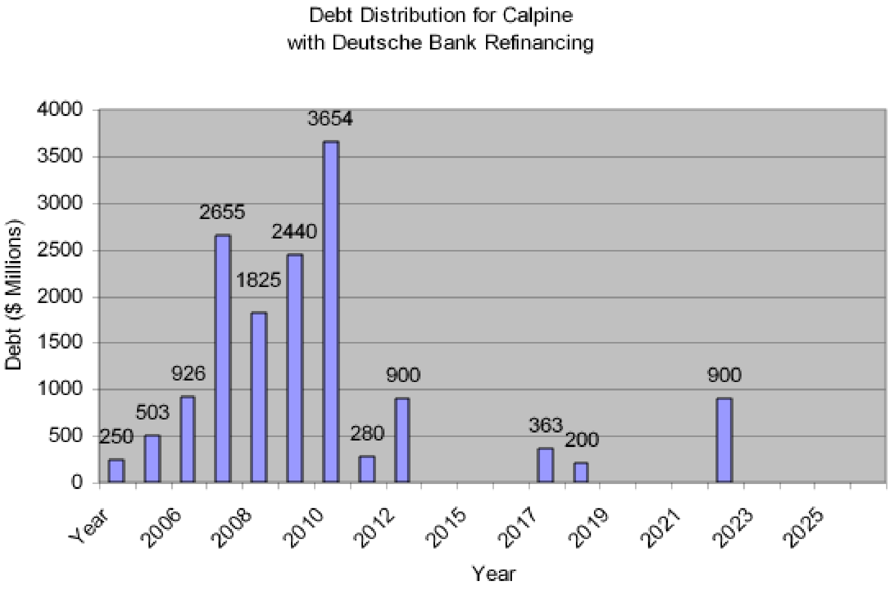
The Calpine - Morgan Stanley Deal

- Calpine needs to refinance USD 8 MM by November 2004
- **Jan. 2004**: Deutsche Bank: no traction on the offering
- **Feb. 2004**: *The Street* thinks Calpine is ”heading South”
- **March 2004**: Morgan Stanley offers a (complex) structured deal
  - A strip of spark spread options on 14 Calpine plants
  - A similar bond offering
- ***How were the options priced?***
  - By Morgan Stanley ?
  - By Calpine ?

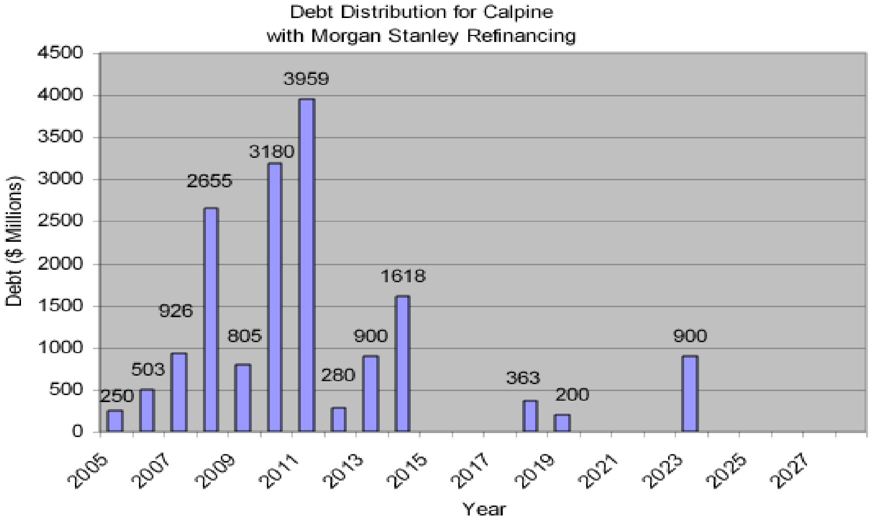
Calpine Debt



Calpine Debt with Deutsche Bank Financing



Calpine Debt with Morgan Stanley Financing



A Possible Model

Assume that Calpine owns **only** one plant

MS guarantees its spark spread will be at least κ for M years

Approach à la **Leland**'s Theory of the **Value of the Firm**

$$V = v - p_0 + \sup_{\tau \leq T} \mathbb{E} \left\{ \int_0^\tau e^{-rt} \overline{\delta}_t \, dt \right\}$$

