Endogenous Liquidity and Defaultable Bonds

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Background: Fundamental vs Liquidity

- **Fundamental** and **liquidity** are interconnected as evident from recent financial crisis
  - Liquidity: funding liquidity, price impact, transaction costs, etc
  - Today's paper: liquidity$\rightleftharpoons$fundamental, two-way feedback
    Liquidity solved jointly with fundamental (default decision)

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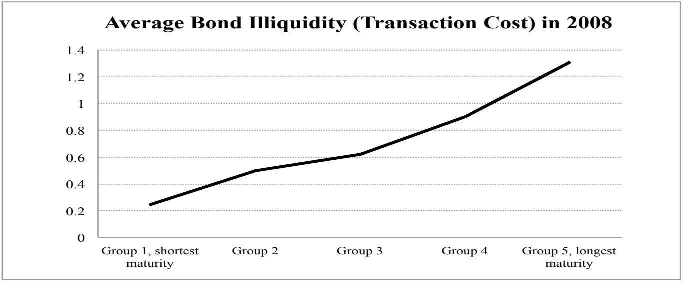
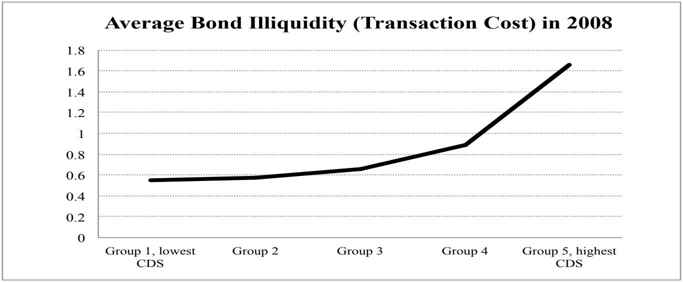
  Corporate bond market much more illiquid than equity market

- **Pattern of bond illiquidity:**
  - OTC transactions have average transaction cost of around 100bps
  - Illiquidity higher for longer time-to-maturity, closer to default
  - Barclays Capital report (2009) shows high correlation between default and liquidity spreads, both time-series and cross-sectional

Motivation: Corporate Bonds





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- **Empirical approach to bond liquidity:**
  - State-of-the-art empirical literature decomposes spreads into independent liquidity and default premium

Mechanism and Results

Building blocks for interaction between fundamental and liquidity:

- How does bond illiquidity arise, and how is it affected by maturity and state of the firm?
  - Over-the-counter market with search friction à la Duffie et al (2005)
- How do corporate decisions interact with secondary market liquidity?
  - Endogenous default à la Leland Toft (1996)

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- How do corporate decisions interact with secondary market liquidity?
  - Endogenous default à la Leland Toft (1996)

Main results:

- Closed-form solution for bond & equity values, default boundary
- Novel liquidity-default spiral, can be quantitatively important for understanding credit spreads
- Ability to target empirical pattern of bond illiquidity, match to credit spreads than can be decomposed into default and liquidity components

Related Literature

Search in asset markets:

- Duffie, Garleanu, Pedersen '05, '07
  OTC search market with simplified 'derivative'

Capital structure models:

- Leland, Toft '96 (LT96)
  Rollover increases exposure of equity holders to fundamental risk
- He, Xiong '12 (HX12)
  Exogenously given secondary market liquidity affects default decision

Empirical literature:

- Bao, Pan, Wang '11; Edwards, Harris, Piwowar '07; Hong, Warga '00; Hong, Warga, Schultz '01; Harris, Piwowar '06; Feldhütter '11

Feedback models:

- Many many more papers...

