Optimal exit strategies for investment projects

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Motivations : Optimal Exit Strategy

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1. ”Patient” firms

   - Continuously estimate the theoretical/fair price

   - Wait the time when the project becomes profitable again or equivalently wait for a buyer at that price.

2. ”Impatient” firms

   - Find a buyer for the project

   - Immediately sell even at a discounted price w.r.t. the theoretical/fair price

3. ”Mixed” strategies

   - Continuously estimate the theoretical/fair price

   - Wait for a buyer at that price

   - If a buyer proposes an ”acceptable” discounted price, immediately sell

Parallelism with finance : Selling an illiquid asset

- ”Impatient” trading : Optimal portfolio selection with transaction costs, Optimal portfolio liquidation,..

  → the trader pays liquidity costs.

- ”Patient” trading : Optimal portfolio liquidation with limit orders (Bayraktar, Ludkowski 2012)

  → the asset is sold at a random time τ (execution and inventory risks).

- ”Mixed” trading : Market making (Guilbaud, Pham 2011)

  → tradeoff between costs and risks

- Model and problem formulation
- Dynamic programming system and properties of v_i
- Logarithmic utility
- Power utility

Basic problem definition

- We consider a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with a filtration $\mathcal{F} = (\mathcal{F}_t)_{t\geq 0}$ satisfying the usual conditions.
- Let W and B be two correlated $(\mathcal{F}_t)$-Brownian motions, with correlation ρ.
- Theoretical/fair value of the firm project evolving according a positive process S, which may be written as $S_t := \exp(X_t)$, where the process X is governed by the following s.d.e.

$$\begin{aligned} dX_t &= \mu(X_t)dt + \sigma(X_t)dB_t \\ X_0 &= x, \end{aligned}$$

where μ and σ are two Lipschitz functions with sub-linear growth conditions.

Termination costs

- **liquidation process** Y, is a mean-reverting process and governed by the following s.d.e.

$$\begin{aligned} dY_t &= \alpha(Y_t)dt + \gamma(Y_t)dW_t, \\ Y_0 &= y, \end{aligned}$$

  where α and γ are locally Lipschitz functions on $\mathbb{R}^+$.

- **Stochastic discount factor** : $(f(Y_t))_{t\geq 0}$, where f is a positive, continuous and decreasing function defined on $\mathbb{R}^+ \rightarrow [0,1]$, and satisfies the following conditions :

$$\begin{aligned} f(0) &= 1 \\ \lim_{y\to\infty} f(y) &= 0 \end{aligned}$$

- **Discount price** : Should the firm manager decide to sell immediately the asset at the discount price, she would obtain a cash-flow of $S_t f(Y_t)$.

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The process Y may be interpreted as a “lack of counterpart” process measuring the degree of “illiquidity” of the assets. In particular, when Y goes to infinity, the discount factor $f(Y)$ goes to zero, i.e. there is no counter-part.

The recovery time

We introduce an external time τ representing the time when the project becomes profitable again or the arrival of a buyer accepting to pay the fair price.

- **Intensity regimes** :
  Let L be a continuous time, time homogenous, irreductible Markov chain, independent of W and B, with $m+1$ states.
  The generator of the chain L under $\mathbb{P}$ is denoted by $A = (\vartheta_{i,j})_{i,j=0,\ldots n}$. Here $\vartheta_{i,j}$ is the constant intensity of transition of the chain L from state i to state j.

- **Break-even time** :
  The arrival time, denoted by τ, is defined as the first jump time of a Cox process with an intensity $(\lambda_{L_t})_{t\geq 0}$.
  τ is independent of W and B and, without lost of generality we assume

$$\lambda_0 \geq \lambda_1 \geq \ldots \geq \lambda_m > 0$$

Utility function

- **Classical assumptions** :
  - $U : \mathbb{R}^+ \to \mathbb{R}$, is non-decreasing, concave and belongs to $\mathcal{C}^2(\mathbb{R}^+)$.
  - U **has the following behavior**

$$\lim_{s\to 0}\ s\,U'(s) \quad < \quad +\infty$$

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- **Supermeanvalued utility** : (Dinkyn, Oksendal) The firm manager is "coherent and rational" : we suppose that U is supermeanvalued w.r.t. S, i.e.

$$U(s) \geq \mathbb{E}^s[U(S_\theta)]$$

for $s \geq 0$ and any stopping time $\theta \in \mathcal{T}$ where $\mathcal{T}$ is the collection of all $\mathcal{F}-$stopping times.

