# Information and Inventories in High-Frequency Trading

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# Introduction

## Outline

# Introduction

## High-Frequency Trading

- High-frequency trading is a game of informational advantage.
  - Informational edge is *small* and *frequent*.
  - *Speed* is necessary to take advantage.
  - The only risk involved is *inventory*.
- Simplest example: *latency arbitrage*.
  - Regular traders observe national best bid and offer prices.
  - HFTs purchase direct feeds to various electronic exchanges.
  - Access to price changes before NBBO is updated.
  - No long-term view on the market. Oscillating positions.
  - Activity looks like white noise to traditional investors.
- This paper: equilibrium model for these features.

# Introduction

## Model in a Nutshell

- Equilibrium between three types of agents:
  - Risk-neutral, competitive market makers.
  - HFTs with perfect one-period look-ahead filtration.
  - Low frequency "noise traders" with exogenous trading motives.
- Equilibrium determined by Stackelberg game.
  - Market makers move first by refilling order book.
  - HFTs and noise traders follow by executing market orders.
  - Same information structure as in literature on optimal price schedules (Glosten, '89, '94; Bernhardt and Hughson, '97; Biais, Martimort, and Rochet, '00).
  - Departs from double auction in Kyle's '85 model and its variants (e.g., Focault, Hombert Rosu, '15, Rosu, '15).
  - Well-suited for high-frequency trading on electronic exchanges. Market makers cannot renege on posted limit orders.
- Allows to study both risk-neutral and inventory-averse HFTs.

# Introduction

## Results in a Nutshell

- Risk-neutral HFTs:
  - Hold martingale inventories. Fluctuate on same time scales as noise traders'.
  - Profits equal a fraction of the price volatility predicted.
  - Equilibrium price is conditional expectation of fundamental value.
  - Equilibrium price impact given by ratio between price and noise trading volatilities.
  - Findings consistent with sequence of one-shot Kyle models.
- With inventory aversion:
  - Autoregressive positions. Converge to zero in the continuous-time limit.
  - Limiting profits remain the same.
  - With sufficient trading speed: information can be monetized with almost no risk.

# Model

## Exogenous Inputs

- One safe asset normalized to one.
- Risky asset with fundamental value $S_T = \int_0^T \sigma_t^S dW_t$.
- Noise trader demand driven by independent Brownian motion:

$$dK_t = \mu_t^K dt + \sigma_t^K dZ_t$$

- To make information structure most transparent:
  - Set up model in discrete time.
  - Discretized processes $X_n^N = X_{Tn/N}$, $\Delta X_n^N = X_n^N - X_{n-1}^N$.
  - Pass to the limit only later to determine equilibrium.
- Market filtration $\mathcal{F}$: generated by $W$ and $Z$.
- HFTs' filtration $\mathcal{G}$: also includes next price move.
  - E.g., latency arbitrage. But no frontrunning of low frequency traders.

# Model

## The trading game

- Market makers move first by posting a baseline price $P_n^N$ and a block shaped order book with height $1/\lambda_n^N$ around it.
- All trades clear together. Price impact shared equally by HFTs and noise traders.
- Risk-neutral HFTs: choose trades $\Delta L_{n+1}^N$ to maximize expected profits:

$$\sum_{n=1}^{N-1} E\left[(P_N^N - P_n^N)\Delta L_{n+1}^N - \frac{\lambda_n^N}{2}\left(\Delta L_{n+1}^N + \Delta K_{n+1}^N\right)\Delta L_{n+1}^N\right]$$

- Pointwise optimization yields optimal strategy:

$$\Delta \hat{L}_{n+1}^N = \frac{E[P_N^N|\mathcal{G}_n] - P_n^N}{\lambda_n^N} - \frac{1}{2}E[\Delta K_{n+1}^N|\mathcal{G}_n]$$

- Difference between private and public forecast. Adjusted for expected noise trading activity.

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# Equilibrium

## No Exploding Inventories

- Competitive market makers: zero expected profits.
- Market makers move first. No filtering required.
  - Short-lived information revealed anyways.
- Baseline price and price impact in equilibrium?
- Already determined by requiring that positions remain bounded in probability in the continuous-time limit.
- This minimal assumption already fixes baseline price as the martingale generated by the fundamental value:

$$P_t = E[S_T|\mathcal{F}_t] = \int_0^t \sigma_s^S dW_s$$

- Corresponding price impact determined by setting expected profits of market makers equal to zero: $\hat{\lambda}_t = \sigma_t^S/\sigma_t^K$.

