A Theoretical and Empirical Comparison of Systemic Risk Measures

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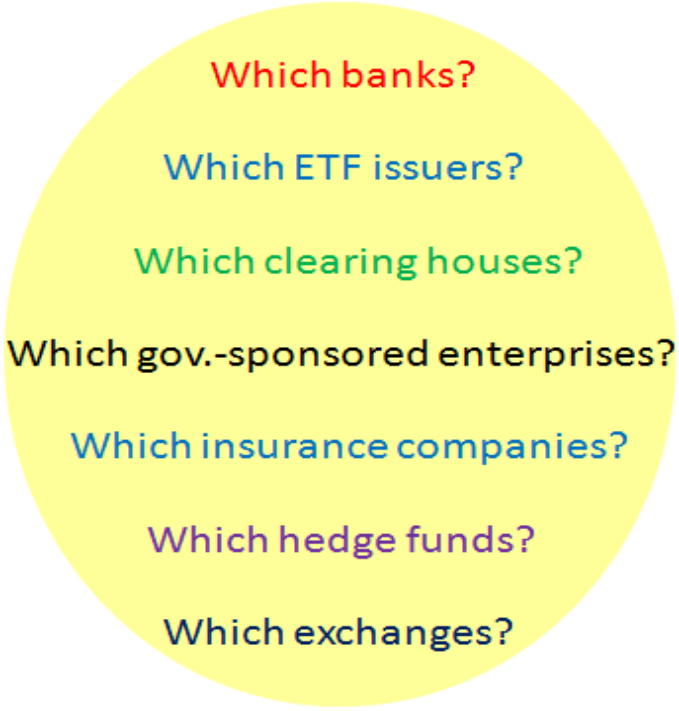
Contribution to Systemic Risk

- Which financial institutions contribute the most to the risk of the system?



- Who are the so-called Systemically Important Financial Institutions (SIFIs)?

Who are the SIFIs?

Definition

"SIFIs are financial institutions whose distress or disorderly failure, because of their size, complexity and systemic interconnectedness, would cause significant disruption to the wider financial system and economic activity."

(2.5%)	Citigroup Deutsche Bank HSBC JP Morgan Chase
(2.0%)	Barclays BNP Paribas
(1.5%)	Bank of America Bank of New York Mellon Credit Suisse Goldman Sachs Mitsubishi UFJ FG Morgan Stanley Royal Bank of Scotland UBS
(1.0%)	Bank of China BBVA Groupe BPCE Group Crédit Agricole ING Bank Mizuho FG Nordea Santander Société Générale Standard Chartered State Street Sumitomo Mitsui FG Unicredit Group Wells Fargo