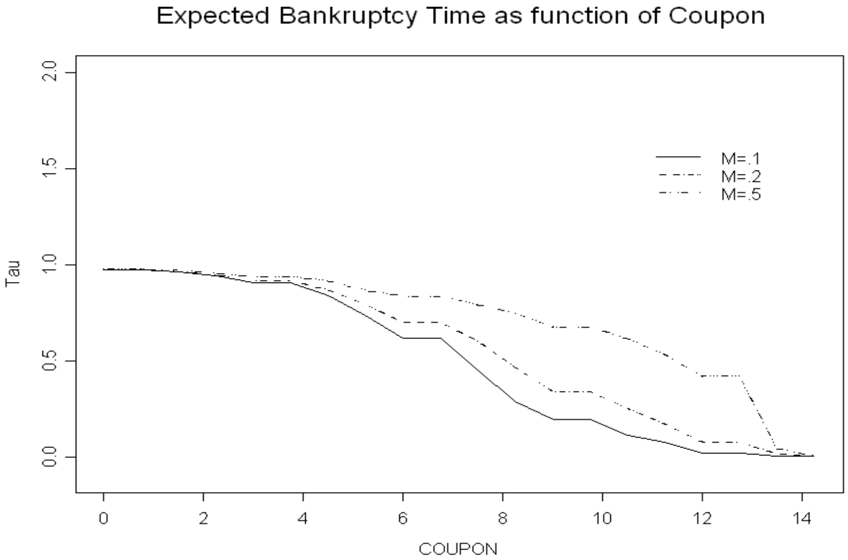
where

$$\overline{\delta}_t = \begin{cases} (P_t - H * G_t - K) \vee \kappa - c_t & \text{if } 0 \leq t \leq M \\ (P_t - H * G_t - K)^+ - c_t & \text{if } M \leq t \leq T \end{cases}$$

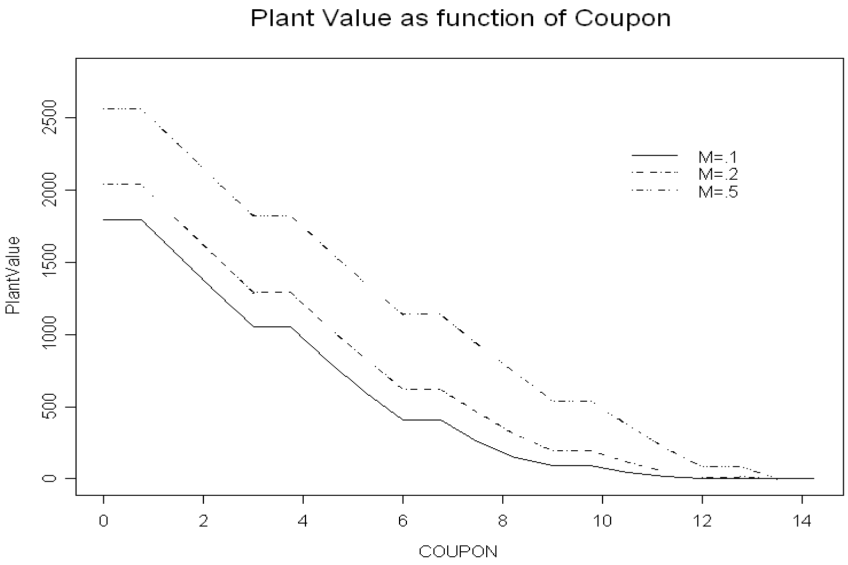
and

- v current value of firm's assets
- p_0 option premium
- M length of the option life
- κ strike of the option
- c_t cost of servicing the existing debt

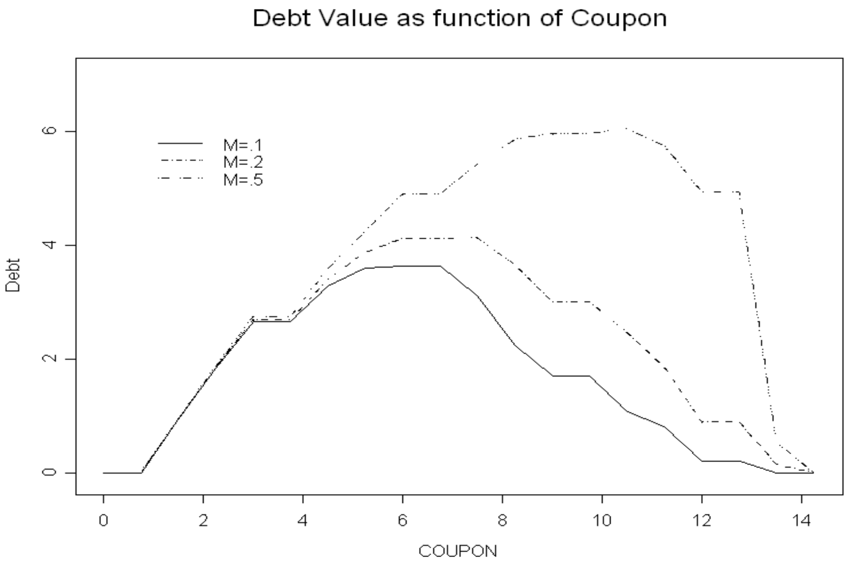
DEFAULT TIME



Plant Value


Plant Value as function of Coupon
PlantValue
0
500
1000
1500
2000
2500
COUPON
0
2
4
6
8
10
12
14
M=.1
M=.2
M=.5

