

FINANCE RESEARCH SEMINAR SUPPORTED BY UNIGESTION

"Demand-Based Security Design"

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Abstract

Uninformed investors facing future carrying cost (liquidity) shocks determine primary market prices for asset-backed securities. A liquidity provider subsequently buys tendered securities in competitive secondary markets. Liquidity provision is distorted by a speculator receiving a private signal regarding cash flow. Optimal structuring minimizes total trading loss and carrying cost discounts demanded by uninformed investors. If uninformed selling is price-insensitive noise-trading, all structurings generate equal discounts. With optimizing uninformed investors, security design is relevant as it alters their secondary market trading, optimally liquefying an otherwise illiquid cash flow stream or de-liquefying an otherwise liquid cash flow stream. The optimal structuring entails tranching. With fixed speculator information, the optimal senior face value entails a tradeoff, with increases in senior face reducing total expected carrying costs but increasing total expected trading losses. This privately optimal structuring is socially suboptimal, as the desire to reduce speculator gains generates excessive illiquidity. With discretionary speculator effort, tranching can either stimulate or deter information production as it can increase or decrease uninformed trading volume. Tranching remains optimal with discretionary speculator effort, but as a deterrent, optimal senior face value is lowered.

Friday, March 1st, 2013, 10.30-12.00

Room 126, 1st floor of the Extranef building at the University of Lausanne

DEMAND-BASED SECURITY DESIGN*

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December 2012

Abstract

Uninformed investors facing future carrying cost (liquidity) shocks determine primary market prices for asset-backed securities. A liquidity provider subsequently buys tendered securities in competitive secondary markets. Liquidity provision is distorted by a speculator receiving a private signal regarding cash flow. Optimal structuring minimizes total trading loss and carrying cost discounts demanded by uninformed investors. If uninformed selling is price-insensitive noise-trading, all structurings generate equal discounts. With optimizing uninformed investors, security design is relevant as it alters their secondary market trading, optimally liquefying an otherwise illiquid cash flow stream or de-liquefying an otherwise liquid cash flow stream. The optimal structuring entails tranching. With fixed speculator information, the optimal senior face value entails a tradeoff, with increases in senior face reducing total expected carrying costs but increasing total expected trading losses. This privately optimal structuring is socially suboptimal, as the desire to reduce speculator gains generates excessive illiquidity. With discretionary speculator effort, tranching can either stimulate or deter information production as it can increase or decrease uninformed trading volume. Tranching remains optimal with discretionary speculator effort, but as a deterrent, optimal senior face value is lowered.

Tranched claims are ubiquitous in asset-backed securities (ABS) markets. In prescient papers, DeMarzo and Duffie (1999) and Biais and Mariotti (2005) show such structures can be understood as an optimal response by a liquidity-motivated primary market issuer who designs claims before observing private information and before choosing the quantity of retained claims. While these

*I thank Ulf Axelson, Sudipto Bhattacharya, Gilles Chemla, Peter DeMarzo, James Dow, Gary Gorton, Denis Gromb, Dmitry Livdan and Hongjun Yan for helpful comments. I also thank seminar participants at Yale, EIEF Rome, Reading, London School of Economics, Columbia, Toulouse School of Economics, LBS, Oxford, Amsterdam, Norwegian School of Management, University of Napoli Federico II, Università Ca' Foscari Venezia, Paris-Dauphine, and the Vienna Graduate School of Finance. This paper subsumes a 2008 working paper titled "Security Design, Liquidity and the Informational Role of Prices."

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theories offer a partial explanation of observed structures, they have limitations. For example, Baker (2009) argues that insufficient attention has been devoted to the role that shifting investor demand plays in the determination of capital structures. Indeed, the financial crisis of 2007/2008 highlighted the value *investors* attach to liquidity, especially the *secondary* market liquidity of securities. How easily can an investor sell in the event of a liquidity shock, and at what price? Moreover, while issuer private information is a concern in some cases, many issuers appear to be as hapless as their investor base. Frequently, it is not issuers but *informed speculators* who are the first to identify problems and exploit superior information in trading against uninformed investors.¹ This suggests the need for a theory of security design based upon investor demand for secondary liquidity, where a paramount concern is the superior information of other investors.

The objective of this paper is to determine the optimal packaging of a fixed stream of cash flows when uninformed investors trade in competitive markets knowing they face liquidity shocks and informed speculation. The setting is as follows. There is a continuum of ex ante identical uninformed investors who determine primary market prices of securities backed by given cash flows. The cash flows are drawn from a continuous distribution with support on the positive real line. The objective of security design is to maximize the uninformed valuation of the cash flows in primary markets. After securities are designed, an unobservable fraction (high or low) of uninformed investors is hit with a private liquidity shock biasing them towards selling. The liquidity shock takes the form of a carrying cost if a security is held until maturity. An uninformed competitive liquidity provider stands ready to buy tendered securities in a secondary market. However, liquidity provision is hindered by the existence of a speculator endowed with a private signal of the true cumulative distribution function (good or bad) governing cash flow. The good (\bar{F}) and bad (\underline{F}) distribution functions have common support with densities satisfying a standard monotone likelihood ratio condition. Payoff mappings on securities must respect limited liability and have monotone payoffs, with non-monotone securities considered as an extension.

The total primary market valuation of the structured cash flow stream is equal to expected cash flow less discounts demanded by uninformed investors to compensate them for future adverse selection costs. The optimal structuring minimizes the discount. In standard noise-trading models, e.g. Holmström and Tirole (1993) and Maug (1998), the primary market discount is just equal to expected uninformed trading losses. But noise-trading frameworks will be shown to be problematic as a basis for theories of security design. In particular, it is shown that if uninformed investors behave as price-inelastic noise-traders in secondary markets, then the primary market discount is the same across all structurings.

¹See Lewis (2010) for examples.

The novel feature of the model is to consider the interesting case where uninformed investors behave as price-sensitive optimizing agents. When each investor optimizes his liquidation decision, the adverse selection discount takes a very different form. Due to correlation in shocks, an uninformed investor hit with a liquidity shock can expect to sell at a price below fundamental value on average. Rather than incur trading losses, the investor can hold until maturity and bear his carrying cost. An optimizing investor sells (holds) a security if his idiosyncratic carrying cost exceeds (falls below) expected trading losses. Consequently, the primary market valuation of a given security is equal to a weighted average of the expected trading losses incurred by high carrying cost investors, who sell when shocked, plus expected carrying costs incurred by low carrying cost investors, who fail to sell despite liquidity shocks.

I consider first a setting with fixed speculator precision where each shocked investor has a carrying cost of c or C , with $c < C = 1$. With such binary carrying costs, the role of security design is to influence the liquidation decision of pivotal uninformed investors with carrying cost c . If c is low, the underlying cash flows would have low liquidity if packaged as a single claim since the pivotal investors would not sell in secondary markets. Here optimal security design serves to *liquefy* a portion of the otherwise illiquid cash flows. If c is high, all uninformed investors would be willing to liquidate a claim on total cash flow. Here optimal security design serves to *de-liquefy* a portion of the otherwise fully liquid cash flows.

It is shown that splitting cash flows into two claims is sufficient for optimality, with the optimal two-claim structure featuring tranching (or prioritization) of the claims. The optimal face value on the senior tranche emerges from the following tradeoff, and this tradeoff carries over with modification to the more complex case where carrying costs are drawn from a continuum. On one hand, an increase in senior face value is costly since it raises the total trading loss discount. The reasoning is as follows. An increase in senior face value results in higher trading losses on the senior tranche, but this is just offset by lower trading losses on the junior tranche when losses are measured per-unit. However, the senior tranche losses hit a wider base since a higher proportion of uninformed investors are willing to sell it given its relatively low information-sensitivity. On the other hand, expected carrying costs are decreasing in senior face value since the less liquid junior tranche becomes smaller. Reflecting this tradeoff, the optimal senior face value is decreasing in speculator signal precision and increasing in the carrying cost c . Finally, it is shown that this privately optimal structuring is socially suboptimal in that higher aggregate welfare can be obtained by adopting either a pass-through structure (in the case of high c) or a higher senior face value (in the case of low c). Intuitively, the private optimum creates socially excessive deadweight carrying costs with the aim of reducing speculator trading gains, but total expected trading gains and losses net to zero and have no direct effect on aggregate welfare.

I next consider a setting where the speculator can increase her signal precision by incurring a convex effort cost. It is first shown that a prioritized two-tranche structure is optimal for implementing a given level of speculator signal precision. The structuring problem then simplifies to determining the optimal senior tranche face value in light of endogenous speculator effort. There are two relevant cases. If c is high then speculator effort is U-shaped in senior face value. Intuitively, if c is high then both tranches have maximal liquidity if the senior face is below an illiquidity threshold. By raising the senior face to the illiquidity threshold, the junior tranche becomes less liquid and effort falls discretely. Incremental increases in the senior face beyond this threshold lead to marginally higher effort as the speculator gains from an increase in the face of the more liquid tranche. With high c it is always optimal to raise the senior face above the illiquidity threshold, but to avoid significantly higher face values in light of the fact that high senior face values map to high speculator effort and high trading loss discounts.

If c is low then speculator effort has an inverted U-shape in senior face value. Intuitively, if c is low then only the senior tranche has the potential to be fully liquid, since a subset of uninformed investors are unwilling to sell the junior tranche. Thus, speculator effort is initially increasing in senior face value as the speculator profits from an increase in the information-sensitivity of the more liquid tranche. However, there is an illiquidity threshold at which even the senior tranche becomes less liquid, at which point speculator gains and effort fall discretely. Again, in light of incentive effects, it is optimal to adopt a lower senior debt face value. Finally, analysis of the two carrying cost scenarios reveals an interesting non-monotonic relationship between a security's information-sensitivity and the intensity of informed trading it attracts. Related information production arguments are contained in Hennessy (2008) and a more recent paper by Farhi and Tirole (2012).

One contribution of the paper is to show that the ABS folk-theorem, that tranching is optimal, carries over to a setting where heterogeneous investor-level liquidity shocks are the primary driver. Moreover, in the binary carrying cost setting, two tranches are sufficient for optimality. Second, the analysis reveals the privately optimal structuring leads to less liquid markets than the socially optimal structuring. A third contribution is to analyze the relationship between securitization structure and incentives for information production, with endogenous trading by dispersed uninformed investors the novel causal mechanism. The fourth contribution, related to the third, is to show how the optimal structure should be modified in light of incentive effects. The final message of the paper is that the entire distribution of prospective investor-level liquidity shocks determines the optimal structuring. This final point is illustrated most clearly when carrying costs vary along a continuum, a case discussed next.

In the continuum case, I confine attention to the bundling of cash flows as two claims. Here too it is shown that tranching is optimal, so the optimal structuring problem reduces to determining the

optimal senior debt face value. The optimal senior face value emerges from the following tradeoff. On one hand, an increase in senior face value is costly since it raises total trading loss discounts. Again, on a per-unit basis an increase in senior tranche losses is just compensated by a decrease in junior tranche losses. But senior tranche losses hit a wider trading base given endogenous liquidation decisions. On the other hand, an increase in senior face value reduces expected carrying costs. The reasoning is as follows. An increase in the size of the senior tranche raises the carrying cost discount on that tranche but lowers the carrying cost discount on the now smaller junior tranche. The latter effect dominates because the average carrying cost for non-liquidating investors is higher for the junior tranche than for the senior tranche given the former has higher information-sensitivity.

This paper is most closely related to DeMarzo and Duffie (1999) and Axelson (2007). The first paper considers optimal pre-design of a security by a liquidity-motivated issuer who acquires private information before deciding how much to sell in primary markets. The latter paper assumes the liquidity-motivated issuer is uninformed, but faces informed investors when the security is auctioned.² Each paper can be viewed as offering a theory of tranching in that the issuer optimally markets a senior debt claim and holds the residual, although in Axelson's model debt is not necessarily the optimal marketed claim. In each model, the optimal senior face value trades off the issuer's carrying cost on retained cash flows against mispricing of the marketed security. A first contrast between the papers is that I consider how security design can be used to influence uninformed trading in order to deter speculator information production. Even with fixed information, there are key differences. In the proposed theory, the entire cash flow is sold in the primary market and all securities are actively traded in the secondary market, so carrying costs and mispricing hit all claims. The novel lever is that the optimal primary market packaging anticipates the optimal idiosyncratic liquidation decisions to be made by uninformed investors. Here there is a tradeoff between carrying costs and trading loss discounts, but one that is less mechanical, being shaped by investors' liquidation decisions. An increase in the senior tranche face value causes the total trading loss discount to rise because it has an endogenously higher uninformed trading base. On the other hand, an increase in senior tranche size causes the total carrying cost discount to fall because the average carrying cost for non-liquidating investors is lower for the senior tranche.

Biais and Mariotti (2005) also consider optimal pre-design of a security by a prospective monopolistic informed seller. In contrast to DeMarzo and Duffie (1999), Biais and Mariotti assume liquidity providers can pre-commit to a payment schedule, allowing for the possibility of cross-subsidies across seller types. Further, they consider that liquidity providers may have market power. Here debt emerges as the optimal marketed security, now having the added advantage of reducing liq-

²DeMarzo (2005) considers an uninformed issuer who can pool assets.

idity provider rents. I do not consider the issue of liquidity provider market power, nor do I allow for pre-commitment to payment schedules. Rather, I consider a competitive secondary market à la Kyle (1985).

Nachman and Noe (1994) analyze optimal security design when the issuer is privately informed at the time the security is designed. They consider a setting with fixed issuer liquidity demand (investment scale) so the equilibrium necessarily entails pooling of issuers. They show that under technical conditions, including monotonicity, debt is the optimal funding source. Intuitively, debt minimizes cross-subsidies from high to low type issuers, as informally argued by Myers and Majluf (1984).

Gorton and Pennacchi (1990) analyze the supply of riskless claims when uninformed investors have a demand for safe storage when facing informed speculators. Uninformed investors exercise effective control over an intermediary's financial structure and carve out a safe debt claim. Dang, Gorton and Holmström (2011) consider an exchange economy with two agents who stand to benefit from bilateral intertemporal trade. Here debt emerges as the socially optimal security in that it minimizes the private incentive to become fully informed at a fixed cost, with information production limiting risk-sharing possibilities.

As in this paper, Boot and Thakor (1993) and Fulghieri and Lukin (2001) allow for the possibility of informed speculation. However, they consider security design by a privately informed issuer-seller. High type issuers benefit from informed speculation since this drives prices closer to fundamentals in pooling equilibria. They show tranching of cash flows can promote information production by relaxing speculator wealth constraints as they trade against pure noise-traders in the levered equity market. I deliberately rule out this causal mechanism by considering a speculator without wealth constraints. The most important difference between the models is that these two papers move uninformed trading across securities exogenously.