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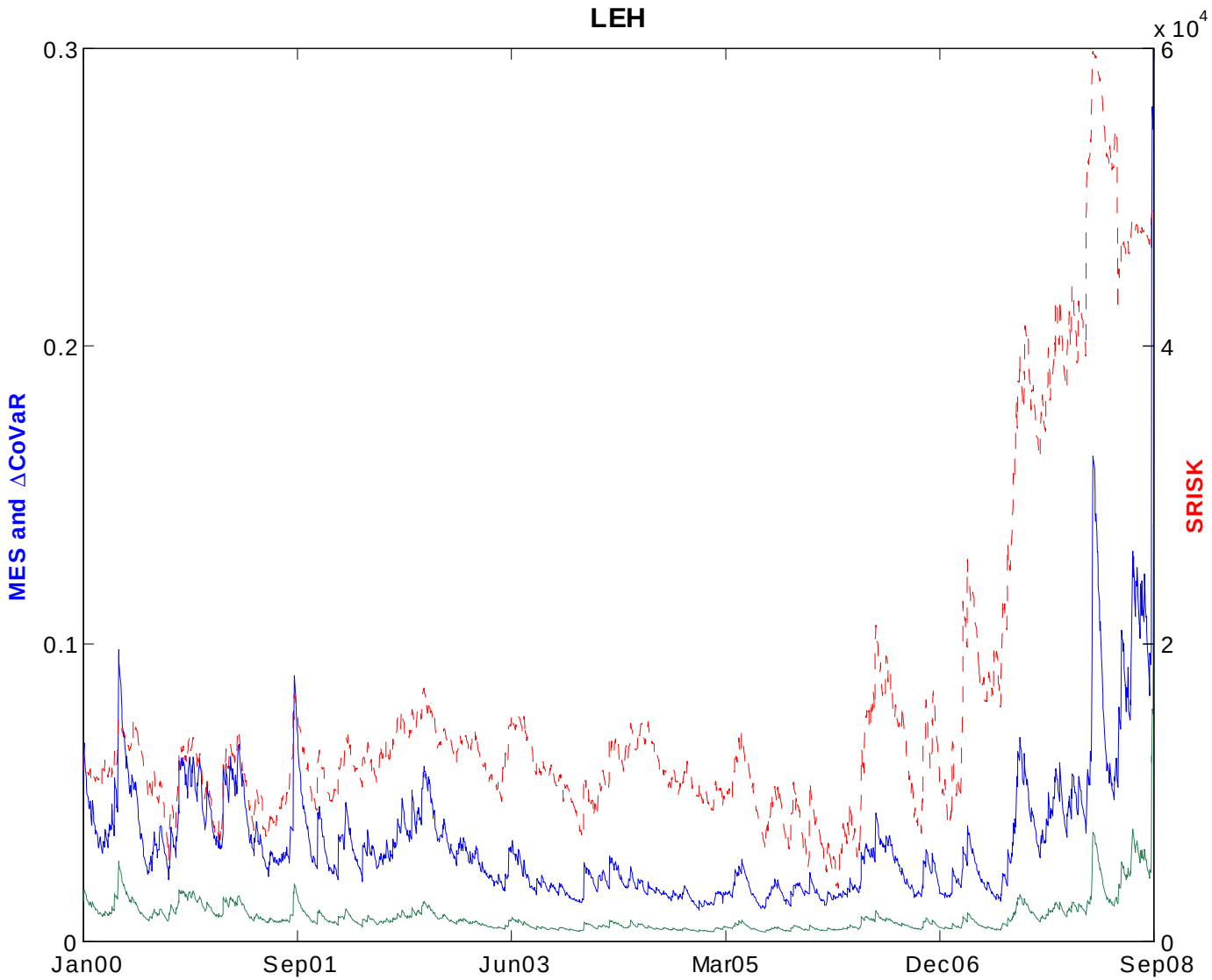
How to Measure Systemic Risk?

1. A first approach relies on proprietary data, such as cross-holdings, interbank lending, derivatives positions, or common exposures to exogeneous sources of risk
   - Gourieroux, Heam and Monfort (2012)
   - Greenwood, Landier and Thesmar (2012)
2. A second approach relies on public market data, such as stock or option prices, or CDS spreads
   - Giglio (2012)

Popular Systemic Risk Measures

Three prominent examples of market data-based measures are:

1. Marginal Expected Shortfall (MES) of Acharya, Pedersen, Philippon, and Richardson (2010)
2. Systemic Risk Measure (SRISK) of Acharya, Engle et Richardson (2012) and Brownlees and Engle (2012)
3. Delta Conditional Value-at-Risk (DCoVaR) of Adrian and Brunnermeier (2011)


LEH
x 10^4
0.3
0.2
0.1
0
6
4
2
0
MES and ΔCoVaR
SRISK
Jan00
Sep01
Jun03
Mar05
Dec06
Sep08

High-Impact Academic Papers

These papers had a high impact both in the academia and on the regulatory debate

1. Dozens of research papers have implemented these systemic risk measures
2. Regularly discussed in the financial press
3. Computed in real-time and publically disclosed on the Internet (V-Lab, New York University)
4. Used by central banks and banking regulators

Goal of this Paper

- We propose a theoretical and empirical comparison of the major market-data based systemic risk measures (MES, SRISK, and DCoVaR) in a common framework
- Uncover the theoretical link between systemic risk measures and standard financial risks (systematic risk, tail risk, correlation, beta)

The Econometrics of Systemic Risk

Methodology and Definitions

Expected Shortfall

ES of the market defined with a threshold C is the expected return of the market conditional on the market return being below C

$$\begin{aligned} ES_{m,t-1}(C) &= \mathbb{E}_{t-1}(r_{mt} \mid r_{mt} < C) \\ &= \sum_{i=1}^{N} w_i \mathbb{E}_{t-1}(r_{it} \mid r_{mt} < C) \end{aligned}$$

Marginal Expected Shortfall

The Marginal Expected Shortfall measure proposed by Acharya, Pedersen, Philippon, and Richardson (2010) extends the ES to measure the contribution of firm i

