

Ambiguity Aversion in Standard and Extended Ellsberg Frameworks: α -Maxmin versus Maxmin Preferences

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7th General AMaMeF and Swissquote Conference 2015

SwissTech Convention Center, EPFL, Switzerland

September 7-10, 2015



University of
Zurich ^{UZH}

Outline

Motivation

Maxmin and α -maxmin expected utility models

Market Model

Standard Ellsberg framework

Extended Ellsberg frameworks

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Ambiguity

Two kinds of uncertainty (distinction due to Knight (1921))

1. *Risk*: all events are associated with obvious probability assignments
2. *Ambiguity* or *Knightian uncertainty*: some events do not have an obvious, unanimously agreeable, probability assignment

Facing *ambiguity*, decision makers may adjust their behaviour:

- ambiguity *averse*: take action robust to ambiguity
- ambiguity *seeking*: take action exposed to ambiguity

Ambiguity sensitive preferences and market phenomena

1. Experimental evidence:

majority of subjects are ambiguity averse, a few ambiguity seeking

2. Theoretical financial market models:

different attitude towards ambiguity are consistent with non participation, portfolio inertia, and excess of volatility of assets returns

⇒

Investors' preferences: *heterogeneous*, well approximated by SEU and ambiguity averse preferences with *different* degree of ambiguity aversion

Ambiguity sensitive behaviors are *inconsistent with SEU theory* ⇒

Several models to model ambiguity sensitive preferences

We focus on: *maxmin* expected utility and *α -maxmin* expected utility

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Maxmin expected utility model Gilboa–Schmeidler (1989)

maxmin *pioneering and workhorse* model to study the implications of ambiguity aversion in portfolio choice and equilibrium asset prices

$$U(w) = \min_{\pi \in \mathcal{C}} \sum_{\sigma \in S} u(w_{\sigma}) \pi_{\sigma}$$

- ▶ \mathcal{C} set of priors *determined jointly by agent's information and personal taste*
- ▶ *ambiguity averse attitude*: portfolio w evaluated by its minimum expected utility
- **tractable optimization problems**: u concave $\Rightarrow U$ concave
- **information and attitude to ambiguity inextricably intertwined in \mathcal{C}** : smaller sets may reflect both better information and less averse ambiguity attitude

α -maxmin expected utility model

α -maxmin expected utility (α -MEU) model

$$U(w) = \alpha \min_{\pi \in \mathcal{C}} \sum_{\sigma \in S} u(w_{\sigma}) \pi_{\sigma} + (1 - \alpha) \max_{\pi \in \mathcal{C}} \sum_{\sigma \in S} u(w_{\sigma}) \pi_{\sigma}, \quad \alpha \in [0, 1]$$

generalizes the *maxmin* model ($\alpha = 1$)

- large spectrum of ambiguity attitude: from the ambiguity aversion of the *maxmin* model to ambiguity seeking of the *maxmax* model ($\alpha = 0$)
- given $\mathcal{C} = \mathcal{C}_{\max} = \{\text{all priors consistent with available information}\}$ the parameter α is a measure the degree of agent's ambiguity aversion: the greater α the more ambiguity averse the agent's preferences
- portfolio optimization problem not concave: U in general not concave despite u concave
- no axiomatization of the model

α -MEU model in the literature

α -MEU model increasingly popular

▶ *decision theory*

▶ *Experimental financial economics*

α -MEU for the estimation of the agent's degree of ambiguity aversion/seeking; *Ahn et al. (2011)*

▶ *Theoretical financial economics*: just a few

competitive financial market with SEU and α -MEU agents;
Bossaerts, Ghirardato Guarnaschelli and Zame (2010) (BGGZ)

Setting of the above papers: *Standard Ellsberg framework BUT* in this setting ambiguity averse α -MEU preferences *reduce to* maxmin preference!

Goal of this paper

Financial economics point of view:

derive the implications of α -MEU model for

- ▶ *portfolio choices*
- ▶ *equilibrium asset prices*

as a function of the degree of ambiguity aversion $\alpha \in (0, 1)$

This allows to

- provide criteria to estimate's agent's ambiguity aversion
- understand the α -MEU model and the preferences that it represents
- contrast α -MEU and *maxmin* model implications

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Setting

Arrow–Debreu complete market with two dates: $t = 0$ and $t = 1$ embedded in an Ellsberg framework

· S finite state space with $m + l$ states of the economy at $t = 1$ states correspond to draws from Ellsberg (1961) urn:

$m \geq 0$ risky states and $l \geq 2$ ambiguous states

· At $t = 0$ investors face *risk* and *ambiguity*:

neither know which state will realize at $t = 1$ (*risk*), nor what is probability of some states (*ambiguity*)

Setting (cont.)

