Spread Options and Implied Correlations
René Carmona, Princeton University

Discussant : Helyette Geman
Director, Commodity Finance Centre, University of London and
ESCP Europe

To be presented at the Swissquote Conference
EPFL, Lausanne – October 20 & 21, 2011

The Literature on Spread Options

→ Margrabe (1978) : Exchange option, a seminal paper
Both assets are driven by geometric Brownian motions
C(T) = Max (0, U(T) – V(T))
Using V as the *numéraire*, no assumption on interest rates is required

→ Kirk (1995)
Spread option paying   C(T) = Max (0, U(T) – hV(T) – k)
When the strike is small, approximation by Margrabe formula

→ Eydeland - G (1998)
Propose to price a power plant as a portfolio of sparkspreads or darkspreads.
In this case, V is multiplied by *h* , the *heat rate* characteristic of the power plant
and provided by the company which has built the plant

→ Carmona and Durrleman (2003)
Exhibit beautiful boundaries for spread option prices leading to good estimates to the exact
    option values, all the more useful as we are essentially in *incomplete* markets when dealing with
    energy

Valuation of Physical Assets in the Energy Industry: the case of a Combined Cycle Gas Turbine

→ Scenario A
- Gas = 3.5 $/MMbtu
- Power = 40 $/MWh
- Heat Rate = 10 MMbtu/MWh
- Profit = 40 - 3.5 x 10 = $5

→ Scenario B
- Gas = 4.5 $/MMbtu

  Do not operate

The ownership of the physical assets amounts to a series of *spark spreads* over the lifetime of the plant

$$\text{Value of the Asset} = \sum_{j=1}^{N} C_j$$

→ Carmona and Sun (2011) represent the commodities prices by geometric Brownian motions with stochastic volatilities

→ Like in Fouque et al (2003), the stochastic volatilities are driven by a « fast scale factor » reprsented by a CIR process with a high speed of mean- reversion and a slow scale volatility factor.

→ Hence, the sources of risk are conveyed by a 4-dimensional Brownian motion W, with correlations between the components. It is transformed into another one under the risk - neutral measure, and the measure change includes the *market prices* of fast and slow volatility risks

→ Then, the same asymptotics as in Fouque et al are used in order to address the complicated partial differetial equation satisfied by the spread option. An approximation formula for the option price is exhibited.

→ Lastly, implied volatilities are computed from options on the individual commodities involved in the spread and an asymptotic formula is derived for the implied correlation.

→ The implied volatility used for the first (rsp second) commodity is chosen as the average of implied volatility data on univariate options

→ This scheme is used to price a power plant as a string of spread options

Some questions/ suggestions

→ 1. Is this averaging procedure performed for each commodity in order to obtain a single implied volatility incorporated in the spread option price satisfactory?

→ 2. Where can we find the market prices of slow and fast volatility risks?
If the commodity market is essentially using Black formula for univariate options, what are the two primitive instruments completing the market? If these are the ATM univariate options, averaging the implied smile/ skew for each commodity in order to recover the implied correlation from the spread option may not be an appropriate choice.
If our focus is the implied correlation, the most consistent approach may be to use the implied volatilities of the ATM options which *complete* the market that has 4 (at least!) sources of randomness.