Definition

The MES is the expected equity loss per dollar invested in a particular financial institution i if the overall market return r_m declines by at least C

$$MES_{it}(C) = \frac{\partial ES_{m,t-1}(C)}{\partial w_i} = \mathbb{E}_{t-1}(r_{it} \mid r_{mt} < C)$$

Systemic Risk (SRISK)

The SRISK measure proposed by Acharya, Engle and Richardson (2012) and Brownlees and Engle (2012) extends the MES to take into account the market cap and the liabilities of the firm

Definition

The SRISK corresponds to the expected capital shortfall of a given financial institution, conditional on a crisis affecting the whole financial system

Systemic Risk (SRISK)

SRISK (Acharya et al., 2012)

The Systemic Risk captures the **expected capital shortage** of a firm given its liabilities and MES:

$$\overbrace{SRISK}^{Capital\ Shortfall} = \underbrace{k}_{Prudential\ Ratio} \overbrace{D_{it}}^{Liability} - (1 - k) \underbrace{W_{it}}_{Market-Value} (1 - LRMES_{it})$$

CoVaR

The third systemic risk measure is the **CoVaR** introduced by Adrian and Brunnermeier (2011)

Definition

The CoVaR corresponds to the VaR of the market return obtained conditionally on an event for firm i

CoVaR

CoVaR

CoVaR is the Value-at-Risk of the market return given a specific event on the firm's returns.

$$\Pr\left(r_{mt} \leq CoVaR_t^{m|r_{it}=VaR_{it}(\alpha)} | r_{it} = VaR_{it}(\alpha)\right) = \alpha$$

ΔCoVaR

The contribution of the institution to systemic risk is the difference between its CoVaR and the CoVaR calculated at the median state.

$$\Delta CoVaR_{it}(\alpha) = CoVaR_t^{m|r_{it}=VaR_{it}(\alpha)} - CoVaR_t^{m|r_{it}=Median(r_{it})}$$

How to Compare Systemic Risk Measures

1. Compare actual risk contributions to estimated risk contributions [NO]
2. See which measure can best reproduce the FSB list of SIFIs [NO]
3. Derive the risk measures in a common framework and try to uncover the driving forces of each systemic risk measure [YES]

A Theoretical Comparison of Systemic Risk Measures

A Theoretical Comparison of Systemic Risk Measures

A Theoretical Comparison of Systemic Risk Measures

We consider the linear market model of Brownlees and Engle (2012):

$$\begin{aligned} r_{mt} &= \sigma_{mt}\varepsilon_{mt} \\ r_{it} &= \sigma_{it}\rho_{it}\varepsilon_{mt} + \sigma_{it}\sqrt{1-\rho_{it}^2}\xi_{it} \end{aligned}$$

$$(\varepsilon_{mt}, \xi_{it}) \sim D$$

where $\nu_t = (\varepsilon_{mt}, \xi_{it})'$ satisfies $\mathbb{E}(\nu_t) = 0$ and $\mathbb{E}(\nu_t \nu_t') = I_2$, and D denotes the bivariate distribution of the standardized innovations

A Theoretical Comparison of Systemic Risk Measures

Proposition 1

In our framework, we have:

$$\begin{aligned} MES_{it}(\alpha) &= \frac{\rho_{it}\,\sigma_{it}}{\sigma_{mt}}\,\mathbb{E}_{t-1}\Big(r_{mt} \mid r_{mt} < VaR_{mt}(\alpha)\Big) \\ &= \beta_{it}\,ES_{mt}(\alpha) \end{aligned}$$

Interpretation

MES is proportional to the **systematic risk** and the coefficient corresponds to the **expected shortfall of market returns**

A Theoretical Comparison of Systemic Risk Measures

Corollary

A corollary of Proposition 1 is that SRISK can be expressed as a linear function of the beta, liabilities, and market capitalization:

$$SRISK_{it} \simeq k\ D_{it} - (1-k)\ \ W_{it}\ \ \exp\left[18 \times \beta_{it} \times ES_{mt}\left(\alpha\right)\right]$$

A Theoretical Comparison of Systemic Risk Measures

Proposition 2

Under A1, we have

$$\Delta CoVaR_{it}(\alpha) = \left(\frac{\rho_{it}\ \sigma_{mt}}{\sigma_{it}}\right) \times \Big(VaR_{it}(\alpha) - VaR_{it}(0.5)\Big)$$

and if the marginal distribution is symmetric around zero:

$$\Delta CoVaR_{it}(\alpha) = \widetilde{\gamma}_{it}\ VaR_{it}(\alpha)$$

Interpretation

ΔCoVaR is proportional to the **linear projection coefficient** of the market return on the firm return and to the **firm tail risk**