An optimal stopping problem with regime switching

- Maximizing the expected utility of the wealth received from the sales of the asset.

Objective function : For $x \in \mathbb{R}$, $y \in \mathbb{R}^+$, $i \in \{0, \dots, m\}$, we set

$$v(i,x,y) \quad := \quad \sup_{\theta \in \mathcal{T}} \mathbb{E}^{i,x,y} \left[h(X_\theta, Y_\theta)\, \mathbb{I}_{\theta \leq \tau} + U(e^{X_\tau})\, \mathbb{I}_{\theta > \tau} \right] ,$$

where $\mathbb{E}^{i,x,y}$ denotes the flow with initial condition $X_0 = x$, $Y_0 = y$ and $L_0 = i$ and $h(x,y) = U(\exp(x) f(y))$.

Dynamic programming system and properties of v_i

- Model and problem formulation
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Objective functions bounds and monotonicity

Proposition

The value functions v_i are non-decreasing in x, non-increasing in y and satisfy

$$\text{Max}\left(h(x,y), \mathbb{E}^x[U(e^{X_\tau})]\right) \leq v(i,x,y) \leq U(e^x) \text{ on } \mathbb{R} \times \mathbb{R}^+ .$$

Proof :

- Monotonicity of U and f.
- Uniqueness of SDEs solution, continuity of processes X and Y (up to $\xi_y := \inf\{t > 0, Y_t^y = 0\}$)
- Supermeanvalued assumption

Remarks

- We have $v(i,x,0) = U(e^x)$: it is optimal to immediately exit from the project when discount factor reaches 1.
- If $\mathbb{E}^x[U(e^{X_\tau})] = U(e^x)$: the optimal policy is to wait until the arrival of the break-even time

Continuity of the value functions

Continuity of the value functions

The value functions v_i are continuous on $\mathbb{R} \times \mathbb{R}^+$ and satisfy :

$$\lim_{(u,y)\to(x,0^+)} v_i(u,y) = v_i(x,0) = U(e^x).$$

Proof

- Continuity of stochastic flows (up to $\inf\{t > 0, Y_t^y = 0\}$).

Lemma

There exists an optimal stopping time $\theta^*_{i,x,y}$ such that

$$v(i,x,y) := \mathbb{E}^{i,x,y}\left[h(X_{\theta^*_{i,x,y}}, Y_{\theta^*_{i,x,y}})\,\mathbb{I}_{\theta^*_{i,x,y}\leq\tau} + U(e^{X_\tau})\,\mathbb{I}_{\theta^*_{i,x,y}>\tau}\right]$$

Moreover, $\theta^*_{i,x,y} \leq \xi_y \wedge \tau$ where $\xi_y := \inf\{t \geq 0; Y_t^y = 0\}$.

- Supermeanvalued utility assumption
- $\lim_{s\to0}\ s\ U'(s) < +\infty$

Viscosity Characterization of value function

Theorem

The value functions v_i, $i \in \{0, \dots, m\}$, are the unique continuous viscosity solutions on $\mathbb{R} \times \mathbb{R}^+$ with growth condition $v_i(x, y) \leq U(e^x)$, and boundary data $\lim_{y \downarrow 0} v_i(x, y) = U(e^x)$, to the system of variational inequalities :

$$\min \Big[-\mathcal{L}v(i, x, y) - \mathcal{G}_i v(., x, y) - \mathcal{J}_i v(., x, y) \, , \, v(i, x, y) - h(x, y) \Big] = 0,$$