Debt Value



Debt Value as function of Coupon
M=.1
M=.2
M=.5
Debt
0
2
4
6
COUPON
0
2
4
6
8
10
12
14

Spread Valuation Mathematical Challenge

$$p = e^{-rT}\mathbb{E}\{(I_2(T) - I_1(T) - K)^+\}$$

- Underlying indexes are spot prices
  - Geometric Brownian Motions ($K = 0$ Margrabe)
  - Geometric Ornstein-Uhlembeck (OK for Gas)
  - Geometric Ornstein-Uhlembeck with jumps (OK for Power)
- Underlying indexes are forward/futures prices
  - HJM-type models with deterministic coefficients

Problem

finding closed form formula and/or fast/sharp approximation for

$$\mathbb{E}\{(\alpha e^{\gamma X_1} - \beta e^{\delta X_2} - \kappa)^+\}$$

for a Gaussian vector (X_1, X_2) of $N(0,1)$ random variables with correlation ρ.

Sensitivities?

Spread Valuation

- $K = 0$ (Easy Case) Exchange Option **Margrabe Formula**
- $K \neq 0$ **Approximations**
  - Fourier Approximations (**Madan, Carr, Dempster, Hurd et. al**)
  - Bachelier approximation (**Alexander, Borovkova**)
  - Zero-strike approximation
  - **Kirk** approximation
  - CD Upper and Lower Bounds (**R.C. - V. Durrleman**)
  - **Bjerksund - Stensland** approximation
  - **Alos-Eydeland-Laurence** approximation for 3 log-normal interests

Can we also approximate the **Greeks** ?

- New **Electricity** pricing formula (**R.C.-Coulon-Schwartz**)
- **Clean** spread (including price of carbon) (**R.C.-Coulon-Schwartz**)

IMPLIED CORRELATION

R.C. - Y. Sun

Given market prices of

- Options on individual underlying interests
- Spread options

INFER / IMPLY a (Pearson) correlation and

- Smiles
- Skews

in the spirit of **implied volatility**

Major Difficulty:

- Data NOT REALLY available !
- Need to rely on trader's observations / speculations

Multi-Scale Stochastic Volatility Model

R.C. - Y. Sun ***à la Fouque-Sircar-Papanicolaou***

$$\begin{cases} dX_t &= \mu_1 X_t dt + X_t f(Z_t) f_1(V_t) dW_t^{(X)}\,, \\ dY_t &= \mu_2 Y_t dt + Y_t f(Z_t) f_2(V_t) dW_t^{(Y)}\,, \\ dZ_t &= \frac{1}{\epsilon}(m - Z_t)dt + \frac{\nu\sqrt{2}}{\sqrt{\epsilon}}\sqrt{Z_t}dW_t^{(Z)}\,, \\ dV_t &= \delta c(V_t)dt + \sqrt{\delta} g(V_t) dW_t^{(V)}\,. \end{cases}$$

- Z_t **Fast scale** volatility factor
- V_t **Slow scale** volatility factor

$$\mathbf{W}_t = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \rho & \sqrt{1-\rho^2} & 0 & 0 \\ \rho_{11} & \widetilde{\rho_{21}} & \sqrt{1-\rho_{11}^2-\widetilde{\rho_{21}}^2} & 0 \\ \rho_{12} & \widetilde{\rho_{22}} & \widetilde{\rho_0} & \sqrt{1-\rho_{12}^2-\widetilde{\rho_{22}}^2-\widetilde{\rho_0}^2} \end{pmatrix} \mathbf{W}_t^0\,.$$

Under Pricing Measure

$$\begin{cases} \mathrm{d}X_t &= rX_t\mathrm{d}t + X_t f(Z_t) f_1(V_t)\mathrm{d}W_t^{(X)*}\,, \\ \mathrm{d}Y_t &= rY_t\mathrm{d}t + Y_t f(Z_t) f_2(V_t)\mathrm{d}W_t^{(Y)*}\,, \\ \mathrm{d}Z_t &= [\frac{1}{\epsilon}(m - Z_t) - \frac{\nu\sqrt{2}}{\sqrt{\epsilon}}\sqrt{Z_t}\Lambda(Z_t, V_t)]\mathrm{d}t + \frac{\nu\sqrt{2}}{\sqrt{\epsilon}}\sqrt{Z_t}\mathrm{d}W_t^{(Z)*}\,, \\ \mathrm{d}V_t &= [\delta c(V_t) - \sqrt{\delta} g(V_t)\Gamma(Z_t, V_t)]\mathrm{d}t + \sqrt{\delta} g(V_t)\mathrm{d}W_t^{(V)*}\,, \end{cases}$$