A Theoretical Comparison of Systemic Risk Measures

Proposition 3

Under A1, the **ratio is not homogenous across financial institutions** and is the product of two terms:

$$\frac{\Delta CoVaR_{it}(\alpha)}{MES_{it}(\alpha)} = \overbrace{\frac{VaR_{it}(\alpha)}{\sigma_{it}^2}}^{\textbf{firm-specific}} \times \overbrace{\frac{\sigma_{mt}^2}{ES_{mt}(\alpha)}}^{\textbf{common}}$$

Interpretation

The fact that this ratio is firm-specific implies that the systemic risk rankings based on the two measures may not be the same

A Theoretical Comparison of Systemic Risk Measures

Proposition 4

A financial institution i is **more systemically risky** that an institution j **according to the MES and the ΔCoVaR measures**, $MES_{it}(\alpha) \leq MES_{jt}(\alpha)$ and $\Delta CoVaR_{it}(\alpha) \leq \Delta CoVaR_{jt}(\alpha)$, if:

$$\rho_{it} \geq \max\left(\rho_{jt}, \frac{\rho_{jt}\sigma_{jt}}{\sigma_{it}}\right)$$

A Theoretical Comparison of Systemic Risk Measures

Proposition 5

A financial institution i is **more systemically** risky than a financial institution j (with the same level of liabilities) **according to the SRISK and the ΔCoVaR measures**, $SRISK_{it}(\alpha) \geq SRISK_{jt}(\alpha)$ and $\Delta CoVaR_{it}(\alpha) \leq \Delta CoVaR_{jt}(\alpha)$, if:

$$\rho_{it} \geq \rho_{jt} \quad \text{and} \quad W_{it} \leq W_{jt} \times \exp\left[18 \times ES_{mt}(\alpha) \times \left(\beta_{jt} - \beta_{it}\right)\right]$$

where W_{it} and W_{jt} denote the market capitalizations of both firms

Summary of the Main Theoretical Findings

1. Strong link between MES and beta (Prop. 1)
2. Strong link between ΔCoVaR and beta (Prop. 2)
3. Rankings of systemic firms can be different (Prop. 3)
4. Rankings of systemic firms are likely to be different (Prop. 4&5)

An Empirical Comparison of Systemic Risk Measures

An Empirical Comparison of Systemic Risk Measures

An Empirical Comparison of Systemic Risk Measures

In practice, the dependence between firm returns can be richer than in our framework. Let's relax it.

A panel of US financial institutions

- Unbalanced panel of 94 companies with daily returns from 01/03/2000 to 12/31/2010.
- Same dataset as in Acharya et al. (2010) and Brownlees and Engle (2012).
- Same estimation techniques as in the original papers

An Empirical Comparison of Systemic Risk Measures

Outside our framework, the conditional expectation $\mathbb{E}_{t-1}\left(\xi_{it} \mid \varepsilon_{mt} < \frac{C}{\sigma_{mt}}\right)$ is not null and has to be estimated

$$\begin{aligned} MES_{it}(C) &= \sigma_{it}\rho_{it}\mathbb{E}_{t-1}\left(\varepsilon_{mt} \mid \varepsilon_{mt} < \frac{C}{\sigma_{mt}}\right) \\ &\quad + \sigma_{it}\sqrt{1-\rho_{it}^2}\mathbb{E}_{t-1}\left(\xi_{it} \mid \varepsilon_{mt} < \frac{C}{\sigma_{mt}}\right) \end{aligned}$$

We use a **nonparametric estimator** (see Scaillet, 2005 for more details) of the tail expectation.