where, for functions $\varphi : \{0, \dots, m\} \times \mathbb{R} \times \mathbb{R}^+ \to \mathbb{R}$ with $\varphi(i, \cdot, \cdot) \in \mathcal{C}^2(\mathbb{R} \times \mathbb{R}^+)$ for all $i \in \{0, \dots, m\}$, we have set

$$\mathcal{L}\phi(x, y) = \mu(x)\frac{\partial \phi}{\partial x} + \alpha(y)\frac{\partial \phi}{\partial y} + \frac{1}{2}\sigma^2(x)\frac{\partial^2 \phi}{\partial x^2} + \rho\gamma(y)\sigma(x)\frac{\partial^2 \phi}{\partial x \partial y} + \frac{1}{2}\gamma^2(y)\frac{\partial^2 \phi}{\partial y^2} \, .$$

and $\mathcal{G}_i$ and $\mathcal{J}_i$ act on functions φ :

$$\begin{aligned} \mathcal{G}_i\varphi(., x, y) &= \sum_{j \neq i} \vartheta_{i,j} \left(\varphi(j, x, y) - \varphi(i, x, y) \right) \\ \mathcal{J}_i\varphi(., x, y) &= \lambda_i \left(U(e^x) - \varphi(i, x, y) \right) . \end{aligned}$$

Proof of uniqueness

Comparaison principle

Let $(\phi_i)_{0\le i\le m}$ (resp. $(\psi_i)_{0\le i\le m}$) a family of continuous subsolution (resp. super solution) of the VI system on $\mathbb{R}\times\mathbb{R}^+$ satisfying the following growth conditions on $\{0,..,m\}\times\mathbb{R}\times\mathbb{R}^+$

$$\text{Max}\left(h(x,y),\mathbb{E}^{i,x}[U(e^{X_\tau})]\right)\le\phi(i,x,y)\ (\text{resp. }\psi(i,x,y))\le U(e^x)\text{ on }\mathbb{R}\times\mathbb{R}^+$$

and $\lim_{y\to 0^+}\phi_i(x,y)\le\lim_{y\to 0^+}\psi_i(x,y)$. We have $\phi\le\psi$ on $\{0,..,m\}\times\mathbb{R}\times\mathbb{R}^+_*$.

Proof.

- **Step 1.** Construction of strict super-solutions to the system with suitable perturbations of ψ_i :
  -
  $$\psi_i^\gamma=(1-\gamma)\psi_i+\gamma\eta_i$$
  - $(\psi_i^\gamma)_{(i=1,\ldots,n)}$ is a strict super-solution to the VI system.
- **Step 2.** It suffices to show (by contradiction) that for all $\gamma\in(0,1)$:

$$\max_{i\in\{0,..,m\}}\ \sup_{\mathbb{R}\times\mathbb{R}^+_*}(\phi_i-\psi_i^\gamma)\le 0,$$

Proof of uniqueness

► Technical point : Construct a strict super solution which dominates ϕ_i and ψ_i.

→ The following function is a strict super solution which dominates $U(e^x)$

$$w(s) = \begin{cases} s^4 + k + A_0 + A_1 s + \frac{1}{2} A_2 s^2 & s \leq 0 \\ s^4 + k + U(e^s) \ln(4+s) & s > 0 \end{cases}$$

where A_0, A_1, A_2 and k are strictly positive constants.

→ Existence of such strict super solution follows from characterization of $\mathbb{E}^{i,x}[U(e^{X_\tau})]$ as solution of a system of PDE.