· $\forall \omega \in S, \exists$ Arrow security traded in the market \Rightarrow agents can form:

▶ *unambiguous* portfolio, i.e. portfolio with no exposure to ambiguity:

$$w = \left(\underbrace{w_R^1, \dots, w_R^m}_{m \text{ risky states}}, \underbrace{\tilde{w}, \dots, \tilde{w}}_{\text{ambiguous states}} \right)$$

▶ *ambiguous* portfolios, i.e. portfolios exposed to ambiguity:

$$w = \left(\underbrace{w_R^1, \dots, w_R^m}_{m \text{ risky states}}, \underbrace{w^{m+1}, \dots, w^{m+l}}_{\text{ambiguous states}} \right)$$

· u (risk aversion) *strictly concave and increasing and differentiable*

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Standard Ellsberg framework ($m = 1, l = 2$)

$$S = \{R, G, B\}$$

$m = 1$ risky state, R , with known probability $\pi_R \in (0, 1)$

$l = 2$ ambiguous states, G and B , with unknown probabilities

Any set of priors consistent with the Ellsberg framework reads

$$\mathcal{D}_{a,b} = \{(\pi_R, q, 1 - q - \pi_R) : q \in [a, b]\}$$

for some given a and b , $0 \leq a \leq b \leq 1 - \pi_R \Rightarrow$

Any α -MEU utility with $\alpha \in [0, 1]$ reads

$$\begin{aligned} U(w) = \pi_R u(w_R) &+ \alpha \min_{q \in [a,b]} [q u(w_G) + (1 - q - \pi_R) u(w_B)] \\ &+ (1 - \alpha) \max_{q \in [a,b]} [q u(w_G) + (1 - q - \pi_R) u(w_B)] \end{aligned}$$

w is a state dependent portfolio $w = (w_R, w_G, w_B) \in \mathbb{R}^3$

Main findings in the Standard Ellsberg framework

- ▷ **Equivalence result:** α -*MEU* preferences are equivalent to:
 - ▷ *maxmin* preferences when $\alpha > 1/2$
 - ▷ *SEU* preferences when $\alpha = 1/2$
 - ▷ *maxmax* preferences when $\alpha < 1/2$

- ▷ **Ambiguity aversion implications on equilibrium asset prices:**
ambiguity aversion does not wash out in equilibrium

Equivalence result

When $\alpha > 1/2$:

any α -MEU utility, $\alpha > 1/2$, set of prior $\mathcal{D}_{a,b}$ is equal to a *unique maxmin* utility with set of priors \mathcal{C} smaller than $\mathcal{D}_{a,b}$, and univocally characterized by α and $\mathcal{D}_{a,b}$:

$$U(w) = \min_{q \in [c,d]} [\pi_R u(w_R) + q u(w_G) + (1 - q - \pi_R) u(w_B)]$$

$$\mathcal{C} = \{(\pi_R, q, 1 - q - \pi_R) : q \in [c, d]\} \subset \mathcal{D}_{a,b}$$

$$c := \alpha a + (1 - \alpha)b \text{ and } d := (1 - \alpha)a + \alpha b$$

decreasing α from 1 to $1/2$ in the α -MEU representation, is equivalent to *shrinking symmetrically* \mathcal{C} in the corresponding *maxmin* representation

Equivalence result (cont.)

Implications of the equivalence result:

- ▶ Standard Ellsberg framework is *not* the right setting to study the α -MEU model: α -MEU preferences cannot be distinguished from maxmin, or maxmax or SEU preferences
- ▶ Experimental studies:
 - ▶ it *clarifies* recent works in the standard Ellsberg framework that uses the α -MEU as generalization of the maxmin model to document a substantial heterogeneity in aversion to ambiguity; e.g. BGGZ (2010), and Ahn *et al.* (2011)
 - ▶ *Same conclusion by using maxmin* varying the size of the set of priors instead of α to measure the degrees of aversion to ambiguity