Option Prices

$$C^{\epsilon,\delta}(x, y, z, v, t) = e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}[h(X_T, Y_T)|X_t = x, Y_t = y, Z_t = z, V_t = v]$$

Pricing PDE (Feynman-Kac)

$$
\begin{aligned}
&\tfrac{1}{2}x^2 f^2(z) f_1^2(v) C_{xx} + \tfrac{1}{2}y^2 f^2(z) f_2^2(v) C_{yy} + \tfrac{\nu^2}{\epsilon} z C_{zz} + \tfrac{1}{2}\delta g^2(v) C_{vv} \\
&+\rho xy f^2(z) f_1(v) f_2(v) C_{xy} + \rho_{11}\tfrac{\nu\sqrt{2}}{\sqrt{\epsilon}} x\sqrt{z} f(z) f_1(v) C_{xz} + \rho_{21}\tfrac{\nu\sqrt{2}}{\sqrt{\epsilon}} y\sqrt{z} f(z) f_2(v) C_{yz} \\
&+\rho_{12} x\sqrt{\delta} g(v) f(z) f_1(v) C_{xv} + \rho_{22} y\sqrt{\delta} g(v) f(z) f_2(v) C_{yv} + \rho_0\sqrt{\tfrac{\delta}{\epsilon}}\nu\sqrt{2}\sqrt{z} g(v) C_{zv} \\
&+[\tfrac{1}{\epsilon}(m-z) - \tfrac{\nu\sqrt{2}}{\sqrt{\epsilon}}\sqrt{z}\Lambda(z,v)] C_z + [\delta c(v) - \sqrt{\delta} g(v)\Gamma(z,v)] C_v + rxC_x + ryC_y \\
&-rC + C_t = 0
\end{aligned}
$$

with terminal condition

$$C^{\epsilon,\delta}(x,y,z,v,T) = h(x,y)$$

Singular Perturbation Theory

Option Price Approximation Formula

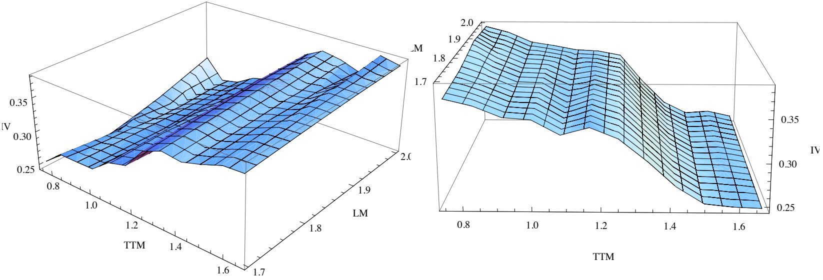
$$C^{\epsilon,\delta} \approx \tilde{C}^{\epsilon,\delta} := C_0 + \sqrt{\epsilon}C_1 + \sqrt{\delta}D_1$$

where coefficients C_0, C_1 and D_1 can be **calibrated** without the full knowledge of the functions f, f_1, f_2, Λ and Γ! (**Fouque-Sircar-Papanicolaou**)

Control of the error: for fixed (x, y, z, v, t), there exists $c > 0$ s.t.

$$\mid C^{\epsilon,\delta} - \widetilde{C}^{\epsilon,\delta} \mid \leq c(\epsilon + \delta + \sqrt{\epsilon\delta})\,.$$

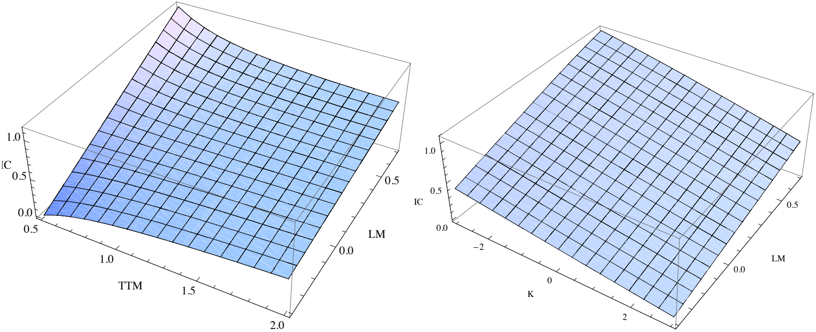
Implied Volatilities



Implied volatility for options on electric power futures maturing in August 2011, and traded in April 2010, May 2010, until April 2011 (left) and for options on natural gas futures maturing in August 2011, and traded in April 2010, May 2010, until April 2011 (right)

Implied Correlation Approximations

$$I^{\epsilon,\delta} \approx \rho + \sqrt{\epsilon} I_1 + \sqrt{\delta} I_2$$



Implied correlation (IC) with fitted parameters and strike $K = 0$ (left) and time-to-maturity $TTM = 1$ year (right)