Stopping and continuation regions

- **Stopping and continuation regions**

$$\begin{aligned} \mathcal{E} &= \left\{(i,x,y) \in \{0,\dots,m\} \times \mathbb{R} \times \mathbb{R}^+ \,|\, v(i,x,y) = h(x,y)\right\} \\ \mathcal{C} &= \{0,\dots,m\} \times \mathbb{R} \times \mathbb{R}^+ \setminus \mathcal{E}. \end{aligned}$$

  We also define the $(i,x)-$sections for every $(i,x) \in \{0,\dots,m\} \times \mathbb{R}$ by

$$\mathcal{E}_{(i,x)} = \{y \in (0,+\infty) \,|\, v(i,x,y) = h(x,y)\} \text{ and } \mathcal{C}_{(i,x)} = \mathbb{R}^+ \setminus \mathcal{E}_{(i,x)}.$$

- **Optimal exit time** : $\theta^*_{ixy} = \inf\left\{u \geq 0 \,|\, \left(L^i_u, X^x_u, Y^y_u\right) \in \mathcal{E}\right\}$.

Properties of the stopping region

Let $(i,x) \in \{0,\dots,m\} \times \mathbb{R}$.

- If $\mathbb{E}^{i,x}[U(e^{X_\tau})] = U(e^x)$, then, for all $y \in \mathbb{R}^+$, $v(i,x,y) = U(e^x)$ and $\mathcal{E}_{(i,x)} = \{0\}$.
- If $\mathbb{E}^{i,x}[U(e^{X_\tau})] < U(e^x)$, then $\exists x_0$ such that $\mathcal{E}_{(i,x_0)} \setminus \{0\} \neq \emptyset$ and $\bar{y}^*(i,x) := \sup \mathcal{E}_{(i,x)} < +\infty$.

- Model and problem formulation
- Dynamic programming system and properties of v_i
- **Logarithmic utility**
- Power utility

Logarithmic utility

Throughout this section we assume that

- $U(s) = \ln(s)$ on $\mathbb{R}_*^+$.
- X and Y solutions of the following SDEs :

$$\begin{aligned} dX_t &= \mu dt + \sigma(X_t) dB_t;\ X_0 = x \\ dY_t &= \kappa\left(\beta - Y_t\right) dt + \gamma\sqrt{Y_t} dW_t;\ Y_0 = y \end{aligned}$$

Remarks

- The supermean value assumption implies that $\mu \leq 0$.
- If $\mu = 0$, we have seen that $v(i,x,y) = U(e^x)$ and $\mathcal{E}_{(i,x)} = \{0\}$

Dimension reduction

Proposition

For $(i, y) \in \{1, ..., m\} \times \mathbb{R}^+$ we define the function :

$$w(i, y) = \sup_{\theta \in \mathcal{T}_{L,W}} \mathbb{E}^{i,y}[\mu(\theta \wedge \tau) + \ln(f(Y_\theta)) \mathbb{I}_{\{\theta \leq \tau\}}],$$

where $\mathcal{T}_{L,W}$ is the set of stopping times with respect to the filtration generated by (L, W). We have

$$v(i, x, y) = x + w(i, y) \text{ on } \{1, ..., m\} \times \mathbb{R} \times \mathbb{R}^+.$$

Proof.

- On $\{0, ..., m\} \times \mathbb{R} \times \mathbb{R}^+$, we have

$$v(i, x, y) = \sup_{\theta \in \mathcal{T}} \mathbb{E}^{i,x,y}[X_{\theta \wedge \tau} + \ln(f(Y_\theta)) \mathbb{I}_{\{\theta \leq \tau\}}].$$

- We prove that $\frac{\partial v}{\partial x}(i, x, y) = 1$ then $v(i, x, y) = x + \phi(i, y)$
- An optimal stopping time is $\theta^*_{ixy} = \inf\{t \geq 0 : \ \phi(L^i_t, Y^y_t) = \ln(f(Y^y_t))\}$, which belongs to $\mathcal{T}_{L,W}$. We obtain $\phi = w$.

No regime switch

Let $i \in \{1, \ldots, m\}$. Throughout this section, we shall assume that $\vartheta_{i,j} = 0 \; \forall i \neq j$ and that there exists $0 < y_i^*$ such that $\mathcal{E}_{(i,x)} = [0, y_i^*]$.