Sep 21, 07 Oil 6 Months implied Volatility
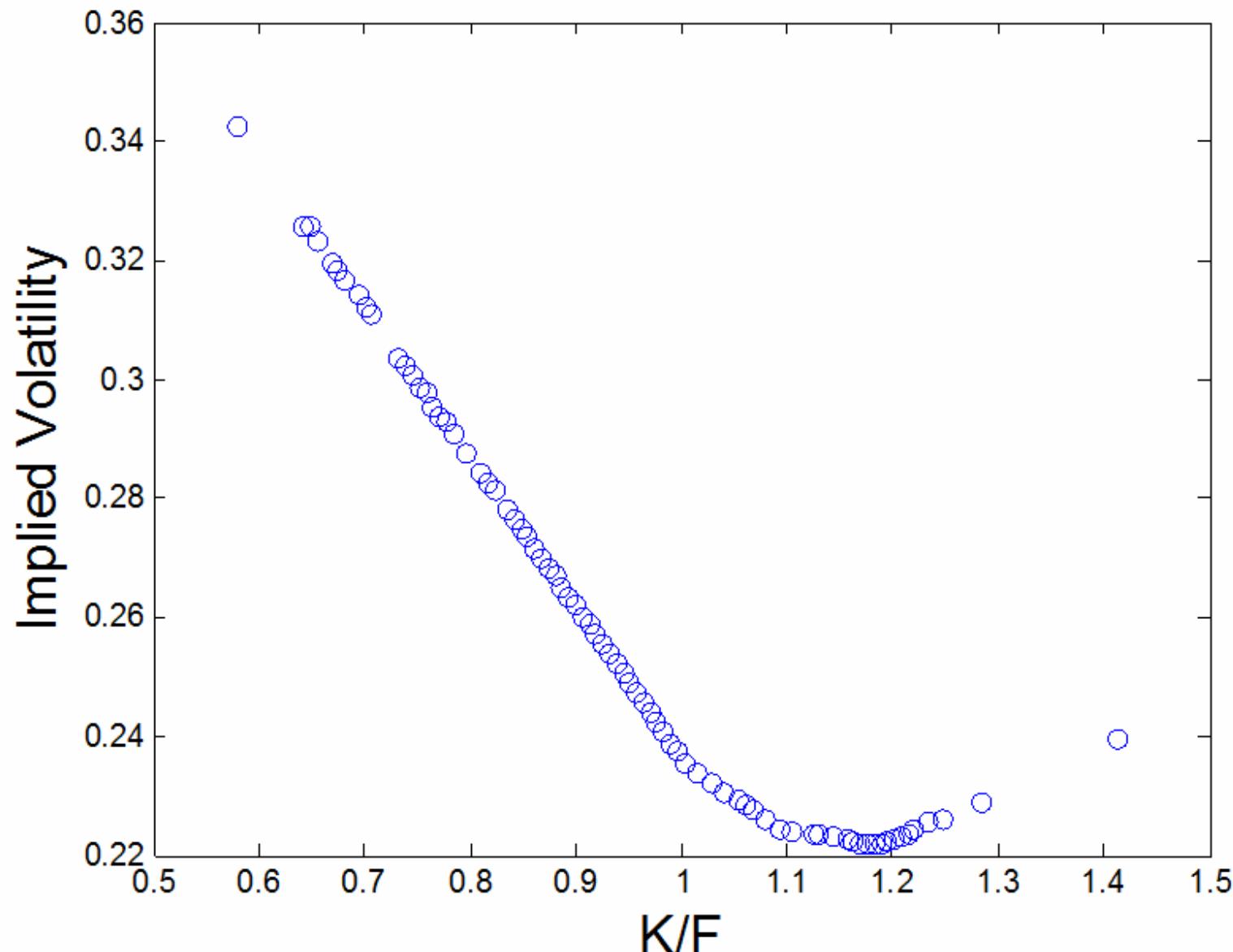
Feb 23, 09 Oil 6 Months