The Model: Basics & Liquidity Shocks

Preferences: Everyone risk-neutral with common discount rate r

Firm:

- Assets produce per-period cash-flow δ_t, $d\delta_t = \mu\delta_t dt + \sigma\delta_t dZ_t^{\mathbb{Q}}$
- Debt in place with aggregate (constant) face value p and coupon c

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Idiosyncratic liquidity shock for bond investors:

- With intensity ξ, jump in individual discount rate to $\bar{r} > r$
- Let H be high-value (r) type, L low-value/liquidity ($\bar{r}$) type
- Idiosyncratic liquidity shock *not* insurable (incomplete market)
- Holding restriction: $\{0, 1\}$ (as in DGP '05)

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Trade:

- Efficient for L types to sell to H types with higher valuation
- D_H and D_L are the values of debt for H/L types taking into account future liquidity shocks/re-trading opportunities/default/maturity

The Model: Illiquid Secondary Bond Market

Search friction in secondary bond market:

- L meets dealers with intensity λ & bargains over sale
- L's outside option (D_L) is waiting for other dealers/default/maturity
- Dealer immediately sells bond on for D_H to H type outside investors
  - For simplicity, frictionless contact with H investors

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Bargaining:

- Nash-Bargaining over surplus from intermediation, $S \equiv D_H - D_L$
- Endogenous price X implements β (L type) and $(1-\beta)$ (dealer) surplus split:

$$\begin{aligned} D_H - X &= (1-\beta)(D_H - D_L) \\ X - D_L &= \beta (D_H - D_L) \end{aligned}$$

The Model: Boundary Conditions - Maturity and Default

Bonds mature at $\tau = 0$:

- At maturity bonds equal to face value, $D_H(\delta, 0) = D_L(\delta, 0) = p$ for $\delta > \delta_B$

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Bonds default at $\delta = \delta_B$:

- Bonds have *equal seniority* in default
- Cash recovery value constant at $\alpha V_B = \alpha \frac{\delta_B}{r-\mu}$ with $\alpha \leq 1$
- Legal delay: Cash-payout αV_B only after an exponential delay with intensity θ
- Post-default trading possible with intermediation intensity λ_B

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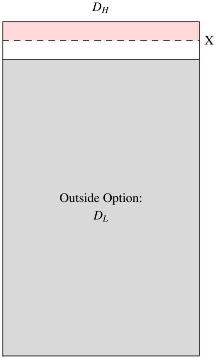
$\Rightarrow$ **Result:** $D_H(\delta_B, \tau) = \alpha_H V_B$, $D_L(\delta_B, \tau) = \alpha_L V_B$. Wedge in *effective* bankruptcy discounts

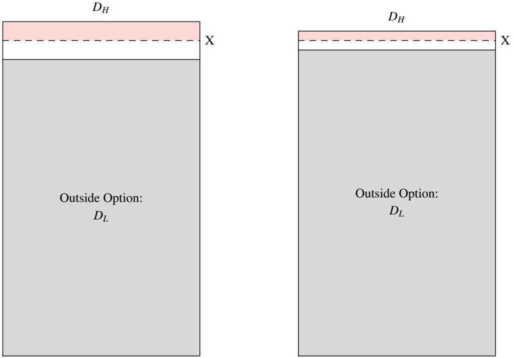
$$\alpha_L < \alpha_H < \alpha$$

The Model: Bargaining



(A) AAA at issuance

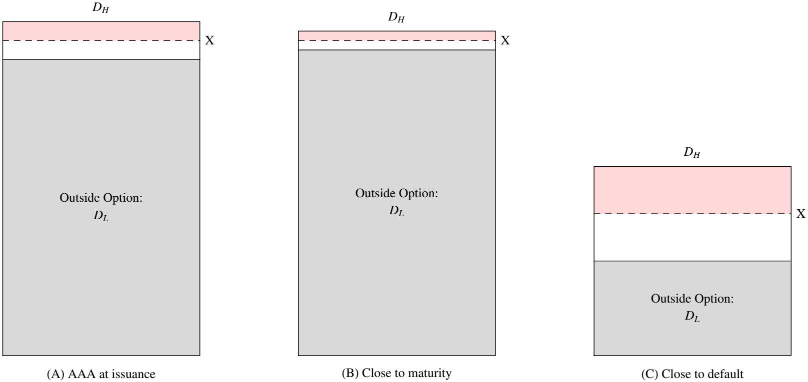
The Model: Bargaining and Maturity



(A) AAA at issuance

(B) Close to maturity

The Model: Bargaining and Default



The Model: Debt Structure, Rollover & Default

Debt structure:

- **Stationary principal & staggered maturity** (as in LT96):
  - Maturity structure evenly staggered (i.e., uniform) on $[0, T]$
  - Maturing bonds reissued with same (c, p, T)
  - Mass $1/T \cdot dt$ of bonds matures every instant

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Rollover:

- Primary market competitive & liquid, so issue at D_H to H types
- Rollover further exposes equity to movement in δ via repricing

$$NetCashFlow_t = \underbrace{\delta_t}_{CF} - \underbrace{(1-\pi)\,c}_{Coupon} + \underbrace{\overbrace{1/T}^{Mass\ maturing}\quad\overbrace{\left[D_H\left(\delta_t, T\right) - p\right]}^{Repricing}}_{Rollover\ gain/loss}$$

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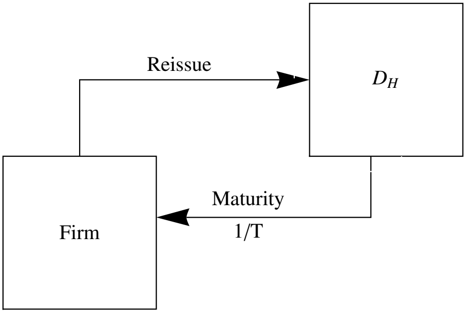
Optimal default:

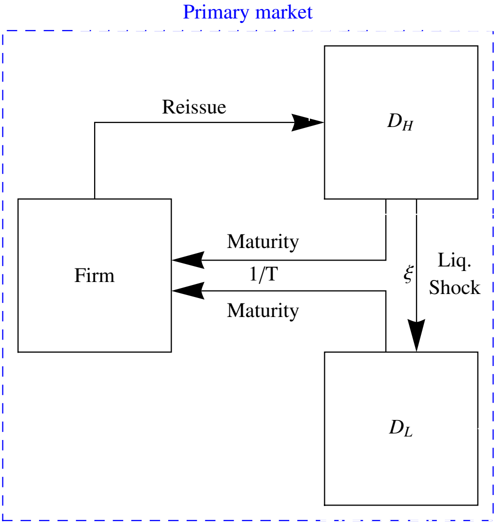
- Equity defaults at δ_B when absorbing further losses unprofitable