Implications of ambiguity aversion on equilibrium asset prices

Financial market populated by $L + M$ investors

- L with **SEU** preferences with prior $\pi = (\pi_R, \pi_G, \pi_B)$
 - M with **ambiguity averse** preferences (maxmin or equivalently α -MEU $\alpha > 1/2$)
1. we derive *equilibrium asset prices* and show through which channels ambiguity aversion impacts equilibrium asset prices
 2. *theoretical* equilibrium asset prices *perfectly match* the *experimental* findings in BGGZ (2010)
- \Rightarrow *Ambiguity aversion does not wash out in equilibrium*

Portfolio choice of SEU and ambiguity averse agents

Well known facts:

- ▶ **SEU agent** optimal portfolio $y = (y_R, y_G, y_B)$ state dependent wealth is ranked opposite to the state-price/state-probability ratios:

$$y_\sigma > y_\nu \Leftrightarrow \frac{p_\sigma}{\pi_\sigma} < \frac{p_\nu}{\pi_\nu}, \quad \sigma, \nu \in \{R, B, G\}$$

- ▶ **ambiguity averse agent** with utility

$$U(w) = \pi_R u(w_R) + \min_{q \in [c, d]} (q u(w_G) + (1 - q - \pi_R) u(w_B)),$$

optimal portfolio $w = (w_R, w_G, w_B)$:

$$\left\{ \begin{array}{l} w_G > w_B \\ w_G < w_B \\ w_G = w_B \text{ (unambiguous)} \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} \frac{p_G}{p_B} < \frac{c}{1 - \pi_R - c} \\ \frac{p_G}{p_B} > \frac{d}{1 - \pi_R - d} \\ \frac{p_G}{p_B} \in \left[\frac{c}{1 - \pi_R - c}, \frac{d}{1 - \pi_R - d} \right] \end{array} \right.$$

The larger the set of priors, the more likely $w_B = w_G$

Market equilibrium: theoretical findings

State-price/state-probability ratios in equilibrium depend on the market total endowment $W = (W_R, W_G, W_B)$

Case of interest:

- ▶ $W_G \neq W_B$, for instance $W_G > W_B$
- ▶ ambiguity averse agents take *unambiguous* portfolio, i.e. $w_G = w_B$

SEU agents have to clear the supply difference $W_G - W_B \Rightarrow \frac{p_B}{\pi_B} > \frac{p_G}{\pi_G}$

Intuition: to induce SEU agents to clear $W_G - W_B$ p_G will be lower and p_B be higher than in a market populated only by SEU agents sharing the same prior

Market equilibrium: theoretical findings (cont.)

Proposition A:

Suppose $W_G > W_B > W_R \Rightarrow$ **two** rankings are possible in equilibrium:

1. $\frac{p_B}{\pi_B} > \frac{p_R}{\pi_R} > \frac{p_G}{\pi_G}$, $y_G > y_R > y_B$ and $w_R < w_G = w_B$
2. $\frac{p_R}{\pi_R} > \frac{p_B}{\pi_B} > \frac{p_G}{\pi_G}$, $y_G > y_B > y_R$ and $w_R < w_G = w_B$

When ranking 1. realizes ($W_G - W_B$ is "large enough") \Rightarrow

- the optimal portfolios of the L SEU agents *do not* rank as the total endowment
- the total endowment is the optimal portfolio of the SEU-representative agent rationalizing the market equilibrium

\Rightarrow SEU-representative agent and SEU agents in the market will rank the state-price/state-probability ratio *differently*

CARA utility $u(z) = 1 - e^{-\delta z} / \delta$

- $\delta = a$ risk aversion of the L SEU agents
- $\delta = b =$ risk aversion of the M ambiguity averse agents

The equilibrium prices resulting is the same as if, with the L SEU,

→ instead of the M ambiguity averse agents we would have

→ M SEU agents with prior $(\pi_R, q, 1 - \pi_R - q)$:

$$q := \pi_G \frac{\pi_G + \pi_B}{\pi_G + \pi_B e^{\frac{a}{L}(W_G - W_B)}}$$

Dependence of q on $\frac{a}{L}(W_G - W_B)$: channel through which ambiguity aversion impacts asset prices

The M ambiguity averse agents do not hold the imbalance $W_G - W_B$, which is left to the L SEU agents to clear:

$$\frac{a}{L}(W_G - W_B) \uparrow \Rightarrow q \downarrow \text{ and } (1 - \pi_R - q) \uparrow \Rightarrow p_G \downarrow \text{ and } p_B \uparrow$$

Matching theoretical and BGGZ's experimental findings

BGGZ (2010): experimental sessions in which a competitive financial market is embedded in the standard Ellsberg framework.