implied Volatility
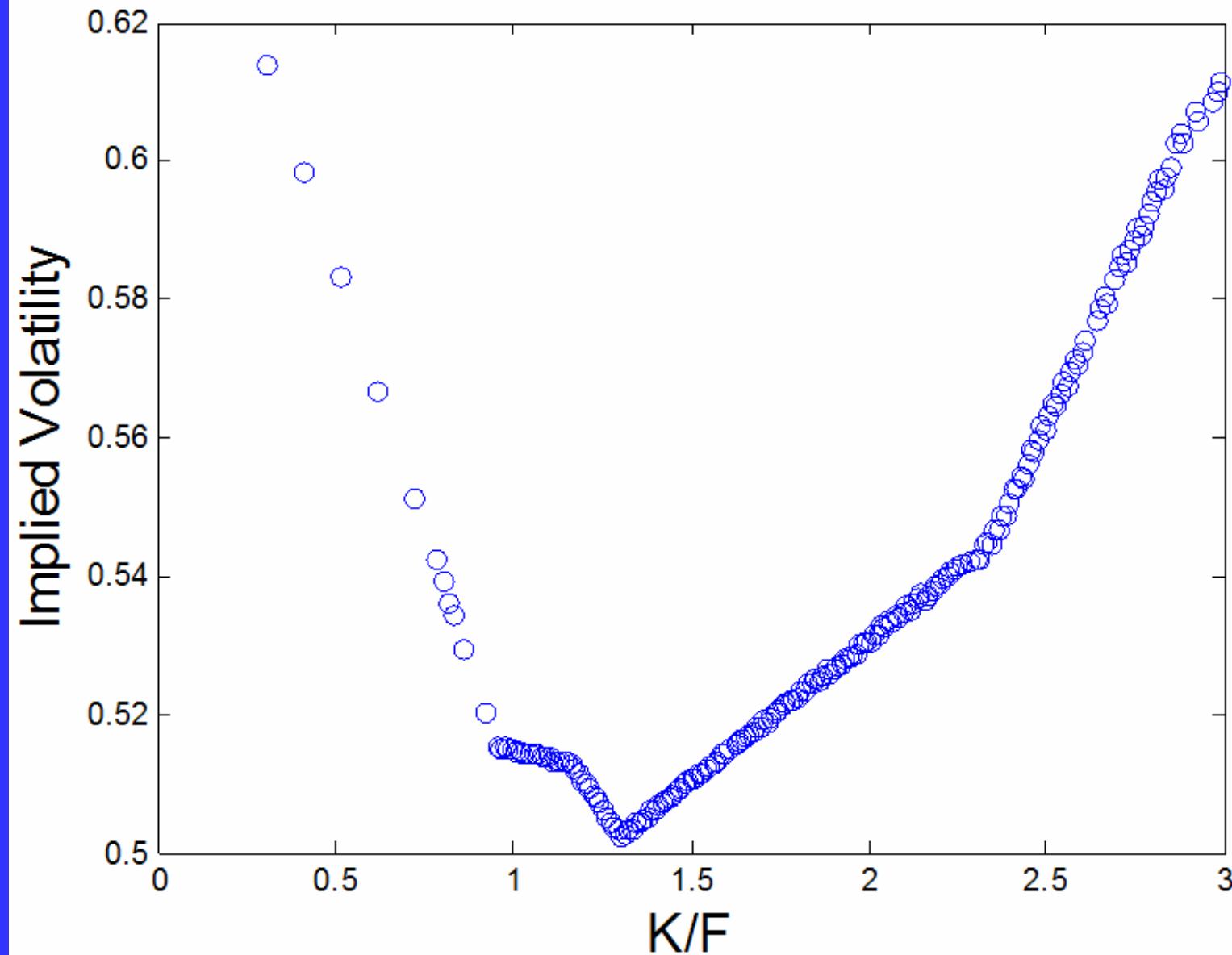
The Intrinsic Value of a Power Plant (G. 2005)