The remainder of the paper is as follows. Section 1 considers a setting with binary carrying costs and fixed speculator signal precision. Section 2 considers the effect of variable signal precision. Section 3 considers a setting where the carrying costs of uninformed investors varies along a continuum. Section 4 considers the role of the monotonicity constraint.

1 Binary Carrying Costs

This section determines optimal security design in the simplest of settings where the degree of informational asymmetry is exogenous and investor carrying costs are either high or low. These two assumptions are relaxed in the subsequent sections.

1.1 Assumptions

There is one asset generating a single verifiable cash flow $x \in \mathcal{X} \equiv [\underline{x}, \bar{x}] \subseteq \mathbb{R}_{++}$ accruing in the model's final period. The c.d.f. governing cash flow is determined by the latent profitability state $\omega \in \{\underline{\omega}, \bar{\omega}\}$. The two states are equiprobable. If the state is $\underline{\omega}$ ($\bar{\omega}$), cash flow is distributed according to the continuously differentiable c.d.f. \underline{F} (\bar{F}) with strictly positive density \underline{f} (\bar{f}) on \mathcal{X} . The densities satisfy the following Monotone Likelihood Ratio (MLR) condition.

$$\textbf{Assumption 1 (MLR)} : \quad \frac{\bar{f}(x_2)}{\underline{f}(x_2)} \geq \frac{\bar{f}(x_1)}{\underline{f}(x_1)} \quad \forall \quad \{(x_1, x_2) \in \mathcal{X} \times \mathcal{X} : x_1 < x_2\}$$

with strict inequality on a set of positive measure.

MLR implies first-order stochastic dominance (FOSD) and monotone hazard rates (MHR):

$$\textbf{FOSD} : \quad \bar{F}(x) \leq \underline{F}(x) \quad \forall \quad x \in \mathcal{X}.$$

$$\textbf{MHR} : \quad \frac{\bar{f}(x)}{1 - \bar{F}(x)} \leq \frac{\underline{f}(x)}{1 - \underline{F}(x)} \quad \forall \quad x \in \mathcal{X}.$$

For an arbitrary measurable mapping $a : \mathcal{X} \rightarrow \mathbb{R}$, let:

$$\bar{\mu}_a \equiv \int_{\underline{x}}^{\bar{x}} a(x) \bar{f}(x) dx$$

$$\underline{\mu}_a \equiv \int_{\underline{x}}^{\bar{x}} a(x) \underline{f}(x) dx.$$

There are three periods: 1, 2, and 3, and two types of investors. Long-term uninformed investors (UI below) are present at all dates, and short-term investors enter in period 2.³ Period 1 is the primary market stage when there is symmetric ignorance. At the start of period 1, the rights to the cash flow are owned by the Issuer, who only values consumption in period 1, so he wants to sell all rights to cash flow. Issuer's objective is to maximize the amount the UI are willing to pay for marketable claims backed by the cash flow. That uninformed investors determine primary market prices follows Holmström and Tirole (1993) and Maug (1998), for example. An alternative motivation, with equivalent results, is to assume the UI own the cash flows initially and are considering repackaging them to maximize the expected value derived from their claims given that their subsequent liquidation decisions will be made non-cooperatively. This may be contrasted with standard primary market models in which a single decisionmaker, the issuer, decides how much to sell of a single pre-designated marketed security.

³Equivalently, one could assume short-term investors are present in period 1 but have limited wealth or high discount rates from period 1 to 2.

Letting \mathcal{A} denote the set of admissible security payoff mappings, we demand that all payoff functions $a \in \mathcal{A}$ satisfy the following limited liability (LL) and monotonicity (MN) constraints:

$$\textbf{Assumption 2 (LL)} : 0 \leq a(x) \leq x \quad \forall x \in \mathcal{X}.$$

$$\textbf{Assumption 3 (MN)} : a \text{ is non-decreasing on } \mathcal{X}.$$

From Assumptions 1 and 3 it follows:

$$\bar{\mu}_a \geq \underline{\mu}_a \quad \forall a \in \mathcal{A}. \tag{1}$$

An extant literature including Innes (1990), Nachman and Noe (1994), DeMarzo and Duffie (1999), DeMarzo (2005), and Axelson (2007) considers that decreasing securities may be inadmissible. One can understand this demand as arising from concerns over two forms of ex post moral hazard. First, agents may engage in unobservable sabotage to reduce cash flow. Second, agents may be able to make unobservable contributions to increase cash flow. If either form of ex post moral hazard is feasible, there will be an equilibrium demand for claim payoffs to be non-decreasing. To see this, note that if a party's claim is decreasing, the party can gain from sabotage. Further, if a party's claim is decreasing, the counterparty can gain from making an unobservable contribution.

Each UI has measure zero. The UI are identical ex ante and face identically distributed liquidity shocks in period 2. These shocks are correlated, leading to fluctuations in aggregate UI trading volume, as in Kyle (1985). In particular, at the start of period 2 each UI privately learns about his own cost to selling at the end of period 2 or holding securities until period 3. A fraction $\gamma \in \{\underline{\gamma}, \bar{\gamma}\}$, with $0 < \underline{\gamma} \leq \bar{\gamma} < 1$, of the UI become impatient in that they face a carrying cost absorbing a fraction $c \in (0, 1)$ of a security's terminal payoff if held until maturity. A fraction $\eta \in \{\underline{\eta}, \bar{\eta}\}$, where $0 < \underline{\eta} < \bar{\eta} < 1$, of the UI become extremely impatient in that they face a carrying cost that will absorb a fraction $C = 1$ of a security's terminal payoff if held until maturity. Since the carrying costs play a critical role, we emphasize these assumptions.

$$\textbf{Assumption 4 (Carrying Costs): } 0 < c < C = 1.$$

The remaining fraction $1 - \gamma - \eta$ of the UI become patient in that they face a fully absorbing cost if they sell at the end of period 2 rather than holding until maturity. The random variables γ and η are determined by a latent liquidity demand state $s \in \{\underline{s}, \bar{s}\}$, with state \bar{s} bringing about $\gamma = \bar{\gamma}$ and $\eta = \bar{\eta}$, and state \underline{s} bringing about $\gamma = \underline{\gamma}$ and $\eta = \underline{\eta}$. The state s is unobservable to any agent. State \bar{s} occurs with probability $\psi \in (0, 1)$. The carrying cost captures in a tractable reduced-form other shocks creating a preference for selling, e.g. pressing consumption needs, a positive NPV outside investment opportunity, or a shock to discount rates. For brevity, UI_c will be used to denote those

UI who have drawn the carrying cost c and UI_C will be used to denote the UI who have drawn the carrying cost C .

At the start of period 2 short-term investors enter, standing ready to carry resources from period 2 to period 3 at a discount rate of zero, being invulnerable to carrying cost shocks. There are two short term investors, a competitive Liquidity Provider (LP) and a speculator. At the start of period 2, the speculator observes a noisy signal of the state ω which is correct with probability $\sigma \in (1/2, 1]$. At the end of period 2, securities can be sold to LP. As in Kyle (1985), the speculator and the UI submit market orders. There is no market segmentation, so LP observes orders in all markets. After observing the aggregate sell orders across markets, LP prices securities at their conditional expected payoff. Note, the adverse selection problem would vanish if LP could simply buy and hold the entire cash flow stream from period 1 to period 3.

Throughout the paper, the variable β denotes the updated belief of LP regarding the probability of the profitability state being $\bar{\omega}$. LP sets the secondary market price of securities according to:

$$P_a = \beta \bar{\mu}_a + (1 - \beta) \underline{\mu}_a \quad \forall \quad a \in \mathcal{A}. \quad (2)$$

In period 3 cash flow is verified and investors are paid.

1.2 The Pass-Through Security

It is convenient to sort the analysis into two cases according to the magnitude of the carrying cost parameter c . This subsection and the next assume

$$\begin{aligned} \text{High } c & : \quad c > \frac{(2\sigma - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma}) \bar{\mu}_x - \underline{\mu}_x}{\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}} \frac{\bar{\mu}_x - \underline{\mu}_x}{\bar{\mu}_x + \underline{\mu}_x} \\ & \Downarrow \\ \frac{\bar{\mu}_x}{\underline{\mu}_x} & < \frac{(2\sigma - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma}) + c[\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}]}{(2\sigma - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma}) - c[\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}]} \end{aligned} \quad (3)$$

Consider first secondary market trading and pricing when the entire cash flow is marketed as a pass-through security. The equilibrium concept is perfect Bayesian equilibrium (PBE) in pure strategies. Let Φ_a denote the indicator function attached to an admissible security A with payoff mapping a , with the indicator function equal to 1 if each UI_C finds it optimal to sell the security in the secondary market rather than holding it until maturity. Since the UI_C always find it optimal to sell, secondary market uninformed selling volume for an arbitrary security can be expressed as:

$$\begin{aligned} u_a & \in \{ \underline{u}_a, \bar{u}_a \}. \\ \underline{u}_a & \equiv \underline{\eta} + \Phi_a \underline{\gamma} \\ \bar{u}_a & \equiv \bar{\eta} + \Phi_a \bar{\gamma}. \end{aligned}$$

If $\Phi_a = 1$ we will say that security A is *liquid* and if $\Phi_a = 0$ we will say that security A is *illiquid*. Note, liquidity and illiquidity are defined in a *relative* sense only since at least η units of each security will be sold by uninformed investors in the secondary market.

Table 1 depicts potential equilibrium-path order configurations in the secondary market for an arbitrary security. In light of the trading pattern of the UI, the only profitable trading strategy for the speculator is to short-sell $(\bar{u} - \underline{u})$ units if she observes a negative signal. This order size confounds the LP regarding the speculator's signal since the aggregate sell order $-\bar{u}$ can arise from either: negative signal and low UI selling volume or positive signal and high UI selling volume. The posited PBE is supported with LP forming the belief that the speculator observed the negative signal in response to any off-equilibrium sell order configuration. That is, $\beta = 1 - \sigma$ off the equilibrium path.

Upon observing the aggregate sell order size $-\underline{u}$, LP knows the speculator observed the signal $\bar{\omega}$ and so forms the belief $\beta = \sigma$. Upon observing the aggregate sell order size $-(2\bar{u} - \underline{u})$, LP knows the speculator observed the signal $\underline{\omega}$ and so forms the belief $\beta = 1 - \sigma$. In each of these cases, orders fully reveal the signal observed by the speculator so securities are priced at their signal-contingent expected payoff. In contrast, LP is confounded upon observing the aggregate sell order of size $-\bar{u}$. When the order flow is non-revealing in this way, LP uses Bayes' rule to form the belief:

$$\beta_n \equiv 1 - \sigma - \psi + 2\sigma\psi. \quad (4)$$

It is conjectured that for high c values the pass-through security will be liquid. To confirm $\Phi_x = 1$, it must be verified each UI_c will indeed find it optimal to sell the pass-through security rather than hold it until maturity. Consider then the selling decision of an individual UI_c . He finds it optimal to sell if his conditional expectation of the price, given personally impatient, exceeds the expected security payoff net of carrying costs. The conditional expectation of the sell price can be computed as the sum over conditional probabilities of each liquidity demand state, given personally impatient, times the expected price in each liquidity demand state. From Table 1 we have:

$$\begin{aligned} E[P_x | IMPATIENT] &= \left[\frac{\psi\bar{\gamma}}{\psi\bar{\gamma} + (1-\psi)\underline{\gamma}} \right] \frac{1}{2} \left[\bar{\mu}_x + \underline{\mu}_x - (1-\psi)(2\sigma-1)(\bar{\mu}_x - \underline{\mu}_x) \right] \\ &+ \left[\frac{(1-\psi)\underline{\gamma}}{\psi\bar{\gamma} + (1-\psi)\underline{\gamma}} \right] \frac{1}{2} \left[\bar{\mu}_x + \underline{\mu}_x + \psi(2\sigma-1)(\bar{\mu}_x - \underline{\mu}_x) \right] \\ &= \frac{1}{2}(\bar{\mu}_x + \underline{\mu}_x) - \frac{1}{2} \left(\frac{(2\sigma-1)\psi(1-\psi)(\bar{\gamma}-\underline{\gamma})}{\psi\bar{\gamma} + (1-\psi)\underline{\gamma}} \right) (\bar{\mu}_x - \underline{\mu}_x). \end{aligned} \quad (5)$$

Equation (5) indicates each UI_c perceives himself as being vulnerable to underpricing since the conditional expectation of price is below the expected payoff. Intuitively, each UI_c views his own exposure to a liquidity shock as suggesting high uninformed liquidity demand ($s = \bar{s}$). And with high

UI selling volume, he expects a lower market clearing price given that LP views high selling volume as suggestive of a negative speculator signal. Consistent with this intuition, expected underpricing is increasing in speculator signal precision, and vanishes if the speculator signal is uninformative.

Each UI_c finds it optimal to sell if

$$\begin{aligned} \frac{1}{2}(\bar{\mu}_x + \underline{\mu}_x) - \frac{1}{2} \frac{(2\sigma - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma})(\bar{\mu}_x - \underline{\mu}_x)}{\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}} &\geq \frac{1}{2}(\bar{\mu}_x + \underline{\mu}_x)(1 - c) \\ &\Downarrow \\ c &\geq \lambda_x \end{aligned} \quad (6)$$

where

$$\lambda_a \equiv \frac{(2\sigma - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma})}{\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}} \frac{\bar{\mu}_a - \underline{\mu}_a}{\bar{\mu}_a + \underline{\mu}_a} \quad \forall a \in \mathcal{A}. \quad (7)$$

The variable λ_a plays a central role, measuring the conditional expectation of security underpricing as a percentage of expected payoff. UI_c are willing to sell only if their personal realized carrying cost exceeds the trading loss measure λ_a . As a short-hand, the variable λ_a is labeled the *trading loss information-sensitivity* (or information-sensitivity, for brevity) of a security. From the perspective of each UI, it is the relevant measure of a security's sensitivity to the latent state variable ω . Apparently, the pass-through security market will be liquid in the present case (Condition (3)).