Back to the Exponential Bid-Stacks

Spread price (for some T) on fuel $v \in I$ (with $w = I \setminus v$) is given by

$$
\begin{aligned}
V_t = {} & \Phi\left(-\xi_8\right) \sum_{i \in I} b_i(\bar{\xi}^i, F_t^i) \Phi\left(\frac{-R_i(\bar{\xi}^i, \bar{\xi}^i)}{\sigma}\right) - HF_t^v \left(1 - \Phi(\xi_7) + \Phi(\xi_6) - \Phi(\xi_5)\right) + \\
& b_v(\mu_z, F_t^v) e^{\frac{1}{2} m_v^2 \sigma_z^2} \Phi_2^{2\times 2}\left(\begin{bmatrix} \bar{\xi}^v & \xi_3 \\ \xi_4 & \xi_2 \end{bmatrix}, \frac{R_v(\mu_z, 0) - m_v^2 \sigma_z^2}{\sigma_{v,z}}; \frac{m_v \sigma_z}{\sigma_{v,z}}\right) + \\
& b_v(\mu_z - \bar{\xi}^w, F_t^v) e^{\frac{1}{2} m_v^2 \sigma_z^2} \Phi_2^{2\times 2}\left(\begin{bmatrix} \xi_8 & \xi_6 \\ \xi_7 & \xi_5 \end{bmatrix}, \frac{-R_v(\mu_z - \bar{\xi}^w, \bar{\xi}^w) + m_v^2 \sigma_z^2}{\sigma_{v,z}}; \frac{-m_v \sigma_z}{\sigma_{v,z}}\right) + \\
& b_w(\mu_z - \bar{\xi}^v, F_t^w) e^{\frac{1}{2} m_w^2 \sigma_z^2} \Phi_2^{2\times 1}\left(\begin{bmatrix} \xi_8 \\ \bar{\xi}^v \end{bmatrix}, \frac{-R_w(\mu_z - \bar{\xi}^v, \bar{\xi}^v) + m_w^2 \sigma_z^2}{\sigma_{w,z}}; \frac{-m_w \sigma_z}{\sigma_{w,z}}\right) - \\
& HF_t^v \Phi_2^{2\times 3}\left(\begin{bmatrix} \xi_7 & \xi_5 & \xi_3 \\ \xi_6 & \xi_4 & \xi_2 \end{bmatrix}, \frac{\tilde{R}_v\left((\log H - \beta - \gamma\mu_z)/\alpha_w\right)}{\sigma_{\gamma z}}; \frac{-\gamma \sigma_z}{\alpha_g \sigma_{\gamma z}}\right) + \\
& (F_t^c)^{\alpha_c} (F_t^g)^{\alpha_g} e^{\eta} \Bigg\{ \Phi_2^{2\times 2}\left(\begin{bmatrix} \bar{\xi}^v & \xi_3 \\ \xi_4 & \xi_2 \end{bmatrix}, \frac{-R_v(\mu_z, 0) - \alpha_w \sigma^2 + \gamma m_v \sigma_z^2}{\sigma_{v,z}}; \frac{-m_v \sigma_z}{\sigma_{v,z}}\right) - \\
& \Phi_2^{2\times 2}\left(\begin{bmatrix} \xi_8 & \xi_6 \\ \xi_7 & \xi_5 \end{bmatrix}, \frac{-R_v(\mu_z - \bar{\xi}^w, \bar{\xi}^w) - \alpha_w \sigma^2 + \gamma m_v \sigma_z^2}{\sigma_{v,z}}; \frac{-m_v \sigma_z}{\sigma_{v,z}}\right) + \\
& \Phi_2^{2\times 1}\left(\begin{bmatrix} \xi_8 \\ \bar{\xi}^v \end{bmatrix}, \frac{R_w(\mu_z - \bar{\xi}^v, \bar{\xi}^v) + \alpha_v \sigma^2 - \gamma m_w \sigma_z^2}{\sigma_{w,z}}; \frac{m_w \sigma_z}{\sigma_{w,z}}\right) - \\
& \Phi_2^{2\times 3}\left(\begin{bmatrix} \xi_7 & \xi_5 & \xi_3 \\ \xi_6 & \xi_4 & \xi_2 \end{bmatrix}, \frac{-\tilde{R}_v\left((\log H - \beta - \gamma\mu_z)/\alpha_w\right) - \alpha_w \sigma^2 - \gamma^2 \sigma_z^2/\alpha_w}{\sigma_{\gamma z}}; \frac{\gamma \sigma_z}{\alpha_w \sigma_{\gamma z}}\right) \Bigg\}
\end{aligned}
$$

with $\tilde{R}_i(z) = z + \log(F_t^j) - \log(F_t^i) - \frac{1}{2}\sigma^2$.

