

# FINANCE RESEARCH SEMINAR SUPPORTED BY UNIGESTION

## "Market Expectations in the Cross Section of Present Values"

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### Abstract

Returns and cash flow growth for the aggregate U.S. stock market are highly and robustly predictable. Using information extracted from the cross section of book-to-market ratios, we find an out-of-sample forecasting R<sup>2</sup> as high as 10% for returns and 25% for dividend growth at the annual frequency. We present a general economic framework linking aggregate market expectations to disaggregated valuation ratios in a dynamic latent factor system. To construct our forecasts we use a regression-based filter to extract factors driving aggregate expected returns and cash flow growth from the cross section of valuation ratios. Our findings suggest that market discount rates are more volatile and less persistent than previously believed.

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# Market Expectations in the Cross Section of Present Values \*

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## Abstract

Returns and cash flow growth for the aggregate U.S. stock market are highly and robustly predictable. Using information extracted from the cross section of book-to-market ratios, we find an *out-of-sample* forecasting  $R^2$  as high as 10% for returns and 25% for dividend growth at the annual frequency. We present a general economic framework linking aggregate market expectations to disaggregated valuation ratios in a dynamic latent factor system. To construct our forecasts we use a regression-based filter to extract factors driving aggregate expected returns and cash flow growth from the cross section of valuation ratios. Our findings suggest that market discount rates are more volatile and less persistent than previously believed.

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Explaining market price behavior of the U.S. capital stock is among the most fundamental challenges facing economists. The present value relationship between prices, discount rates and future cash flows has proved a valuable lens for understanding stock price variation. It reveals that price changes are wholly driven by fluctuations in investors' expectations of future returns and cash flow growth. Understanding asset prices amounts to understanding the dynamic behavior of these expectations.

The most common approach to measuring aggregate return and cash flow expectations is predictive regression. As suggested by the present value relation, research has found the aggregate price-dividend ratio to be among the most informative predictive variables. Typical in-sample estimates find that about 10% of annual return variation (1% of annual dividend growth variation) can be explained by forecasts based on the aggregate book-to-market ratio, but find little or no out-of-sample predictive power.<sup>1</sup> In this paper we show that reliance on aggregate quantities drastically understates the degree of value ratios' predictive content for both returns and cash flow growth, and hence understates the volatility of investor expectations. Our estimates suggest that as much as 10% of the *out-of-sample* variation in annual market returns (25% for dividend growth), and somewhat more of the in-sample variation, can be explained by the cross section of past disaggregated value ratios.

To harness the disaggregated information we represent the cross section of asset-specific book-to-market ratios as a dynamic latent factor model. We relate these disaggregated value ratios to aggregate expected market returns and cash flow growth. Our model highlights the idea that the same dynamic state variables driving aggregate expectations also govern the dynamics of the entire panel of asset-specific valuation ratios. This representation allows us to exploit rich cross-sectional information to extract precise estimates of market expectations.

Our approach attacks a challenging problem in empirical asset pricing: How does one exploit a wealth of predictors in relatively short time series? If the predictors number near or

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<sup>1</sup>See Cochrane (2005) and Kojien and Van Nieuwerburgh (2011) for surveys of return and cash flow predictability evidence using the aggregate price-dividend ratio. Similar results obtain from forecasts based on the aggregate book-to-market ratio.

more than the number of observations, the standard ordinary least squares (OLS) forecaster is known to be poorly behaved or nonexistent (see Huber (1973)). Our solution is to use partial least squares (PLS, Wold (1975)), which is a simple regression-based procedure designed to parsimoniously forecast a single time series using a large panel of predictors. We use it to construct a *univariate* forecaster for market returns (or cash flow growth) that is a linear combination of assets' valuation ratios. The weight of each asset in this linear combination is based on the covariance of its value ratio with the forecast target. Much of our analysis relies on results from Kelly and Pruitt (2011), who derive properties of PLS in the factor model setting that apply directly to the asset pricing model considered here.

Using data from 1930-2009, PLS forecasts based on the cross section of portfolio-level book-to-market ratios achieve an out-of-sample predictive  $R^2$  as high as 9.9% for annual market returns, 25.4% for annual dividend growth and 0.8% for monthly market returns (in-sample  $R^2$  of 32.9%, 44.5% and 4.1%, respectively). Since we construct a single factor from the cross section, our results can be directly compared with univariate forecasts from the many alternative predictors that have been considered in the literature. In contrast to our results here, previously studied predictors typically perform well in-sample but become insignificant out-of-sample, often performing worse than forecasts based on the historical mean return (Goyal and Welch (2008)).

Our estimates shed new light on the dynamic processes for expected returns and cash flow growth rates. We find that the volatility of expected one-year returns since 1955 is 6.9% based on out-of-sample estimates (6.7% from in-sample estimates), nearly 80% higher than the discount rate volatility estimated from the aggregate book-to-market ratio. We also find much less persistence in expected returns, with an autocorrelation of 0.33, contrasted against discount rate persistence of 0.84 based on the aggregate book-to-market ratio. The evidence for expected market cash flow growth is similar. This degree of variability in short term expectations is difficult to reconcile with fundamentals-based structural asset pricing models, which generate persistent, low volatility fluctuations in expected market returns

(e.g. Campbell and Cochrane (1999) and Bansal and Yaron (2004)).

We establish the robustness of our main findings in a number of ways. First, we evaluate various degrees of portfolio aggregation and find similar results whether we use six, 25 or 100 size and book-to-market sorted portfolios, with forecast performance that increases in the number of portfolios. Second, applying our method to individual stocks rather than portfolios corroborates our main results. Using the entire cross section of CRSP stocks (a cross section of several thousand value ratios), we find an out-of-sample one month return forecasting  $R^2$  of nearly 1.6%. Third, sensitivity analysis of out-of-sample predictive performance to different subsamples shows that our results are robust to virtually any choice of sample split after 1955. Fourth, our results are also robust to data from outside the U.S. We forecast returns on the value-weighted aggregate world portfolio (excluding the U.S.) by applying PLS to an international cross section of non-U.S., country-level valuation ratios, available beginning in 1975. We find an out-of-sample predictive  $R^2$  of 2.3% (5.3% in-sample) at the monthly frequency, corroborating our results in U.S. data. Lastly, we find similar results when applying PLS to the cross section of portfolio price-dividend ratios rather than book-to-market ratios.

Why do disaggregated prices produce such accurate forecasts? To illustrate the advantages of cross section information, consider a simple CAPM example.<sup>2</sup> In particular, suppose one period expected market returns  $\mu_t$  and expected return on equity  $g_t$  are the two common factors in the economy, and the book-to-market ratio of any asset  $i$  is

$$v_{i,t} = a_i - b_{i,\mu}\mu_t + b_{i,g}g_t + e_{i,t} \tag{1}$$

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<sup>2</sup>The present value system in Equations 1 and 2 obtains as a special case of the model in Section I. It arises in an economy where  $\mu_t$  and  $g_t$  each follow an AR(1), individual expected returns obey an exact one factor model as in the CAPM,  $\mu_{i,t} = \mu_{i,0} + c_{i,\mu}\mu_t$ , and individual expected return on equity obeys a one factor model,  $g_{i,t} = g_{i,0} + c_{i,g}g_t + e_{i,t}$ . This special case is similar to the formulation of Polk, Thompson and Vuolteenaho (2006).

while the aggregate book-to-market ratio is

$$v_t = a - b_\mu \mu_t + b_g g_t. \quad (2)$$

Equation 2 highlights the predictive relationship between  $v_t$ , realized market returns ( $r_{t+1} = \mu_t + \epsilon_{t+1}^r$ ) and return on equity ( $\Delta cf_{t+1} = g_t + \epsilon_{t+1}^d$ ).<sup>3</sup> However, it also evokes the limitations of the aggregate system. Predictive regressions of  $r_{t+1}$  on  $v_t$  take the form

$$\mathbb{E}_t[r_{t+1}|v_t] = \hat{a} + \hat{b}v_t = \hat{a} + \hat{b}(b_\mu \mu_t + b_g g_t)$$

and thus are unduly influenced by information about expected return on equity. The reciprocal problem arises in forecasting  $\Delta cf_{t+1}$ . To overcome this difficulty, researchers have taken present value approaches that account for the joint relationship among  $v_t$ ,  $\mu_t$  and  $g_t$  (see Cochrane (2008b), Lettau and Van Nieuwerburgh (2008), van Binsbergen and Koijen (2010)). While this begins to disentangle the link between prices and expectations, these joint systems continue to rely solely on aggregate variables. Because both  $\mu_t$  and  $g_t$  are latent, each adds noise to the signal extraction problem of the other.<sup>4</sup> If there exist other signals for  $\mu_t$  and  $g_t$  in the economy, incorporating them will improve estimates of the latent expectations. This is how disaggregated valuation ratios in Equation 1 become a valuable information source as long as each  $v_{i,t}$  provides a non-redundant signal for  $\mu_t$  and  $g_t$ .

PLS conveniently reduces the many available signals to an optimal forecast with a series of ordinary least squares regressions. The first stage consists of “reverse” regressions in which individual valuation ratios are regressed on the forecast target. Next, in each time period, we run second stage cross-sectional regressions of assets’ valuation ratios on regression coefficients estimated in the first stage. In the final stage, aggregate return or cash flow

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<sup>3</sup>In Vuolteenaho’s (2002) book-to-market identity, cash flow growth enters as return on equity. We later discuss how this system relates to the Campbell and Shiller (1988) price-dividend identity, where cash flows enter in the form of dividend growth.

<sup>4</sup>This remains true despite the absence of measurement error in the aggregate book-to-market expression, as pointed out by Fama and French (1988).

growth realizations are regressed on the fitted factors from the second stage, delivering our final filtered estimates for unobservable return and growth expectations. The final-stage predictor is a discerningly-constructed linear combination of disaggregated value ratios that parsimoniously incorporates information from individual valuation ratios into predictions of future aggregate returns and cash flow growth.

The preceding CAPM example is also useful to develop intuition for how PLS works. Each  $v_{i,t}$  is a function of only the *expected* portion of returns and cash flows and is uncorrelated with their unanticipated future shocks. Therefore, first stage time series regression coefficients of  $v_{i,t}$  on  $r_{t+1}$  and  $\Delta cf_{t+1}$  (which serve as observable proxies for the latent factors  $\mu_t$  and  $g_t$ ) describe how each asset's valuation ratio depends on the true factors  $\mu_t$  and  $g_t$ . When the coefficients  $b_{i,\mu}$  and  $b_{i,g}$  differ across  $i$ , fluctuations in  $\mu_t$  and  $g_t$  cause the cross section of value ratios to fan out and compress over time. The first-stage coefficient estimates provide a map from the cross-sectional distribution of  $v_{i,t}$ 's to the latent factors. Second-stage cross section regressions of  $v_{i,t}$  on first-stage coefficients use this map to estimate the factors at each point in time. Because the first-stage regression takes an errors-in-variables form, second-stage estimates of the latent expectations  $(\mu_t, g_t)'$  have a multiplicative bias. Since OLS forecasts are invariant to scalar multiples of regressors, the third-stage regression of realized returns or growth on the estimated factors delivers consistent estimates of  $\mu_t$  and  $g_t$ .

The earliest predecessors of this study use predictive regressions based on the aggregate price-dividend ratio, including Rozeff (1984), Campbell and Shiller (1988) and Fama and French (1988). A large subsequent literature has evolved testing the predictive relation based on aggregate valuation ratios, including Stambaugh (1986, 1999), Hodrick (1992), Goetzmann and Jorion (1993), Nelson and Kim (1993), Kothari and Shanken (1997), Lewellen (2004), Paye and Timmermann (2006) and Pástor and Stambaugh (2009a).

More directly, our paper builds on recent literature that exploits the present value relation to identify market expectations for returns and dividends, including van Binsbergen and Koijen (2010), Ghosh and Constantinides (2010), Ferreira and Santa-Clara (2010),

Cochrane (2008a,b), Pástor, Sinha, and Swaminathan (2008), Rytchkov (2008), Campbell and Thompson (2008), Lettau and Van Nieuwerburgh (2008), Ang and Bekaert (2007), Lettau and Ludvigson (2005), Brennan and Xia (2005) and Menzly, Santos and Veronesi (2002). While these papers focus on aggregate present value models, the key to our analysis is incorporating cross-sectional information. Vuolteenaho (2002), Hansen, Heaton, and Li (2008), Pástor and Veronesi (2003, 2006), Kiku (2006) and Kelly (2011) also model valuation ratios for individual assets, though we are the first to exploit a factor structure in value ratios to form market return and cash flow forecasts.

The economics literature mainly relies on principal components (PCs) to condense information from the large cross section into a small number of predictive factors *before* estimating a linear forecast, an approach exemplified in the macro-forecasting literature by Stock and Watson (2002) and Bai and Ng (2006). PC forecasts based on macroeconomic indicators have recently been applied in the context of stock return prediction by Ludvigson and Ng (2007). The key difference between principal components and partial least squares is their method of dimension reduction. PLS condenses the cross section according to *covariance with the forecast target* and chooses a linear combination of predictors that is optimal for forecasting. On the other hand, PC condenses the cross section according to *covariance within the predictors*. The principal components that best describe predictor variation are not necessarily the factors most useful for forecasting, and therefore PCs can produce suboptimal forecasts (see Kelly and Pruitt (2011) for a detailed discussion). As we show, principal components have little forecasting success in our present value setting.