Having verified each UI_c will sell as conjectured, consider now the expected trading gain of the speculator, denoted G . From Table 1 it follows her expected gain is:

$$\begin{aligned} G &= \frac{\bar{u}_x - \underline{u}_x}{2} \left[\begin{aligned} &(1 - \sigma)(1 - \psi)[\beta_n \bar{\mu}_x + (1 - \beta_n)\underline{\mu}_x - \bar{\mu}_x] + (1 - \sigma)\psi[(1 - \sigma)\bar{\mu}_x + \sigma\underline{\mu}_x - \bar{\mu}_x] \\ &+ \sigma(1 - \psi)[\beta_n \bar{\mu}_x + (1 - \beta_n)\underline{\mu}_x - \underline{\mu}_x] + \sigma\psi[(1 - \sigma)\bar{\mu}_x + \sigma\underline{\mu}_x - \underline{\mu}_x] \end{aligned} \right] \\ &= \frac{1}{2}(2\sigma - 1)\psi(1 - \psi)(\bar{\mu}_x - \underline{\mu}_x)[\bar{\gamma} - \underline{\gamma} + \bar{\eta} - \underline{\eta}]. \end{aligned} \quad (8)$$

The *primary market valuation* of the cash flows will be denoted V . It is equal to expected cash flow less expected trading losses (speculator trading gains) less expected carrying costs. Under the pass-through security, carrying costs are zero and the primary market discount is equal to expected trading losses.

$$\text{High } c: V_{PT} = \frac{1}{2}(\bar{\mu}_x + \underline{\mu}_x) - \frac{1}{2}(2\sigma - 1)\psi(1 - \psi)(\bar{\mu}_x - \underline{\mu}_x)[\bar{\gamma} - \underline{\gamma} + \bar{\eta} - \underline{\eta}]. \quad (9)$$

1.3 Optimal Structuring: High Carrying Costs

This subsection considers how the primary market valuation V can be increased when carrying costs satisfy Condition (3). Recall, the trading loss information-sensitivity measure λ_x defined in equation (7) determined the willingness of each UI_c to sell the pass-through security. With this in

mind, suppose cash flow is split into, say, two securities L and I with corresponding payoff mappings $(l, i) \in \mathcal{A} \times \mathcal{A}$. Following the same argument as that leading to equation (6), each UI_c finds it optimal to sell security L if $\lambda_l \leq c$ and to hold security I if $\lambda_i \geq c$. That is, the decision to sell a security again entails a comparison of expected trading loss with idiosyncratic carrying costs. Indeed, this simple causal mechanism is central throughout the model: Security design redistributes trading loss information-sensitivity of total cash flow across claims, altering secondary market trading by UI.

The following identity is useful:

$$\sum_j a_j = A \Rightarrow \sum_j \left[\frac{\bar{\mu}_{a_j} + \underline{\mu}_{a_j}}{\bar{\mu}_A + \underline{\mu}_A} \right] \lambda_{a_j} = \lambda_A; \quad \{a_j \in \mathcal{A}\}. \quad (10)$$

Identity (10) states that the weighted average of the trading loss information-sensitivities of a basket of securities is just equal to the trading loss information-sensitivity of a single security combining them. This identity leads to the following useful lemma.

Lemma 1 *Any primary market valuation attainable with three or more securities is attainable with no more than two securities.*

Lemma 1 follows directly from identity (10). To see this, note that if there are three or more securities, they can be sorted into two baskets, those that are liquid (in that UI_c are willing to sell them) and those that are illiquid (in that UI_c prefer to hold them until maturity). The basket of liquid (illiquid) securities can be combined to form a single liquid (illiquid) security. Total carrying costs and speculator trading gains will remain unchanged, leaving the primary market valuation unchanged.

In light of the preceding lemma, the remainder of this section confines attention to two securities without loss of generality. The value attainable with two securities will then be compared with that attainable under a pass-through security. It follows from equation (10) that when cash flow is split into two securities:

$$\left(\frac{\bar{\mu}_l + \underline{\mu}_l}{\bar{\mu}_x + \underline{\mu}_x} \right) \lambda_l + \left(\frac{\bar{\mu}_i + \underline{\mu}_i}{\bar{\mu}_x + \underline{\mu}_x} \right) \lambda_i = \lambda_x. \quad (11)$$

From the preceding equation it is apparent that repackaging of cash flow cannot change the weighted average of trading loss information-sensitivities. Structuring simply redistributes trading loss information-sensitivity.

To identify the optimal structuring, consider first cases such that both securities remain fully liquid. We claim that

$$\forall (i, l) \in \mathcal{A} \times \mathcal{A} : \frac{(2\sigma - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma})}{\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}} \max \left\{ \frac{\bar{\mu}_l - \underline{\mu}_l}{\bar{\mu}_l + \underline{\mu}_l}, \frac{\bar{\mu}_i - \underline{\mu}_i}{\bar{\mu}_i + \underline{\mu}_i} \right\} < c, \quad V(i, l) = V_{PT}. \quad (12)$$

The demonstration of equation (12) is as follows. Under the stated inequality, each UI_c prefers to sell both securities. The speculator masks herself by shorting $\bar{\gamma} - \underline{\gamma} + \bar{\eta} - \underline{\eta}$ units of both securities upon observing a negative signal. Order flow is fully revealing and non-revealing in the same manner as in Table 1. Carrying costs are zero and the total speculator trading gain is the same as under the pass-through security. It follows that all structurings satisfying condition (12) result in the primary market valuation V_{PT} in equation (9).

Equation (12) implies that a necessary condition for achieving a primary market valuation greater than V_{PT} is for one of the securities, say security I , to be structured such that it will become illiquid in that each UI_c will be unwilling to sell it. But note, inducing illiquidity may not be feasible since trading loss information-sensitivity is bounded above with:

$$\lambda_a < \frac{(2\sigma - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma})}{\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}} \quad \forall \quad a \in \mathcal{A}. \quad (13)$$

From equation (12) it follows:

$$c \geq \frac{(2\sigma - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma})}{\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}} \Rightarrow V(i, l) = V_{PT} \quad \forall \quad (i, l) \in \mathcal{A} \times \mathcal{A}. \quad (14)$$

The next remark is weaker than the preceding equation but useful nevertheless in light of the common noise-trading assumption adopted in many models.

Remark 1 *If secondary market selling by uninformed investors is price-insensitive ($c = C = 1$), all structurings of the cash flow result in the same primary market valuation V_{PT} (equation (9)).*

In light of equation (14), the rest of this subsection considers the optimal structuring for the remaining case where c is sufficiently low such that it is possible to induce illiquidity into some security market. The structuring problem consists of finding the optimal liquid and illiquid securities. To this end we impose the constraints $\lambda_i \geq c$ and $\lambda_l \leq c$. Under Condition (3), the second constraint is slack.

$$\text{High } c: \lambda_i \geq c \Rightarrow \lambda_l < c. \quad (15)$$

The primary market valuation under a mix of liquid and illiquid securities is:

$$\begin{aligned} V(i, l) \equiv & \frac{1}{2}(\bar{\mu}_x + \underline{\mu}_x) - \frac{1}{2}[\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}](\bar{\mu}_i + \underline{\mu}_i)c \\ & \text{Expected Cash Flow} \qquad \qquad \qquad \text{Carrying Costs} \\ & - \frac{1}{2}(2\sigma - 1)\psi(1 - \psi)(\bar{\mu}_l - \underline{\mu}_l)(\bar{\gamma} - \underline{\gamma} + \bar{\eta} - \underline{\eta}) - \frac{1}{2}(2\sigma - 1)\psi(1 - \psi)(\bar{\mu}_i - \underline{\mu}_i)(\bar{\eta} - \underline{\eta}). \\ & \text{Trading Losses on Liquid Claim} \qquad \qquad \qquad \text{Trading Losses on Illiquid Claim} \end{aligned} \quad (16)$$

Dropping constants from equation (16), the problem is to solve:

$$\max_{l \in \mathcal{A}} \int_{\underline{x}}^{\bar{x}} [\underline{\kappa}(c, \sigma, \psi, \underline{\gamma}, \bar{\gamma})\underline{f}(x) - \bar{\kappa}(c, \sigma, \psi, \underline{\gamma}, \bar{\gamma})\bar{f}(x)]l(x)dx \quad (17)$$

where

$$\begin{aligned}\underline{\kappa}(c, \sigma, \psi, \underline{\gamma}, \bar{\gamma}) &\equiv (2\sigma - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma}) + c[\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}] \\ \bar{\kappa}(c, \sigma, \psi, \underline{\gamma}, \bar{\gamma}) &\equiv (2\sigma - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma}) - c[\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}]\end{aligned}$$

subject to the constraint that security I will not be sold by UI_c :

$$NSI : \frac{\psi(1 - \psi)(2\sigma - 1)(\bar{\gamma} - \underline{\gamma})(\bar{\mu}_x - \bar{\mu}_l) - (\underline{\mu}_x - \underline{\mu}_l)}{\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}} \geq c. \quad (18)$$

It is worth noting the NSI constraint can be written as

$$NSI : \frac{\bar{\mu}_i}{\underline{\mu}_i} \geq \frac{\psi(1 - \psi)(2\sigma - 1)(\bar{\gamma} - \underline{\gamma}) + c[\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}]}{\psi(1 - \psi)(2\sigma - 1)(\bar{\gamma} - \underline{\gamma}) - c[\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}]}. \quad (19)$$

The second formulation of the NSI constraint illustrates that in order for security I to be illiquid, its expected payoff must be sufficiently sensitive to the state ω . Moreover, the required sensitivity is increasing in c . Intuitively, if c is high, UI_c have a strong demand for immediacy. Deterring them from selling requires exposing them to higher expected trading losses.

The NSI constraint can also be written as:

$$NSI : \underline{\kappa}(c, \sigma, \psi, \underline{\gamma}, \bar{\gamma})\underline{\mu}_l - \bar{\kappa}(c, \sigma, \psi, \underline{\gamma}, \bar{\gamma})\bar{\mu}_l \geq \underline{\kappa}(c, \sigma, \psi, \underline{\gamma}, \bar{\gamma})\underline{\mu}_x - \bar{\kappa}(c, \sigma, \psi, \underline{\gamma}, \bar{\gamma})\bar{\mu}_x. \quad (20)$$

This last form of the NSI constraint is of interest for two reasons. First, substituting equation (20) into the primary market valuation equation (16) reveals:

$$V(i, l) = V_{PT} + \frac{1}{2}[\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}](\bar{\mu}_i + \underline{\mu}_i)(\lambda_i - c) \quad (21)$$

From the preceding equation it follows that the two security structure dominates the pass-through structure if the NSI constraint is slack. The second point worth noting is that the left side of equation (20) for the NSI constraint is actually objective function for this program. Thus, the optimal structuring can be determined in two steps. We first solve a relaxed program (denoted RP1) ignoring the NSI constraint. If the solution to RP1 satisfies NSI then it is the optimal structure. Otherwise, it is not possible to induce illiquidity into any security (NSI cannot be satisfied) implying the primary market valuation is V_{PT} for all structurings.

In solving RP1 we begin by forming the Lagrangian:

$$\mathcal{L}(l) = \int_{\underline{x}}^{\bar{x}} [\underline{\kappa}(c, \sigma, \psi, \underline{\gamma}, \bar{\gamma})f(x) - \bar{\kappa}(c, \sigma, \psi, \underline{\gamma}, \bar{\gamma})\bar{f}(x)] l(x)dx. \quad (22)$$

From the LL and MN constraints it follows any $l \in \mathcal{A}$ is absolutely continuous. Further, the derivative

$$l' \equiv \delta$$

is well-defined, with $\delta \in [0, 1]$, except on a subset of \mathcal{X} with Lebesgue measure zero. Using integration by parts, the Lagrangian can be rewritten as:

$$\mathcal{L}(l) = \int_{\underline{x}}^{\bar{x}} [\bar{\kappa}(c, \sigma, \psi, \underline{\gamma}, \bar{\gamma})\bar{F}(x) - \underline{\kappa}(c, \sigma, \psi, \underline{\gamma}, \bar{\gamma})\underline{F}(x)] \delta(x) dx + 2l(\bar{x})[\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}]c. \quad (23)$$

The optimal contract in RP1 entails maximizing $\mathcal{L}(l)$ with respect to l . Solving this standard optimal control problem, the appendix establishes the following lemma.

Lemma 2 *The optimal structuring in Relaxed Program 1 splits cash flow into a relatively liquid senior debt claim and a relatively illiquid junior claim.*

Next it must be determined whether the solution to RP1 satisfies the neglected NSI constraint. To this end, the following lemma is useful.

Lemma 3 *The trading loss information-sensitivity of a senior debt security (λ_{sr}) with face value θ and the residual junior security (λ_{jr}) are increasing in θ . The trading loss information-sensitivity of the junior security exceeds that of a claim to total cash flow (λ_x) while that of the senior debt is less than that of a claim to total cash flow.*

Since the information-sensitivity of the junior claim is increasing in senior debt face value, a necessary and sufficient condition for the feasibility of satisfying the NSI constraint is:

$$\lim_{\theta \uparrow \bar{x}} \lambda_{jr}(\theta) = \frac{(2\sigma - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma}) \frac{\bar{f}(\bar{x}) - \underline{f}(\bar{x})}{\bar{f}(\bar{x}) + \underline{f}(\bar{x})}}{\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}} > c. \quad (24)$$

We then have the following proposition.

Proposition 1 *If carrying costs are high, satisfying Condition (3), all structurings of the cash flow result in a total primary market valuation of V_{PT} (equation (9)) if*

$$c \geq \frac{(2\sigma - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma}) \frac{\bar{f}(\bar{x}) - \underline{f}(\bar{x})}{\bar{f}(\bar{x}) + \underline{f}(\bar{x})}}{\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}}.$$

Otherwise, the privately optimal structuring splits cash flow into a relatively illiquid junior claim and a risky yet relatively liquid senior debt claim with face value θ^ such that*

$$\frac{1 - \bar{F}(\theta^*)}{1 - \underline{F}(\theta^*)} = \frac{(2\sigma - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma}) + c[\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}]}{(2\sigma - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma}) - c[\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}]}. \quad (25)$$

The first part of the proposition states that if the uninformed demand for immediacy is sufficiently high, the primary market valuation of the cash flows is invariant to its packaging. Intuitively,

under the stated condition all impatient UI find it optimal to sell any security they hold, so expected trading losses and carrying costs cannot be modified. Conversely, if c is sufficiently low, it is possible to design a residual junior claim that becomes relatively illiquid in secondary markets, so security design has the potential to influence the ex ante primary market valuation. Finally, the proposition states that the optimal structure tranches securities. The intuition is as follows. In equilibrium, one security will have higher liquidity than the other, in that atomistic UI will be more willing to sell it in secondary markets. Amongst monotone claims, debt is preferred as the most-heavily-traded security in competitive markets as it minimizes expected uninformed investor losses per unit of revenue.