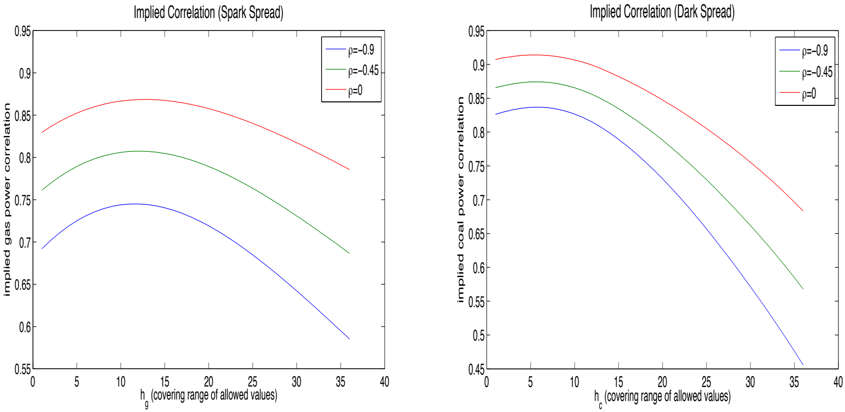
Applications

- Computation of higher **moments** and covariances:

$$\left(\text{e.g.,} \quad \mathbb{E}_t[P_T^2], \quad \mathbb{E}_t[P_T C_T], \quad \mathbb{E}_t[P_T G_T]\right)$$

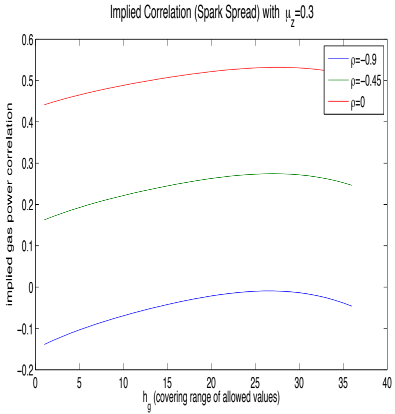
- **Sensitivities** (Greeks).
- **Calibration** to power forwards (with fuel forwards as inputs).
- **Spikes** (and negative prices) is possible
- Extension to **carbon** markets possible: merit order affected by allowance price, and accumulated emissions also driven by merit order.
- **Key advantage:** Structural link between electricity and fuel prices.

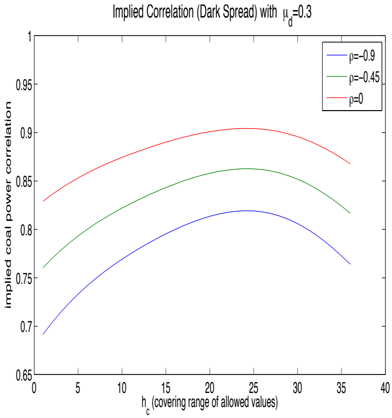
IMPLIED CORRELATION



Implied Correlation for high demand, - Spark (left) - Dark (right)

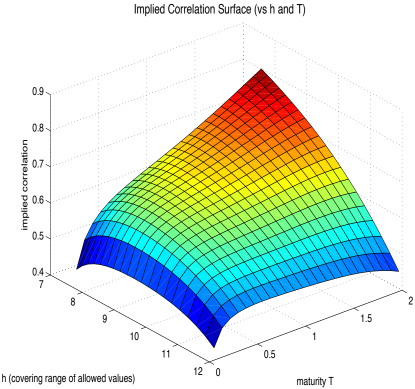
IMPLIEDCORRELATION II

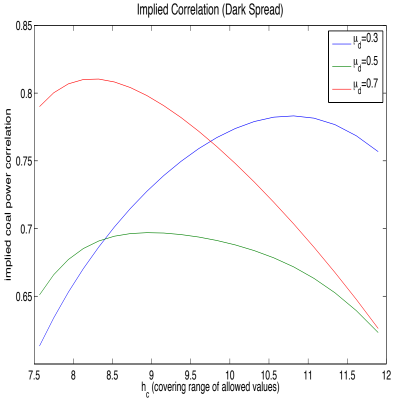




Implied Correlation - low demand - Spark (left), Dark (right)