PLS is reminiscent of two-pass regression tests of cross-sectional beta-pricing models (see Fama and MacBeth (1973), Shanken (1992) and Jagannathan and Wang (1998)). Both techniques rely on cross-sectional dispersion among financial variables to infer market risk prices. There are two key differences between our three-pass regression approach and two-pass return tests. First, our cross section is constituted of valuation ratios rather than returns. Second, we string together period-by-period estimates from second-stage regressions to construct our



key predictor variable, as opposed to averaging second-stage output over time to find a single risk price. Our approach is also related to Polk, Thompson and Vuolteenaho (2006) who use a CAPM-motivated two-pass approach to forecasting returns, and to Chowdhry, Roll, and Xia (2005), who construct estimates of the latent inflation time series from the cross section of returns using two-pass regression.

In the next section we present an economic framework for the cross section of present values. Section II introduces partial least squares and relevant results from Kelly and Pruitt (2011). In Section III we present empirical findings, compare alternative methodologies and discuss our results. We present our conclusions in Section IV. Technical assumptions and other details are relegated to the appendix.

## I The Cross-Sectional Present Value System

We assume that one-period expected log returns and log cash flow growth rates across assets and over all horizons are linear in a set of common factors<sup>5</sup>

$$\begin{aligned}\mu_{i,t} &= \mathbb{E}_t[r_{i,t+1}] = \gamma_{i,0} + \boldsymbol{\gamma}'_i \mathbf{F}_t \\ g_{i,t} &= \mathbb{E}_t[\Delta c f_{i,t+1}] = \delta_{i,0} + \boldsymbol{\delta}'_i \mathbf{F}_t + \varepsilon_{i,t}.\end{aligned}\tag{3}$$

Equation 3 states that, conditional on time  $t$  information, expected one-period returns and growth rates are driven solely by the  $(K_F \times 1)$  vector of factors  $\mathbf{F}_t$  that are common across valuation ratios.

We assume that assets' expected returns are determined by systematic factors and possess

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<sup>5</sup>Factor models are analytically tractable and are sufficiently general to subsume a wide range of models considered in the asset pricing literature. Asset pricing models, both theoretical and empirical, link individual expected returns to aggregate expected returns either directly, as in the CAPM (Sharpe (1964), Lintner (1965), Treynor (1961)) and Fama-French model (Fama and French (1993)), or indirectly via common state variables, as in Merton's (1973) ICAPM. Similarly, theoretical models commonly assume a factor structure in dividend growth (Connor (1984), Bansal and Viswanathan (1993), and Bansal, Dittmar, and Lundblad (2005), among others).

no idiosyncratic behavior.<sup>6</sup> This restriction is not imposed for individual asset's expected dividend growth, which may possess an idiosyncratic component,  $\varepsilon_{i,t}$ . The aggregate market obeys the same structure, with no  $i$  subscripts and no idiosyncrasies:

$$\begin{aligned}\mu_t &= \gamma_0 + \boldsymbol{\gamma}'\mathbf{F}_t \\ g_t &= \delta_0 + \boldsymbol{\delta}'\mathbf{F}_t.\end{aligned}\tag{4}$$

The factor loading vectors  $\boldsymbol{\gamma}$  and  $\boldsymbol{\delta}$  summarize how market expectations respond to movements in the underlying economic factors. We assume that at least  $K_f \leq K_F$  of the factors receive non-zero loadings from the market (i.e. have non-zero elements in  $\boldsymbol{\gamma}$  or  $\boldsymbol{\delta}$ ). If  $K_f \lesssim K_F$ , the remaining  $K_F - K_f$  factors are *irrelevant* for explaining market expectations. In this case, the total set of valuation ratio factors contains elements that are inessential – maybe even pernicious – to the goal of forecasting either market returns or cash flow growth. We discuss below how PLS successfully filters out the impact of these irrelevant factors, while the method of principal components cannot be guaranteed to do so.

To emphasize the parsimony of our approach we will focus on  $K_f = 1$ , the case in which a single factor drives returns or cash flow growth (though our approach generalizes to multiple factors). We do not need to make any assumptions about the total number of common factor among value ratios ( $K_F$ ), since target-irrelevant factors will be filtered out by PLS.

Realized returns and growth rates are equal to their conditional expectations plus an unforecastable shock:

$$\begin{aligned}r_{i,t+1} &= \mu_{i,t} + \eta_{i,t+1}^r \\ \Delta cf_{i,t+1} &= g_{i,t} + \eta_{i,t+1}^c.\end{aligned}$$

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<sup>6</sup>This assumption can be relaxed. Allowing for an orthogonal idiosyncratic component in firms' expected returns has no impact on the development or implementation of our approach.

Finally, we assume that the factor vector evolves as a first order vector autoregression<sup>7</sup>

$$\mathbf{F}_{t+1} = \mathbf{\Lambda}_1 \mathbf{F}_t + \boldsymbol{\xi}_{t+1}. \quad (5)$$

The above structure may be embedded in the linearized present value formula of Vuolteenaho (2002). This accounting-based identity relates an asset's log book-to-market ratio to future discount rates and earnings growth

$$v_{i,t} = \frac{\kappa_i}{1 - \rho_i} + \sum_{j=1}^{\infty} \rho_i^{j-1} \mathbb{E}_t[-r_{i,t+j} + \Delta cf_{i,t+j}].$$

where  $v_{i,t}$  is the log book-to-market ratio of stock  $i$ ,  $r_{i,t+j}$  is its log return,  $\Delta cf_{i,t+j}$  is its return on equity (ROE), and  $\kappa_i$  and  $\rho_i$  are linearization constants. ROE is defined as<sup>8</sup>

$$ROE_{t+j} = \log \left( 1 + \frac{\text{earnings}_{t+j}}{\text{book equity}_{t+j-1}} \right).$$

This weighted sum of expected one-period returns and growth rates over all future horizons, combined with (3) and (5), reduces to the following expression for the valuation ratio

$$v_{i,t} = \phi_{i,0} + \boldsymbol{\phi}'_i \mathbf{F}_t + \varepsilon_{i,t}, \quad (6)$$

where formulas for  $\phi_{i,0}$ ,  $\boldsymbol{\phi}_i$  and  $\varepsilon_{i,t}$  are provided in Appendix A.1. Equations 4 and 6 unify disaggregated valuation ratios and aggregate expectations via a common factor model. They also provide a framework for utilizing cross section information to achieve our ultimate goal of precisely estimating conditional expected market returns and cash flow growth.

An alternative to Vuolteenaho's (2002) present value system is the well known Campbell and Shiller (1988) present value identity, which relates the log price-dividend ratio of an asset

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<sup>7</sup>That  $\mathbf{F}_t$  is a first order process is without loss of generality since any higher order vector autoregression can be written as a VAR(1).

<sup>8</sup>Vuolteenaho (2002) represents this identity in terms of excess returns. We use his identity exactly, though we represent it in terms of returns rather than excess returns.

to its future discount rates and dividend growth. The Campbell-Shiller identity also falls into the framework of Equations 3-6 when  $v_{i,t}$  is the log price-dividend ratio,  $r_{i,t+j}$  is the log return, and  $\Delta cf_{i,t+j}$  is log dividend growth. However, working with individual assets' price-dividend ratios can be problematic. Fama and French (2000) document a steep downward trend in the fraction of U.S. firms paying dividends, with only 20.8% of firms classified as dividend payers in 1999. Because price-dividend ratios are undefined for the majority of stocks, our analysis focuses on the cross section of book-to-market ratios. However, we do consider the performance of certain price-dividend ratios in a robustness check.

## II Estimation

In this section we outline our empirical methodology, which is based on Kelly and Pruitt's (2011) generalization of partial least squares. Interested readers can refer to that paper for detailed econometric development and proofs of results stated below. Assumptions underlying the stated results are explained here and made precise in Appendix A.2.

### II.A Setup

To ease the algebraic development we first establish notation. Partial least squares forecasts use two sets of inputs. The first input is the forecasting target, which in general takes the form  $y_{t+1} = \beta_0 + \boldsymbol{\beta}' \mathbf{F}_t + \eta_{t+1}$ . We will focus primarily on two targets, aggregate market returns  $r_m$  and cash flow growth  $\Delta cf_m$ , implying

$$y_{t+1} = \begin{cases} \gamma_0 + \boldsymbol{\gamma}' \mathbf{F}_t + \eta_{t+1}^r & \text{if } y_{t+1} = r_{m,t+1} \\ \delta_0 + \boldsymbol{\delta}' \mathbf{F}_t + \eta_{t+1}^c & \text{if } y_{t+1} = \Delta cf_{m,t+1}. \end{cases}$$

Defining  $\mathbf{F}_{(T \times K_F)} = [\mathbf{F}_1, \mathbf{F}_2, \dots, \mathbf{F}_T]'$ , the matrix representation of  $y_{t+1}$  is

$$\begin{aligned} \mathbf{y}_{(T \times 1)} &= [y_2, y_3, \dots, y_{T+1}]' \\ &= \boldsymbol{\nu}\beta_0 + \mathbf{F}\boldsymbol{\beta} + \boldsymbol{\eta} \end{aligned}$$

where  $\beta_0, \boldsymbol{\beta}$  are defined in the obvious way for either  $r_{m,t+1}$  or  $\Delta cf_{m,t+1}$ .

The second input to PLS is the cross section of book-to-market ratios  $v_{i,t} = \phi_{i,0} + \boldsymbol{\phi}'_i \mathbf{F}_t + \varepsilon_{i,t}$  ( $i = 1, \dots, N$ ). These are arranged into the vector  $\mathbf{x}_t = (v_{1,t}, \dots, v_{N,t})'$  and stacked as

$$\begin{aligned} \mathbf{X}_{(T \times N)} &= [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_T]' \\ &= \boldsymbol{\nu}\boldsymbol{\phi}'_0 + \mathbf{F}\boldsymbol{\Phi}' + \boldsymbol{\varepsilon} \end{aligned}$$

with  $\boldsymbol{\phi}_0 = [\phi_{1,0}, \phi_{2,0}, \dots, \phi_{N,0}]'$  and  $\boldsymbol{\Phi}_{(N \times K_F)} = [\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, \dots, \boldsymbol{\phi}_N]'$ .

## II.B The Estimator

PLS can be implemented by the following series of ordinary least squares regressions. In the first stage, for each asset  $i$  we run a time series regression of its book-to-market ratio on the forecast target

$$v_{i,t} = \hat{\phi}_{i,0} + \hat{\boldsymbol{\phi}}'_i y_{t+1} + e_{i,t}.$$

The loading estimate  $\hat{\boldsymbol{\phi}}_i$  describes the sensitivity of each  $v_{i,t}$  to the latent factor driving the forecast target.

In the second stage, for each period  $t$ , we run a cross-sectional regression of assets' book-to-market ratios on their loadings estimated in the first stage

$$v_{i,t} = \hat{c}_t + \hat{\mathbf{F}}'_t \hat{\boldsymbol{\phi}}_i + w_{i,t}.$$

Here, the first stage loadings become the independent variables, and the latent factors  $\mathbf{F}_t$

are the coefficients to be estimated. The first two stages exploit the factor nature of the system to draw inferences about the underlying factors. As the factors fluctuate over time, the cross section of valuation ratios fans out or compresses. If the true factor loadings  $\phi_i$  were known, we could consistently estimate the latent factor time series by simply running cross section regressions of  $v_{i,t}$  on  $\phi_i$  period-by-period. Since  $\phi_i$  is unknown, the first-stage regression coefficients provide a preliminary description of how each  $v_{i,t}$  depends on  $\mathbf{F}_t$ . This first stage regression sketches a map from the cross-sectional distribution of value ratios to the latent factors. Second-stage cross section regressions of  $v_{i,t}$  on first-stage coefficients use this map to produce estimates of the factors at each point in time.

The third step in the filter runs a predictive regression of realized returns or cash flow growth rates on the lagged factors estimated in the second stage. This final regression is the culmination of the multi-asset present value system. It parsimoniously combines information from individual assets' valuation ratios to arrive at a prediction of future aggregate returns or cash flow growth. The ultimate predictor,  $\hat{\mathbf{F}}_t$ , is a discerningly-constructed linear combination of disaggregated valuation ratios that collapses the cross section system to its fundamental driving factors. The  $R^2$  from the final step regression summarizes the predictive power embodied in the cross section of valuation ratios.

Kelly and Pruitt (2011) provide a one-step representation of this algorithm:

$$\hat{\mathbf{y}} = \boldsymbol{\nu}\bar{y} + \mathbf{J}_T \mathbf{X} \mathbf{J}_N \mathbf{X}' \mathbf{J}_T \mathbf{y} (\mathbf{y}' \mathbf{J}_T \mathbf{X} \mathbf{J}_N \mathbf{X}' \mathbf{J}_T \mathbf{X} \mathbf{J}_N \mathbf{X}' \mathbf{J}_T \mathbf{y})^{-1} \mathbf{y}' \mathbf{J}_T \mathbf{X} \mathbf{J}_N \mathbf{X}' \mathbf{J}_T \mathbf{y}, \quad (7)$$

where  $\mathbf{J}_L \equiv \mathbf{I}_L - L^{-1} \boldsymbol{\nu}_L \boldsymbol{\nu}_L'$ ,  $\mathbf{I}_L$  is the  $L$ -dimensional identity matrix and  $\boldsymbol{\nu}_L$  is a  $L$ -vector of ones.  $\mathbf{J}$  matrices are present since each regression step is run including a constant regressor.