Schematic Representation: Leland Toft 1996



Above analysis outside default

Schematic Representation: The Primary Market



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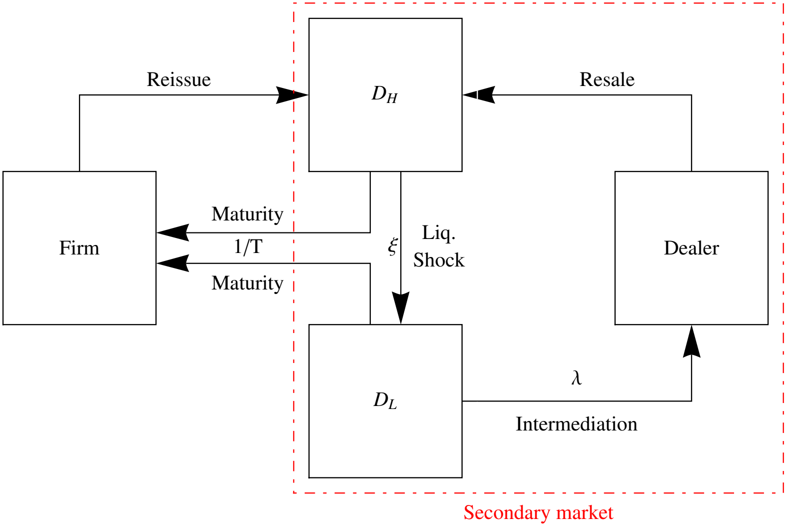
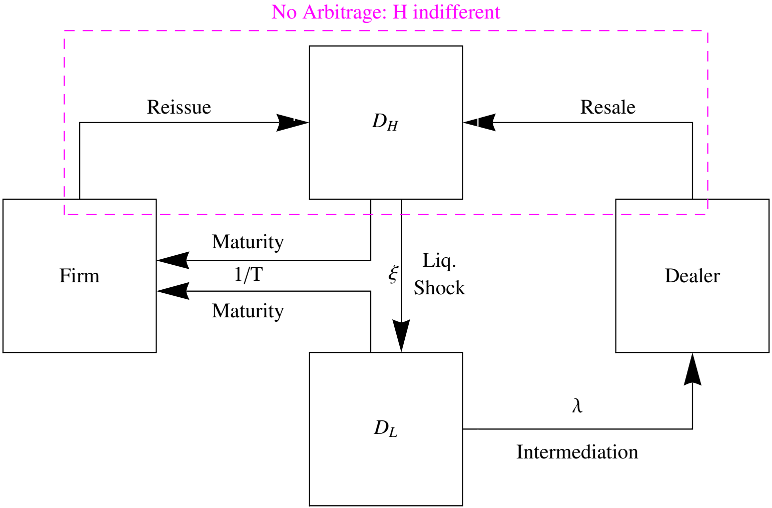
Schematic Representation: The Secondary Market



Above analysis outside default

Schematic Representation: No Arbitrage



Above analysis outside default

Solution: Equity, Debt & Bankruptcy Boundary

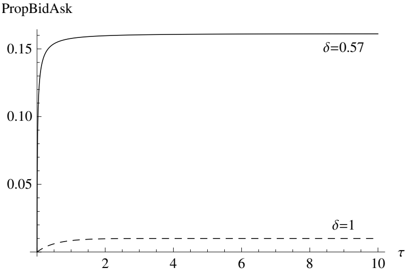
Closed form solutions for all important objects:

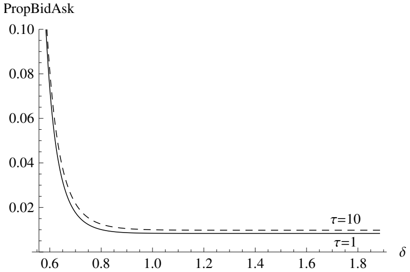
Debt D_H, D_L: mixture of distorted LT96 solutions

Equity E: solved directly as no 'adding up' as in LT96

Optimal default boundary δ_B

Bond Liquidity: Relative Bid-Ask Spread





Consistent with empirical pattern:
BA spread lower for shorter-term bonds and higher quality bonds

Liquidity and Default: Feedback Loop

Counterfactual: Fixed illiquidity / transaction cost

- Fixed transaction cost k (bid-ask spread of $\frac{k}{1-k/2}$) with immediate sale after shock (as in Amihud Mendelson '86, He Xiong '12)
- Our model: pro-cyclical liquidity, i.e., liquidity dries up as fundamental δ worsens
- Thought experiment to get feedback:
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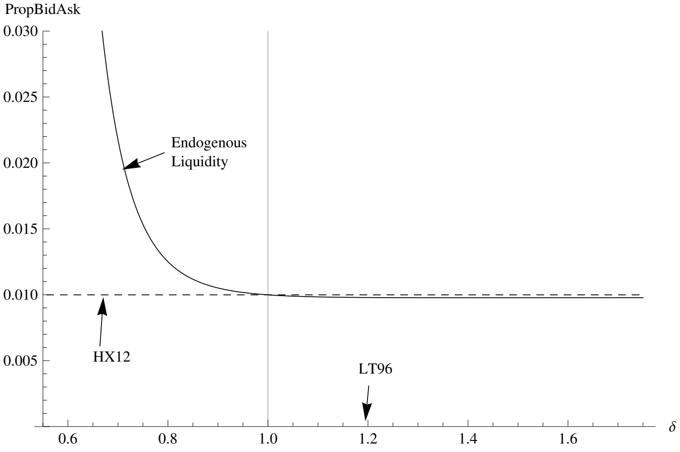
Parameters: normalize $\delta_0 = 1$