The optimal senior face value weighs two effects. First, an increase in senior face value raises total expected trading losses. Intuitively, increases in senior face value raise per-unit trading losses on the senior tranche but lower per-unit trading losses on the junior tranche by an equal amount. However, as shown in equation (16), senior tranche trading losses hit a wider trading base given endogenous secondary market selling by the UI. The benefit of increasing the senior face value is that it lowers expected carrying costs. In the present binary carrying cost setting, this effect is mechanical. An increase in senior tranche size lowers the size of the junior tranche, and the junior tranche is the only tranche where carrying costs are generated. Anticipating, the two effects become more subtle in the case of a continuum of carrying costs.

Proposition 1 shows that the optimal senior face value depends on: the densities describing cash flows; the speculator's signal precision; and the probability, intensity, and breadth of liquidity shocks hitting the UI_c . In the present setting, the measures for the UI_C ($\underline{\eta}, \bar{\eta}$) are irrelevant to the optimal design. Applying the implicit function theorem to equation (25) one obtains the following comparative static properties of the senior face value:

$$\begin{aligned}\frac{\partial \theta^*}{\partial \sigma} &= -\frac{4c\psi(1-\psi)(\bar{\gamma}-\underline{\gamma})[\psi\bar{\gamma}+(1-\psi)\underline{\gamma}]}{[(2\sigma-1)\psi(1-\psi)(\bar{\gamma}-\underline{\gamma})-c(\psi\bar{\gamma}+(1-\psi)\underline{\gamma})]^2} \frac{[1-\underline{F}(\theta^*)]^2}{[1-\bar{F}(\theta^*)]\underline{f}(\theta^*)-[1-\underline{F}(\theta^*)]\bar{f}(\theta^*)} < 0 \\ \frac{\partial \theta^*}{\partial c} &= \frac{2(2\sigma-1)\psi(1-\psi)(\bar{\gamma}-\underline{\gamma})[\psi\bar{\gamma}+(1-\psi)\underline{\gamma}]}{[(2\sigma-1)\psi(1-\psi)(\bar{\gamma}-\underline{\gamma})-c(\psi\bar{\gamma}+(1-\psi)\underline{\gamma})]^2} \frac{[1-\underline{F}(\theta^*)]^2}{[1-\bar{F}(\theta^*)]\underline{f}(\theta^*)-[1-\underline{F}(\theta^*)]\bar{f}(\theta^*)} > 0.\end{aligned}$$

1.4 Optimal Structuring: Low Carrying Costs

The preceding subsection determined the optimal structuring of cash flows if carrying costs are high enough to satisfy Condition (3). This subsection considers the optimal structuring when carrying

costs are low enough to satisfy the following condition.

$$\begin{aligned} \text{Low } c & : c < \frac{(2\sigma - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma})}{\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}} \frac{\bar{\mu}_x - \underline{\mu}_x}{\bar{\mu}_x + \underline{\mu}_x} \\ & \Downarrow \\ \frac{\bar{\mu}_x}{\underline{\mu}_x} & > \frac{(2\sigma - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma}) + c[\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}]}{(2\sigma - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma}) - c[\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}]} \end{aligned} \quad (26)$$

Consider first the value obtained under a pass-through structure. Under the maintained Condition (26), $c < \lambda_x$ so each UI_c finds it optimal to incur carrying costs on the pass-through security rather than selling. The primary market valuation is therefore equal to expected cash flow less expected carrying costs incurred by the UI_c less the total trading losses of the UI_c , who sell both tranches.

$$\text{Low } c: V_{PT} = \frac{1}{2}(\bar{\mu}_x + \underline{\mu}_x) - \frac{1}{2}[\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}]c(\bar{\mu}_x + \underline{\mu}_x) - \frac{1}{2}(2\sigma - 1)\psi(1 - \psi)(\bar{\mu}_x - \underline{\mu}_x)(\bar{\eta} - \underline{\eta}). \quad (27)$$

Consider now the optimal structuring of the cash flows. Without loss of generality attention can be confined to two securities, one liquid and the other illiquid, since equation (10) implies baskets of liquid (illiquid) securities can be combined to form a single liquid (illiquid) security, resulting in the same primary market valuation. The objective is to maximize the primary market valuation in equation (16), which reduces to the objective function in equation (17), subject to securities L and I being liquid and illiquid as posited. Thus, we demand $\lambda_l \leq c$ and $\lambda_i \geq c$. The former constraint, ensuring security L is liquid, can be written as:

$$SL : \lambda_l \leq c \Leftrightarrow \underline{\kappa}(c, \sigma, \psi, \underline{\gamma}, \bar{\gamma})\underline{\mu}_l - \bar{\kappa}(c, \sigma, \psi, \underline{\gamma}, \bar{\gamma})\bar{\mu}_l \geq 0. \quad (28)$$

And equation (11) implies the NSI constraint is slack since:

$$\text{Low } c: \lambda_l \leq c \Rightarrow \lambda_i > c. \quad (29)$$

Note, the feasible set for this program is non-empty since any riskless senior debt claim with face value no greater than \underline{x} satisfies the SL constraint. And since the left side of the SL constraint is just the objective function itself, any structuring improving upon riskless debt leaves the SL constraint slack. It follows that in the present setting, with low carrying costs, the full security design program is identical in form to RP1. And we have the following proposition.

Proposition 2 *If carrying costs are low, satisfying Condition (26), the privately optimal structuring entails splitting cash flow into a relatively illiquid junior claim and a risky yet relatively liquid senior debt claim with face value θ^* such that*

$$\frac{1 - \bar{F}(\theta^*)}{1 - \underline{F}(\theta^*)} = \frac{(2\sigma - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma}) + c[\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}]}{(2\sigma - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma}) - c[\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}]}.$$

1.5 The Socially Optimal Structuring

Although the primary objective of this paper is to determine the privately optimal structuring, the model also demonstrates a divergence between private and social preferences regarding the design of ABS. To illustrate this, consider again a social planner placing equal weight on all agents. Social welfare is equal to the primary market valuation plus the speculator's expected trading gain. In turn, this is equal to expected cash flow less expected carrying costs.

$$SW = SW_{FB} - \frac{1}{2}[\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}][\bar{\mu}_i + \underline{\mu}_i]c. \quad (30)$$

In light of the preceding equation, consider first the socially optimal structuring in the case where c is high, satisfying Condition (3). Here the socially optimal structuring achieves zero carrying costs (first-best) by marketing the cash flow as a fully liquid pass-through security. In contrast, as described in Proposition 1, the privately optimal structure introduces some degree of illiquidity into securities markets whenever feasible with the goal of reducing uninformed trading losses. But from the perspective of aggregate social welfare uninformed trading losses are of no concern since they constitute a transfer to the speculator.

Consider next the social planner's preferred structuring when c is low, satisfying Condition (26). The planner's objective is to minimize deadweight carrying costs, subject to the constraint that each impatient UI indeed finds it optimal to sell the posited liquid claim. The planner's program can be written as:

$$\max_{l \in \mathcal{A}} \int_{\underline{x}}^{\bar{x}} [f(x) + \bar{f}(x)] l(x) dx \quad (31)$$

subject to the SL constraint in equation (28).

In solving the planner's program we begin by forming the Lagrangian with the multiplier m here associated with the SL constraint:

$$\mathcal{L}(l, m) = \int_{\underline{x}}^{\bar{x}} [f(x) + \bar{f}(x)] l(x) dx + m \left[\int_{\underline{x}}^{\bar{x}} (\underline{\kappa}(c, \sigma, \psi, \underline{\gamma}, \bar{\gamma})f(x) - \bar{\kappa}(c, \sigma, \psi, \underline{\gamma}, \bar{\gamma})\bar{f}(x)) l(x) dx \right]. \quad (32)$$

Using integration by parts, the Lagrangian can be rewritten as:

$$\mathcal{L}(l, m) = - \int_{\underline{x}}^{\bar{x}} [(1 + m\underline{\kappa})\underline{F}(x) + (1 - m\bar{\kappa})\bar{F}(x)] \delta(x) dx + 2[1 + mc(\psi\bar{\gamma} + (1 - \psi)\underline{\gamma})]l(\bar{x}). \quad (33)$$

The objective is to maximize $\mathcal{L}(l, m)$ over $l \in \mathcal{A}$. Solving this standard optimal control problem, the appendix establishes the following proposition which contrasts private and social objectives in security design.

Proposition 3 *If c is high, satisfying Condition (3), the socially optimal structuring is a pass-through security ensuring all impatient investors sell. The privately optimal structuring creates an illiquid junior claim that a subset of impatient investors do not sell. If c is low, satisfying Condition (26), the socially optimal structuring splits cash flow into an illiquid junior claim and liquid senior claim with face value θ_s^* satisfying, $\lambda_{sr}(\theta_s^*) = c$. The socially optimal senior tranche size exceeds the private optimum.*

The above proposition illustrates that the privately optimal level of liquidity (uninformed security sales) is socially suboptimal. This arises from an inherent conflict between private and social objectives in security design. The socially optimal structuring maximizes liquidity. In contrast, the privately optimal structuring sacrifices liquidity in order to reduce speculator trading gains. However, from a social welfare perspective, transfers from the uninformed to the speculator are of no consequence since they net to zero.

2 Endogenous Speculator Information

This section considers the role that security design plays in influencing incentives for information production. We consider now that the speculator can exert costly effort in order to increase her signal precision along a continuum. Specifically, the speculator is endowed with a noisy signal correct with probability $\underline{\sigma} \in (1/2, 1)$. She can increase her signal precision by exerting effort at cost $E : \Sigma \rightarrow \mathbb{R}_+$, where $\Sigma \equiv [\underline{\sigma}, 1]$. We adopt

$$\begin{aligned} \textbf{Assumption 5} & : E(\sigma) \equiv \frac{1}{2}\xi(\sigma - \underline{\sigma})^2 \\ & : \xi \geq \frac{(\bar{\mu}_x - \underline{\mu}_x)(\bar{\gamma} - \underline{\gamma} + \bar{\eta} - \underline{\eta})}{2(1 - \underline{\sigma})}. \end{aligned}$$

The assumed lower bound on the effort cost parameter ξ implies the incentive compatible speculator effort is responsive to marginal increases in prospective trading gains.

Before analyzing the optimal structuring, it is worth revisiting key conclusions that carry over from the prior section. First, Table 1 continues to describe order combinations for each security. Second, equations (9) and (16) measure the primary market valuation, but now σ varies endogenously across alternative structures. Third, the argument demonstrating Lemma (1) remains valid, so without loss of generality attention can be confined to no more than two securities. Finally, the λ -contingent selling rule of each UI remains the same.

2.1 High Carrying Costs and Endogenous Information

This subsection considers optimal security design when the carrying cost c is high. To this end, let σ_{pt} denote the speculator's incentive compatible signal precision if she were to face a fully liquid pass-through security market in which $u = \eta + \gamma$. For this subsection, assume the following inequality is satisfied.

$$\begin{aligned} \text{High } c & : c > \frac{(2\sigma_{pt} - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma})\bar{\mu}_x - \underline{\mu}_x}{\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}} \frac{\bar{\mu}_x - \underline{\mu}_x}{\bar{\mu}_x + \underline{\mu}_x} \\ & \Downarrow \\ \frac{\bar{\mu}_x}{\underline{\mu}_x} & < \frac{(2\sigma_{pt} - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma}) + c[\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}]}{(2\sigma_{pt} - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma}) - c[\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}]} \end{aligned} \quad (34)$$

Consider first the speculator's incentive compatible signal precision. Momentarily, let σ denote the signal precision posited by the LP in pricing securities. In choosing her signal precision, the speculator treats LP beliefs and prices as fixed as she chooses $\tilde{\sigma} \in \Sigma$. From Table 1 it follows the speculator's total expected trading gain across the two securities markets is:

$$\begin{aligned} G(\tilde{\sigma}, \sigma) & = \frac{\bar{\gamma} - \underline{\gamma} + \bar{\eta} - \underline{\eta}}{2} \left[(1 - \tilde{\sigma})(1 - \psi)[\beta_n \bar{\mu}_l + (1 - \beta_n) \underline{\mu}_l - \bar{\mu}_l] + \tilde{\sigma}(1 - \psi)[\beta_n \bar{\mu}_l + (1 - \beta_n) \underline{\mu}_l - \underline{\mu}_l] \right. \\ & \quad \left. + (1 - \tilde{\sigma})\psi[(1 - \sigma)\bar{\mu}_l + \sigma\underline{\mu}_l - \bar{\mu}_l] + \tilde{\sigma}\psi[(1 - \sigma)\bar{\mu}_l + \sigma\underline{\mu}_l - \underline{\mu}_l] \right] \\ & \quad + \frac{\bar{\eta} - \underline{\eta}}{2} \left[(1 - \tilde{\sigma})(1 - \psi)[\beta_n \bar{\mu}_i + (1 - \beta_n) \underline{\mu}_i - \bar{\mu}_i] + \tilde{\sigma}(1 - \psi)[\beta_n \bar{\mu}_i + (1 - \beta_n) \underline{\mu}_i - \underline{\mu}_i] \right. \\ & \quad \left. + (1 - \tilde{\sigma})\psi[(1 - \sigma)\bar{\mu}_i + \sigma\underline{\mu}_i - \bar{\mu}_i] + \tilde{\sigma}\psi[(1 - \sigma)\bar{\mu}_i + \sigma\underline{\mu}_i - \underline{\mu}_i] \right] \\ & = \frac{1}{2}(\bar{\gamma} - \underline{\gamma})(\bar{\mu}_l - \underline{\mu}_l) [\psi(1 - \psi)(2\sigma - 1) + \tilde{\sigma} - \sigma] + \frac{1}{2}(\bar{\eta} - \underline{\eta})(\bar{\mu}_x - \underline{\mu}_x) [\psi(1 - \psi)(2\sigma - 1) + \tilde{\sigma} - \sigma]. \end{aligned} \quad (35)$$

The IC signal precision (σ_{ic}) equates the marginal increase in expected trading gains with the marginal effort cost:

$$G_1(\sigma_{ic}, \sigma) = E'(\sigma_{ic}) \Rightarrow \sigma_{ic} = \underline{\sigma} + \frac{(\bar{\mu}_l - \underline{\mu}_l)(\bar{\gamma} - \underline{\gamma}) + (\bar{\mu}_x - \underline{\mu}_x)(\bar{\eta} - \underline{\eta})}{2\xi}. \quad (36)$$

It follows from the preceding equation that under the fully liquid pass-through security the incentive compatible signal precision is:

$$\text{High } c : \sigma_{pt} = \underline{\sigma} + \frac{(\bar{\mu}_x - \underline{\mu}_x)(\bar{\gamma} - \underline{\gamma} + \bar{\eta} - \underline{\eta})}{2\xi}. \quad (37)$$

In equilibrium, LP correctly infers the signal precision and the speculator's expected trading gain is:

$$G(\sigma, \sigma) = \frac{1}{2}(2\sigma - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma} + \bar{\eta} - \underline{\eta})(\bar{\mu}_l - \underline{\mu}_l) + \frac{1}{2}(2\sigma - 1)\psi(1 - \psi)(\bar{\eta} - \underline{\eta})(\bar{\mu}_i - \underline{\mu}_i). \quad (38)$$

It is worth emphasizing from equation (38) that speculator gains reflect a larger trading base in the more liquid security market. Returning to the pass-through structure, we have the following implied valuation.

$$\text{High } c: V_{PT} = \frac{1}{2}(\bar{\mu}_x + \underline{\mu}_x) - \frac{1}{2}(2\sigma_{pt} - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma} + \bar{\eta} - \underline{\eta})(\bar{\mu}_x - \underline{\mu}_x). \quad (39)$$

Having considered the valuation attainable under a pass-through structure, the overall optimal structuring is determined in two steps. In the first step, we consider the optimal structuring for implementing specific feasible $\sigma \in \Sigma$. In the second step, the optimal σ is chosen from the feasible set, here denoted Σ_F .