An Empirical Comparison of Systemic Risk Measures

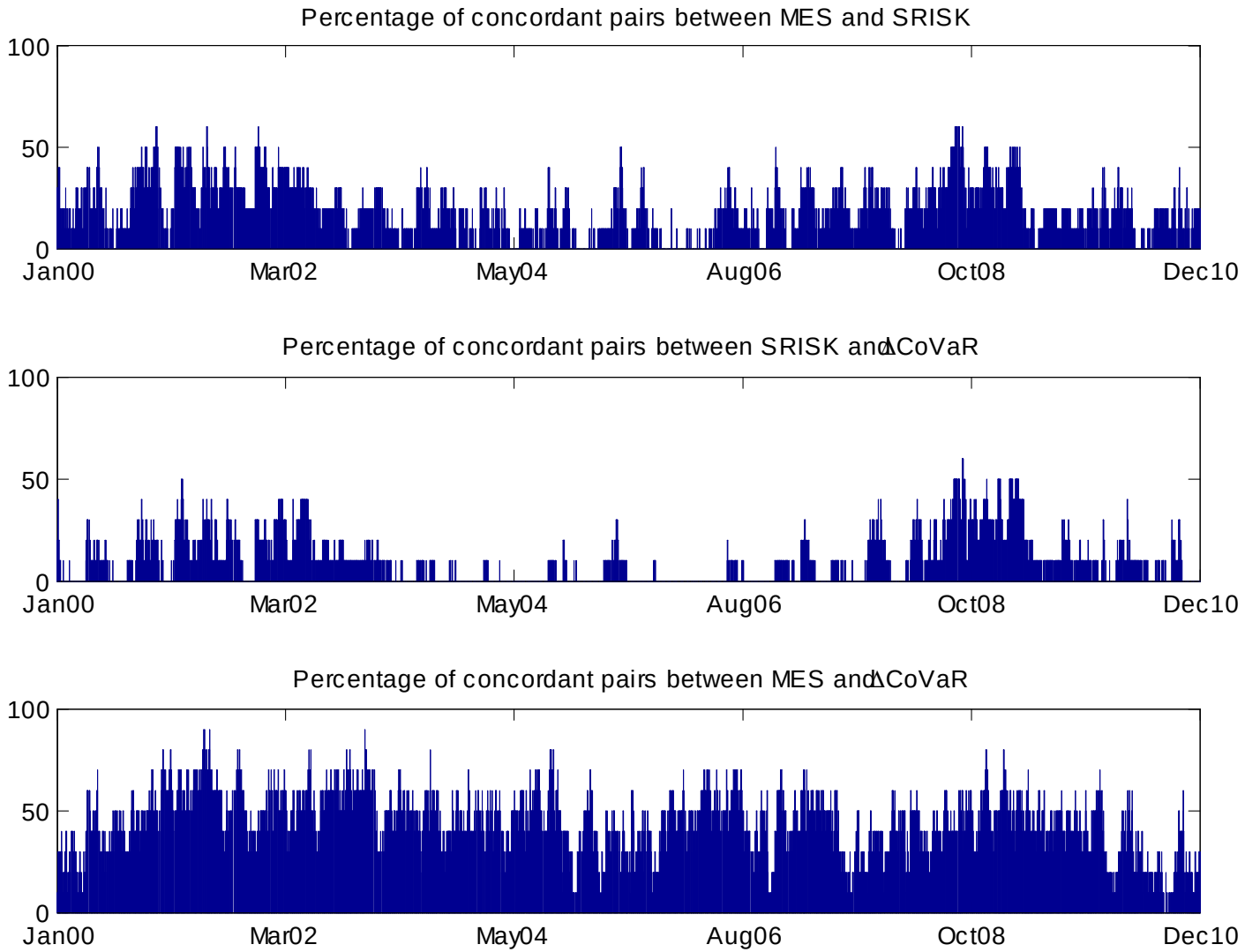
SIFI or not SIFI?

Different Systemic Risk Measures, Different SIFIs

Table: Systemic Risk Rankings: 12/31/2010

Rank	MES	SRISK	ΔCoVaR
1	MBI	BAC	HRB
2	AIG	C	MI
3	MI	JPM	BEN
4	CBG	MS	CIT
5	RF	AIG	WU
6	LM	MET	AIZ
7	JNS	PRU	AXP
8	HRB	HIG	JNS
9	BAC	SLM	NYB
10	UNM	LNC	MTB

Different Systemic Risk Measures, Different SIFIs



An Empirical Comparison of Systemic Risk Measures

MES and SRISK

Systemic or Systematic Risk?