Their findings:

- ▶ individuals are divided between SEU and ambiguity averse
- ▶ *experimental evidence that ambiguity aversion matters for equilibrium prices*

Our theoretical equilibrium prices fully explain and theoretically justify their experimental findings

BGGZ interpret their experimental findings using market with SEU and ambiguity averse α -MEU investors

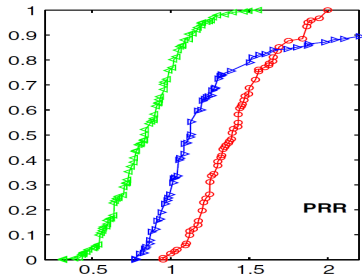
BGGZ do not derive the equilibrium asset prices, only provide conjectures

Matching theoretical and BGGZ's experimental findings

(cont.)

Empirical distribution functions of state-price/state-probability ratios from BGGZ (2010) when $W_G = 272$, $W_B = 162$, and $W_R = 81$.

p_R/π_R ; p_B/π_B ; p_G/π_G (Figure 8, right panel, in BGGZ (2010))



Two experimental rankings: $\frac{p_B}{\pi_B} > \frac{p_R}{\pi_R} > \frac{p_G}{\pi_G}$ and $\frac{p_R}{\pi_R} > \frac{p_B}{\pi_B} > \frac{p_G}{\pi_G}$

These are exactly the theoretical rankings 1. and 2. in *Proposition A*

Matching theoretical and BGGZ's experimental findings

(cont.)

BGGZ (2010) *only expect ranking 1.* $p_B/\pi_B > p_R/\pi_R > p_G/\pi_G$

Proposition A shows that Ranking 2. prevails when $W_B - W_R$ is large enough to imply an SEU optimal portfolios $y_G > y_B > y_R$

Proposition A suggests a possible explanation of why in the experiment prices do not settle in favor of Ranking 1. or Ranking 2.

W_R , W_G , and W_B in the experiment are close to the point at which the change from Ranking 1. or Ranking 2 takes place

Proposition A predicts: to observe a clean separation of the two rankings, the aggregate wealth W_B should be chosen closer to W_R or W_G , respectively

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Extended Ellsberg framework ($m \geq 0$, $l \geq 3$)

$m = 1$ risky state, R , with known probability $\pi_R \in (0, 1)$

$l \geq 3$ ambiguous states; $A := S \setminus \{R\}$ set of ambiguous states

Equivalence result does not hold anymore: α -MEU preference do not reduce to maxmin/SEU/maxmax \Rightarrow we study the α -MEU model

To study the α -MEU as function of α we *have to* fix the set of priors of the α -MEU model \Rightarrow we choose

$$\begin{aligned} \mathcal{C}_{\max} &= \{ \text{all priors consistent with the uncertainty of the setting} \} \\ &= \{ \text{all priors } \pi : \pi(R \text{ realizes}) = \pi_R \} \end{aligned}$$

because this choice:

- ▶ α is a measure the **agent's degree of aversion toward ambiguity**
- ▶ **makes meaningful the comparison between α - \mathcal{C}_{\max} -MEU and maxmin preferences** by providing a utility specification common to the two classes of models

α - \mathcal{C}_{\max} -MEU model

With set of priors \mathcal{C}_{\max} , the α -MEU utility reduces to

α - \mathcal{C}_{\max} -MEU utility:

$$U(w) = \pi_R u(w_R) + (1 - \pi_R) [\alpha u(w_{\min}^A) + (1 - \alpha) u(w_{\max}^A)]$$

$w_{\min}^A := \min_{\sigma \in A} w_{\sigma}$ smallest wealth allocated among the l ambiguous states in the portfolio $w \in \mathbb{R}^{m+l}$

$w_{\max}^A := \max_{\sigma \in A} w_{\sigma}$ largest wealth allocated among the l ambiguous states in the portfolio $w \in \mathbb{R}^{m+l}$

Remark: α - \mathcal{C}_{\max} -MEU is concave if and only if $\alpha = 1$

Main findings: Extended Ellsberg framework

- ▷ Portfolio choice of the C_{\max} -MEU as function of α
 - pin down the different attitude (*seeking* and *averse*) towards ambiguity expressed by the C_{\max} -MEU
 - disentangle ambiguity seeking from ambiguity averse agents, and among the last ones, α - C_{\max} -MEU from maxmin agents
- ▷ ambiguity seeking attitudes may prevent the existence of the market equilibrium