Consider then optimal implementation of some posited σ . The objective is to choose a payoff mapping for the relatively liquid security $l \in \mathcal{A}$ to maximize the objective function specified in equation (17) subject to the constraints that: the signal precision σ is incentive compatible (IC); the posited illiquid security satisfies $\lambda_i(\sigma) \geq c$ (NSI); and the posited liquid security satisfies $\lambda_l(\sigma) \leq c$ (SL). Since here $\lambda_x(\sigma) < \lambda_x(\sigma_{pt}) < c$, it follows from equation (11) that satisfaction of the NSI constraint implies satisfaction of the SL constraint. So the relevant constraints for the full structuring problem are NSI and the following incentive constraint:

$$IC: \bar{\mu}_l - \underline{\mu}_l = \frac{2\xi(\sigma - \underline{\sigma}) - (\bar{\mu}_x - \underline{\mu}_x)(\bar{\eta} - \underline{\eta})}{\bar{\gamma} - \underline{\gamma}}. \quad (40)$$

The third specification of the NSI constraint (equation (20)) shows the NSI constraint can be satisfied if and only if it is satisfied by the solution to a relaxed program which ignores it. So we begin by solving a relaxed program (denoted RP2) ignoring the NSI constraint and subsequently check whether the neglected constraint is satisfied. If the solution to RP2 satisfies the neglected constraint then it is optimal for the implementation of the specified σ . Otherwise, the specified σ is not implementable.

The set of feasible signal precisions in RP2 is itself limited by the IC constraint as follows:

$$RP2: \Sigma_F \subseteq \left[\underline{\sigma} + \frac{(\bar{\mu}_x - \underline{\mu}_x)(\bar{\eta} - \underline{\eta})}{2\xi}, \sigma_{pt} \right]. \quad (41)$$

In solving RP2 we begin by forming the following Lagrangian, with the multiplier m here corresponding to the IC constraint:

$$\begin{aligned} \mathcal{L}(l, m) &= \int_{\underline{x}}^{\bar{x}} [\underline{\kappa}(c, \sigma, \psi, \underline{\gamma}, \bar{\gamma})\underline{f}(x) - \bar{\kappa}(c, \sigma, \psi, \underline{\gamma}, \bar{\gamma})\bar{f}(x)] l(x) dx \\ &+ m \left[\int_{\underline{x}}^{\bar{x}} [\bar{f}(x) - \underline{f}(x)] l(x) dx - \frac{2\xi(\sigma - \underline{\sigma}) - (\bar{\mu}_x - \underline{\mu}_x)(\bar{\eta} - \underline{\eta})}{\bar{\gamma} - \underline{\gamma}} \right]. \end{aligned} \quad (42)$$

From the LL and MN constraints it follows any $l \in \mathcal{A}$ is absolutely continuous. Further, the derivative $l' \equiv \delta$ is well-defined, with $\delta \in [0, 1]$, except on a subset of \mathcal{X} with Lebesgue measure zero. Using integration by parts, the Lagrangian can be rewritten as:

$$\begin{aligned} \mathcal{L}(l, m) &= \int_{\underline{x}}^{\bar{x}} [(\bar{\kappa}(c, \sigma, \psi, \underline{\gamma}, \bar{\gamma}) - m)\bar{F}(x) - (\underline{\kappa}(c, \sigma, \psi, \underline{\gamma}, \bar{\gamma}) - m)\underline{F}(x)]\delta(x)dx \\ &\quad + 2l(\bar{x})c[\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}] - m \left[\frac{2\xi(\sigma - \underline{\sigma}) - (\bar{\mu}_x - \underline{\mu}_x)(\bar{\eta} - \underline{\eta})}{\bar{\gamma} - \underline{\gamma}} \right]. \end{aligned} \quad (43)$$

The objective is to maximize $\mathcal{L}(l, m)$ over $l \in \mathcal{A}$. Solving this standard optimal control problem we obtain the following lemma which again shows tranching is optimal.

Lemma 4 *The optimal structuring in Relaxed Program 2 splits cash flow into a relatively liquid senior debt claim and a relatively illiquid junior claim. Senior debt with face value \underline{x} is optimal to implement the minimal signal precision*

$$\sigma_1 \equiv \underline{\sigma} + \frac{(\bar{\mu}_x - \underline{\mu}_x)(\bar{\eta} - \underline{\eta})}{2\xi}.$$

To implement higher σ , the senior debt face must be $\hat{\theta}$, where

$$\int_{\underline{x}}^{\hat{\theta}} [F(x) - \bar{F}(x)]dx = \frac{2\xi(\sigma - \underline{\sigma}) - (\bar{\mu}_x - \underline{\mu}_x)(\bar{\eta} - \underline{\eta})}{\bar{\gamma} - \underline{\gamma}}. \quad (44)$$

Having characterized the solution to RP2, it is necessary to determine whether the candidate structuring satisfies the neglected NSI constraint. If the neglected constraint is satisfied, the candidate design is indeed optimal for implementing the posited σ . If it is not, then the σ under consideration is not implementable. With this in mind, we rewrite the NSI constraint as:

$$NSI : \lambda_{jr}(\hat{\theta}(\sigma), \sigma) = \lambda_{jr}(\underline{x}, \sigma_1) + \int_{\sigma_1}^{\sigma} \left[\frac{\partial}{\partial \theta} \lambda_{jr}(\hat{\theta}(\tilde{\sigma}), \tilde{\sigma}) \tilde{\theta}'(\tilde{\sigma}) + \frac{\partial}{\partial \sigma} \lambda_{jr}(\hat{\theta}(\tilde{\sigma}), \tilde{\sigma}) \right] d\tilde{\sigma} \geq c. \quad (45)$$

Since the integrand in the preceding equation is positive, it follows that in order for the NSI constraint is to be satisfied (and for σ to be implementable), the posited σ must be sufficiently high in relation to c .

Since λ_{jr} is increasing in the senior debt face value, the highest possible trading loss information-sensitivity on the junior claim is:

$$\lim_{\theta \uparrow \bar{x}} \lambda_{jr}(\theta, \sigma_{pt}) = \frac{(2\sigma_{pt} - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma})\bar{f}(\bar{x}) - \underline{f}(\bar{x})}{\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}} \frac{\bar{f}(\bar{x}) - \underline{f}(\bar{x})}{\bar{f}(\bar{x}) + \underline{f}(\bar{x})}. \quad (46)$$

Therefore, a necessary and sufficient condition for $\Sigma_F \neq \{\sigma_{pt}\}$ is for NSI be slack in the limit, or:

$$c < \frac{(2\sigma_{pt} - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma})\bar{f}(\bar{x}) - \underline{f}(\bar{x})}{\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}} \frac{\bar{f}(\bar{x}) - \underline{f}(\bar{x})}{\bar{f}(\bar{x}) + \underline{f}(\bar{x})}.$$

This establishes the following lemma.

Lemma 5 *If c is high, satisfying Condition (34), the set of feasible incentive compatible signal precisions is $\{\sigma_{pt}\}$ if*

$$c \geq \frac{(2\sigma_{pt} - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma}) \frac{\bar{f}(\bar{x}) - \underline{f}(\bar{x})}{\bar{f}(\bar{x}) + \underline{f}(\bar{x})}}{\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}}.$$

Otherwise, Σ_F is the compact interval $[\sigma^{\min}, \sigma_{pt}]$, where

$$\sigma^{\min} \equiv \min_{\sigma \geq \sigma_1} : \lambda_{jr}(\hat{\theta}(\sigma), \sigma) \geq c.$$

Having characterized the feasible set and the optimal securities for implementing each $\sigma \in \Sigma_F$, consider next the question of the overall optimal structure. Attention is confined to the interesting case where $\Sigma_F \neq \{\sigma_{pt}\}$, since otherwise security design cannot influence the primary market valuation. Rather than optimize directly over $\sigma \in \Sigma_F$, it is more convenient to optimize over senior debt face values, recognizing that for all face values above the illiquidity threshold the incentive compatible signal precision is defined by the function:

$$\sigma(\theta) \equiv \underline{\sigma} + \frac{[\bar{\mu}_{sr}(\theta) - \underline{\mu}_{sr}(\theta)](\bar{\gamma} - \underline{\gamma}) + (\bar{\mu}_x - \underline{\mu}_x)(\bar{\eta} - \underline{\eta})}{2\xi}. \quad (47)$$

The optimal face value with discretionary effort is

$$\begin{aligned} \theta^{**} &\in \arg \max_{\theta \in \Theta_F} V(\theta) \\ \Theta_F &\equiv \left[\hat{\theta}(\sigma^{\min}), \bar{x} \right] \\ V(\theta) &\equiv \frac{1}{2}(\bar{\mu}_x + \underline{\mu}_x) - \frac{1}{2}[\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}]c[\bar{\mu}_{jr}(\theta) + \underline{\mu}_{jr}(\theta)] \\ &\quad - \frac{1}{2}[2\sigma(\theta) - 1]\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma} + \bar{\eta} - \underline{\eta})[\bar{\mu}_{sr}(\theta) - \underline{\mu}_{sr}(\theta)] - \frac{1}{2}[2\sigma(\theta) - 1]\psi(1 - \psi)(\bar{\eta} - \underline{\eta})(\bar{\mu}_{jr} - \underline{\mu}_{jr}). \end{aligned} \quad (48)$$

Since V is continuous and Θ_F a closed bounded interval, it follows from Weierstrass' Theorem a maximum point exists. Differentiating the objective function yields:

$$\begin{aligned} V' &= -\frac{1}{2}\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma}) \left[(2\sigma - 1)(\bar{\mu}'_{sr}(\theta) - \underline{\mu}'_{sr}(\theta)) \right] - \frac{1}{2}[\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}]c[\bar{\mu}'_{jr}(\theta) + \underline{\mu}'_{jr}(\theta)] \\ &\quad - \psi(1 - \psi)[(\bar{\gamma} - \underline{\gamma})(\bar{\mu}_{sr}(\theta) - \underline{\mu}_{sr}(\theta)) + (\bar{\eta} - \underline{\eta})(\bar{\mu}_x - \underline{\mu}_x)]\sigma'(\theta). \end{aligned} \quad (49)$$

In the preceding equation it is worth noting that endogenous increases in speculator signal precision lead to higher expected trading losses across both tranches. This suggests that the optimal senior face value will be lower in light of such incentive considerations. To confirm this intuition, we note first that if the optimal face value is interior, it must satisfy the first-order condition:

$$\begin{aligned} &\frac{1}{2}\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma}) \left[[2\sigma - 1][\bar{\mu}'_{sr} - \underline{\mu}'_{sr}] \right] + \psi(1 - \psi) \left[(\bar{\gamma} - \underline{\gamma})(\bar{\mu}_{sr} - \underline{\mu}_{sr}) + (\bar{\eta} - \underline{\eta})(\bar{\mu}_x - \underline{\mu}_x) \right] \sigma' \\ &\quad \text{Marginal Trading Losses} \\ &= -\frac{1}{2}[\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}]c[\bar{\mu}'_{jr} + \underline{\mu}'_{jr}]. \\ &\quad \text{Marginal Carrying Costs} \end{aligned} \quad (50)$$

Above we see that when the speculator has discretion to increase her effort, there is a higher marginal trading loss cost to increasing the senior debt face value. Specifically, increasing the senior debt face value raises the incentive compatible signal precision, which raises the expected trading losses of the UI_c on the senior tranche, as well as the expected trading losses of the UI_C across both tranches. Computing σ' and rearranging terms, the first-order condition can be written as:

$$\frac{1 - \overline{F}(\theta_{int}^{**})}{1 - \underline{F}(\theta_{int}^{**})} = \frac{\psi(1 - \psi)(\overline{\gamma} - \underline{\gamma})(4\sigma - 1 - 2\underline{\sigma}) + c[\psi\overline{\gamma} + (1 - \psi)\underline{\gamma}]}{\psi(1 - \psi)(\overline{\gamma} - \underline{\gamma})(4\sigma - 1 - 2\underline{\sigma}) - c[\psi\overline{\gamma} + (1 - \psi)\underline{\gamma}]}. \quad (51)$$

Consistent with the argument that speculator effort raises the marginal cost of increasing senior debt face value, it can be verified that the face value satisfying the first-order condition when the speculator has discretion to increase effort above $\underline{\sigma}$ (equation (51)) is strictly less than the solution to the first-order condition when speculator signal precision is fixed at $\underline{\sigma}$ (equation (25)).

If the optimal senior face value is not interior, it must be that the minimum feasible speculator precision is to be implemented, with $\theta^{**} = \hat{\theta}(\sigma^{\min})$. To see this, note that \overline{x} cannot be the optimal face value since $V(\overline{x}) = V_{PT}$ and $V(\theta) > V_{PT}$ for all other points in Θ_F . Based upon an analysis of both interior and potential corner solutions, the appendix establishes that optimal face value with discretionary speculator effort is lower than when speculator effort is fixed.

Proposition 4 [*Endogenous Information Case*] *If carrying costs are high, satisfying Condition (34) then all structurings of the cash flow result in a total primary market valuation of V_{PT} if*

$$c \geq \frac{(2\sigma_{pt} - 1)\psi(1 - \psi)(\overline{\gamma} - \underline{\gamma}) \overline{f}(\overline{x}) - \underline{f}(\overline{x})}{\psi\overline{\gamma} + (1 - \psi)\underline{\gamma}} \frac{\overline{f}(\overline{x}) - \underline{f}(\overline{x})}{\overline{f}(\overline{x}) + \underline{f}(\overline{x})}.$$

Otherwise, the optimal structuring splits cash flow into a relatively illiquid junior claim and a relatively liquid senior debt claim with face value strictly less than the optimal face value when signal precision is fixed at $\underline{\sigma}$.