Proposition

y_i^* is the solution of the following equation

$$\frac{g(y_i^*) - \frac{\mu}{\lambda_i}}{g'(y_i^*)} = -\frac{\gamma^2}{2\lambda_i} \frac{\Psi\left(\frac{\lambda_i}{\alpha}, \frac{2\alpha\beta}{\gamma^2}, \frac{2\alpha}{\gamma^2} y_i^*\right)}{\Psi\left(\frac{\lambda_i}{\alpha} + 1, \frac{2\alpha\beta}{\gamma^2} + 1, \frac{2\alpha}{\gamma^2} y_i^*\right)} .$$

the function $w(i, \cdot)$ is given by

$$w(i, y) = \begin{cases} g(y) & y \leq y_i^* \\ \dfrac{g(y_i^*) - \frac{\mu}{\lambda_i}}{\Psi\left(\frac{\lambda_i}{\alpha}, \frac{2\alpha\beta}{\gamma^2}, \frac{2\alpha}{\gamma^2} y_i^*\right)} \Psi\left(\frac{\lambda_i}{\alpha}, \frac{2\alpha\beta}{\gamma^2}, \frac{2\alpha}{\gamma^2} y\right) + \frac{\mu}{\lambda_i} & y > y_i^* \end{cases}$$

where Ψ denotes the confluent hypergeometric function of second kind

Two regimes

We assume that $m = 1$, $\vartheta_{0,1}\vartheta_{1,0} \neq 0$ and that, for $i \in \{0, 1\}$, there exists $y_i^* > 0$ such that $\mathcal{E}_{(i,s)} = [0, y_i^*]$.

Proposition

We can show that $y_0^* \leq y_1^*$

Let Λ be the matrix

$$\Lambda = \begin{pmatrix} \lambda_0 + \vartheta_{0,1} & -\vartheta_{0,1} \\ -\vartheta_{1,0} & \lambda_1 + \vartheta_{1,0} \end{pmatrix}$$

As $\vartheta_{0,1}\vartheta_{1,0} > 0$ it is easy to check that Λ has two eigenvalues $\widetilde{\lambda}_0$ and $\widetilde{\lambda}_1 < \widetilde{\lambda}_0$. Let $\widetilde{\Lambda} = P^{-1}\Lambda P$ be the diagonal matrix with diagonal $(\widetilde{\lambda}_0, \widetilde{\lambda}_1)$. The transition matrix P is denoted by

$$P = \begin{pmatrix} p_0^0 & p_1^0 \\ p_0^1 & p_1^1 \end{pmatrix}.$$

Two regimes

Proposition

The function $w(1,\cdot)$ is given by

$$w(1,y) = \begin{cases} g(y) & y \in [0, y_1^*] \\ p_0^1 \left[\widehat{e}\Psi\left(\frac{\widetilde{\lambda}_0}{\alpha}, \frac{2\alpha\beta}{\gamma^2}, \frac{2\alpha}{\gamma^2}x \right) + \frac{\mu}{\widetilde{\lambda}_0} \right] & y \in (y_1^*, \infty) \\ \qquad + p_1^1 \left[\widehat{f}\Psi\left(\frac{\widetilde{\lambda}_1}{\alpha}, \frac{2\alpha\beta}{\gamma^2}, \frac{2\alpha}{\gamma^2}y \right) + \frac{\mu}{\widetilde{\lambda}_1} \right] & \end{cases}$$

where Ψ denotes the confluent hypergeometric function of second kind, $\mathcal{I}$ is a particular solution to the non-homogeneous confluent differential equation.