This procedure is consistent: It asymptotically recovers the latent expectations of aggregate market returns and cash flow growth as the number of predictors and time series observation both become large.<sup>9</sup> In particular, Theorems 1 and 4 in Kelly and Pruitt (2011)

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<sup>9</sup>Because the first-stage regression takes an errors-in-variables form, coefficients estimated in the first and second stage are biased. The third regression step accounts for this bias, and removes this effect from the ultimate forecast of  $y$ , since least squares fitted values are invariant to scalar multiples of regressors or

show that  $\hat{\beta}_0 + \beta' \hat{F}_t$  is normally distributed around  $\mathbb{E}_t[y_{t+1}]$  under the assumptions in Appendix A.2 as  $N, T \rightarrow \infty$ . These assumptions are quite weak. The key assumption is that log book-to-market ratios obey a linear factor structure, which is consistent with a range of theories for conditional expected returns (assuming that ROE is also linear in its factors). The remaining assumptions are largely technical, and impose that second moments are finite and probability limits are well-behaved, that there is limited time series and cross-sectional autocorrelation among the residuals  $\boldsymbol{\eta}$  and  $\boldsymbol{\varepsilon}$ , and that unanticipated shocks to returns and cash flow growth are asymptotically orthogonal to past expectations.

The general version of our theory accommodates multiple factors in both returns and cash flow growth. In the interest of parsimony, and to highlight the power of our approach compared to the large set of alternative univariate predictors, we assume that returns and cash flow expectations are each driven by a single factor (though the return factor may be different than the growth rate factor). Extending our analysis to extract additional factors from the cross section of valuation ratios transforms our third step forecasts into multivariate predictive regressions, rather than univariate predictions, and can potentially improve forecastability beyond what we document below.

Importantly, Kelly and Pruitt (2011) prove that this procedure remains consistent even if there are additional factors that drive the cross section of value ratios but do not impact expected market returns or dividend growth (that is, even if  $K_F > K_f$ ). They also show that consistency obtains regardless of where the target-relevant factor stands in the principal component ordering for the value ratio cross section. Note that this is not generally the case for the method of principal components. For example, the first principal component explains the most covariance among the value ratios, regardless of that component's relationship  $\mu_t$  or  $g_t$ . Only if this component happens to also be the factor driving variation in the forecast target will the first component produce a consistent forecast. Kelly and Pruitt demonstrate precisely how PLS avoids this problem by extracting only the forecast-relevant factors, while

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additive constants. See the consistency argument in Kelly and Pruitt (2011) for details.

discarding any irrelevant factors.

## II.C In-Sample Versus Out-of-Sample Implementation

Throughout our empirical analysis we consider both in-sample and out-of-sample approaches to implementing our forecasts. To outline the differences in information sets it is convenient to work within the three-stage construction of the filter rather than the direct formulation of (7).

The basic implementation, which uses all available information, is a purely in-sample estimation. First-stage regressions use the full time series of data to estimate factor loadings. Second-stage regressions construct the predictive factor at time  $t$  as a weighted sum of time  $t$  book-to-market ratios, where the weights are based on first stage regression coefficients. Third-stage predictive regressions are also run in-sample.

In our in-sample analysis, it is possible that first-stage regressions introduce a small sample bias in our predictors since first-stage factor coefficients are based on the full time series. This is analogous to small sample bias in standard OLS predictive regression (cf. Stambaugh (1986) and Nelson and Kim (1993)), which enters into forecasts via estimated predictive coefficients.<sup>10</sup> Consider, for instance, OLS forecasts of  $r_{t+1}$  on some predictor  $z_t$ , where both  $r$  and  $z$  are mean zero. The in-sample estimated coefficient is  $\hat{b}_T = (\frac{1}{T} \sum_{t=0}^{T-1} r_{t+1} z_t) / (\frac{1}{T} \sum_{t=0}^{T-1} z_t^2)$ . The forecast for  $r_{t+1}$  is given by  $\hat{b}_T z_t$ , thus the targeted observation is directly contributing  $1/T$  of the total data to the parameter estimate in its own forecast. For small  $T$ , this can favor false detection of predictability. However, this is *not* a look-ahead bias because it vanishes as  $T$  becomes large. Instead it is strictly a small sample phenomenon.<sup>11</sup> Our annual forecasts consist of overlapping observations spanning 80 years, while our monthly forecasts provide 960 non-overlapping time series observations. While neither of these sample sizes is

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<sup>10</sup>Small sample bias is not unique to PLS, but arises in effectively any forecasting methodology, including OLS, principal components, and in the preliminary parameter estimation step of a Kalman-filtered state space.

<sup>11</sup>An example of look-ahead bias would be using a predictor that is constructed as a centered moving average of the forecast target. In this case, forward-looking information in the predictor favors false detection of predictability, and this bias does not attenuate even in very large samples.



particularly small, we nonetheless take the possibility of small sample bias very seriously.

Thus, our second implementation is a pure (recursive) out-of-sample analysis, and these results serve as the focus of our empirical analysis. The procedure we use is a standard out-of-sample estimation scheme which has been well-studied in the literature (e.g. Goyal and Welch (2008)). The main idea is to run first- and third-stage estimations on training samples that exclude the return or cash flow observation ultimately forecast. We split the full  $T$ -period sample at date  $\tau$ , using the first  $\tau$  observations as training sample and the last  $T - \tau$  observations as the test sample. We estimate first-stage factor loadings using observations  $\{1, \dots, \tau\}$ . Then, for each period  $t \in \{1, \dots, \tau\}$ , we estimate the time  $t$  value of our predictor variable using the cross section of valuation ratios at  $t$  and first-stage coefficients (which are based on data  $\{1, \dots, \tau\}$ ). We then estimate the predictive coefficient in a third-stage forecasting regression of realized returns (or cash flow growth) for periods  $\{2, \dots, \tau\}$  on the factor extracted from  $\{1, \dots, \tau - 1\}$ . Finally, our out-of-sample forecast of the  $\tau + 1$  return is the product of the third-stage predictive coefficient and the time  $\tau$  second-stage result. This procedure is then repeated recursively (next using only data from  $\{1, \dots, \tau + 1\}$  to construct a forecast for the return at  $\tau + 2$ , and so on) until the entire sample of length  $T$  has been exhausted.

All of our analyses are performed using data at the monthly frequency. The out-of-sample procedure just described may be applied to one month forecasts without modification. Annual out-of-sample forecasts require additional care to account for overlap in monthly observations. Hence for annual returns we use the training sample  $\{1, \dots, \tau\}$  to estimate all model coefficients for the purpose of forecasting the annual return through  $\tau + 12$ . The resulting annual horizon forecasts are genuinely out-of-sample.

Inference for in-sample one month forecasts is based on the asymptotic distributions for PLS estimates derived in Kelly and Pruitt (2011). For one year forecasts, we use overlapping data and must adjust our standard errors to reflect the dependence that this introduces into forecast errors. We do this in three ways. First, we calculate Kelly and Pruitt (2011)

standard errors using only the non-overlapping forecast errors from December of each year. This avoids the overlapping observations problem and, with 80 data points, still provides a reasonable sample size for approximating the theoretical asymptotic test statistic distribution.<sup>12</sup> As a second alternative, we calculate Hodrick (1992) standard errors using all overlapping observations. This approach explicitly accounts for the moving average structure that overlap introduces into residuals. Third, we report Newey-West (1985) standard errors with twelve lags to account for overlap-induced serial correlation among residuals. In our empirical analysis, results for all of these test statistics are very similar. Our exposition focuses on the Kelly and Pruitt in-sample standard errors since our results show that these are typically the most conservative.

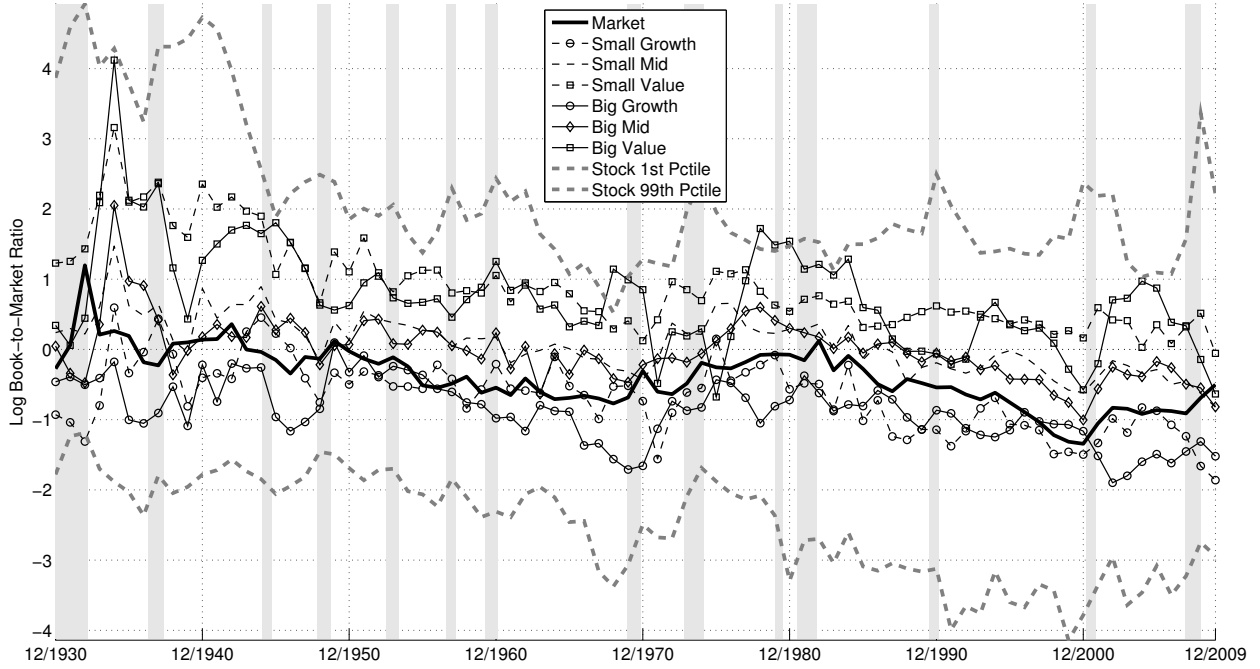
PLS in-sample tests have no direct out-of-sample counterpart (this is also true of OLS). Instead, we conduct out-of-sample inference with the “encompassing” forecast test ENC-NEW derived by Clark and McCracken (2001). This statistic has become widely used in the forecasting literature, and tests the null hypothesis that two predictors provide the same out-of-sample forecasting performance. When we report this statistic, we are testing the denoted predictor versus the historical mean of the target series.<sup>13</sup> We report significance levels as found from Clark and McCracken’s (2001) appendix tables, where critical values for the 10%, 5% and 1% levels are provided. The notation “ $< x$ ” represents the smallest significance level  $x$  for which the encompassing test statistic exceeds the critical value. When evaluating overlapping forecast errors (as we do for annual return, dividend growth and earnings growth forecasts) we use Newey-West standard errors with twelve lags to consistently estimate the appropriate asymptotic variance in the denominator of ENC-NEW, as suggested in Clark and McCracken (2005). Out-of-sample results reported in tables are based on a 1980 sample split, save for the international data which are split at 1995 owing to its much more recent

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<sup>12</sup>We can calculate Kelly and Pruitt standard errors using non-overlapping forecast errors corresponding to any month January through December. We report the  $p$ -value for the median of 12  $t$ -statistics constructed for each of the possible year-end months, therefore covering every non-overlapping set of forecast errors. In practice any choice of year-end month yields very similar statistics so that the median is representative.

<sup>13</sup>This test compares our time  $\tau$  forecast of the  $\tau + 1$  ( $\tau + 12$  for annual returns) realization against the forecast based on the target variable’s mean estimated through time  $\tau$ .

Figure 1: BOOK-TO-MARKET RATIOS



*Notes:* The figure shows year-end log book-to-market ratios from 1930-2009 for the aggregate U.S. stock market, six size and value-sorted portfolios from Fama and French (1993) (available on Kenneth French’s website), and the first and 99<sup>th</sup> percentile (each year) for individual U.S. stock book-to-market ratios (data from CRSP and Compustat). NBER recession dates are represented by the shaded area.

start date. We also report out-of-sample results over a wide range of alternative split dates to demonstrate the robustness of our findings to different test samples.

To evaluate forecasting fit, we calculate the predictive  $R^2 = 1 - \frac{\sum_t (y_t - \hat{y}_t)^2}{\sum_t (y_t - \bar{y})^2}$ , which for our PLS forecasts is equal to the  $R^2$  of the third stage univariate regression. The out-of-sample  $R^2$  lies in the range  $(-\infty, 1]$ , where a negative number means that a predictor provides a less accurate forecast than the target’s historical mean.

## III Empirical Results

### III.A Data

Our central empirical analysis examines market return and cash flow growth predictability by applying partial least squares to different cross sections of valuation ratios. We use book-to-market ratios for Fama and French's (1993) size and value-sorted portfolios (in which U.S. stocks are divided into six, 25 or 100 portfolios).