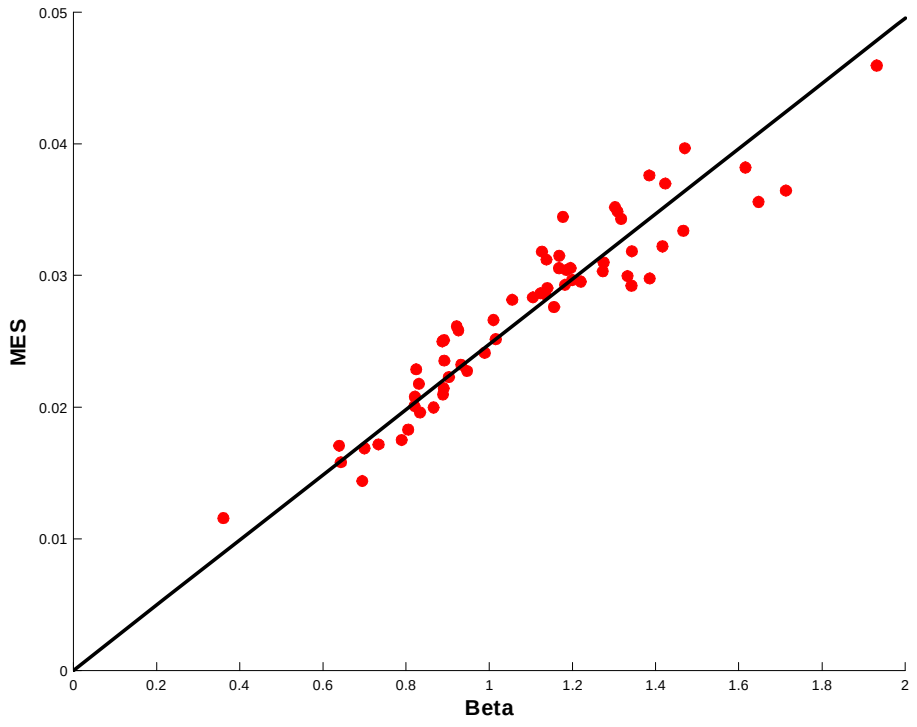
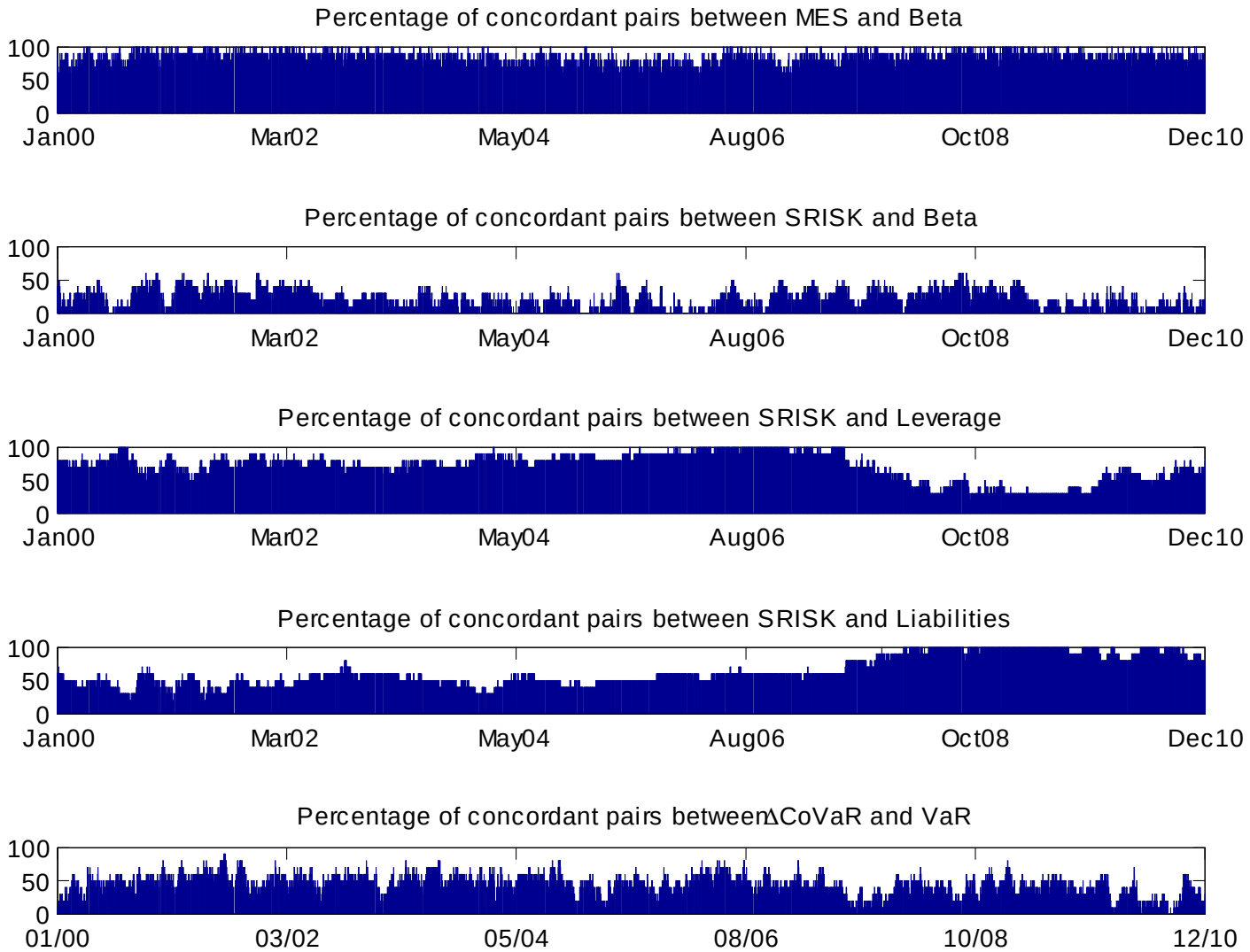