- Calibrate so at δ_0 bid-ask is 100bps
- Benchmark of HX12: $k = 99.5bps$ (so 100bps bid-ask spread)
- Benchmark of LT96: $k = 0$ (no illiquidity)
- Effective bankruptcy discounts $\alpha_H = 67\%$ and $\alpha_L = 55\%$

Liquidity and Default: Pro-cyclical Liquidity

Pro-cyclical liquidity:

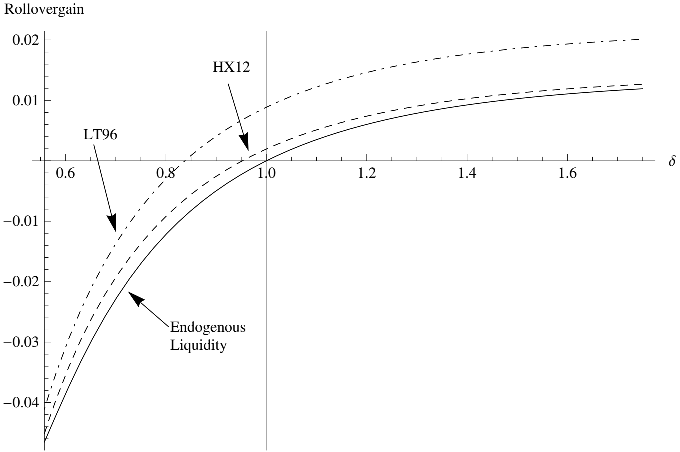
- Illiquidity increases as distance to default shrinks
- Illiquidity non-zero for large δ / AAA-rated bonds



Liquidity and Default: Rollover Losses & Default

Rollover loss amplified:

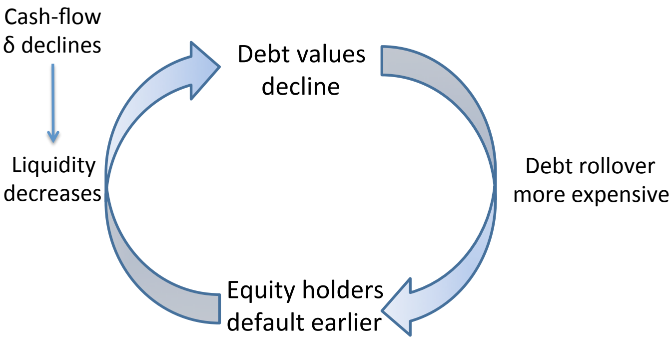
- Possible future illiquidity depresses primary market price $D_H(\delta, T)$
- Higher rollover losses for every δ lead to earlier default



Liquidity and Default: Full Feedback Loop

Equilibrium feedback loop:

- Compare to counterfactual *constant* transaction costs



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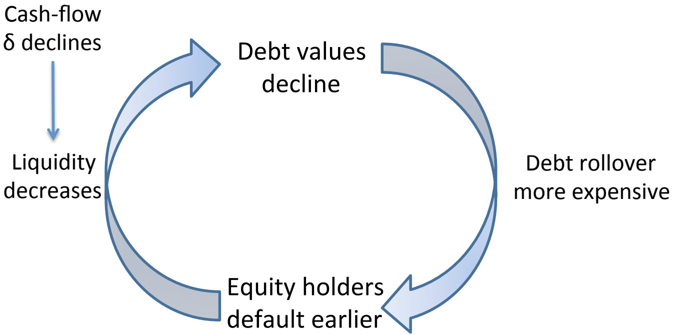