Many authors, including Fama and MacBeth (1973), Miller and Scholes (1982), Fama and French (1988) and Polk, Thompson and Vuolteenaho (2006), have highlighted the difficulties in working noisy firm-level accounting variables (book values and cash flows). Fama and MacBeth prescribe the use of portfolios to reduce the impact of individual stock noise on information extraction, which motivates our focus on the cross section of portfolios rather than individual stocks for our main analysis. However, we also analyze the robustness of our main results to several other cross sections. One alternative we explore is indeed the cross section of individual stock data, at which point we also discuss in more detail the tradeoffs associated with portfolio data versus that of individual firms. We also consider price-dividend ratios for size and value-sorted portfolios in place of book-to-market ratios. Finally, we take our analysis to international data, using the country-level portfolio valuation ratios of Fama and French (1998). Our focus is on the 1930-2009 sample for U.S. data. The international sample is available only from 1975-2009. Individual stock data are from CRSP and Compustat. U.S. and international portfolio data are from Kenneth French's website. Alternative predictors are obtained from Amit Goyal's website.

Figure 1 plots log book-to-market ratios of the aggregate U.S. stock market and six Fama-French size and value-sorted portfolios in December of each year. It also shows the 1<sup>st</sup> and 99<sup>th</sup> percentile of individual U.S. stock book-to-market ratios by year. Disaggregated book-to-market ratios exhibit more variability than that of the market portfolio, with a standard deviation of 0.53 on average across the six Fama-French portfolios, versus 0.42

for the market. The portfolio series as well as the individual stock percentiles show that cross sectional dispersion among book-to-market ratios varies dramatically over time. The interquartile range of stock-level log book-to-market ratios reaches its maximum of 1.8 during the Great Depression, falls to 0.7 shortly after World War II, and rises again to 1.5 during the technology boom of the late 1990s. Variation in ordering, dispersion, and comovement of book-to-market ratios is the key predictive information that PLS exploits to measure market expectations.

## III.B Market Return Predictability

### III.B.1 Forecasting with Portfolio Book-to-Market Ratios

Our main empirical analysis evaluates the predictability of aggregate market returns using the cross section of book-to-market ratios. We directly estimate our model of the cross section system described in Section I. This model emphasizes the low dimension predictive structure underlying the many-predictor cross section, which motivates our estimation of the model via PLS. Table I presents return forecasting results based on six, 25 and 100 book-to-market ratios of size and value-sorted portfolios of U.S. stocks (Fama and French (1993)). We consider two different forecasting horizons – one month and one year – and report findings for both in-sample and out-of-sample forecasts.

The left half of Table I shows that a single factor extracted via PLS demonstrates a striking degree of predictability for one year returns. The in-sample implementation generates a predictive  $R^2$  reaching 25.0% and 32.9% based on 25 and 100 book-to-market ratios, respectively ( $p < 0.001$  in both cases). When only six portfolios are used, the in-sample forecasting relation is only marginally significant ( $R^2 = 3.84\%$ ,  $p = 0.126$ ), highlighting the information gains from using finer portfolio divisions. Out-of-sample PLS forecasts based on portfolio book-to-market ratios are similarly powerful, delivering an  $R^2$  of 5.9%, 9.8% and 9.9% for six, 25 and 100 portfolios. These are large economic magnitudes for out-of-sample prediction, comparable to in-sample results from commonly studied predictors such as the

Table I: MARKET RETURN PREDICTIONS (1930-2009)

|                | One Year Forecasts |             |               |          | One Month Forecasts |             |
|----------------|--------------------|-------------|---------------|----------|---------------------|-------------|
|                | $R^2$ (%)          | $p$ (KP/CM) | $p$ (Hodrick) | $p$ (NW) | $R^2$ (%)           | $p$ (KP/CM) |
| 6 Portfolios   |                    |             |               |          |                     |             |
| In-Sample      | 3.84               | 0.126       | 0.119         | 0.062    | 0.51                | 0.136       |
| Out-of-Sample  | 5.85               | < 0.050     | -             | -        | 0.44                | < 0.100     |
| 25 Portfolios  |                    |             |               |          |                     |             |
| In-Sample      | 25.04              | < 0.001     | < 0.001       | < 0.001  | 2.07                | < 0.001     |
| Out-of-Sample  | 9.79               | < 0.010     | -             | -        | 0.60                | < 0.010     |
| 100 Portfolios |                    |             |               |          |                     |             |
| In-Sample      | 32.87              | < 0.001     | < 0.001       | < 0.001  | 4.08                | < 0.001     |
| Out-of-Sample  | 9.88               | < 0.010     | -             | -        | 0.76                | < 0.010     |

*Notes:* Results of PLS forecasts of one year and one month market returns. The sets of predictor variables are six, 25 and 100 book-to-market ratios of size and value-sorted portfolios of U.S. stocks from Fama and French (1993). In-sample results are for the 1930-2009 sample. Our out-of-sample procedure splits the sample in 1980, uses the pre-1980 period as a training window, and recursively forecasts returns beginning in January 1980 (results from a wide range of alternative sample splits are shown in Figure 3). We report in-sample and out-of-sample forecasting regression  $R^2$  in percent. We also report  $p$ -values of three different in-sample test statistics. The first is the asymptotic predictive loading  $t$ -statistic from Kelly and Pruitt (2011, denoted “KP” in the table), calculated on every non-overlapping set of residuals as described in the text. For annual returns we also report Hodrick (1992) and Newey-West (1985)  $t$ -statistic  $p$ -values. For out-of-sample tests we report  $p$ -values for Clark and McCracken’s (2001, denoted “CM” in the table) ENC-NEW encompassing test statistic. This tests the null hypothesis of no forecast improvement over the historical mean. For annual returns we follow Clark and McCracken (2005) and use Newey-West standard errors with twelve lags to consistently estimate the appropriate asymptotic variance.

aggregate price-dividend ratio. Each of these out-of-sample results is statistically significant at the 5% level or better based on Clark-McCracken tests.

The last two columns report forecasting results for one month returns. The monthly in-sample  $R^2$  reaches 2.1% and 4.1% based on a single linear combination of 25 or 100 portfolio book-to-market ratios, respectively. Out-of-sample one month return forecasts are significant at the 1% level or better for 25 and 100 portfolios, with an  $R^2$  reaching as high as 0.8%. At the monthly frequency, an out-of-sample  $R^2$  of 0.8% has large economic significance. A heuristic calculation suggested by Cochrane (1999) shows that the Sharpe ratio ( $s^*$ ) earned by an active investor exploiting predictive information (summarized by the regression  $R^2$ ),

and the Sharpe ratio ( $s_o$ ) earned by a buy-and-hold investor, are related by  $s^* = \sqrt{\frac{s_o^2 + R^2}{1 - R^2}}$ . Campbell and Thompson (2008) estimate a monthly equity buy-and-hold Sharpe ratio of 0.108 using data back to 1871. Therefore, an out-of-sample predictive  $R^2$  of 0.8% implies that an active investor exploiting our approach could achieve a Sharpe ratio improvement of roughly 30% over a buy-and-hold investor, using only real-time information in the form of portfolio book-to-market ratios.

How do our market return forecasts compare with predictors proposed in earlier literature? Table II compares the predictive accuracy of our approach with an extensive collection of alternative predictors that have been considered in the literature. In particular, we explore forecasts from 16 predictors studied in a recent return predictability survey by Goyal and Welch (2008). The table considers both in-sample and out-of-sample forecasts of market returns over horizons of one year and one month from each regressor individually. Among the alternatives, the best univariate forecasts at the annual horizon (Panel A) are achieved by the consumption-wealth ratio (*cay*, Lettau and Ludvigson (2001)), which delivers an in-sample  $R^2$  of 14.9% and an out-of-sample  $R^2$  of 2.5%, showing statistically significant predictive power at the 1% level.<sup>14</sup> Other successful out-of-sample predictors include the cross section premium (*csp*) of Polk, Thompson and Vuolteenaho (2006), the term spread (*tms*), the long term government bond return (*ltr*), and the aggregate log earnings-to-price ratio (*ep*). The first row of the column shows that even the success of *cay* is dominated by the single PLS factor extracted from portfolio-level book-to-market ratios.

Table II also reports forecasting results using the first three principal components extracted from the cross section of 25 portfolio book-to-market ratios.<sup>15</sup> Principal components fail to demonstrate any significant return forecasting power in-sample or out-of-sample. In the finance and economics literature, principal components (PC) has become the *de facto*

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<sup>14</sup>Specifically, this is the *cayp* variable which Goyal and Welch (2008) discuss at some length, analyzing how the construction of the variable uses information from the full sample. The highest frequency at which *cay* is available is quarterly, therefore we take each observation to represent the quarter's last month observation and treat the other months as missing.

<sup>15</sup>Principal components results are very similar when extracted from 100 portfolios.

Table II: MARKET RETURN PREDICTIONS: COMPARISON WITH COMMON ALTERNATIVE PREDICTORS

|                 | Panel A: One Year Forecasts |                |                |             | Panel B: One Month Forecasts |             |                |             |                |
|-----------------|-----------------------------|----------------|----------------|-------------|------------------------------|-------------|----------------|-------------|----------------|
|                 | In-Sample                   |                | Out-of-Sample  |             | In-Sample                    |             | Out-of-Sample  |             |                |
|                 | $R^2$ (%)                   | $p$ (Hodrick)  | $p$ (NW)       | $R^2$ (%)   | $p$ (CM)                     | $R^2$ (%)   | $p$ (NW)       | $R^2$ (%)   | $p$ (CM)       |
| <b>100 PtfS</b> | <b>32.87</b>                | < <b>0.001</b> | < <b>0.001</b> | <b>9.88</b> | < <b>0.010</b>               | <b>4.08</b> | < <b>0.001</b> | <b>0.76</b> | < <b>0.010</b> |
| dfy             | 0.31                        | 0.674          | 0.664          | -0.13       | -                            | 0.05        | 0.778          | -0.39       | -              |
| inf             | 0.00                        | 0.853          | 0.747          | -0.31       | -                            | 0.04        | 0.641          | -0.14       | -              |
| svar            | 0.00                        | 0.995          | 0.650          | -0.13       | -                            | 0.09        | 0.654          | -0.80       | -              |
| csp             | 0.36                        | 0.401          | 0.492          | 1.29        | < 0.050                      | 0.61        | 0.043          | -0.11       | -              |
| de              | 0.80                        | 0.527          | 0.525          | -5.59       | -                            | 0.09        | 0.574          | -1.54       | -              |
| lty             | 0.79                        | 0.351          | 0.279          | -0.73       | -                            | 0.02        | 0.671          | -0.87       | -              |
| tms             | 1.10                        | 0.195          | 0.151          | 0.41        | < 0.100                      | 0.12        | 0.266          | -0.20       | -              |
| tbl             | 0.16                        | 0.680          | 0.661          | -3.74       | -                            | 0.00        | 0.962          | -1.03       | -              |
| dfr             | 0.00                        | 0.553          | 0.614          | -0.27       | -                            | 0.15        | 0.499          | 1.06        | < 0.010        |
| pd              | 3.57                        | 0.090          | 0.062          | -3.08       | -                            | 0.20        | 0.304          | -7.44       | -              |
| dy              | 3.84                        | 0.080          | 0.047          | -6.31       | -                            | 0.32        | 0.200          | -0.23       | -              |
| ltr             | 0.89                        | 0.002          | < 0.001        | 0.89        | < 0.050                      | 0.37        | 0.067          | -5.16       | -              |
| ep              | 8.04                        | < 0.001        | < 0.001        | 1.35        | < 0.010                      | 0.47        | 0.044          | -3.48       | -              |
| bm              | 9.99                        | 0.002          | < 0.001        | -18.67      | -                            | 0.81        | 0.096          | -3.58       | -              |
| ntis            | 8.19                        | 0.081          | 0.071          | -56.44      | -                            | 0.52        | 0.110          | -2.13       | -              |
| cay             | 14.93                       | 0.004          | < 0.001        | 2.48        | < 0.010                      | 1.59        | 0.009          | -2.32       | -              |
| pc1             | 1.96                        | 0.182          | 0.115          | -2.19       | -                            | 0.16        | 0.204          | -0.62       | -              |
| pc2             | 0.08                        | 0.811          | 0.790          | -0.32       | -                            | 0.00        | 0.857          | -0.03       | -              |
| pc3             | 0.02                        | 0.960          | 0.882          | -0.55       | -                            | 0.03        | 0.764          | 0.00        | -              |

Notes: Results of PLS forecasts of one year and one month market returns from 100 book-to-market ratios of size and value-sorted portfolios of U.S. stocks from Fama and French (1993); and results for alternative predictors taken from Goyal and Welch (2008) with data updated through 2009. These alternative predictors are the default yield spread (dfy), the inflation rate (inf), stock variance (svar), the cross-section premium (csp), the dividend payout ratio (de), the long term yield (lty), the term spread (tms), the T-bill rate (tbl), the default return spread (dfr), the price-dividend ratio (pd), the dividend yield (dy), the long term rate of returns (ltr), the earning price ratio (ep), the book to market ratio (bm), the investment to capital ratio (ik), the net equity expansion ratio (ntis), the percent equity issuing ratio (eqis), and the *ex post* consumption-wealth-income ratio (cay) which Goyal and Welch refer to as “cayp.” Additionally we consider the first three principal components extracted from the 25 portfolio book-to-market ratios and find no significant improvement when all three predictors are used together. In-sample results are for the 1930-2009 sample wherever possible. Our out-of-sample procedure splits the sample in 1980, uses the pre-1980 period as a training window, and recursively forecasts returns beginning in January 1980 (results from a wide range of alternative sample splits are shown in Figure 3). We report in-sample and out-of-sample forecasting regression  $R^2$  in percent. We also report  $p$ -values of two different in-sample test statistics: Hodrick (1992) and Newey-West (1985)  $t$ -statistic  $p$ -values. For monthly returns, the Newey-West standard error is identical to the White robust standard error. For out-of-sample tests we report  $p$ -values for Clark and McCracken’s (2001) ENC-NEW encompassing test statistic. This tests the null hypothesis of no forecast improvement over the historical mean. For annual returns we follow Clark and McCracken (2005) and use Newey-West standard errors with twelve lags to consistently estimate the appropriate asymptotic variance.