Before concluding this subsection, it is worthwhile to briefly discuss predictions regarding the determinants of informed trading intensity when the carrying cost c is high, satisfying Condition (34). Informed trading intensity is captured by the incentive compatible signal precision. Consider first how incentive compatible signal precision varies with the face value of senior debt. First, for senior face value sufficiently low, both security markets will be liquid and the incentive compatible signal precision will be maximal, equal to that attained under a pass-through structure. However, there is a critical senior face value at which the junior claim becomes illiquid, with the UI_c failing to sell. At this threshold, the incentive compatible signal precision falls discretely. From this point onwards, as senior face value is increased, total informed trading gains increase as unit trading gains are shifted to the more liquid senior market, inducing higher signal precision. In the limit, as senior

face value tends to \bar{x} , incentive compatible signal precision tends to the maximal level induced under a pass-through structure. Thus, informed speculator gains and signal precision are both U-shaped in senior debt face value when c is high.

In light of these effects, consider ranking claims by standard measures of riskiness or information-sensitivity. Consider the following paired ABS capital structures (with the junior claim labeled equity): (1) low risk debt paired with low levered equity; (2) medium risk debt paired with medium levered equity; and (3) high risk debt paired with high levered equity. In the case of high carrying costs, the theory predicts: highly informed speculation across markets in the first case, since both markets are liquid and informed trading gains are maximal; poorly informed speculation across markets in the second case, as the junior market becomes relatively illiquid; and highly informed speculation across markets in the latter case, as the high risk junior debt market offers large trading gains. Note, this implies a non-monotonic relationship between a security's information-sensitivity and the intensity of informed trading it attracts.

2.2 Low Carrying Costs and Endogenous Information

This subsection examines optimal structuring when information production is endogenous, with carrying costs low enough to satisfy the following condition.

$$\begin{aligned}
 \text{Low } c & : c < \frac{(2\underline{\sigma} - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma})\bar{\mu}_x - \underline{\mu}_x}{\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}} \frac{\bar{\mu}_x - \underline{\mu}_x}{\bar{\mu}_x + \underline{\mu}_x} & (52) \\
 & \Downarrow \\
 \frac{\bar{\mu}_x}{\underline{\mu}_x} & > \frac{(2\underline{\sigma} - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma}) + c[\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}]}{(2\underline{\sigma} - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma}) - c[\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}]}
 \end{aligned}$$

As in the previous subsection, the optimal structuring can be determined in two steps. In the first step, we consider the optimal security design for inducing specific feasible $\sigma \in \Sigma$. In the second step, the optimal σ is chosen from the feasible set, a set now denoted Σ'_F .

Consider an arbitrary σ conjectured to be feasible. The objective is to choose a payoff mapping for the relatively liquid security $l \in \mathcal{A}$ to maximize the objective function in equation (17) subject to the constraints that: the signal precision σ is incentive compatible (IC); the posited illiquid security satisfies $\lambda_i(\sigma) \geq c$ (NSI); and the posited liquid security satisfies $\lambda_l(\sigma) \leq c$ (SL). Since here $\lambda_x(\sigma) \geq \lambda_x(\underline{\sigma}) > c$, it follows from equation (11) that satisfaction of the SL constraint implies satisfaction of the NSI constraint. So the relevant restrictions are the SL constraint in equation (28) and the incentive constraint in equation (40). But note the left side of equation (28) is just the objective function for this program. So, we proceed by solving a relaxed program (again RP2) that accounts for the IC constraint but ignores the SL constraint. If the solution to RP2 satisfies

the neglected SL constraint then it is indeed the optimal contract for inducing the posited σ level. Otherwise, the posited σ is not implementable.

From Lemma 4 we know the optimal liquid security in RP2 is senior debt with face value set according to equation (44). To ensure the posited σ is indeed implementable, the solution to RP2 must satisfy the neglected SL constraint. If it is not, the σ under consideration is not implementable. With this in mind, we rewrite the SL constraint as:

$$SL : \lambda_{sr}(\hat{\theta}(\sigma), \sigma) = \int_{\sigma_1}^{\sigma} \left[\frac{\partial}{\partial \theta} \lambda_{sr}(\hat{\theta}(\tilde{\sigma}), \tilde{\sigma}) \hat{\theta}'(\tilde{\sigma}) + \frac{\partial}{\partial \sigma} \lambda_{sr}(\hat{\theta}(\tilde{\sigma}), \tilde{\sigma}) \right] d\tilde{\sigma} \leq c. \quad (53)$$

Since the integrand in the preceding equation is positive, it follows that in order for the SL constraint to be satisfied, the posited σ must be sufficiently low in relation to c .

The above analysis yields the following lemma.

Lemma 6 *If c is low, satisfying Condition (52), the set of feasible incentive compatible signal precisions is the compact interval $[\sigma_1, \sigma^{\max}]$, where*

$$\sigma^{\max} \equiv \max_{\sigma \in \Sigma} : \lambda_{sr}(\hat{\theta}(\sigma), \sigma) = c.$$

Rather than optimize directly over $\sigma \in \Sigma'_F$, it is more convenient to optimize over senior debt face values. The optimal face value satisfies:

$$\begin{aligned} \theta^{**} &\in \arg \max_{\theta \in \Theta'_F} V(\theta) \\ \Theta'_F &\equiv [\underline{x}, \hat{\theta}(\sigma^{\max})]. \end{aligned} \quad (54)$$

Since V is continuous and Θ'_F a closed bounded interval, it follows from Weierstrass' Theorem a maximum point exists. The maximum point cannot be \underline{x} since the value function is increasing at this point. Thus, the maximum point is either the illiquidity threshold $\hat{\theta}(\sigma^{\max})$ or an interior solution satisfying equation (50)). The appendix establishes the following proposition.

Proposition 5 *[Endogenous Information Case] If carrying costs are low, satisfying Condition (52) the optimal structuring splits cash flow into a relatively illiquid junior claim and a relatively liquid senior debt claim with face value strictly less than the optimal face value when signal precision is fixed at $\underline{\sigma}$.*

Taken together, Propositions 4 and 5 imply that with discretionary speculator effort, optimal senior face value is reduced with the goal of reducing speculator effort and concomitant uninformed trading losses. It is also worth observing that the primary market valuation is lower when discretionary speculator effort is feasible. Further, if the speculator could be prevented from acquiring any

informative signal, the primary market valuation would equal expected cash flow, with no discount for illiquidity or trading losses. This observation is consistent with the argument of Dang, Gorton and Holmström (2011) that ignorance is bliss in economies where the sole market imperfection is asymmetric information.

Before concluding this subsection it is worthwhile to briefly discuss predictions regarding the determinants of informed trading intensity (σ) when carrying costs are low, satisfying Condition (52). In this case, the junior tranche is always illiquid ($\Phi_{jr} = 0$), while the senior tranche is liquid ($\Phi_{sr} = 1$) provided that its face value is sufficiently low. Therefore, in the case of low c if σ is plotted as a function of the senior face value it will have an inverted U-shape, with σ initially increasing in the face value of the liquid senior tranche but falling discretely once the senior tranche becomes illiquid. Taken together with the results for high c we see that tranching can either decrease or increase incentives for information production depending on c and the senior face value.

Based on these observations, consider the following paired ABS capital structures (with the junior claim labeled equity): (1) low risk debt paired with low levered equity; (2) medium risk debt paired with medium levered equity; and (3) high risk debt paired with high levered equity. The theory predicts that with low carrying costs there will be: poorly informed speculation across markets in the first case, since only the low risk senior market is liquid; highly informed speculation across markets in the second case, as the risky yet liquid senior market affords higher trading gains; and very poorly informed speculation across markets in the latter case, as both markets become illiquid. In this case there is also a non-monotonic relationship between a security's information-sensitivity and the intensity of informed trading it attracts.

2.3 Numerical Examples

This subsection illustrates comparative static properties of the model via numerical examples. The assumed parameter values are: $\psi = 0.5$; $\underline{\gamma} = \underline{\eta} = 0.10$; $\bar{\gamma} = \bar{\eta} = 0.20$; $\underline{\sigma} = .90$; $\underline{x} = 100$; $\bar{x} = 200$; $\bar{\mu}_x = 199.4$; $\underline{\mu}_x = 150.5$. The effort cost parameter ξ is fixed to ensure incentive compatible signal precision never exceeds 1. Figures 1A-1C consider high carrying costs ($c = 5\%$) so that if cash flows were to be marketed as a pass-through security, it would be liquid. Figures 2A-2C consider instead $c = 1\%$, which implies a pass-through security would be illiquid. Panel A plots the primary market valuation V as a function of senior face value under three scenarios: Fixed σ ; Variable σ ; and Variable σ with greater UI_C trading ($\bar{\eta} = 0.30$). The last of the three scenarios captures anticipation of extreme demands for immediacy for a subset of investors. Panel B plots the speculator signal precision corresponding to each scenario. Panel C illustrates the effect of an increase in the information-sensitivity of the underlying cash flows across the three scenarios, with $\underline{\mu}_x$ reset to 110.5.

The results on optimal senior face value are shown in Table 2.

Figures 1, 2

Table 2 here.

First note that across the three scenarios optimal senior face value is lower for $c = 1\%$ than for $c = 5\%$. Intuitively, for higher UI_c carrying costs, it is optimal to reduce the size of the illiquid junior tranche via increases in senior tranche face value. The second point worth of noting is that optimal senior face value is lower if the speculator has discretion to increase signal precision above $\underline{\sigma}$. As shown in Panels 1B and 2B, reductions in senior face value can be understood as mitigating speculative information production, an effect capitalized as decreases in the trading loss discount across tranches.

The model also allows one to think in a structured way various demand-side shocks to ABS markets. For example, suppose heightened concern over future financial panics manifests itself as an increase in the expected measure of investors having extreme demands for immediacy ($\bar{\eta} = 0.30$). The first obvious effect of such a shock is a deeper primary market discount, as shown in Panel A. However, the extent of the discount can be attenuated by varying the structuring appropriately. From Panel B it is apparent that such a shock would actually manifest itself as an increase in the incentive for speculative information production, since there is higher expected uninformed trading across both markets. Conversely, the issuer's incentive to mitigate information production is heightened. Intuitively, investors with extreme demand for immediacy will incur trading losses across both tranches, so deterring speculative information production becomes a paramount concern for the issuer. The optimal issuer response to such a liquidity shock is a reduction in senior tranche face value, which leads to a reduction in both σ and the trading loss discount across the two tranches.

Another potential shock to ABS markets is a decrease in expected cash flows. This scenario is captured in Panel C ($\underline{\mu}_x = 110.5$). In this case, the information-sensitivity of the underlying cash flows is higher, so concerns over trading loss discounts become the paramount concern. To reduce this discount, the optimal senior face value is reduced significantly. In fact, in the case of low UI_c carrying costs ($c = 1\%$), the optimal senior tranche is nearly riskless.

A final set of effects worth noting is the role of security design in determining speculator signal precision. As discussed above, in the case of high UI_c carrying costs, signal precision is U-shaped in the senior face value, falling discretely at the threshold where the junior tranche becomes illiquid, with a corresponding upward jump in the asset valuation at this same point. In the case of low UI_c carrying costs, signal precision has an inverted U-shape, falling discretely at the threshold where

even the senior tranche becomes illiquid, with a corresponding upward jump in the asset valuation at this same point.

3 Heterogeneous Liquidity Shocks: The Continuum Case

For simplicity, previous sections assumed binary carrying costs satisfying Assumption 4. This subsection considers instead still greater heterogeneity in interim-period carrying costs. Aside from increased realism, this extension illustrates clearly how the optimal structuring depends upon the entire distribution of prospective investor-level carrying costs. Moreover, the link between structuring, trading losses and carrying costs becomes more subtle with more heterogeneous carrying costs.

For simplicity, assume σ is fixed. We now impose an assumption that cash flows are split into no more than two claims, since the argument in Lemma 1 proving two claims are sufficient in the binary case does not apply to a continuum of carrying costs. Regarding carrying costs, the following assumptions are imposed. Again, UI are identical ex ante, implying identical primary market valuations. At the interim date, a fraction $\gamma \in \{\underline{\gamma}, \bar{\gamma}\}$ of UI will be impatient, facing an *idiosyncratic* carrying cost c if a security is held until maturity. The remaining measure $1 - \gamma$ of the UI will be patient, facing a transaction cost consuming all proceeds if a security is sold prior to maturity. The carrying costs have support $[0, 1]$ with the cross-sectional distribution of c having an atomless continuously differentiable c.d.f. G with positive density g .

Each impatient UI will update beliefs and follow the same trading rule as in the prior sections, selling a security only if its λ is less than his idiosyncratic carrying cost c . This implies the endogenous liquidity of each security is decreasing in its trading loss information-sensitivity. Specifically, for each security, state-contingent uninformed trading now takes the form:

$$\begin{aligned} u_a &\in \{\underline{u}_a, \bar{u}_a\} \\ \underline{u}_a &\equiv \underline{\gamma}[1 - G(\lambda_a)] \\ \bar{u}_a &\equiv \bar{\gamma}[1 - G(\lambda_a)]. \end{aligned} \tag{55}$$

Once again, the speculator will mask her trading by shorting $\bar{u}_a - \underline{u}_a$ of each security. Thus, Table 1 depicts potential order flow configurations and the pricing rule used by LP. The two securities are indexed by their payoff functions i and l , and we adopt the convention that security l is the more liquid security in that $\lambda_i \geq \lambda_l$.