→ It is the one bankers from project finance will wish to see before lending money to a utility which wants to expand its production

→ It consists in . defining the residual lifetime of the plant
. eliminating (essentially) all randomness from the revenues by selling the electricity forward and buying the natural gas in the forward market as far in the future as possible
. identifying a proper discount factor in order to compute the Discounted Cash Flows (DCF)

➔ In this case, the key elements in the valuation are
a) the two forward curves
b) the term structure of correlations between the pairs of same maturity forward contracts

PJM Western Hub
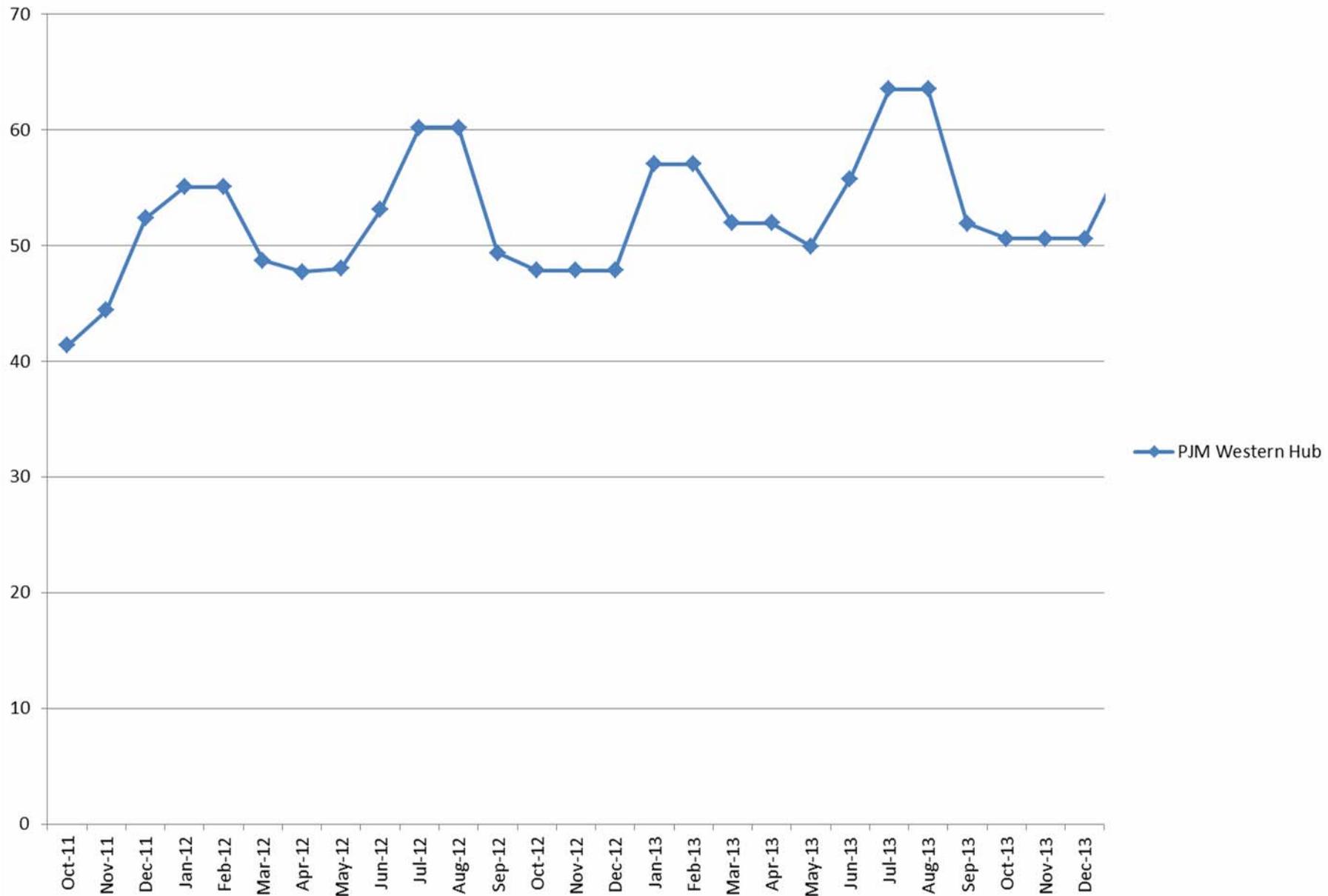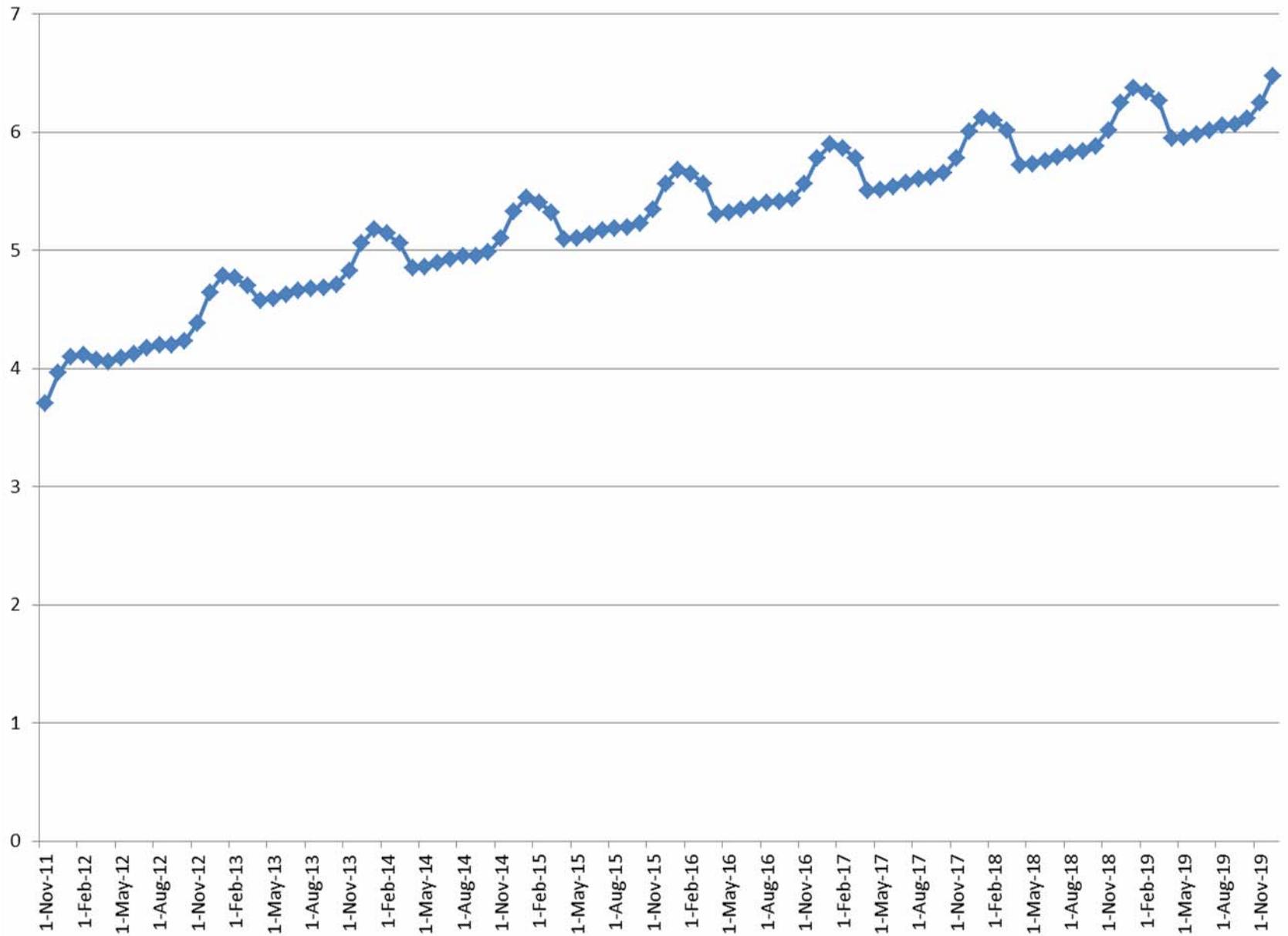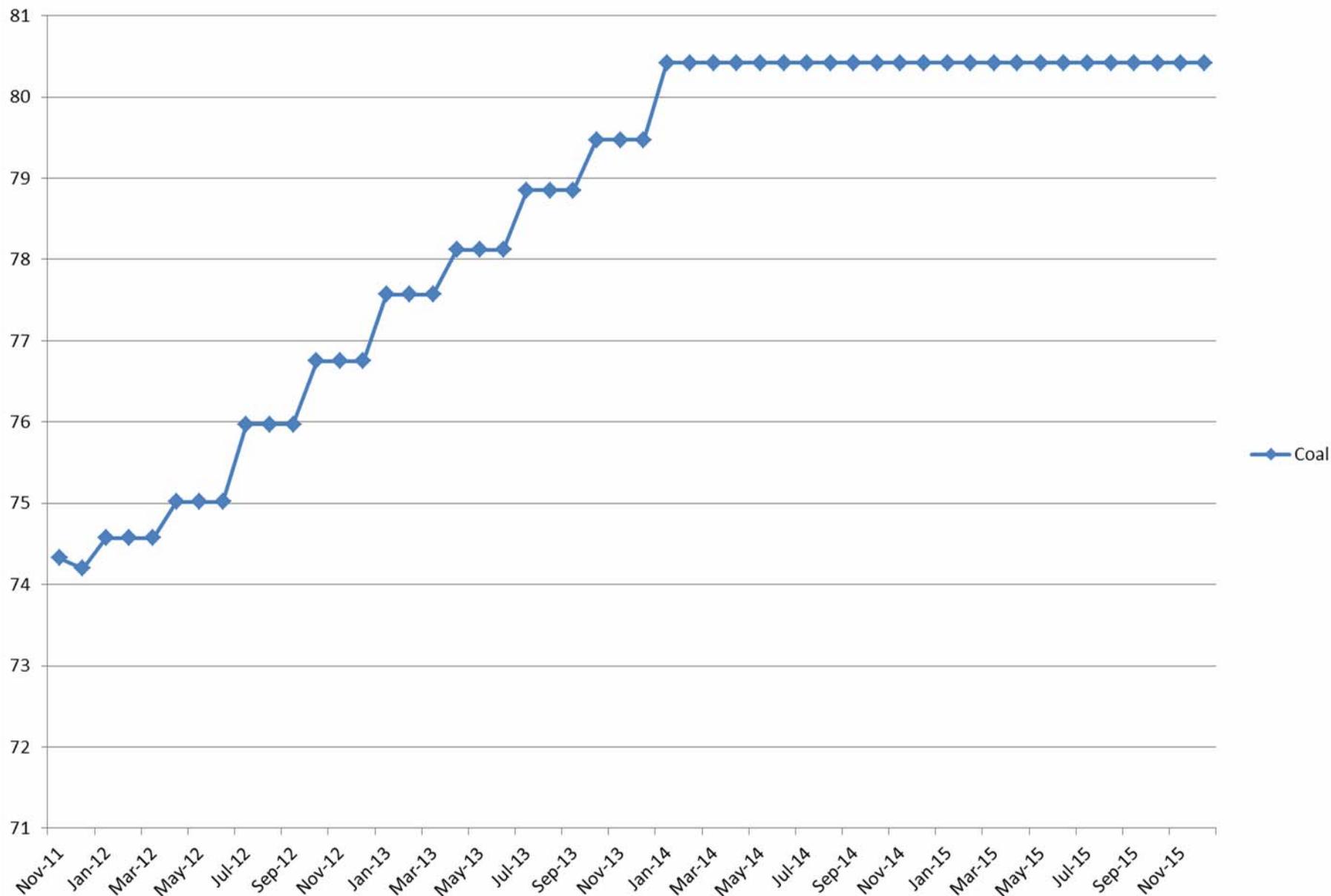
Coal