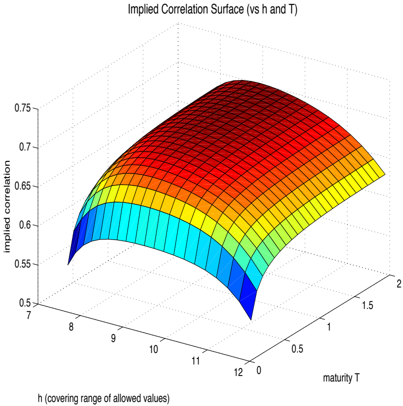
IMPLIEDCORRELATION III

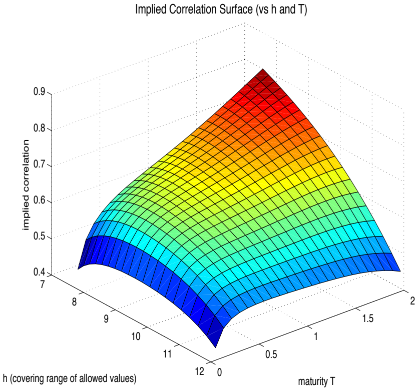




Implied correlation varying $\bar{\xi}^g$ (left) – Implied correlation varying μ_d (right), Dark (right)

Implied Correlations Surface IV

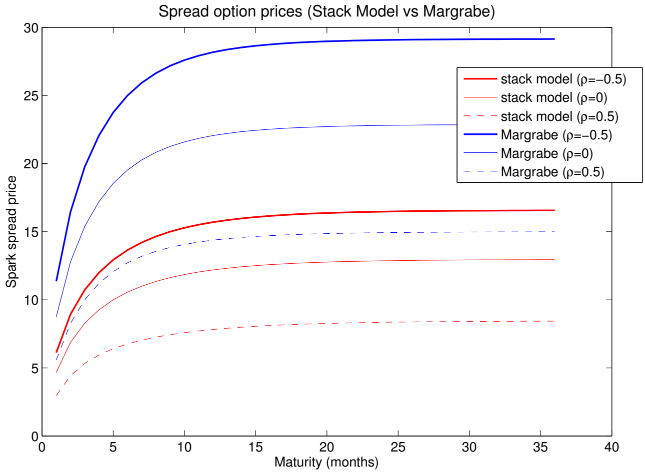




Implied correlation surfaces, for symmetric case (left) and for spark spread in case of coal in contango, gas in backwardation (right)

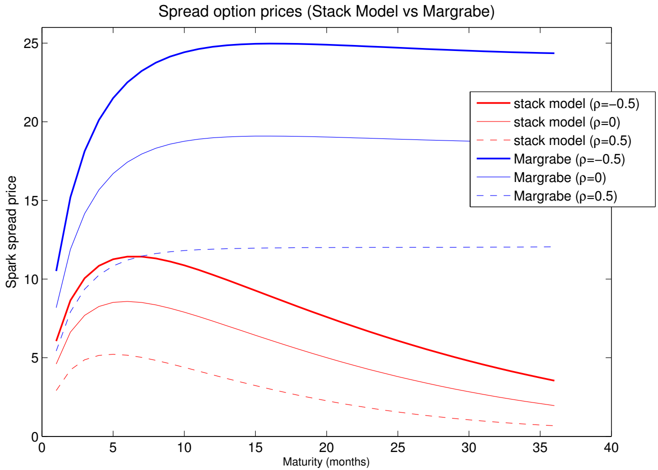
Comparison with Log-Normal (Margrabe) I

The bid stack model (without spikes) typically prices spread options lower than the Margrabe formula due to strong structural link. (We first match means and variances in the two approaches.)



Comparison with Log-Normal (Margrabe) II

In addition, the stack model automatically adjusts to information about likely future merit order changes. (Here we choose gas forwards in contango, coal in backwardation.)