The primary market valuation is equal to expected cash flow less expected carrying costs incurred by impatient UI holding until maturity less expected trading losses of the UI who sell:

$$\begin{aligned}
V \equiv & \underbrace{\frac{1}{2}(\bar{\mu}_x + \underline{\mu}_x)}_{\text{Expected Cash Flow}} - \underbrace{\frac{1}{2}[\psi\bar{\gamma} + (1-\psi)\underline{\gamma}](\bar{\mu}_i + \underline{\mu}_i)}_{\text{Carrying Costs Less Liquid Claim}} \int_0^{\lambda_i} cg(dc) - \frac{1}{2}[\psi\bar{\gamma} + (1-\psi)\underline{\gamma}](\bar{\mu}_l + \underline{\mu}_l) \int_0^{\lambda_l} cg(dc) \\
& - \underbrace{\frac{1}{2}(2\sigma-1)\psi(1-\psi)(\bar{\mu}_i - \underline{\mu}_i)(\bar{\gamma} - \underline{\gamma})[1 - G(\lambda_i)]}_{\text{Trading Losses Less Liquid Claim}} - \underbrace{\frac{1}{2}(2\sigma-1)\psi(1-\psi)(\bar{\mu}_l - \underline{\mu}_l)(\bar{\gamma} - \underline{\gamma})[1 - G(\lambda_l)]}_{\text{Trading Losses More Liquid Claim}}.
\end{aligned}$$

Before proceeding it is instructive to contrast the preceding equation's primary market valuation for the continuum case with the valuation in the binary case, equation (16). Note that in the continuum case, trading losses and carrying costs are incurred on both claims, with both losses depending upon the optimized liquidation thresholds.

Pinning down the optimal security design is more involved in the continuum case. As a first step, it is convenient to integrate by parts and eliminate all constants from the primary market valuation equation, so the maximand may be expressed as:

$$(\bar{\mu}_x + \underline{\mu}_x) \int_0^{\lambda_i} G(c)dc - (\bar{\mu}_l + \underline{\mu}_l) \int_{\lambda_l}^{\lambda_i} G(c)dc. \quad (56)$$

From the preceding equation it follows that an optimal structuring minimizes $\bar{\mu}_l + \underline{\mu}_l$ subject to achieving a posited pair of information-sensitivities, say $(\lambda_l^0, \lambda_i^0)$. Solving this optimal control problem, the appendix establishes the following lemma.

Lemma 7 *If carrying costs are drawn from a continuum, the optimal two claim structuring consists of a relatively illiquid junior tranche and a relatively liquid senior debt claim.*

Having established the optimality of tranching, consider next determination of the optimal senior face value. Before doing so, it is useful to anticipate some of the tradeoffs. Consider an increase in the senior face value. On a per-unit basis, the increase in trading losses suffered on the senior tranche is just compensated by a decrease in trading losses on the junior tranche. However, the net effect must be an increase in total trading losses since uninformed trading volume is higher on the senior tranche due to its lower information-sensitivity. Consider next the effect of an increase in senior face value on expected carrying costs. When the senior face value is increased, the information-sensitivity of the senior tranche increases, so a greater proportion of impatient UI hold rather than sell. Since the senior tranche is also larger, it is apparent that expected carrying costs connected with the senior tranche increase. When the senior face value is increased, the information-sensitivity of the junior tranche also increases, so more UI incur carrying costs on this tranche as well. However, here there is a competing effect since the junior tranche is smaller, which lowers carrying costs ceteris paribus.

Let us consider these effects more formally. The objective is to find the senior face value minimizing the primary market discount:

$$\begin{aligned} \min_{\theta \in [0, \bar{x}]} & \frac{1}{2} [\psi \bar{\gamma} + (1 - \psi) \underline{\gamma}] (\bar{\mu}_{jr} + \underline{\mu}_{jr}) \int_0^{\lambda_{jr}} cg(c) dc + \frac{1}{2} [\psi \bar{\gamma} + (1 - \psi) \underline{\gamma}] (\bar{\mu}_{sr} + \underline{\mu}_{sr}) \int_0^{\lambda_{sr}} cg(c) dc \\ & + \frac{1}{2} (2\sigma - 1) \psi (1 - \psi) (\bar{\gamma} - \underline{\gamma}) (\bar{\mu}_{jr} - \underline{\mu}_{jr}) [1 - G(\lambda_{jr})] + \frac{1}{2} (2\sigma - 1) \psi (1 - \psi) (\bar{\gamma} - \underline{\gamma}) (\bar{\mu}_{sr} - \underline{\mu}_{sr}) [1 - G(\lambda_{sr})]. \end{aligned} \quad (57)$$

The derivative of the discount with respect to the senior face value is:

$$\begin{aligned} & \frac{1}{2} [\psi \bar{\gamma} + (1 - \psi) \underline{\gamma}] \left[(\bar{\mu}_{jr} + \underline{\mu}_{jr}) \lambda_{jr} g(\lambda_{jr}) \lambda'_{jr} + (\bar{\mu}'_{jr} + \underline{\mu}'_{jr}) \int_0^{\lambda_{jr}} cg(c) dc \right] \\ & + \frac{1}{2} [\psi \bar{\gamma} + (1 - \psi) \underline{\gamma}] \left[(\bar{\mu}_{sr} + \underline{\mu}_{sr}) \lambda_{sr} g(\lambda_{sr}) \lambda'_{sr} + (\bar{\mu}'_{sr} + \underline{\mu}'_{sr}) \int_0^{\lambda_{sr}} cg(c) dc \right] \\ & + \frac{1}{2} (2\sigma - 1) \psi (1 - \psi) (\bar{\gamma} - \underline{\gamma}) \left[(1 - G(\lambda_{jr})) (\bar{\mu}'_{jr} - \underline{\mu}'_{jr}) - (\bar{\mu}_{jr} - \underline{\mu}_{jr}) g(\lambda_{jr}) \lambda'_{jr} \right] \\ & + \frac{1}{2} (2\sigma - 1) \psi (1 - \psi) (\bar{\gamma} - \underline{\gamma}) \left[(1 - G(\lambda_{sr})) (\bar{\mu}'_{sr} - \underline{\mu}'_{sr}) - (\bar{\mu}_{sr} - \underline{\mu}_{sr}) g(\lambda_{sr}) \lambda'_{sr} \right]. \end{aligned} \quad (58)$$

The first line in the preceding equation captures the effect of an increase in senior face value on junior tranche carrying costs. Here we see competing effects. On one hand, there is an increase in carrying costs due to more impatient UI holding onto the junior tranche. On the other hand, the junior tranche itself becomes smaller. The second line captures the effect on senior tranche carrying costs. Here the two effects work in the same direction, serving to increase carrying costs associated with the senior tranche. The third line captures a decline in trading losses on the junior tranche since it offers lower losses on a unit basis, and UI trading volume in the junior tranche declines. The fourth line reveals competing effects on the senior tranche trading loss since it offers higher losses on a unit basis, but UI trading volume declines.

Intuition suggests that a number of terms in the preceding equation should cancel since optimal liquidations on the part of the impatient UI implies zero first-order loss to inducing marginal investors to hold rather than sell. Indeed, from the definition of λ_{jr} it follows that:

$$\frac{1}{2} [\psi \bar{\gamma} + (1 - \psi) \underline{\gamma}] (\bar{\mu}_{jr} + \underline{\mu}_{jr}) \lambda_{jr} g(\lambda_{jr}) \lambda'_{jr} = \frac{1}{2} (2\sigma - 1) \psi (1 - \psi) (\bar{\gamma} - \underline{\gamma}) (\bar{\mu}_{jr} - \underline{\mu}_{jr}) g(\lambda_{jr}) \lambda'_{jr} \quad (59)$$

and similarly for the senior claim. Canceling these terms in equation (58), and using the fact that $\mu'_{jr} = -\mu'_{sr}$ we obtain the following expression of the derivative of the primary market discount with respect to senior debt face value:

$$\frac{1}{2} (2\sigma - 1) \psi (1 - \psi) (\bar{\gamma} - \underline{\gamma}) [G(\lambda_{jr}) - G(\lambda_{sr})] (\bar{\mu}'_{sr} - \underline{\mu}'_{sr}) - \frac{1}{2} [\psi \bar{\gamma} + (1 - \psi) \underline{\gamma}] \left[\int_{\lambda_{sr}}^{\lambda_{jr}} cg(c) dc \right] (\bar{\mu}'_{sr} + \underline{\mu}'_{sr}). \quad (60)$$

Equation (60) illustrates most clearly the tradeoffs associated with increasing senior face value. On one hand, increasing the senior face raises total trading loss discounts. Intuitively, as the senior

face is raised, there is an increase in the per-unit trading loss on the senior tranche, and an offsetting decrease in the per-unit trading loss on the junior tranche. Critically, the trading losses on the senior tranche hit a larger uninformed trading base given that a higher fraction of the uninformed are willing to sell it, given its lower information-sensitivity. This effect is captured by the first term in the preceding equation. On the other hand, as the senior face is increased, the total carrying cost discount falls. Intuitively, reducing the size of the junior tranche reduces expected carrying costs since the average carrying cost on the junior tranche is higher than on the senior tranche. Note, both of these effects result from optimized liquidation decisions by uninformed investors.

It is apparent from equation (60) that riskless debt cannot be optimal since the primary market discount is locally decreasing in senior face value for riskless debt. And the optimal senior face cannot be \bar{x} because such a claim is actually equity, and here too the discount decreases if one were to carve out a riskless debt claim. So the optimal senior face value must be interior. The following proposition summarizes.

Proposition 6 *If carrying costs are drawn from a continuum, the optimal two claim structuring consists of a relatively illiquid junior tranche and a relatively liquid senior debt claim with face value θ^* satisfying*

$$\frac{1 - \bar{F}(\theta^*)}{1 - \underline{F}(\theta^*)} = \frac{(2\sigma - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma})[G(\lambda_{jr}) - G(\lambda_{sr})] + [\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}] \left[\int_{\lambda_{sr}}^{\lambda_{jr}} cg(c)dc \right]}{(2\sigma - 1)\psi(1 - \psi)(\bar{\gamma} - \underline{\gamma})[G(\lambda_{jr}) - G(\lambda_{sr})] - [\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}] \left[\int_{\lambda_{sr}}^{\lambda_{jr}} cg(c)dc \right]}.$$

It is instructive to compare the preceding optimality condition with the optimality condition in the binary case (equation (25)). The key difference is that when we consider a rich realistic depiction of investor heterogeneity, the optimal structuring depends upon the entire distribution of prospective carrying cost shocks. When viewed from this perspective, the penchant of structurers to attempt understanding the motivations and concerns of their buy-side clientele roster is less surprising.

4 Non-Monotone Securities

An important assumption in the preceding sections was that security payoffs must be nondecreasing. Although this assumption can be justified based upon concern over ex post moral hazard, it is worth considering whether debt remains the optimal liquid security when decreasing securities are admitted. For brevity, attention is confined to the simplest setting where: carrying costs are binary; speculator signal precision is fixed; and the carrying cost c is low. We recall that if c is low, UI_c are unwilling to sell a pass-through security, so some other security must serve as a source of interim liquidity for these investors.

It is straightforward to show that debt is not the optimal liquid security if decreasing payoffs are admitted. To demonstrate this, it is sufficient to confine attention to a liquid security payoff function l such that $\bar{\mu}_l \geq \underline{\mu}_l$. Note, this set of payoff functions includes the senior debt claim which was optimal within the narrower set of nondecreasing securities. Note further that for payoff functions l such that $\bar{\mu}_l \geq \underline{\mu}_l$, the speculator's trading strategy, the UI_c selling strategies, and the primary market valuation V remain as above.

The objective is to find a payoff function l for the posited liquid security to maximize the primary market valuation in equation (16) (which simplifies to equation (17)) subject to: limited liability (LL); $\bar{\mu}_l \geq \underline{\mu}_l$; and the SL constraint in equation (28), with the NSI constraint being redundant. It is apparent that SL is slack since safe debt satisfies the constraint, and any security raising the objective further leaves this constraint slack. We treat this as a standard optimal control problem, with l a constrained control. The integral constraint $\bar{\mu}_l \geq \underline{\mu}_l$ is incorporated as a state variable constraint:

$$\begin{aligned} K(\underline{x}) &= 0 \\ K'(x) &= l(x) (\bar{f}(x) - \underline{f}(x)) \\ K(\bar{x}) &\geq 0. \end{aligned}$$

The Hamiltonian for this problem is:

$$H(x, K, l, \pi) = [\underline{\kappa} \underline{f}(x) - \bar{\kappa} \bar{f}(x)] l(x) + \pi(x) l(x) (\bar{f}(x) - \underline{f}(x)). \quad (61)$$

From Pontryagin's Maximum Principle, an optimal control policy l^* maximizes the Hamiltonian, with the corresponding co-state variable π^* being a piecewise continuously differentiable function satisfying the following co-state equation at all points where the control l^* is continuous:

$$\pi' = -H_K = 0. \quad (62)$$

Thus π^* is a constant, with the transversality condition demanding $\pi^* \geq 0$. Substituting this back into the Hamiltonian we obtain:

$$H(x, K, l, \pi^*) = [(\underline{\kappa} - \pi^*) \underline{f}(x) - (\bar{\kappa} - \pi^*) \bar{f}(x)] l(x). \quad (63)$$

Thus the optimal control has a bang-bang solution with:

$$\begin{aligned} \frac{\bar{f}(x)}{\underline{f}(x)} &\leq \frac{\underline{\kappa} - \pi^*}{\bar{\kappa} - \pi^*} \Rightarrow l^*(x) = x \\ \frac{\bar{f}(x)}{\underline{f}(x)} &> \frac{\underline{\kappa} - \pi^*}{\bar{\kappa} - \pi^*} \Rightarrow l^*(x) = 0. \end{aligned} \quad (64)$$

The control policy l^* can be implemented with a down-and-out security, as in Innes (1990) for example. And it is apparent that such a security dominates a standard debt claim as a source of interim liquidity for UI_c if decreasing claims are admitted. Intuitively, non-monotone securities allow investors to raise funds with lower expected underpricing. For example, a non-monotone claim can be risky, but be structured so as to have zero information-sensitivity if the drop-off point is chosen appropriately.

5 Conclusions

This paper developed a demand-side theory of security design, where the key parameters determining the optimal structure are those describing the distribution of liquidity shocks hitting uninformed investors. Uninformed investors have a demand for liquidity, but liquidity provision is hindered by informed speculation. This exposes the uninformed to underpricing in the event they sell securities. Consequently, the uninformed may prefer to incur carrying costs rather than sell. This implies that with optimizing investors, the adverse selection discount on a given security is not equal to expected trading losses, in general. Rather, the primary market discount is equal to the minimum of expected trading losses and carrying costs. A novel theory of optimal security design emerges as one recognizes that alternative structures can influence the secondary market trading of the uninformed as they weigh trading losses and carrying costs on a security by security basis.