# Equilibrium

## Comparison to Kyle ‘85

- Discrete-time quantities different because of information structure.
- Distinction vanishes in the continuous-time limit.
- Our model is consistent with a series of one-period Kyle models.
  - Cf. Admati and Pfleiderer ‘88, Foucault et al. ‘15, Rosu ‘15.
  - Equilibrium only determined by current volatilitites.
  - Without long-lived information, insiders cannot time predictable trends as in Collin-Dufresne and Vos ‘14.
- But since no filtering is needed in our setting, nonlinear strategies can be treated as well.
- For example: inventory aversion.

# Inventory Aversion

## Criterion

- Empirical studies (Kirilenko et al. '14, SEC '10): HFTs characterized by high volume *and* low inventories.
- Risk-neutral case: HFTs' and noise traders' positions vary on the same time scales.
- As a remedy: add explicit inventory penalty $\gamma^N$:

$$E\left[\sum_{n=1}^{N-1}\left((P_N^N - P_n^N)\Delta L_{n+1}^N - \frac{\lambda_n^N}{2}\left(\Delta L_{n+1}^N + \Delta K_{n+1}^N\right)\Delta L_{n+1}^N - \frac{\gamma^N}{2}(L_{n+1}^N)^2\right)\right]$$

- Penalty for buy-and-hold should be proportional to time held $\rightsquigarrow \gamma^N = \gamma/N$.
- Equilibrium?
- Tractable solutions?

# Inventory Aversion

## Equilibrium

- Inventory averse HFTs do not exploit mispricings as ruthlessly.
- No-exploding inventory condition no longer uniquely determines equilibrium price.
- But in reality, market makers do not know HFTs' inventory aversion..
- If they are worried that at least one sufficiently risk-tolerant insider exists, they have to quote the martingale baseline price.
- Even with this choice, more complex preferences have to be tackled using dynamic programming.
  - Only possible in concrete models.
- Here: simplest specification. Brownian motions:

$$S_T = \sigma^S W_T, \qquad K_t = \sigma^K Z_t$$

# Inventory Aversion

## Dynamic Programming

- Quadratic ansatz as in Garleanu and Pedersen '13 leads to closed-form solution.
- For frequent trading, positions follow autoregressive process of order one:

$$\hat{L}^N_{n+1} = -\sqrt{\frac{\gamma^N}{\lambda}}\hat{L}^N_n + \frac{\sigma^S}{\lambda}\Delta W^N_{n+1} + O(N^{-1})$$

- Speed of inventory management: tradeoff between inventory aversion $\gamma^N$ and trading cost $\lambda$.
- Same constant also shows up in other liquidation and optimization problems with linear price impact (Almgren and Chriss '01; Moreau, M-K, Soner '15).
- But here: mean reversion speed of order $O(N^{-1/2})$ rather than $O(N^{-1})$ as for a discretized OU process.

# Inventory Aversion

## Continuous-Time Limit

- Consequence: "infinite trading speed" in the continuous-time limit.
- HFTs' positions converge to zero in $L^2(P)$.
- But expected profits do not!
  - At the leading order: same performance as without inventory management.
  - Losses due to inventory management only visible in the first-order correction term of order $O(N^{-1/2})$.
- Apparent contradiction to Rosu '15, who finds nontrivial losses for both "fast" ($O(1)$) and "slow" ($O(N^{-1})$) inventory management in a series of one-shot Kyle models.
- Resolved by noticing that neither of his ad hoc policies uses the optimal trading speed of order $O(N^{-1/2})$.

# Summary and Outlook

- Summary:
  - Equilibrium model for information asymmetries in high-frequency trading.
  - Risk-neutral HFTs hold martingale inventories fluctuating on the same time scales as noise traders'.
  - Inventory aversion leads to vanishing positions yielding approximately the same returns in the continuous-time limit.
  - Information can be monetized with very little risk.
- Outlook:
  - Add information about noise trader order flow obtained by frontrunning. Should lead to HFT strategies alternating between market and limit orders.
  - Add strategic low frequency traders, i.e., institutional investors. Do these benefit from a transaction tax?