α - \mathcal{C}_{\max} -MEU portfolios choice

$p \in \mathbb{R}^{1+I}$ state price vector

$p_{\min}^A := \min_{\eta \in A} p_{\eta}$ lowest price among the ambiguous states prices

Characteristic of the α - \mathcal{C}_{\max} -MEU portfolios choice:

→ *only two types of optimal portfolios, portfolio inertia at both portfolios:*

1. if $\alpha \geq 1 - \frac{p_{\min}^A}{1-p_R} \Rightarrow$ optimal portfolio *unambiguous* and *unique*:

$$w = (w_R, w, \dots, w, \dots, w) \text{ for some } w \in \mathbb{R}$$

2. if $\alpha < 1 - \frac{p_{\min}^A}{1-p_R} \Rightarrow$ optimal portfolio *ambiguous* and

$$w = (w_R, \underline{w}, \dots, \bar{w}, \dots, \underline{w}) \text{ for some } \bar{w} > \underline{w}$$

$w = \text{unambiguous} + \text{bet of } (\bar{w} - \underline{w}) \text{ on a cheapest ambiguous states}$

ambiguous optimal portfolio might be not unique: number equals number of ambiguous states with cheapest price

Exposure to ambiguity of the ambiguous portfolio

▷ *Exposure to ambiguity of the ambiguous portfolio*

$$w = (w_R, \underline{w}, \dots, \bar{w}, \dots, \underline{w}), \text{ is equal to } \bar{w} - \underline{w} > 0$$

→ the larger α

- ▶ the smaller $\bar{w} - \underline{w}$
 - ▶ the smaller demand for the ambiguous portfolio
- $\alpha = 1 \Rightarrow$ the optimal portfolio is unambiguous

→ the more u is concave, the smaller $\bar{w} - \underline{w}$

▷ *Allocation between risky and the ambiguous states:*

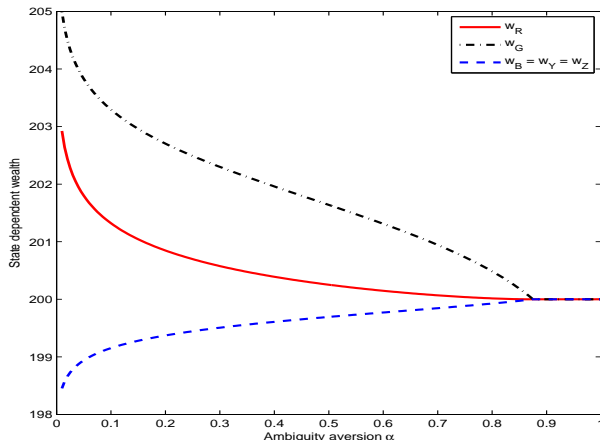
the larger α the smaller $\bar{w} - w_R$

When $\alpha \uparrow \frac{I-1}{I}$, optimal portfolio tends to the unambiguous and optimal allocation is the same of an SEU with prior $\tilde{\pi} = (\pi_R, \frac{1-\pi_R}{I}, \dots, \frac{1-\pi_R}{I})$

Impact of ambiguity aversion α on portfolio choice, $l = 4$

$$S = \{R\} \cup A, A = \{G, B, Y, Z\}.$$

$$w = (w_R, w_G, w_B, w_Y, w_Z) = (w_R, \bar{w}, \underline{w}, \underline{w}, \underline{w})$$

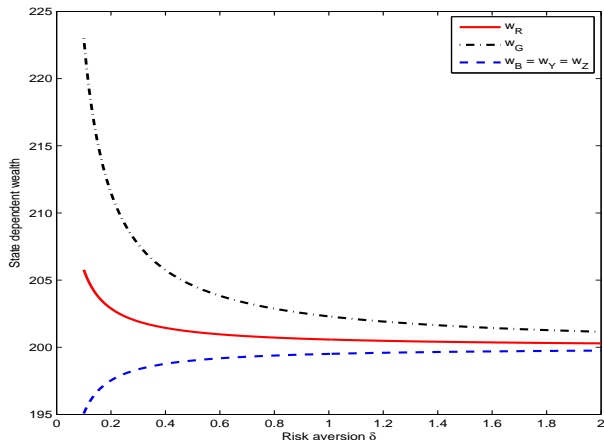


State dependent wealth impacted by the *degree of ambiguity aversion* α

Impact of risk aversion on portfolio choice, $l = 4$

$$S = \{R\} \cup A, A = \{G, B, Y, Z\}.$$

$$w = (w_R, w_G, w_B, w_Y, w_Z) = (w_R, \bar{w}, \underline{w}, \underline{w}, \underline{w})$$