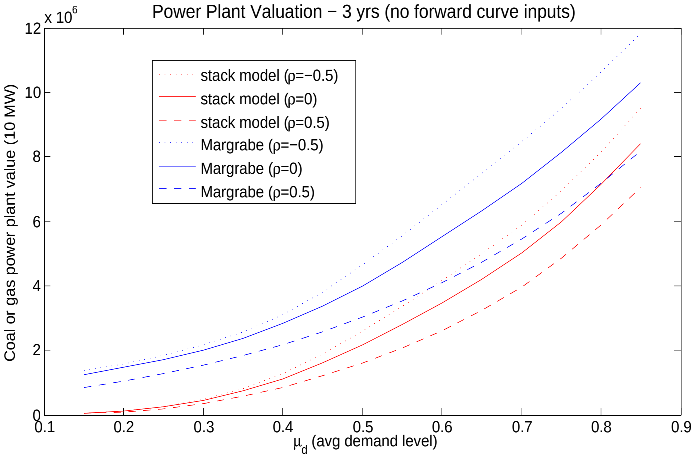
Power Plant Valuation Revisited

- Choose Bid-Stack
- Choose NG Forward Curve
- Choose Coal Forward Curve

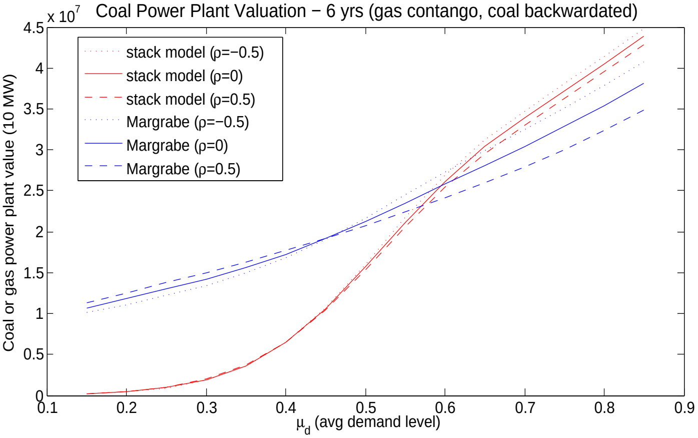
Examples

- 3 or 6 yrs tolling agreement
- 10MW
- No O&M ($K = 0$) Compare to Price from Margrabe formula
- Plot Value as function of mean demand μ_d

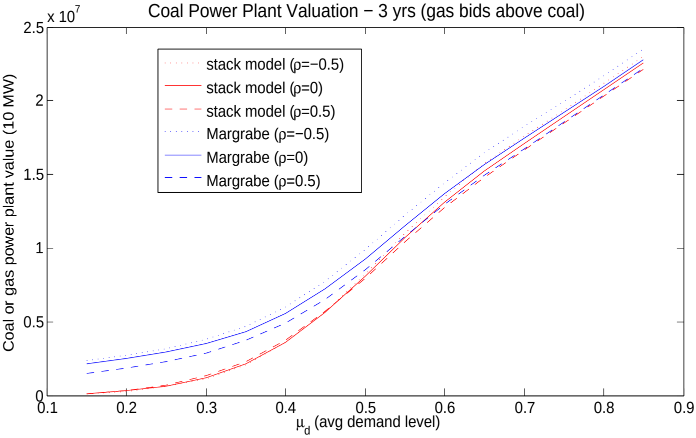
POWER PLANT VALUATION EX#1



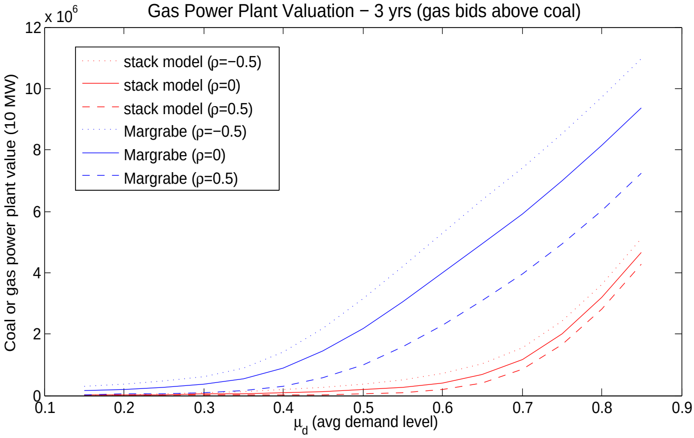
Power Plant Valuation Ex#2



Power Plant Valuation Ex#3



Power Plant Valuation Ex#4



More Physical Asset Valuation

Stochastic (Control) Optimization to Take Full Advantage of the Optionality

- **Physical Asset:** Fossil Fuel Power Plant, Oil Refinery, Pipeline, Gas Storage Facility, Hydro, . . .
- **Owner** (of the asset or a tolling contract)
  - Decides **when** and **how** to use the asset (e.g. run the power plant)
  - Has someone else do the leg work
- **Optimal Switching R.C - M. Ludkovski**
- **Extensions**
  - Accomodate **outages**, switch separation
  - Duality upper bounds (**Meinshausen-Hambly**)
  - More (rigorous) **Mathematical Analysis**
    - **Porchet-Touzi** (BSDEs)
    - **Forsythe-Ware** (Numeric scheme to solve HJB QVI)
    - **Bernhart-Pham** (reflected BSDEs)
    - **Bouchard-Warin** (numerics of reflected BSDEs)
  - **Financial Hedging**: Extending the Analysis Adding Access to a Financial Market (indifference pricing)

Revisiting Old Issues: the Clean Spark Spread

R.C. - M. Coulon - D. Schwarz (in preparation)

Given

- $P(t)$ sale price of 1 MWhr of electricity
- $G(t)$ price of 1 MBtu natural gas
- $A(t)$ price of an allowance for 1 ton of CO_2 equivalent

compute

$$e^{-rT}\mathbb{E}\{(P(T) - H_{eff}G(T) - e_G A(T))^+\}$$

where e_G is the emission coefficient of the technology.

Requires

- Joint model for $\{(P(t), G(t), A(t)\}_{0 \le t \le T}$