Correlation : Copper and Crude Oil (www.commoditymodels.com)
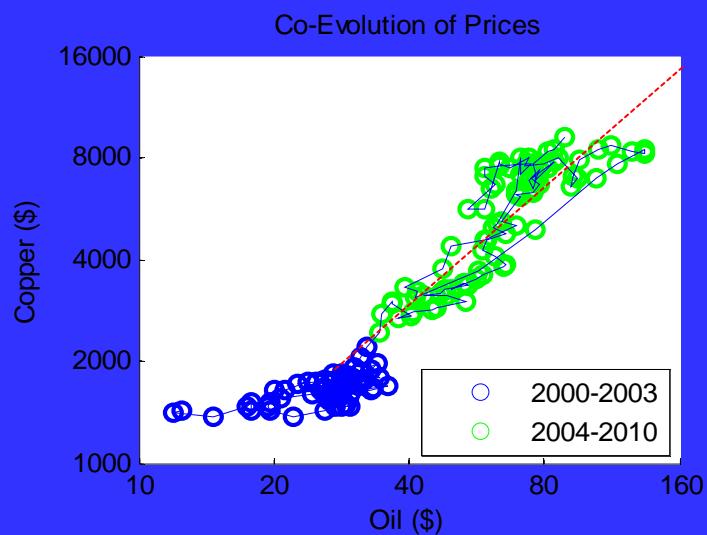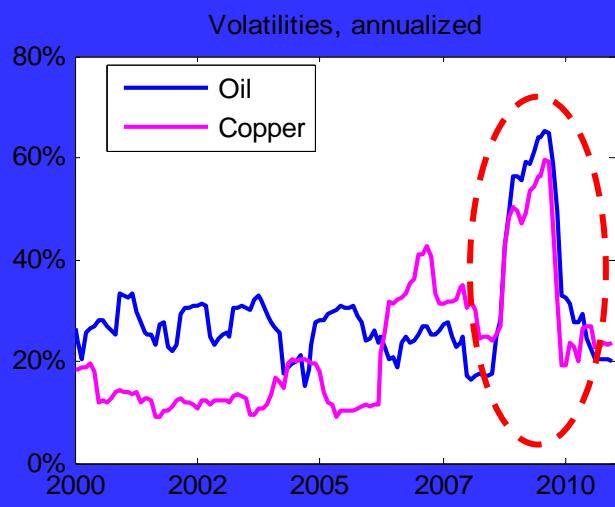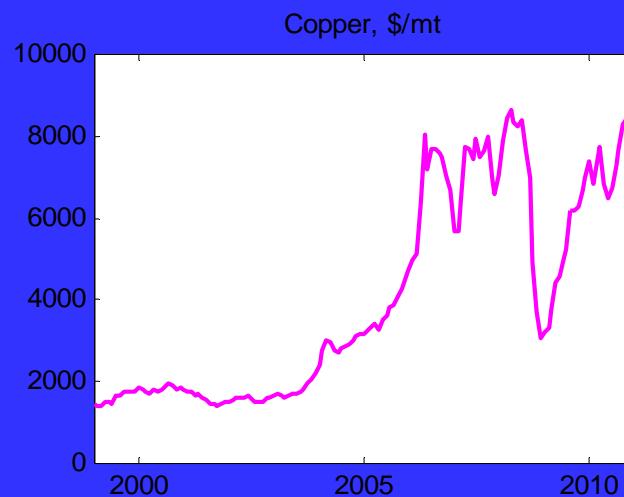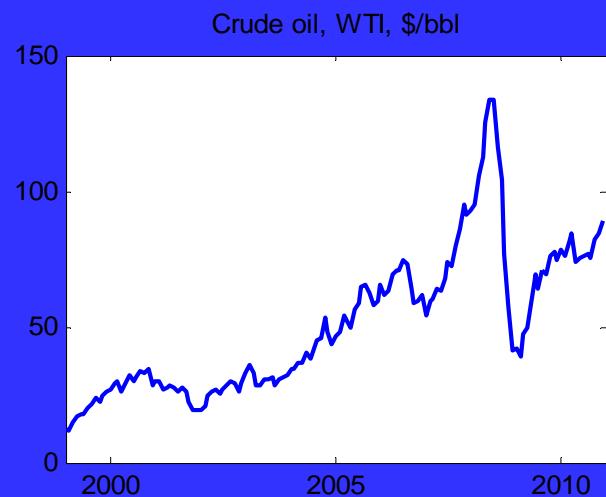
Correlation : Steel and Coal
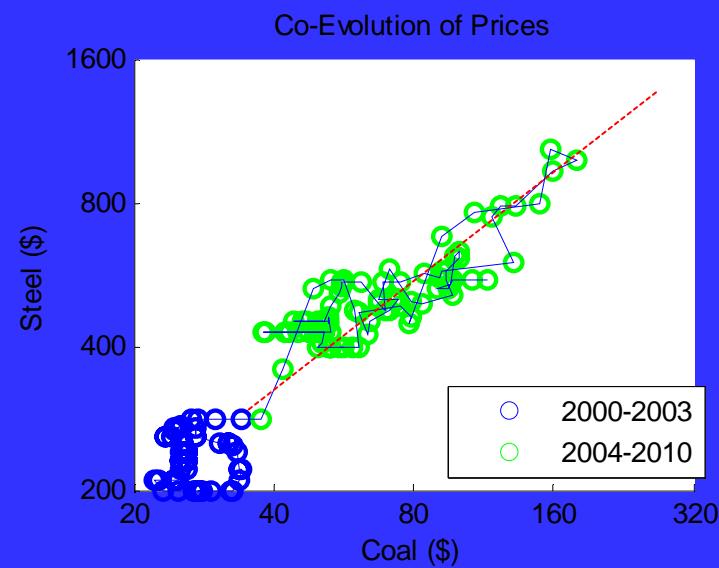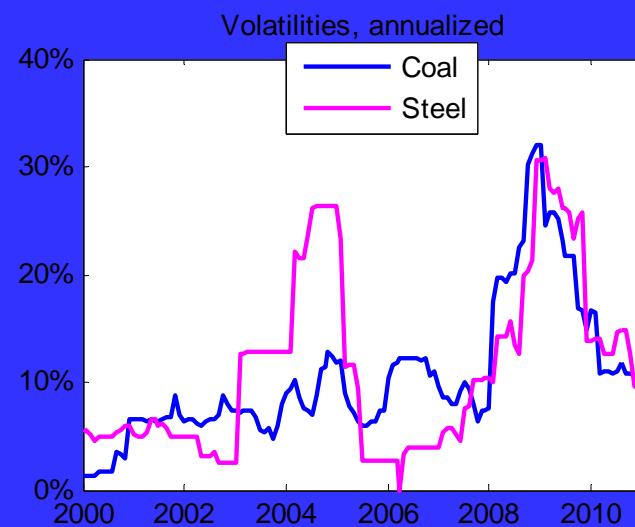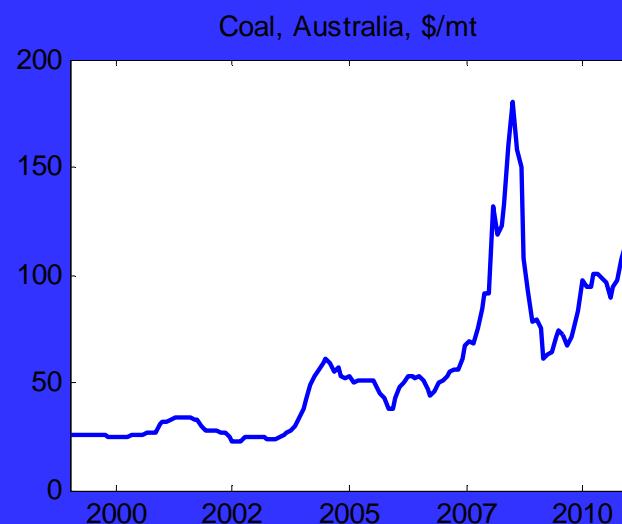
Metals vs. Energy :
A Regime Shift in Prices and Correlation

Energy Index

Minerals Index

Volatilities, annualized

Correlation (rolling 12 months window)

Co-Evolution of Prices

Sources : World Bank Energy Index (oil, gas, coal) vs Minerals Index (aluminum, copper, tin, lead, nickel, zinc, iron ore)

Co-Evolution of Prices

Sources : World Bank Energy Index (oil, gas, coal) vs Minerals Index (aluminum, copper, tin, lead, nickel, zinc, iron ore)