Figure: **Strong Relationship between MES and beta**

Systemic or Systematic Risk?



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DCoVaR

An Empirical Comparison of Systemic Risk Measures

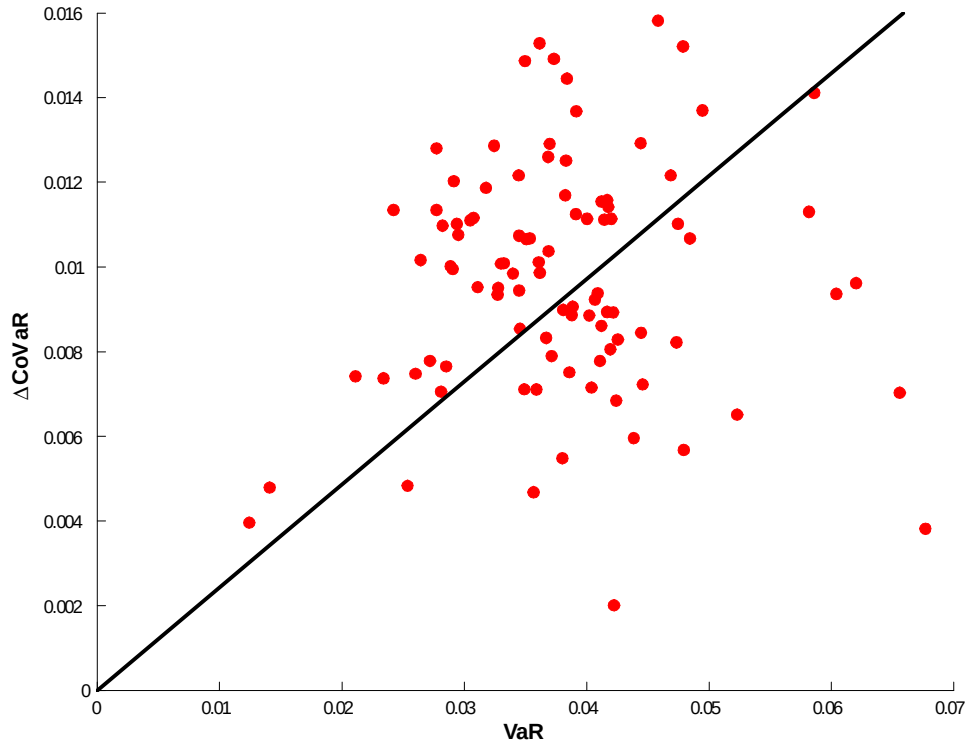

Figure: **ΔCoVaR is not Equivalent to VaR in the Cross-Section**

An Empirical Comparison of Systemic Risk Measures

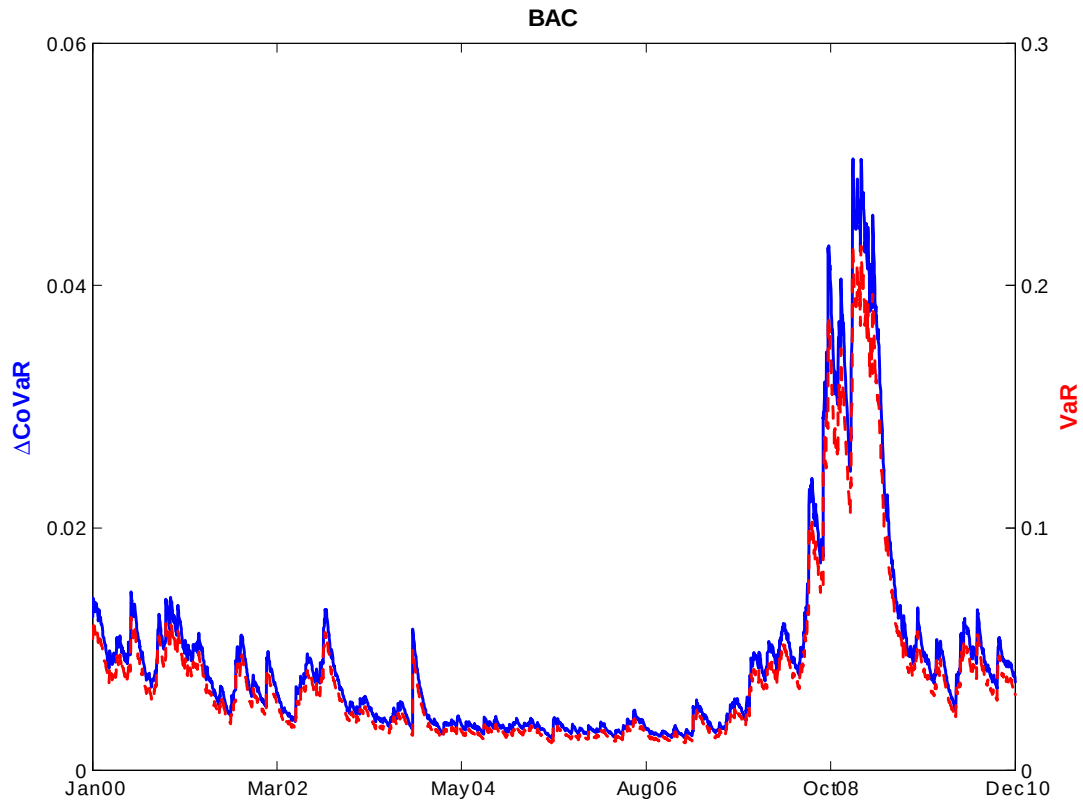

Figure: **ΔCoVaR is Equivalent to VaR in Time Series**

An Empirical Comparison of Systemic Risk Measures

Regression Analysis

Regression Analysis

$$MES_{i,t} = a + b \cdot X_{i,t} + e_{i,t}$$

	Cross-section					
MES	MV	LTQ	LVG	beta	VaR	all
average R^2	0.0071	0.0403	0.2591	0.9571	0.7968	0.9837
min R^2	0.0000	0.0000	0.0137	0.7198	0.3972	0.9433
max R^2	0.0452	0.1852	0.7883	0.9946	0.9785	0.9986
std R^2	0.0086	0.0416	0.1477	0.0319	0.1100	0.0105

Regression Analysis

$$SRISK_{i,t} = a + b \cdot X_{i,t} + e_{i,t}$$

	Cross-section					
SRISK	MV	LTQ	LVG	beta	VaR	all
average R^2	0.3197	0.8341	0.1840	0.1173	0.0592	0.9932
min R^2	0.0085	0.2569	0.0110	0.0034	0.0022	0.9807
max R^2	0.5759	0.9952	0.4103	0.3331	0.2269	0.9995
std R^2	0.1073	0.1279	0.0757	0.0661	0.0445	0.0036

Regression Analysis

$$\Delta CoVaR_{it} = a + b \cdot X_{i,t} + e_{i,t}$$

	Time series					
ΔCoVaR	MV	LTQ	LVG	beta	VaR	all
average R^2	0.3235	0.1870	0.3642	0.2645	1.0000	1.0000
min R^2	0.0022	0.0000	0.0001	0.0000	1.0000	1.0000
max R^2	0.8478	0.7876	0.7453	0.9799	1.0000	1.0000
std R^2	0.2244	0.1766	0.2178	0.2339	0.0000	0.0000

Conclusion

We have shown that:

- Different systemic risk measures identify different SIFIs
- Firm rankings based on systemic risk estimates mirror rankings obtained by sorting firms on market risk or liabilities
- One-factor linear models explain between 83% and 100% of the variability of the systemic risk estimates
- Standard systemic risk measures fail to capture the multiple facets of systemic risk

Future Research

- Given the very nature of systemic risk, future research on systemic risk should combine various sources of information, including balance-sheet data, proprietary data on positions, and market data (hybrid approach)
- Future research on systemic risk should also consider the definition of the perimeter of the financial system