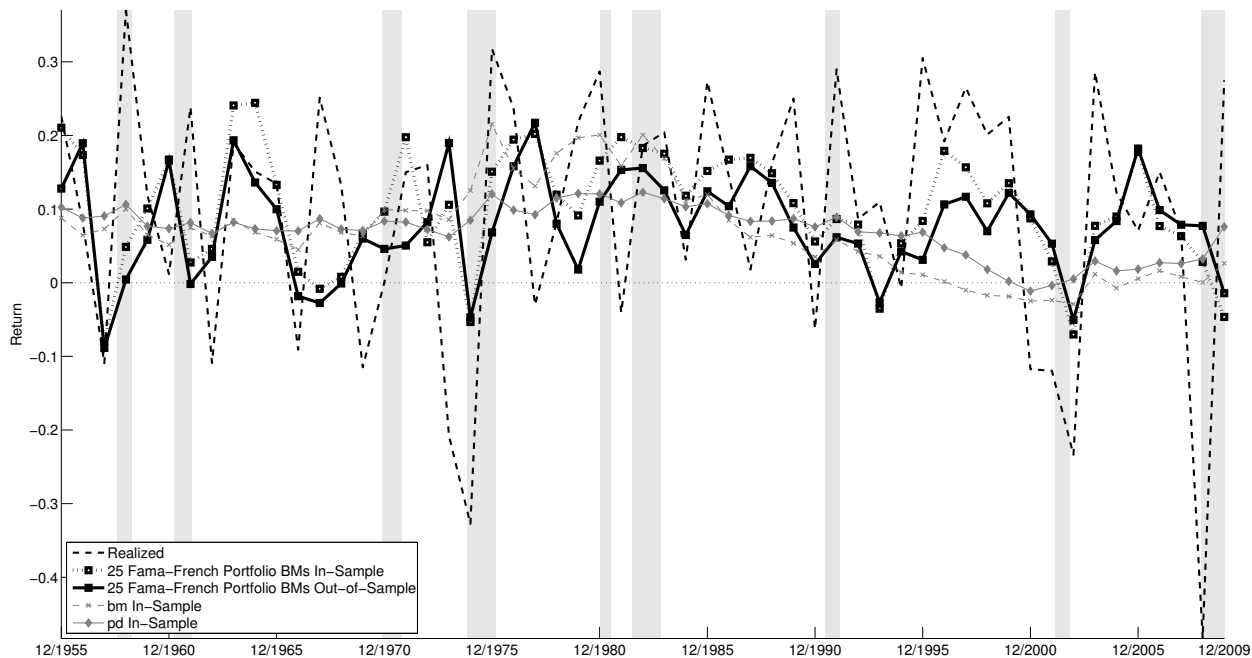


method for condensing large cross sections of predictors into a small number of predictive factors. PC suffers from an important shortcoming in our application: The components that best describe variation among the predictors are not necessarily the factors most useful for predicting next period’s aggregate return. PLS, on the other hand, extracts predictive factors that are optimal for forecasting. If there is a common factor among the predictors that is useful for forecasting returns, but that describes a relatively small amount of the variation within the predictors (that is, it is a low ranking principal component), PC can fail to detect that factor. PLS differs in that it only identifies *forecast-relevant* factors, ignoring factors that may be pervasive among predictors but useless for forecasting. For an in depth discussion of PLS versus PC forecast properties, we refer readers to Kelly and Pruitt (2011).

Predictions of one month returns (Table II, Panel B) tell the same story as annual forecasts. Our procedure is the dominant in-sample univariate predictor ( $R^2 = 4.1\%$ ,  $p < 0.001$ ), with only *cay* and the log earnings-price ratio (*ep*) as the other predictors with in-sample forecasting power that is significant at least at the 5% level. Out-of-sample, only our procedure ( $R^2 = 0.8\%$ ,  $p < 0.01$ ) and the default rate (*dfr*) provide significant positive results, but the in-sample predictive power of *dfr* is tiny and insignificant ( $p = 0.5$ ). In summary, our PLS factor derived from the cross section of book-to-market ratios is the only predictor to exhibit significant performance both in-sample and out-of-sample for one month returns.

Estimates of the expected annual return based on our cross-sectional approach are plotted in Figure 2, and are compared against fitted values from regressions on aggregate valuation ratios. Our estimated annual expected return process differs from other estimates in qualitatively important ways. Table III presents the volatility and persistence of expected market returns estimated from predictive regressions based on the aggregate market book-to-market ratio, compared to estimates based on a single PLS factor extracted from the cross section of 25 portfolio book-to-market ratios. We find that expected returns are nearly twice as volatile as previously estimated. We find expected return volatility of 10.8% at the annual frequency based on in-sample estimates, suggesting that about one-half of the annual vari-

Figure 2: MARKET RETURN PREDICTIONS



*Notes:* The figure shows annual realized returns for the aggregate U.S. stock market (Realized), both in-sample and out-of-sample forecasts from our PLS factor extracted from 25 book-to-market ratios of size and value-sorted portfolios of U.S. stocks from Fama and French (1993) (25 Fama-French Portfolio BMs In-Sample/Out-of-Sample), and in-sample forecasts from predictive regressions on the aggregate book-to-market (bm In-Sample) and aggregate price-dividend (pd In-Sample) ratios. NBER recession dates are represented by the shaded area.

ation in stock market value during this period is attributable to fluctuations in investor expectations. Examining the 1955-2009 subsample allows us to compare with out-of-sample estimates from our approach. During this period we see that in-sample and out-of-sample expected return estimates have similar volatility, nearly 7% per annum, which is about 80% higher than in-sample estimates based on aggregate value ratios and accounts for about 40% of annual stock market variation.

Our estimates also show that annual expected returns mean revert more quickly than previously believed. We find an autocorrelation between 0.199 and 0.445, compared to aggregate value ratios that imply a persistence as high as 0.912. In light of the volatility and lack of serial correlation in realized returns, higher volatility and lower persistence of our one-year expected returns contribute to return forecasts that are substantially more accurate than alternative forecasts. The close agreement across in-sample and out-of-sample

Table III: VOLATILITY AND AUTOCORRELATION OF EXPECTED MARKET RETURN ESTIMATES

|                               | 1930-2009 |       | 1955-2009 |        |
|-------------------------------|-----------|-------|-----------|--------|
|                               | Vol (%)   | AC(1) | Vol (%)   | AC(1)  |
| Realized Returns              | 20.2      | 0.032 | 17.5      | -0.113 |
| Aggregate Price-Dividend      | 4.0       | 0.889 | 5.1       | 0.908  |
| Aggregate Book-to-Market      | 6.5       | 0.839 | 3.9       | 0.912  |
| 25 Portfolios (in-sample)     | 10.8      | 0.332 | 6.7       | 0.445  |
| 25 Portfolios (out-of-sample) | -         | -     | 6.9       | 0.199  |

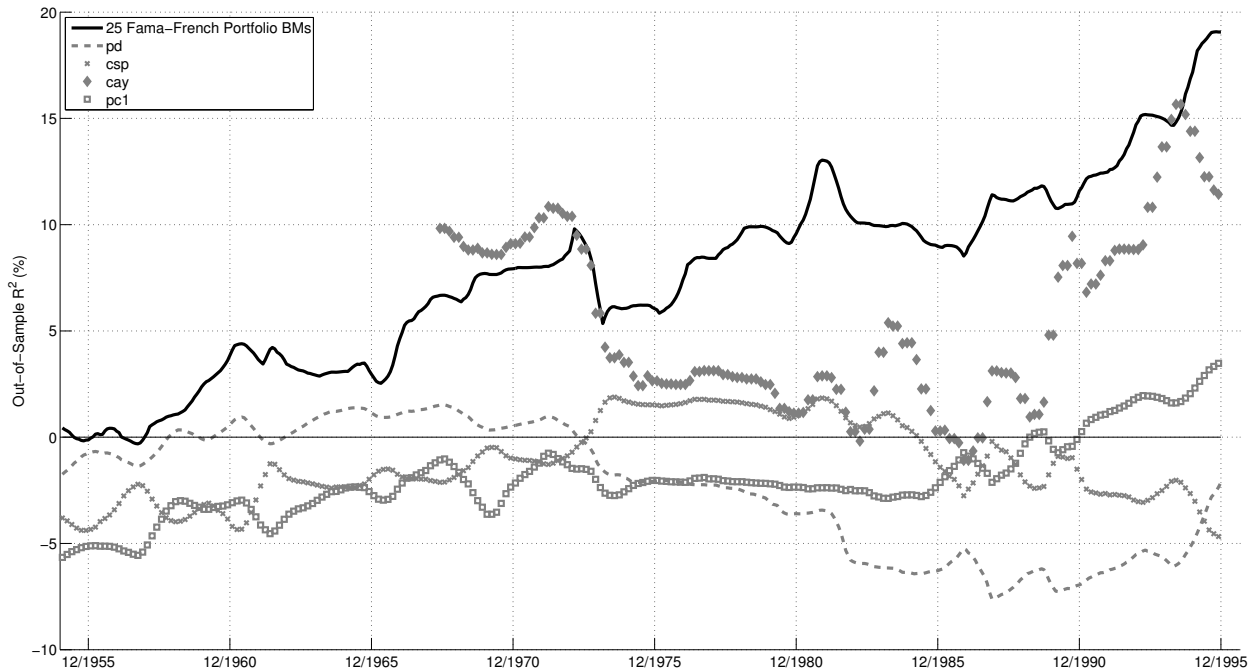
*Notes:* Volatility (standard deviation) and first-order autocorrelations for realized aggregate U.S. stock market annual returns, and expected annual returns estimated using the aggregate price-dividend ratio in-sample, the aggregate book-to-market ratio in-sample or our PLS factor extracted either in-sample or out-of-sample from 25 book-to-market ratios of size and value-sorted portfolios of U.S. stocks from Fama and French (1993). Out-of-sample expected returns begin only in 1955.

estimates imply that this conclusion is a genuine feature of market prices rather than an artifact of statistical overfit. Our results point to market expectation dynamics that are quite different than the persistent expected returns generated by standard consumption-based asset pricing paradigms such as habit persistence or long run risks, models that are typically calibrated to match the much weaker empirical return predictability generated by the aggregate price-dividend ratio.

### III.B.2 Varying Out-of-Sample Sample Splits

Our first robustness check recognizes the following: We have reported out-of-sample forecasting tests based on a 1980 sample-split date, but recent forecast literature suggests that sample splits themselves can be data-mined (cf. Hansen and Timmermann (2011) and Inoue and Rossi (2011)). To demonstrate the robustness of out-of-sample forecasts to alternative sample splits, Figure 3 plots out-of-sample annual return predictive  $R^2$  as a function of sample-split date for a variety of predictors. The earliest sample split we consider is January 1955, which uses less than one third of the data (25 out of 80 years) as a training

Figure 3: OUT-OF-SAMPLE  $R^2$  BY SAMPLE SPLIT DATE, ONE YEAR RETURNS



Notes: Out-of-sample  $R^2$  across sample split dates. Forecasts are based on a single PLS factor from 25 book-to-market ratios of size and value-sorted portfolios of U.S. stocks from Fama and French (1993); the aggregate price-dividend ratio; the cross-section premium (csp); the *ex post* consumption-wealth-income ratio (cay); and the first principal component extracted from the 25 Fama-French portfolio book-to-market ratios (pc1).

sample. The latest split we consider is 1995, which uses a long training sample and limited test sample. The figure shows our procedure consistently outperforms alternative predictors across sample splits. Note that the aggregate book-to-market ratio is not shown since its out-of-sample  $R^2$  consistently falls below -10%. Forecasts using *cay* are competitive in only a small subset of the sample splits. The first principal component (“pc1” in the figure) has consistently poor out-of-sample performance, as do forecasts that use the first three principal components simultaneously (not shown due to close similarity with results from the first principal component). The remaining predictors fail to consistently demonstrate out-of-sample predictability across various split dates. An attractive feature of our estimator is that its out-of-sample  $R^2$  shows a gradual, steady increase as the training sample expands. This suggests that PLS successfully learns from new information being fed into the procedure as more data becomes available, allowing it to more effectively counteract sample noise.