Two regimes

Proposition

The function $w(0,\cdot)$ is given by

$$w(0,y) = \begin{cases} g(y) & y \in [0, y_0^*] \\ \widehat{c}\Phi\left(\frac{\lambda_0+\vartheta_{0,1}}{\alpha}, \frac{2\alpha\beta}{\gamma^2}, \frac{2\alpha}{\gamma^2}y\right) + \widehat{d}\Psi\left(\frac{\lambda_0+\vartheta_{0,1}}{\alpha}, \frac{2\alpha\beta}{\gamma^2}, \frac{2\alpha}{\gamma^2}y\right) & y \in (y_0^*, y_1^*] \\ \quad +\mathcal{I}\left(\frac{2\alpha}{\gamma^2}, \beta, -2\frac{\lambda_0+\vartheta_{0,1}}{\gamma^2}, 2\frac{\vartheta_{0,1}g(\cdot)+\mu}{\gamma^2}\right)(y) & \\ p_0^0\left[\widehat{e}\Psi\left(\frac{\widetilde{\lambda}_0}{\alpha}, \frac{2\alpha\beta}{\gamma^2}, \frac{2\alpha}{\gamma^2}y\right) + \frac{\mu}{\widetilde{\lambda}_0}\right] & y \in (y_1^*, \infty) \\ \quad +p_1^0\left[\widehat{f}\Psi\left(\frac{\widetilde{\lambda}_1}{\alpha}, \frac{2\alpha\beta}{\gamma^2}, \frac{2\alpha}{\gamma^2}y\right) + \frac{\mu}{\widetilde{\lambda}_1}\right] & \end{cases}$$

where Φ and Ψ denote respectively the confluent hypergeometric function of first and second kind, $\mathcal{I}$ is a particular solution to the non-homogeneous confluent differential equation.

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- **Power utility**

Power utility

Throughout this section we assume that

- $U(s) = s^a$ on $\mathbb{R}_*^+$ with $0 < a \leq 1$.
- There exists μ and σ in $\mathbb{R}$ s.t. X is solution of the following sde :

$$dX_t = \mu dt + \sigma dB_t$$

Remarks

- The supermean value assumption implies that $\mu a + \frac{\sigma^2}{2} a^2 \leq 0$.
- If $\mu a + \frac{\sigma^2}{2} a^2 = 0$, we have seen that $v(i,x,y) = U(e^x)$ and $\mathcal{E}_{(i,x)} = \{0\}$

Dimension reduction

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For $(i, y) \in \{1, \ldots, m\} \times \mathbb{R}^+$ we define the function :

$$u(i, y) = \sup_{\theta \in \mathcal{T}_{L,W}} \mathbb{E}^{i,y}[e^{(\mu + (1-\rho^2)\frac{\sigma^2}{2})(\theta \wedge \tau) + \rho \sigma W_{\theta \wedge \tau}} \left(\mathbb{I}_{\{\theta > \tau\}} + g(Y_\theta)\mathbb{I}_{\{\theta \leq \tau\}}\right)].$$

We have

$$v(i, x, y) = e^{ax} u(i, y) \text{ on } \{1, \ldots, m\} \times \mathbb{R} \times \mathbb{R}^+.$$

$(u(i, \cdot))_{0 \leq i \leq m}$ are the unique continuously differentiable viscosity solutions of the system of equation :

$$\min \Big[- \tilde{\mathcal{L}} u(i, y) - \lambda_i (1 - u(i, y)) - \sum_{j \neq i} \vartheta_{i,j} \left(u(j,\, y) - u(i,\, y)\right) \,,\, u(i, y) - g(y) \Big] = 0\,.$$

where we have set $g(y) = (f(y))^a$ and, for $\phi \in \mathcal{C}^1(\mathbb{R}^+)$,

$$\tilde{\mathcal{L}}\phi(y) = \frac{1}{2}\gamma^2 y \frac{\partial^2 \phi}{\partial y^2} + \left[\alpha(\beta - y) + \rho\sigma\gamma a\sqrt{y}\right] \frac{\partial \phi}{\partial y} + \left[\frac{\sigma^2 a^2}{2} + \mu a\right]\phi(y).$$

Conclusions

- Study of a optimal exit strategy from a firm project
- Mathematical characterization of the objective function
- Explicit solutions for specific utility functions (power and logarithm)

Thank you for your attention

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