- Default is just *one* channel to affect fundamental
  - Simple extension: endogenous investment by equity to improve asset-in-place creates feedback of illiquidity on cash-flows

Maturity: Rollover Risk vs Liquidity Provision

Negative: Short-term debt leads to earlier default

- Higher rollover frequency increases equity's exposure to δ

$$\mathit{Rollover\ gain/loss}_t = \underbrace{1/T}_{\mathit{Rollover\ frequency}} \times \underbrace{\left[D_H\left(\delta_t, T\right) - p\right]}_{\mathit{Repricing}}$$

- Higher exposure to δ leads to higher default boundary δ_B

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Positive: Short-term debt provides liquidity

- Short maturity improves bargaining outcome between seller & dealer
- Issuing to H types more frequently improves allocative efficiency as it 'recycles' L types to H types quicker (lower SS mass of L holdings)

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$\Rightarrow$ Finite maturity $T^* < \infty$ optimal if moderate initial leverage;
T^* lower the less liquid secondary market (i.e. the lower λ)

Current Work: Aggregate Shocks & Serious Calibration

Advantage of structural model:

- Added discipline of **jointly** matching credit spreads and liquidity

Changes to model:

- Sacrifice deterministic maturity, use random maturity to handle shifts in aggregate state while maintaining tractability:
  - **Good** period with normal cash-flows and well intermediated OTC markets
  - **Bad / Crisis** period with shock to intermediation intensity (financial crisis), riskier cash-flows, and higher price of risk (Chen 2010)

Implementation:

- Extract α_H, α_L from bond ultimate recovery and trading prices at default (Moody's Default & Recovery Database)
- Target bid-ask spread to one observed in data, match total credit spreads of bonds of different ratings
- *Decompose* credit-spreads into default-, liquidity- and interaction terms, and see how they vary cross-sectionally and across states

Model-Based Decomposition: Methodology

- Model allows to decompose *total credit spread* in more refined way:
  - "**Pure Default**": Yield of a defaultable bond free from liquidity frictions with adjusted default boundary reflecting improved secondary market liquidity (both before and after default)
  - "**Liquidity Driven Default**": Yield of a defaultable bond free from liquidity frictions with original default boundary minus "Pure Default"
  - "**Pure Liquidity**": Yield of a default free bond subject to the same liquidity frictions
  - "**Default Driven Liquidity**": The residual
- None of the above parts are directly observable from data: We need a structural model to construct this decomposition
- The decomposition scheme is designed to quantify the interaction between liquidity and default

Model Based Decomposition: Superior Grade

	State G	State B	Change (in bps)	Change (%)
Total Credit Spread	84.73	124.13	39.39	100.00
Pure Default	22.46	40.16	17.70	44.92
Liquidity Driven Default	9.04	14.87	5.83	14.80
Pure Liquidity	45.59	53.68	8.27	20.98
Default Driven Liquidity	7.64	15.25	7.60	19.30

Table : Model Based Decomposition: Superior Grade Bonds

Model Based Decomposition: Investment Grade

	State G	State B	Change (in bps)	Change (in %)
Total Credit Spread	196.82	288.77	91.95	100.00
Pure Default	86.20	139.63	53.43	58.11
Liquidity Driven Default	24.63	33.14	8.51	9.26
Pure Liquidity	56.69	67.03	10.34	11.24
Default Driven Liquidity	29.29	48.97	19.67	21.39

Table : Model Based Decomposition: Investment Grade Bonds

Model Based Decomposition: Junk Grade

	State G	State B	Change (in bps)	Change (in %)
Total Credit Spread	396.09	574.54	178.45	100.00
Pure Default	210.46	319.81	109.35	61.28
Liquidity Driven Default	48.08	63.47	15.39	8.62
Pure Liquidity	74.74	88.49	13.76	7.71
Default Driven Liquidity	62.81	102.76	39.96	22.39

Table : Model Based Decomposition: Junk Grade Bonds

What did we learn from this decomposition?