Table IV: CROSS-VALIDATION OUT-OF-SAMPLE  $R^2$ 

|                |          | Size of Test Window in Years |       |       |       |       |
|----------------|----------|------------------------------|-------|-------|-------|-------|
|                |          | 1                            | 2     | 3     | 5     | 10    |
| 6 Portfolios   | CV $R^2$ | -4.71                        | -2.91 | -2.43 | -3.48 | -2.99 |
| 25 Portfolios  | CV $R^2$ | 4.85                         | 6.62  | 7.19  | 4.63  | 2.33  |
| 100 Portfolios | CV $R^2$ | 3.74                         | 4.72  | 5.67  | 6.67  | 11.14 |

*Notes:* Percentage  $R^2$  for annual returns from an subsample cross-validation procedure (described in Section III.B.3). Results from PLS forecasts of annual market returns from six, 25 or 100 book-to-market ratios of size and value-sorted portfolios of U.S. stocks from Fama and French (1993). This summarizes the out-of-sample predictive accuracy of PLS across all possible subsamples of a given window size.

### III.B.3 Subsamples and Cross-Validation

The attractiveness of our benchmark recursive out-of-sample procedure is that it strictly relies only on information available to analysts in real time. One limitation of this test is that it always trains on an early portion of the sample and tests on a later portion. If there are differences in predictability between early and late parts of the sample, this can be missed by the recursive, forward-looking approach. Cross-validation breaks this strict timing. It allows us to evaluate the performance of our approach across many different subsamples, which may include estimating parameters on late sample observations and testing on early observations.<sup>16</sup> That is, cross-validation is an out-of-sample procedure that does not require temporal ordering of training and test samples.

We begin by partitioning the data into  $T - 12$  subsamples of length  $K$  months. For the subsample indexed by date  $t$ , we perform the full three-pass PLS procedure using all data *except* that for periods  $t$  through  $t + K$ . Parameter estimates are based on the training sample  $\{1, \dots, t - 1, t + K + 1, \dots, T\}$ , including weights in the PLS factor construction and the final stage predictive coefficient. These parameter estimates are then used to construct

<sup>16</sup>Stock and Watson (2011) advocate cross-validation in the context of macroeconomic forecasting by a similar rationale and use a similar scheme.

the PLS factor at time  $t$  and forecast the annual return realized in period  $t + 12$ .<sup>17</sup> This is repeated for each subsample, and hence out-of-sample forecast errors for all  $t = 1, \dots, T - 12$  are aggregated to calculate a cross-validation  $R^2$ , denoted CV  $R^2$  and reported in Table IV.<sup>18</sup> For all subsample window lengths, our single PLS factor forecasts from 25 and 100 portfolios produces a positive cross-validation  $R^2$ .<sup>19</sup> This sensitivity analysis suggests that our main out-of-sample predictability results are representative of the behavior of value ratios and market returns across a wide range of subsamples.

### III.B.4 Forecasts From Individual Stock Valuation Ratios

We next investigate the usefulness of information in individual stock valuation ratios for predicting market returns. We tackle the formidable task of applying our simple PLS forecasting approach directly to stock-level valuation ratios of all CRSP stocks from 1930-2009.

Polk, Thompson and Vuolteenaho (2006) is the benchmark study for combining individual firm data into a market return prediction. Their paper emphasizes the difficulty in working with noisy firm-level data, such as book value or cash flows, which may distort valuation ratios and complicate extraction of market expectations from the cross section. They address this with a series of pre-filtering adjustments to firms' valuation ratios and robust statistics. These include relying on ordinal ranks rather than cardinal values, value-weighting observations, and censoring extreme observations.

One alternative solution to this problem, as suggested by Fama and MacBeth (1973) and done in our main analysis above, is to combine individual stocks into portfolios. In this

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<sup>17</sup>That the first forecast occurs at  $t + 12$  reflects care taken to ensure that no information from the testing window is used in parameter estimation. This accounts for overlapping monthly observations of annual returns, as discussed in Section II.C.

<sup>18</sup>This procedure produces a unique forecast error for each data point. We use the calculation  $CV R^2 = 1 - \frac{\sum_t (y_t - \hat{y}_t)^2}{\sum_t (y_t - \bar{y})^2}$ , where  $\bar{y}$  is mean return in the training sample corresponding to forecast  $\hat{y}_t$ . As in the recursive case, this  $R^2$  has the interpretation of measuring out-of-sample predictive power relative to that of the training sample mean return.

<sup>19</sup>Statistical tests for cross-validation are not well-known in the economics forecasting literature. A natural alternative for such a test is the Clark and McCracken's (2001) statistic developed for recursive forecasting. Treating this as a cross-validation test, we find that out-of-sample forecasts from 25 and 100 portfolios are statistically significant at the 5% level or better in all cases.

section, we are interested in drilling deeper to understand how much we can learn about aggregate market expectations directly from individual stock data.

Miller and Scholes (1982) and Fama and French (1988) propose a different approach to dealing with noise in individual stock valuation ratios. They suggest that, rather than using infrequent and potentially mis-measured balance sheet data, it may be beneficial to omit fundamentals information entirely and focus only on the price portion of the valuation ratio. In addition to applying our PLS approach to raw firm-level book-to-market ratios (which will be excessively noisy according to the arguments in Polk et al. and others), we also apply our method to a value ratio that omits balance sheet data altogether, per the recommendation of Miller-Scholes and Fama-French. This ratio, called the individual stock price-to-moving-average (PMA) ratio, divides a firm’s month-end share price by the moving average of its monthly prices over the previous three years. We then use the log of this ratio as  $v_{i,t}$  in system (6).

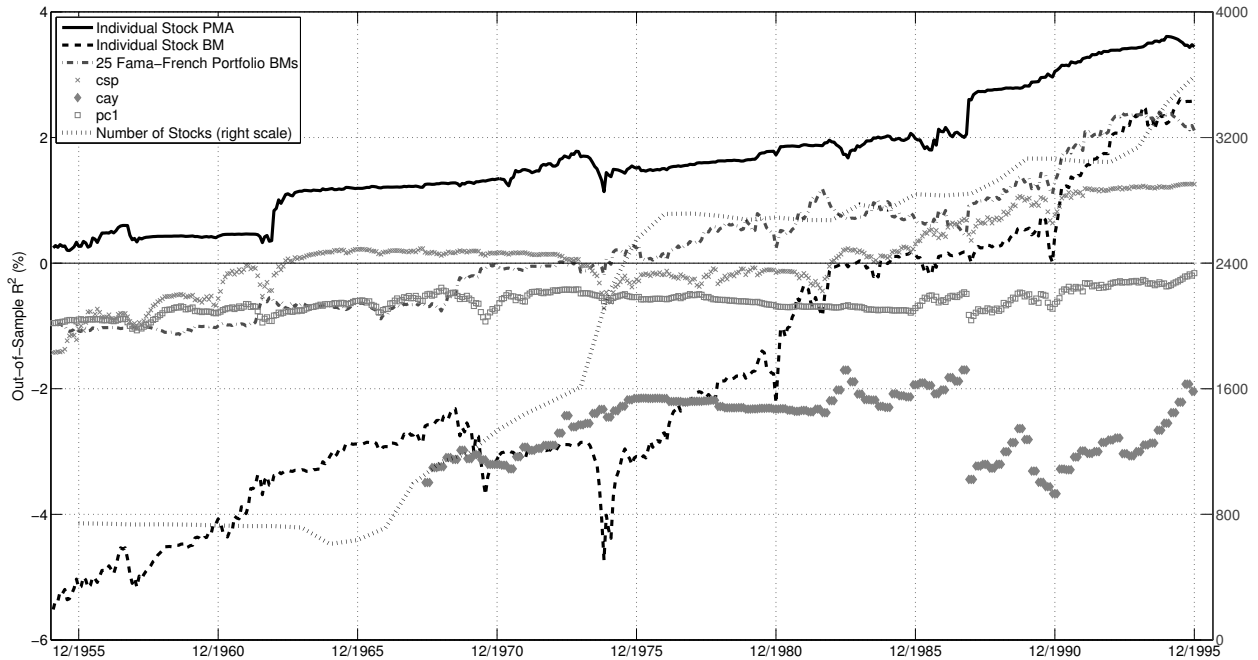
Individual stock data comprises a severely unbalanced panel. To address this, as well as to address potential parameter instability at the stock level, we estimate the model using a rolling five-year estimation window, and only include stocks that have no missing observations in the estimation window.

We focus on monthly returns to directly compare with the benchmark of Polk et al., who study monthly forecasts. Figure 4 reports the out-of-sample monthly return forecasting  $R^2$  from individual stock PMA ratios across a range of sample splits. It also plots the  $R^2$  for Polk et al.’s *csp* variable. Our single PLS factor extracted from the cross section of individual stock PMA ratios consistently produces a positive out-of-sample  $R^2$ , rising above 1% in the mid-1960s and exceeding 2% per month by the mid-1980s. It uniformly dominates *csp*, as well as forecasts from *cay* and the first component of 25 Fama-French portfolios’ book-to-market ratios.<sup>20</sup> For comparison, we also plot results from the PLS factor extracted from 25 portfolio book-to-market ratios.

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<sup>20</sup>PCs extracted from the cross section of individual stock value ratios produce substantially weaker forecasts and are not shown.

Figure 4: OUT-OF-SAMPLE  $R^2$  BY SAMPLE SPLIT DATE, ONE MONTH RETURNS



Notes: Out-of-sample  $R^2$  across sample split dates (left scale). For forecasts of one month market returns from: a single PLS factor from the entire cross section of U.S. stocks' price-to-moving-average ratios (Individual Stock PMA) or book-to-market ratios (Individual Stock BM); a single PLS factor from 25 book-to-market ratios of size and value-sorted portfolios of U.S. stocks from Fama and French (1993) (25 Fama-French Portfolio BMs); and alternative predictors taken from Goyal and Welch (2008) – the cross-section premium (csp) and the *ex post* consumption-wealth-income ratio (cay); and the first principal component extracted from the 25 portfolio book-to-market ratios (pc1). The number of stocks available in the cross-section is shown on the right scale.

$R^2$ 's for a single PLS factor extracted from individual stock book-to-market ratios are also plotted in Figure 4. For small training samples these forecasts produce a negative  $R^2$ , presumably due to the noisiness of firm-level balance sheet data. In early sample splits the book-to-market ratios are not only dominated by forecasts based solely on firm-level prices, but also by *csp*, whose clever modifications mitigate the influence of noise in balance sheet data. Just as in Figure 3, the individual stock book-to-market ratio  $R^2$  series trends upwards as the training window expands. Across sample splits, not only is more time series data being used to train the procedure, but the number of available individual stocks is also increasing. Figure 4 shows (on the right-hand scale) that the number of stocks grows steadily from around 800 in 1955 to around 3600 in 1995. The  $R^2$  for the PLS factor of firm-level book-to-market ratios increases more rapidly than other forecasters as the training sample



is expands, suggesting that PLS forecasts learn relatively quickly as firms' book-to-market ratios become available and parameter estimates sharpen, rapidly overcoming the initial noise-induced  $R^2$  deficit. By the mid-1980s the raw book-to-market  $R^2$  series intersects that of *csp* and begins to provide better forecasts. However, it never reaches the high degree of predictability demonstrated by our PLS approach applied to the cross section of firm-level valuation ratios constructed solely from stock prices.

We conclude from this analysis that our forecasting approach continues to demonstrate strong predictive power when we rely on the cross section of individual stock data.

### III.B.5 Forecasting with Price-Dividend Ratios

As another robustness check, we consider the ability of a cross section of alternative valuation ratios to forecast market returns. In Section I, we note that the Campbell and Shiller (1988) present value identity produces a factor model for the cross section of log price-dividend ratios in direct analogy with Equation 6 under similar assumptions. Thus far, our analysis has focused on book-to-market ratios to avoid the lack of dividend payments (and hence undefined price-dividend ratios) for a substantial fraction of U.S. firms.<sup>21</sup> While concerns about declining numbers of dividend-paying firms are partly mitigated by portfolio aggregation, in many cases portfolios can be dominated by non-dividend payers, resulting in an erratic and highly inflated price-dividend ratio for that portfolio.<sup>22</sup> In order to develop well-behaved portfolio price-dividend ratios, we form our own sets of six, 25 and 100 portfolios on the basis of underlying firms' market equity and book-to-market ratios, with the key difference that we exclude non-dividend paying firms. When forming portfolios, we only assign a stock to a portfolio in month  $t$  if it paid positive dividends in the twelve months prior to  $t$ .<sup>23</sup> This greatly increases the fraction of firms in our portfolios with well-defined

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<sup>21</sup>The fraction of firms that paid dividends in 1946, 1980 and 2008 was 86%, 64% and 36%, though these fractions are substantially higher, 97%, 93% and 76%, when weighted by market capitalization.

<sup>22</sup>An earlier draft of this paper documents this behavior in detail. These results are available upon request.

<sup>23</sup>We use simultaneous two-way sorts, rebalance portfolios monthly, and most importantly, strictly preclude look-ahead information in portfolio construction, as is the case in the original Fama and French (1993) portfolios.

price-dividend ratios, while continuing to condition portfolio formation only on past publicly available information. We refer to this sample as “past dividend payers.” Dividend paying behavior is highly persistent among U.S. firms, so that a firm having paid dividends in the past twelve months strongly predicts that it will pay dividends in the subsequent twelve months.