State dependent wealth impacted by the *degree of risk aversion* δ , CARA

α - \mathcal{C}_{\max} -MEU and maxmin attitude towards ambiguity

$I \geq 3$

- ▶ *ambiguity seeking*: α - \mathcal{C}_{\max} -MEU agents with $\alpha \in (0, \frac{I-1}{I})$
- ▶ **not ambiguity seeking**
 - ▶ any α - \mathcal{C}_{\max} -MEU agents with $\alpha \in [\frac{I-1}{I}, 1)$
 - ▶ any **maxmin** agent with a set of priors including $\tilde{\pi} :=$ prior assigning equal probability to all ambiguous states

Remark: α - \mathcal{C}_{\max} -MEU agent with $\alpha = \frac{I-1}{I}$ is not equivalent – not even observationally – to an ambiguity neutral SEU agent

Lack of equilibrium with ambiguity seeking investors

$$S = \{R, G, B, Y\}, A = \{G, B, Y\} \quad (m = 1, l = 3)$$

- SEU agent with prior $\tilde{\pi}$
- maxmin agent with set of priors \mathcal{C} , $\tilde{\pi} \in \mathcal{C}$
- α - \mathcal{C}_{\max} -MEU agent with $\alpha \in (0, 1)$

Market with 2 agents. Total endowment $W \in \mathbb{R}^{1+3}$, $W_G = W_B = W_Y$

1. *Market agents: SEU + ambiguity averse* (e.g. α - \mathcal{C}_{\max} -MEU $\alpha \in [\frac{l-1}{l}, 1)$ or maxmin) \Rightarrow **equilibrium exists:**

- $p \in \mathbb{R}^{1+3}$ with $p_G = p_B = p_Y = \frac{1-p_R}{3}$
- SEU optimal portfolio $y \in \mathbb{R}^{1+3}$: $y_G = y_B = y_Y$
- ambiguity averse optimal portfolio $w \in \mathbb{R}^{1+3}$: $w_G = w_B = w_Y$

Lack of equilibrium with ambiguity seeking investors

2. *Market agents*: SEU + ambiguity seeking α - \mathcal{C}_{\max} -MEU agent ($\alpha \in (0, \frac{I-1}{I})$) \Rightarrow equilibrium does not exist:

· ambiguity seeking agent:

always chooses portfolio exposed to ambiguity with a strictly larger wealth on one cheapest ambiguous states:

$w \in \mathbb{R}^{1+3}$: $w_{\sigma} = \bar{w} > \underline{w}$, $w_{\eta} = \underline{w}$ for any $\eta \in A = \{G, B, Y\} \setminus \{\sigma\}$ where σ cheapest ambiguous state in equilibrium, i.e.

$$p_{\sigma} \leq p_{\eta}, \text{ for any } \eta \in A = \{G, B, Y\} \setminus \{\sigma\} \Rightarrow \quad (5.1)$$

· SEU agent: to clear the market has to hold a portfolio $y \in \mathbb{R}^{1+3}$:

$y_{\sigma} < y_{\eta} = y_{\nu}$ for any $\eta \in A = \{G, B, Y\} \setminus \{\sigma\}$

SEU agent chooses this portfolio only if equilibrium prices satisfy

$$p_{\sigma} > p_{\eta} = p_{\nu}, \text{ for any } \eta \in A = \{G, B, Y\} \setminus \{\sigma\} \quad (5.2)$$

which is incompatible with (5.1)

Conclusion

α -MEU model increasing popular generalization of the maxmin model

- ▶ Standard Ellsberg framework (two ambiguous states):
 - ▶ we show that *ambiguity averse α -MEU and maxmin preferences are equivalent*
 - ▶ we derive theoretical implications of ambiguity aversion on equilibrium asset prices. These are strikingly consistent with experimental findings in BGGZ (2010) \Rightarrow *ambiguity aversion matters does not wash out in equilibrium*
- ▶ Extended Ellsberg framework (at least three ambiguous states):
 - ▶ equivalence result does not hold anymore
 - ▶ we characterize optimal portfolio of α - \mathcal{C}_{\max} -MEU agent. Several differences with the maxmin optimal portfolio (only two types of portfolios, portfolio inertia, demand not unique, ...)
 - ▶ pin down ambiguity attitude of α - \mathcal{C}_{\max} -MEU agent
 - ▶ show that ambiguity seeking behaviour may prevent existence of equilibrium