- Liquidity driven default is quantitatively important, especially in bad times and for risky bonds
- Default driven (endogenous) liquidity is as important as pure liquidity (search frictions) for risky bonds
- Increase in default driven illiquidity responsible for most of the contribution of liquidity to credit spread when the economy switches to bad state

Conclusion

Fully solved non-stationary dynamic search model:

- Closed form solution for debt, equity, default boundary

Liquidity-default spiral:

- Lower liquidity in secondary market lowers the distance to default, which further lowers liquidity in secondary market,...

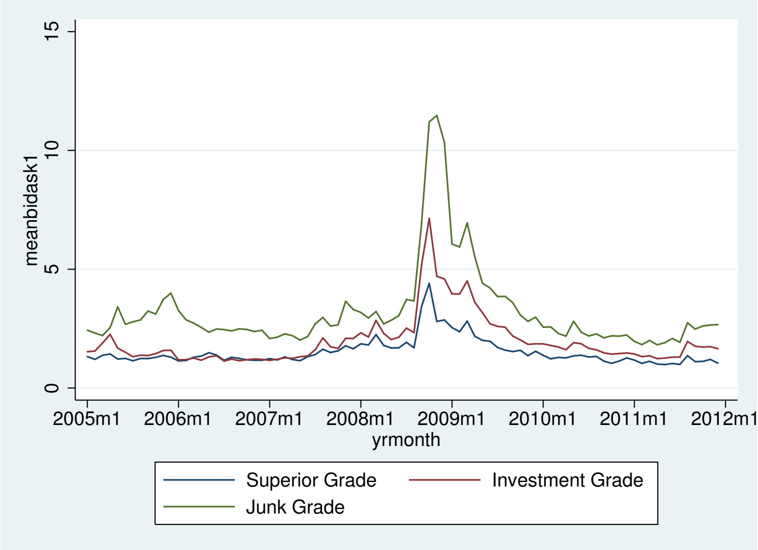
What about adverse selection?

- Definitely reasonable but challenging. Probably generates similar empirical illiquidity pattern
- For understanding the role of liquidity in credit spreads, search framework (simple, easy to be integrated) delivers first-order effects

Empirical implementation:

- Targeting liquidity, we match bond credit spreads and are then able to decompose into liquidity and default components

Future work: Aggregate Shocks & Serious Calibration



TRACE implied bid-ask spread (in %, Bao et al 2011) by year and by rating class

Solution: Derivation of Closed-Forms

Debt D_H, D_L:

- Mix of two distorted LT96 solutions

$$
\begin{aligned}
rD_H(\delta,\tau) &= \underbrace{\mathcal{A}^{\delta} D_H(\delta,\tau)}_{\textit{CF dynamics}} - \underbrace{\frac{\partial D_H}{\partial \tau}(\delta,\tau)}_{\textit{Maturity}} + c + \underbrace{\xi\left[D_L(\delta,\tau) - D_H(\delta,\tau)\right]}_{\textit{Liqidity shock}} \\
\bar{r} D_L(\delta,\tau) &= \underbrace{\mathcal{A}^{\delta} D_L(\delta,\tau)}_{\textit{CF dynamics}} - \underbrace{\frac{\partial D_L}{\partial \tau}(\delta,\tau)}_{\textit{Maturity}} + c + \underbrace{\lambda\left[X(\delta,\tau) - D_L(\delta,\tau)\right]}_{\textit{Secondary market}}
\end{aligned}
$$

Equity E:

- No 'adding up' as in LT96, solve for equity via ODE *directly*

$$
r \cdot E(\delta) = \mathcal{A}^{\delta} E(\delta) + \underbrace{\delta}_{\textit{CF}} - \underbrace{(1-\pi)\,c}_{\textit{Coupon}} + \underbrace{1/T\left[D_H(\delta, T) - p\right]}_{\textit{Rollover gain/loss}}
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Optimal default boundary δ_B:

- Unique fixed-point δ_B from smooth pasting

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