Market return forecasts based on a single PLS factor extracted from the cross section of portfolio price-dividend ratios demonstrates strong predictive accuracy, on par with our results using book-to-market ratios. In-sample annual return  $R^2$ 's for six, 25 and 100 portfolios are 8.3%, 10.6% and 33.9%, and Kelly-Pruitt, Hodrick and Newey-West  $t$ -statistics are all significant at least at the 0.1% level. Out-of-sample  $R^2$ 's are 13.7%, 4.8% and 9.6% respectively, all significant at least at the 1% level according to the Clark-McCracken test. The out-of-sample  $R^2$  from 25 price-dividend ratios across various split dates behaves very similarly to those shown for 25 book-to-market ratios in Figure 3.

### III.B.6 Forecasting Outside the U.S.

Our last robustness check asks whether our return predictability results hold internationally. To do so, we forecast returns on the value-weighted aggregate world portfolio (excluding the U.S.) by applying PLS to an international cross section of non-U.S., country-level valuation ratios. Monthly data for country-level portfolios are available from Ken French's website beginning in 1975, following the construction described in Fama and French (1998). This sample, based on data from MSCI, sorts equities from each country into a high value and low value portfolio. Countries covered in the sample are Austria, Australia, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Ireland, Italy, Japan, Malaysia, Netherlands, Norway, New Zealand, Singapore, Spain, Sweden, Switzerland and UK. We use cum- and ex-dividend returns on these portfolios to calculate price-dividend ratios of the two portfolios in each country, resulting in a cross section of 42 portfolios price-dividend ratios.<sup>24</sup> This

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<sup>24</sup>French's data includes price and dividend data at the monthly frequency, and aggregate country book-to-market ratios at the annual frequency. Because the sample begins, at the earliest, in 1975 (some countries

cross section is used to forecast the return on an international equity index (excluding the U.S.), which is a value-weighted portfolio of country-level index returns in these 21 countries (portfolio weights are determined by a country’s weight in the MSCI EAFE index). Because the data begin in 1975, we consider only monthly returns and take 1995 to be our benchmark out-of-sample split date (though we consider robustness across alternative splits).

We find that the world equity index return is highly predictable by country-level value ratios. The monthly out-of-sample  $R^2$  is 2.3%, for which Clark and McCracken’s (2001) test statistic is significant at least at the 1% level. Figure 5 shows that this strong out-of-sample performance is robust to a wide range of sample splits, and gradually increases with the length of the training sample as in the U.S. data. In-sample analysis produces an  $R^2$  of 5.3% with a Kelly-Pruitt  $t$ -stat significant at least at the 0.1% level. The success of a single PLS factor drawn from the cross section of value ratios in an international sample lends further confidence to the robustness of our findings.

### III.C Cash Flow Growth Predictions

Thus far we have focused on forecasts of aggregate market returns. Asset prices depend not only on discount rates, but also on expectations about assets’ future cash flows. Hence it is important for our understanding of asset pricing to also investigate how much information valuation ratios contain about the market’s expectations of future cash flow growth. The Vuolteenaho identity incorporates cash flow growth in terms of return on equity, while the Campbell-Shiller identity depends on dividend growth. Our analysis focuses on forecasting dividend growth and earnings growth, since these quantities have been at the center of growth forecasting in the asset pricing literature (see Ball and Watts (1972), Campbell and Shiller

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have an even shorter sample), for meaningful out-of-sample analysis we conduct our forecasting analysis at the monthly frequency and therefore rely on price-dividend ratios rather than book-to-market ratios as our predictors. A comparison of *annual* price-dividend and book-to-market ratios suggests that the two series are highly similar at the country level. The median correlation between the two ratios is 86% (mean of 78%) across the 21 countries we study. In-sample forecasts of non-overlapping annual returns based on country-level book-to-market ratios generates an  $R^2$  of 8.2%, versus an  $R^2$  of 9.8% using price-dividend ratio data at the same frequency.

Figure 5: OUT-OF-SAMPLE  $R^2$  BY SAMPLE SPLIT DATA, ONE MONTH INTERNATIONAL RETURNS



Notes: Out-of-sample  $R^2$  across sample split dates. For forecasts of one month international stock returns from a single PLS factor from 42 book-to-market ratios of high- and low-value portfolios for 21 countries from Fama and French (1998). See Section III.B.6 for the list of countries.

(1988), Cochrane (1992), Fama and French (2000), Lettau and Ludvigson (2005), Kojien and Van Nieuwerburgh (2011) and Lacerda and Santa-Clara (2010)).

Aggregate dividend growth is calculated from the universe of CRSP stocks and aggregate earnings growth data is calculated from Standard and Poor’s data on Robert Shiller’s website. Our analysis focuses on annual cash flow growth data in order to avoid spurious predictability arising from well-known within-year cash flow seasonality. Table V reports results from our PLS approach to forecasting annual aggregate U.S. dividend or earnings growth based on six, 25 and 100 Fama-French portfolio book-to-market ratios. Across all portfolio sorts and cash flow measures, the in-sample and out-of-sample results are positive and statistically significant. For dividend growth, the in-sample  $R^2$  is between 21% and 44%, with an out-of-sample  $R^2$  between 6% and 25%. Earnings growth forecasts produce an in-sample  $R^2$  between 9% and 29% with an out-of-sample  $R^2$  around 3%.<sup>25</sup> Figure 6 plots the out-of-sample  $R^2$  for dividend growth forecasts, comparing our procedure to forecasts based on

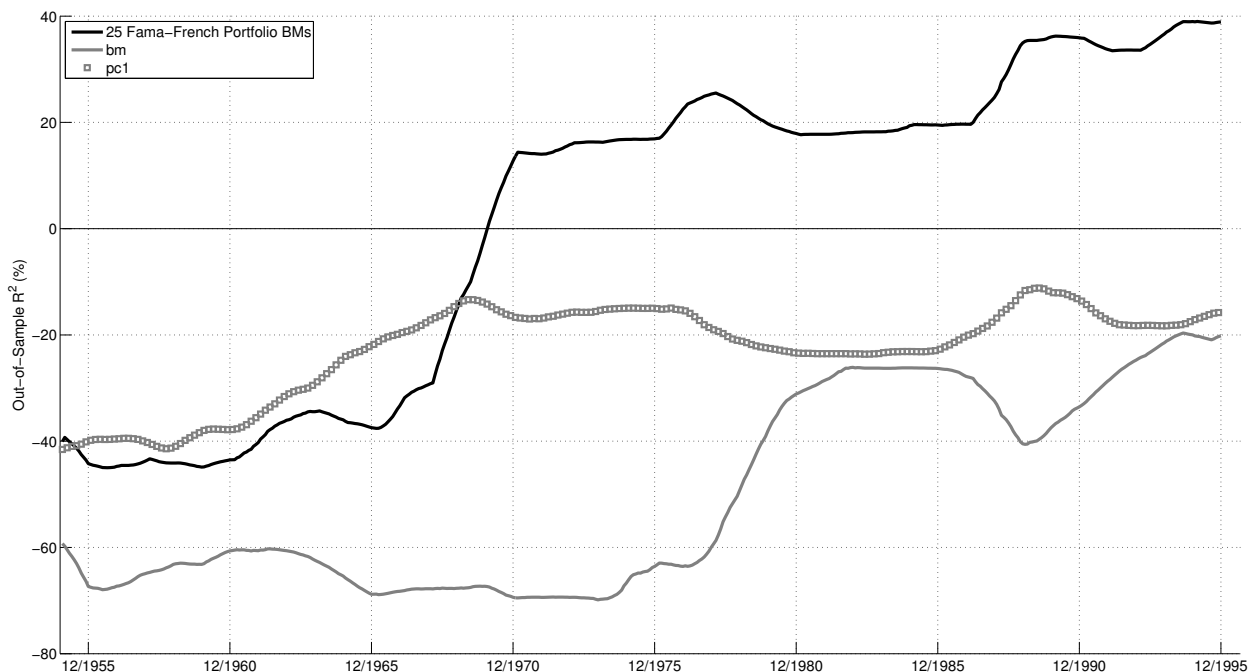
<sup>25</sup>Our annual dividend growth series is calculated assuming interim dividend payments are reinvested at the risk free rate. Alternative strategies that reinvest in the market portfolio convolute market return and dividend growth dynamics. We refer readers to van Binsbergen and Kojien (2010) for a detailed discussion of this point.

Table V: MARKET CASH FLOW PREDICTIONS (1930-2009)

|                                 | $R^2$ (%) | $p$ (KP/CM) | $p$ (Hodrick) | $p$ (NW) |
|---------------------------------|-----------|-------------|---------------|----------|
| <b>Panel A: Dividend Growth</b> |           |             |               |          |
| 6 Portfolios                    |           |             |               |          |
| In-Sample                       | 21.12     | 0.002       | 0.002         | < 0.001  |
| Out-of-Sample                   | 25.44     | < 0.010     | -             | -        |
| 25 Portfolios                   |           |             |               |          |
| In-Sample                       | 20.58     | 0.005       | 0.002         | 0.001    |
| Out-of-Sample                   | 19.53     | < 0.010     | -             | -        |
| 100 Portfolios                  |           |             |               |          |
| In-Sample                       | 44.29     | < 0.001     | < 0.001       | < 0.001  |
| Out-of-Sample                   | 6.13      | < 0.010     | -             | -        |
| <b>Panel B: Earnings Growth</b> |           |             |               |          |
| 6 Portfolios                    |           |             |               |          |
| In-Sample                       | 8.68      | 0.002       | < 0.001       | < 0.001  |
| Out-of-Sample                   | 2.76      | < 0.100     | -             | -        |
| 25 Portfolios                   |           |             |               |          |
| In-Sample                       | 11.98     | < 0.001     | < 0.001       | < 0.001  |
| Out-of-Sample                   | 2.63      | < 0.050     | -             | -        |
| 100 Portfolios                  |           |             |               |          |
| In-Sample                       | 28.88     | < 0.001     | < 0.001       | < 0.001  |
| Out-of-Sample                   | 3.49      | < 0.050     | -             | -        |

*Notes:* Results of PLS forecasts of one year aggregate dividend or earnings growth for the U.S. stock market. The sets of predictor variables are six, 25 and 100 book-to-market ratios of size and value-sorted portfolios of U.S. stocks from Fama and French (1993). In-sample results are for the 1930-2009 sample. Our out-of-sample procedure splits the sample in 1980, uses the pre-1980 period as a training window, and recursively forecasts growth rates beginning in January 1980 (results from a wide range of alternative sample splits are shown in Figure 6). We report in-sample and out-of-sample forecasting regression  $R^2$  in percent. We also report  $p$ -values of three different in-sample test statistics. The first is the asymptotic predictive loading  $t$ -statistic from Kelly and Pruitt (2011), calculated on every non-overlapping set of residuals as described in the text, and also Hodrick (1992) and Newey-West (1985)  $t$ -statistic  $p$ -values. For out-of-sample tests we report  $p$ -values for Clark and McCracken's (2001) ENC-NEW encompassing test statistic. This tests the null hypothesis of no forecast improvement over the historical mean. We follow Clark and McCracken (2005) and use Newey-West standard errors with twelve lags to consistently estimate the appropriate asymptotic variance.

Figure 6: OUT-OF-SAMPLE  $R^2$  BY SAMPLE SPLIT DATA, ANNUAL DIVIDEND GROWTH

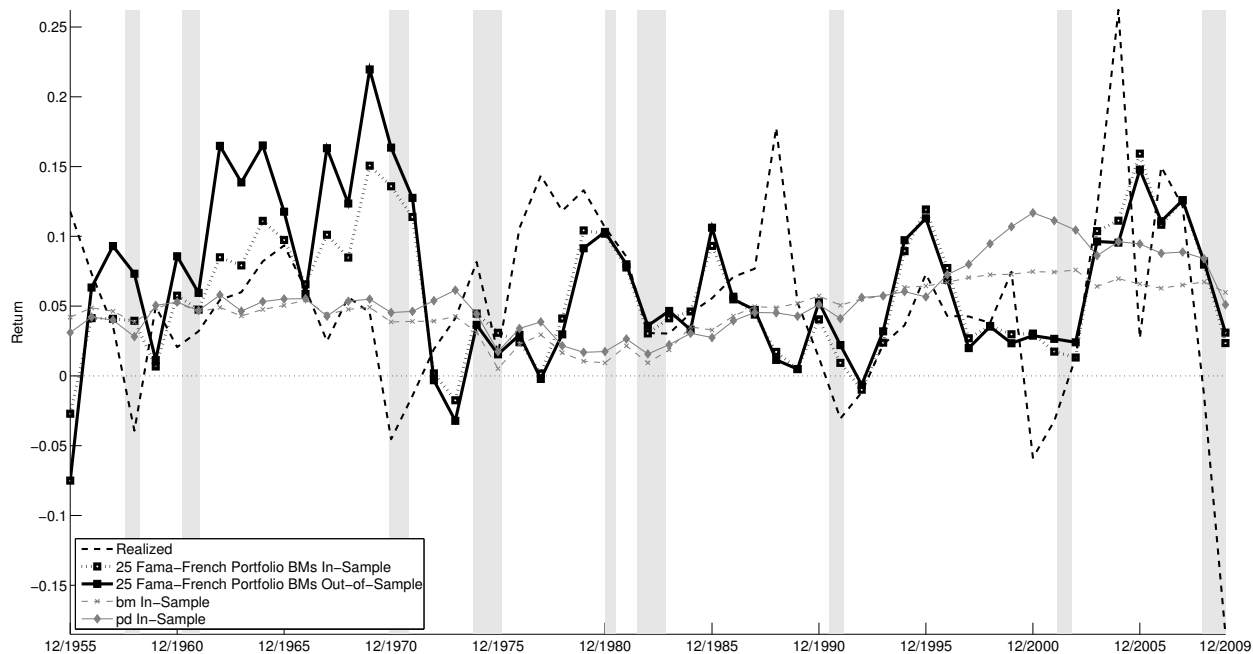


*Notes:* Out-of-sample  $R^2$  across sample split dates. For forecasts of annual aggregate dividend growth from a single PLS factor from 25 book-to-market ratios of size and value-sorted portfolios of U.S. stocks from Fama and French (1993) (25 Fama-French Portfolio BMs). We also plot results from forecasts based on the aggregate book-to-market ratio (bm) and the first principal component extracted from the 25 portfolio book-to-market ratios (pc1).

the aggregate book-to-market ratio and the first principal component extracted from the cross section of book-to-market ratios. As we showed with returns, the strong out-of-sample predictive results from our procedure are robust across sample split and dominate aggregate value ratios and principal components. Figure 7 plots the in-sample and out-of-sample fitted dividend growth series alongside fits from aggregate valuation ratios. Not only do in-sample PLS estimates suggest a much more volatile series for conditional expected annual dividend growth, this is true out-of-sample as well, whose fits are nearly identical to in-sample estimates.

Only recently have more sophisticated econometric approaches begun to identify predictability in aggregate dividend growth, as in van Binsbergen and Kojien (2010). The cross section of book-to-market ratios identifies similarly large dividend growth forecastability, and we document the robustness of this fact out-of-sample. We also document new evidence

Figure 7: DIVIDEND GROWTH PREDICTIONS



*Notes:* The figure shows year-end (December) realized dividend growth for the aggregate U.S. stock market (Realized), both in-sample (PLS IS) and out-of-sample (PLS OOS) forecasts of aggregate growth from a univariate PLS factor extracted from 25 Fama-French book-to-market ratios of size and value-sorted portfolios of U.S. stocks from Fama and French (1993), and in-sample forecasts from predictive regressions on the aggregate book-to-market (bm IS) and aggregate price-dividend (pd IS) ratios. NBER recession dates are represented by the shaded area.

of robust predictability in aggregate U.S. earning growth.

## IV Conclusion

We derive a dynamic latent factor model representation for the cross section of asset valuation ratios. The same factors that drive these present values also determine aggregate expectations of market returns and cash flow growth, enabling us to use rich cross-sectional information in constructing forecasts. To analyze these latent processes, we use the method of partial least squares and recent econometric results on its behavior in factor model settings. By extracting information from disaggregate valuation ratios we are able to construct remarkably accurate forecasts of returns and cash flow growth rates both in-sample and out-of-sample. The resulting estimates suggest that market expectations are much more volatile

and less autocorrelated than shown in previous literature, facts that stand in contrast to the far more stable and persistent conditional expectations implied by standard models of asset prices. Our results are robust across a variety of cross sections, out-of-sample procedures and hold in both U.S. and international data. The cross section of valuation ratios, as present value identities imply, hold a wealth of information about investor expectations. A deeper understanding of economic fundamentals that drive these valuation ratios is a promising avenue for future research.

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# A Appendix

## A.1 Derivation of Present Value System

$$\begin{aligned}
v_{i,t} &= \frac{\kappa_i}{1 - \rho_i} + \sum_{j=0}^{\infty} \rho_i^j (-r_{i,t+j+1} + \Delta c f_{i,t+j+1}) \\
&= \frac{\kappa_i}{1 - \rho_i} + \sum_{j=0}^{\infty} \rho_i^j \mathbb{E}_t (-\mu_{i,t+j} + g_{i,t+j}) \\
&= \frac{\kappa_i}{1 - \rho_i} + \sum_{j=0}^{\infty} \rho_i^j \mathbb{E}_t [ -(\gamma_{i,0} + \boldsymbol{\gamma}'_i \mathbf{F}_{t+j}) + (\delta_{i,0} + \boldsymbol{\delta}'_i \mathbf{F}_{t+j}) + \varepsilon_{i,t+j} ] \\
&= \frac{\kappa_i - \gamma_{i,0} + \delta_{i,0}}{1 - \rho_i} + \sum_{j=0}^{\infty} \rho_i^j (\mathbb{E}_t [\boldsymbol{\iota}' \boldsymbol{\Upsilon}'_i \mathbf{F}_{t+j} + \varepsilon_{i,t+j}]) \\
&= \frac{\kappa_i - \gamma_{i,0} + \delta_{i,0}}{1 - \rho_i} + \sum_{j=0}^{\infty} \rho_i^j \boldsymbol{\iota}' \boldsymbol{\Upsilon}'_i \boldsymbol{\Lambda}'_1 \mathbf{F}_t + \varepsilon_{i,t} \\
&= \frac{1}{1 - \rho_i} (\kappa_i - \gamma_{i,0} + \delta_{i,0}) + \boldsymbol{\iota}' \boldsymbol{\Upsilon}'_i (I - \rho_i \boldsymbol{\Lambda}_1)^{-1} \mathbf{F}_t + \varepsilon_{i,t} \\
&= \phi_{i,0} + \boldsymbol{\phi}'_i \mathbf{F}_t + \varepsilon_{i,t}
\end{aligned}$$

where we have defined  $\boldsymbol{\Upsilon}_i = (\boldsymbol{\gamma}_i, \boldsymbol{\delta}_i)$ ,  $\boldsymbol{\iota} = (-1, 1)'$ ,  $\boldsymbol{\phi}'_i = \boldsymbol{\iota}' \boldsymbol{\Upsilon}'_i (I - \rho_i \boldsymbol{\Lambda}_1)^{-1}$ , and  $\phi_{i,0} = \frac{1}{1 - \rho_i} (\kappa_i - \gamma_{i,0} + \delta_{i,0})$ .

## A.2 Partial Least Squares Assumptions

We adapt the assumptions of Kelly and Pruitt (2011) for their analysis of the three-pass regression filter, a generalization of partial least squares. That paper has extensive discussion of the assumptions. There is a final condition in that paper that is trivially satisfied by our application because we use the forecast target itself as our proxy variable in the first-pass regressions.

**Assumption 1** (Factor Structure). *The data are generated by the following:*

$$\begin{aligned}
\mathbf{v}_t &= \phi_0 + \boldsymbol{\Phi} \mathbf{F}_t + \boldsymbol{\varepsilon}_t & y_{t+h} &= \beta_0 + \boldsymbol{\beta}' \mathbf{F}_t + \eta_{t+h} \\
\mathbf{V} &= \boldsymbol{\iota} \phi'_0 + \mathbf{F} \boldsymbol{\Phi}' + \boldsymbol{\varepsilon} & \mathbf{y} &= \boldsymbol{\iota} \beta_0 + \mathbf{F} \boldsymbol{\beta} + \boldsymbol{\eta}
\end{aligned}$$

where  $\mathbf{F}_t = (\mathbf{f}'_t, \mathbf{g}'_t)'$ ,  $\boldsymbol{\Phi} = (\boldsymbol{\Phi}_f, \boldsymbol{\Phi}_g)$ , and  $\boldsymbol{\beta} = (\boldsymbol{\beta}'_f, \mathbf{0}')'$  with  $|\boldsymbol{\beta}_f| > \mathbf{0}$ .  $K_f > 0$  is the dimension of vector  $\mathbf{f}_t$ ,  $K_g \geq 0$  is the dimension of vector  $\mathbf{g}_t$ , and  $K = K_f + K_g$ .

**Assumption 2** (Factors, Loadings and Residuals). *Let  $M < \infty$ . For any  $i, s, t$*

1.  $\mathbb{E} \|\mathbf{F}_t\|^4 < M$ ,  $T^{-1} \sum_{s=1}^T \mathbf{F}_s \xrightarrow[T \rightarrow \infty]{p} \boldsymbol{\mu}$  and  $T^{-1} \mathbf{F}' \mathbf{J}_T \mathbf{F} \xrightarrow[T \rightarrow \infty]{p} \boldsymbol{\Delta}_F$
2.  $\mathbb{E} \|\phi_i\|^4 \leq M$ ,  $N^{-1} \sum_{j=1}^N \phi_j \xrightarrow[T \rightarrow \infty]{p} \bar{\phi}$ ,  $N^{-1} \boldsymbol{\Phi}' \mathbf{J}_N \boldsymbol{\Phi} \xrightarrow[N \rightarrow \infty]{p} \mathcal{P}$  and  $N^{-1} \boldsymbol{\Phi}' \mathbf{J}_N \phi_0 \xrightarrow[N \rightarrow \infty]{p} \mathbf{P}_1$ <sup>26</sup>
3.  $\mathbb{E}(\varepsilon_{it}) = 0$ ,  $\mathbb{E}|\varepsilon_{it}|^8 \leq M$
4.  $\mathbb{E}(\boldsymbol{\omega}_t) = \mathbf{0}$ ,  $\mathbb{E} \|\boldsymbol{\omega}_t\|^4 \leq M$ ,  $T^{-1/2} \sum_{s=1}^T \boldsymbol{\omega}_s = \mathcal{O}_p(1)$  and  $T^{-1} \boldsymbol{\omega}' \mathbf{J}_T \boldsymbol{\omega} \xrightarrow[N \rightarrow \infty]{p} \boldsymbol{\Delta}_\omega$
5.  $\mathbb{E}_t(\eta_{t+h}) = \mathbb{E}(\eta_{t+h} | y_t, F_t, y_{t-1}, F_{t-1}, \dots) = 0$ ,  $\mathbb{E}(\eta_{t+h}^2) = \delta_\eta < \infty$  for any  $h > 0$ , and  $\eta_t$  is independent of  $\phi_i(m)$  and  $\varepsilon_{i,s}$ .

**Assumption 3** (Dependence). *Let  $x(m)$  denote the  $m^{\text{th}}$  element of  $\mathbf{x}$ . For  $M < \infty$  and any  $i, j, t, s, m_1, m_2$*

1.  $\mathbb{E}(\varepsilon_{it} \varepsilon_{js}) = \sigma_{ij,ts}$ ,  $|\sigma_{ij,ts}| \leq \bar{\sigma}_{ij}$  and  $|\sigma_{ij,ts}| \leq \tau_{ts}$ , and

<sup>26</sup>  $\|\phi_i\| \leq M$  can replace  $\mathbb{E} \|\phi_i\|^4 \leq M$  if  $\phi_i$  is non-stochastic.

$$(a) \quad N^{-1} \sum_{i,j=1}^N \bar{\sigma}_{ij} \leq M$$

$$(c) \quad N^{-1} \sum_{i,s} |\sigma_{ii,ts}| \leq M$$

$$(b) \quad T^{-1} \sum_{t,s=1}^T \tau_{ts} \leq M$$

$$(d) \quad N^{-1} T^{-1} \sum_{i,j,t,s} |\sigma_{ij,ts}| \leq M$$

$$2. \quad \mathbb{E} \left| N^{-1/2} T^{-1/2} \sum_{s=1}^T \sum_{i=1}^N [\varepsilon_{is} \varepsilon_{it} - \mathbb{E}(\varepsilon_{is} \varepsilon_{it})] \right|^2 \leq M$$

$$3. \quad \mathbb{E} \left| T^{-1/2} \sum_{t=1}^T F_t(m_1) \omega_t(m_2) \right|^2 \leq M$$

$$4. \quad \mathbb{E} \left| T^{-1/2} \sum_{t=1}^T \omega_t(m_1) \varepsilon_{it} \right|^2 \leq M.$$

**Assumption 4** (Central Limit Theorems). *For any  $i, t$*

$$1. \quad N^{-1/2} \sum_{i=1}^N \phi_i \varepsilon_{it} \xrightarrow{d} \mathcal{N}(0, \mathbf{\Gamma}_{\Phi\varepsilon}), \text{ where } \mathbf{\Gamma}_{\Phi\varepsilon} = \text{plim}_{N \rightarrow \infty} N^{-1} \sum_{i,j=1}^N \mathbb{E}[\phi_i \phi_j' \varepsilon_{it} \varepsilon_{jt}]$$

$$2. \quad T^{-1/2} \sum_{t=1}^T \mathbf{F}_t \eta_{t+h} \xrightarrow{d} \mathcal{N}(0, \mathbf{\Gamma}_{F\eta}), \text{ where } \mathbf{\Gamma}_{F\eta} = \text{plim}_{T \rightarrow \infty} T^{-1} \sum_{t=1}^T \mathbb{E}[\eta_{t+h}^2 \mathbf{F}_t \mathbf{F}_t'] > 0$$

$$3. \quad T^{-1/2} \sum_{t=1}^T \mathbf{F}_t \varepsilon_{it} \xrightarrow{d} \mathcal{N}(0, \mathbf{\Gamma}_{F\varepsilon,i}), \text{ where } \mathbf{\Gamma}_{F\varepsilon,i} = \text{plim}_{T \rightarrow \infty} T^{-1} \sum_{t,s=1}^T \mathbb{E}[\mathbf{F}_t \mathbf{F}_s' \varepsilon_{it} \varepsilon_{is}] > 0.$$

**Assumption 5** (Normalization).  $\mathcal{P} = \mathbf{I}$ ,  $\mathbf{P}_1 = \mathbf{0}$  and  $\mathbf{\Delta}_F$  is diagonal, positive definite, and each diagonal element is unique.