Maximizing the primary market valuation is equivalent to minimizing the sum of expected uninformed trading losses and carrying costs. In general, the optimal structuring entails tranching claims, with senior debt the primary source of secondary market liquidity. With fixed speculator information precision, the optimal senior face value entails a tradeoff. An increase in senior face value increases the total trading loss discount since unit profits are shifted to the less information-sensitive security where there is a higher level of uninformed trading. On the other hand, an increase in senior face value reduces total carrying costs, since there is a decrease in the size of the junior tranche, where average carrying costs are higher for non-sellers. This privately optimal structuring was shown to be socially suboptimal, as the desire to reduce speculator gains generates socially excessive carrying costs. With discretionary effort, tranching can either stimulate or deter speculator information production as it can increase or decrease uninformed trading volume. Tranching remains optimal with endogenous speculator information production, but as a deterrent optimal senior face is lowered.

A methodological contribution of the paper is to consider optimal security design in a competitive Kyle-type setting. This analysis suggests it may be productive to move microstructure-based models of corporate finance and security design away from assuming the uninformed are pure noise-traders.

Indeed, novel and interesting theories of corporate finance may emerge if one considers that all agents optimize.

PROOFS

Lemma 2: Relaxed Program 1

The objective is to maximize $\mathcal{L}(l)$ over $l \in \mathcal{A}$. Consider first $l \in \mathcal{A}$ that are piecewise continuously differentiable. The Hamiltonian is:

$$H(x, l, \delta, \pi) = [\bar{\kappa}\bar{F}(x) - \underline{\kappa}F(x)] \delta(x) + \pi(x)\delta(x). \quad (65)$$

From Pontryagin's Maximum Principle, an optimal control policy δ^* maximizes the Hamiltonian, with the co-state variable π^* being a piecewise continuously differentiable function satisfying the following co-state equation at all points where the control δ^* is continuous:

$$\pi' = -H_l = 0. \quad (66)$$

Thus π^* is a constant, with the transversality condition demanding

$$\pi^* = 2c[\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}]. \quad (67)$$

Substituting this back into the Hamiltonian we obtain

$$H(x, l, \delta, \pi^*) = [\bar{\kappa}\bar{F}(x) - \underline{\kappa}F(x) + 2c(\psi\bar{\gamma} + (1 - \psi)\underline{\gamma})] \delta(x). \quad (68)$$

Maximizing H over admissible δ , a candidate optimal control policy is:

$$\delta^* = 1_{\{x \in \mathcal{X}: \bar{\kappa}\bar{F}(x) - \underline{\kappa}F(x) + 2c(\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}) \geq 0\}}. \quad (69)$$

From the transversality condition it follows LL binds at \underline{x} so the state variable l can be expressed as

$$l^*(x) = \underline{x} + \int_{\underline{x}}^x \delta^*(\tilde{x}) d\tilde{x}. \quad (70)$$

By construction, the Hamiltonian is linear in (l, δ) implying Mangasarian's sufficiency conditions are satisfied. Thus, the proposed δ^* is an optimal control and l^* maximizes $\mathcal{L}(l)$ on the space of piecewise continuously differentiable functions in \mathcal{A} . It follows from the Stone-Weierstrass Theorem this space is dense in \mathcal{A} endowed with the sup norm. And we know $\mathcal{L}(l)$ is continuous in l in this topology. Thus, the proposed state variable l^* maximizes $\mathcal{L}(l)$ amongst all $a \in \mathcal{A}$.

We next claim the proposed control policy implies the optimal liquid security is senior debt. Consider that $\delta^* = 1$ (and zero otherwise) for all $x \in \mathcal{X}$ such that:

$$\begin{aligned} \bar{\kappa}\bar{F}(x) - \underline{\kappa}F(x) + 2c(\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}) &\geq 0 \\ &\Downarrow \\ \frac{1 - \bar{F}(x)}{1 - \underline{F}(x)} &\leq \frac{\underline{\kappa}}{\bar{\kappa}}. \end{aligned} \quad (71)$$

Since MLR implies MHR, the left side of the preceding equation is non-decreasing so the optimal control switches from 1 to 0 at θ^* solving:

$$\frac{1 - \overline{F}(\theta^*)}{1 - \underline{F}(\theta^*)} = \frac{\underline{\kappa}}{\overline{\kappa}}. \blacksquare \quad (72)$$

Lemma 3: Information-Sensitivity of Claims

The information-sensitivity of the junior security is:

$$\lambda_{jr}(\theta) \equiv \frac{(2\sigma - 1)\psi(1 - \psi)(\overline{\gamma} - \underline{\gamma}) \overline{\mu}_{jr}(\theta) - \underline{\mu}_{jr}(\theta)}{\psi\overline{\gamma} + (1 - \psi)\underline{\gamma}} \frac{\overline{\mu}_{jr}(\theta) - \underline{\mu}_{jr}(\theta)}{\overline{\mu}_{jr}(\theta) + \underline{\mu}_{jr}(\theta)}. \quad (73)$$

The information-sensitivity of the junior claim increases in the face value of senior riskless debt since

$$\theta \leq \underline{x} \Rightarrow \lambda_{jr}(\theta) \equiv \frac{(2\sigma - 1)\psi(1 - \psi)(\overline{\gamma} - \underline{\gamma})}{\psi\overline{\gamma} + (1 - \psi)\underline{\gamma}} \frac{\overline{\mu}_x - \underline{\mu}_x}{\overline{\mu}_x + \underline{\mu}_x - 2\theta}. \quad (74)$$

Consider next information-sensitivity for $\theta \in (\underline{x}, \overline{x})$. Differentiating with respect to θ one obtains:

$$\lambda'_{jr}(\theta) = \frac{(2\sigma - 1)\psi(1 - \psi)(\overline{\gamma} - \underline{\gamma})}{\psi\overline{\gamma} + (1 - \psi)\underline{\gamma}} \frac{2[\underline{\mu}_{jr}\overline{\mu}'_{jr} - \overline{\mu}_{jr}\underline{\mu}'_{jr}]}{[\overline{\mu}_{jr} + \underline{\mu}_{jr}]^2}. \quad (75)$$

Thus

$$\begin{aligned} \lambda'_{jr}(\theta) &\geq 0 & (76) \\ &\Downarrow \\ \frac{\overline{\mu}_{jr}(\theta)}{\overline{\mu}'_{jr}(\theta)} &\leq \frac{\underline{\mu}_{jr}(\theta)}{\underline{\mu}'_{jr}(\theta)} \\ &\Downarrow \\ \frac{\int_{\theta}^{\overline{x}} (x - \theta) \overline{f}(x) dx}{[1 - \overline{F}(\theta)]} &\geq \frac{\int_{\theta}^{\overline{x}} (x - \theta) \underline{f}(x) dx}{[1 - \underline{F}(\theta)]} \\ &\Downarrow \\ \int_{\theta}^{\overline{x}} x \left(\frac{\overline{f}(x)}{1 - \overline{F}(\theta)} \right) dx &\geq \int_{\theta}^{\overline{x}} x \left(\frac{\underline{f}(x)}{1 - \underline{F}(\theta)} \right) dx. \end{aligned}$$

The last inequality follows from the fact that the conditional densities in brackets also have the MLR property.

Consider next the information-sensitivity of the senior security. For $\theta \in (\underline{x}, \bar{x})$ we have:

$$\begin{aligned}
\lambda'_{sr}(\theta) &\geq 0 & (77) \\
&\Downarrow \\
\frac{\bar{\mu}_{sr}(\theta)}{\bar{\mu}'_{sr}(\theta)} &\leq \frac{\underline{\mu}_{sr}(\theta)}{\underline{\mu}'_{sr}(\theta)} \\
&\Downarrow \\
\frac{\int_{\underline{x}}^{\theta} x \bar{f}(x) dx + \theta[1 - \bar{F}(\theta)]}{[1 - \bar{F}(\theta)]} &\leq \frac{\int_{\underline{x}}^{\theta} x \underline{f}(x) dx + \theta[1 - \underline{F}(\theta)]}{[1 - \underline{F}(\theta)]} \\
&\Downarrow \\
\int_{\underline{x}}^{\theta} x \left(\frac{\bar{f}(x)}{1 - \bar{F}(\theta)} \right) dx &\leq \int_{\underline{x}}^{\theta} x \left(\frac{\underline{f}(x)}{1 - \underline{F}(\theta)} \right) dx
\end{aligned}$$

To establish the last inequality above it is sufficient to prove that for arbitrary $x \in (\underline{x}, \theta)$

$$\frac{\bar{f}(x)}{1 - \bar{F}(\theta)} \leq \frac{\underline{f}(x)}{1 - \underline{F}(\theta)}. \quad (78)$$

Consider then that from MLR we have

$$\underline{x} < x_0 < \theta < x_1 < \bar{x} \Rightarrow \underline{f}(x_0) \bar{f}(x_1) \geq \underline{f}(x_1) \bar{f}(x_0). \quad (79)$$

And thus

$$\begin{aligned}
\underline{f}(x_0) \int_{\theta}^{\bar{x}} \bar{f}(x_1) dx_1 &\geq \bar{f}(x_0) \int_{\theta}^{\bar{x}} \underline{f}(x_1) dx_1 & (80) \\
&\Downarrow \\
\underline{f}(x_0)[1 - \bar{F}(\theta)] &\geq \bar{f}(x_0)[1 - \underline{F}(\theta)].
\end{aligned}$$

Finally, the last statement in the lemma follows from the identity (10). Rearranging we obtain:

$$\lambda_{sr} = \lambda_x + [\lambda_{jr} - \lambda_x] \left[\frac{\bar{\mu}_{jr} + \underline{\mu}_{jr}}{\bar{\mu}_{sr} + \underline{\mu}_{sr}} \right] \leq \lambda_x. \blacksquare \quad (81)$$

Proposition 3: Social Planner's Problem

The Hamiltonian for the social planner's problem is:

$$H(x, l, \delta, \pi, m) = - [(1 + m\underline{\kappa})\underline{F}(x) + (1 - m\bar{\kappa})\bar{F}(x)] \delta(x) + \pi(x)\delta(x). \quad (82)$$

From Pontryagin's Maximum Principle, an optimal control policy δ^* maximizes the Hamiltonian, with the co-state variable π^* being a piecewise continuously differentiable function satisfying the following co-state equation at all points where the control δ^* is continuous:

$$\pi' = -H_l = 0. \quad (83)$$

Thus π^* is a constant, with the transversality condition demanding

$$\pi^* = 2 + 2mc(\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}). \quad (84)$$

Substituting this back into the Hamiltonian we have:

$$H(x, l, \delta, \pi^*, m) = [2 + 2mc(\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}) - (1 + m\underline{\kappa})\underline{F}(x) - (1 - m\bar{\kappa})\bar{F}(x)]\delta(x). \quad (85)$$

Thus, a candidate optimal control policy is:

$$\delta^* = 1_{\{x \in \mathcal{X}: 2 + 2mc(\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}) - (1 + m\underline{\kappa})\underline{F}(x) - (1 - m\bar{\kappa})\bar{F}(x) \geq 0\}}. \quad (86)$$

From the transversality condition it follows LL binds at \underline{x} so the implied state variable is:

$$l^*(x) = \underline{x} + \int_{\underline{x}}^x \delta^*(\tilde{x}) d\tilde{x}. \quad (87)$$

By construction, the Hamiltonian is linear in (l, δ) implying Mangasarian's sufficiency conditions are satisfied. Thus, the proposed δ^* is an optimal control and l^* maximizes $\mathcal{L}(l, m)$ on the space of piecewise continuously differentiable functions in \mathcal{A} . It follows from the Stone-Weierstrass Theorem this space is dense in \mathcal{A} endowed with the sup norm. And we know $\mathcal{L}(l, m)$ is continuous in l in this topology. Thus, the proposed state variable l^* maximizes $\mathcal{L}(l, m)$ amongst all $a \in \mathcal{A}$.

We next claim the proposed control policy implies the socially optimal liquid security is debt. Consider that $\delta^* = 1$ (and 0 otherwise) for all $x \in \mathcal{X}$ such that:

$$(1 + m\underline{\kappa})[1 - \underline{F}(x)] \geq -(1 - m\bar{\kappa})[1 - \bar{F}(x)]. \quad (88)$$

The term on the right side of the preceding inequality must be positive otherwise the inequality would always hold, and the optimal liquid security would be a pure pass-through. But this contradicts the fact that a pass-through security would be illiquid in the present setting. Thus, $\delta^* = 1$ (and 0 otherwise) for all $x \in \mathcal{X}$ such that:

$$\frac{1 - \bar{F}(x)}{1 - \underline{F}(x)} \leq -\frac{1 + m\underline{\kappa}}{1 - m\bar{\kappa}}. \quad (89)$$

Since MLR implies MHR, the left side of the preceding equation is non-decreasing so the optimal control switches from 1 to 0, implying debt is the optimal liquid security. Finally, note the objective function is increasing in debt face value, but so too is the information-sensitivity of the posited liquid debt claim. Thus, the socially optimal face value is such that the SL constraint is binding. ■

Lemma 4: Relaxed Program 2

Consider first implementation of the minimal signal precision in the feasible set for RP2, denoted σ_1 . From the IC constraint it follows that the liquid security must satisfy $\bar{\mu}_l = \underline{\mu}_l$. Within this set, carrying costs on the illiquid claim are minimized by setting the face value of the liquid senior claim to \underline{x} . Consider next the program RP2 for $\sigma \in (\sigma_1, \sigma_{pt})$. Consider first $l \in \mathcal{A}$ that are piecewise continuously differentiable. The Hamiltonian is:

$$H(x, l, \delta, \pi, m) = [(\bar{\kappa} - m)\bar{F}(x) - (\underline{\kappa} - m)\underline{F}(x)]\delta(x) + \pi(x)\delta(x). \quad (90)$$

From Pontryagin's Maximum Principle, an optimal control policy δ^* maximizes the Hamiltonian, with the co-state variable π^* being a piecewise continuously differentiable function satisfying the following co-state equation at all points where the control δ^* is continuous:

$$\pi'_l = -H_l = 0. \quad (91)$$

Thus π^* is a constant, with the transversality condition demanding

$$\pi^* = 2c[\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}]. \quad (92)$$

Substituting this back into the Hamiltonian we obtain

$$H(x, l, \delta, \pi^*, m) = [2c(\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}) + (\bar{\kappa} - m)\bar{F}(x) - (\underline{\kappa} - m)\underline{F}(x)]\delta(x). \quad (93)$$

Maximizing H over admissible δ , a candidate optimal control policy is:

$$\delta^* = 1_{\{x \in \mathcal{X}: 2c[\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}] + (\bar{\kappa} - m)\bar{F}(x) - (\underline{\kappa} - m)\underline{F}(x) \geq 0\}}. \quad (94)$$

Further, from the transversality condition it follows LL binds at \underline{x} so the implied state variable is:

$$l^*(x) = \underline{x} + \int_{\underline{x}}^x \delta^*(\tilde{x})d\tilde{x}. \quad (95)$$

By construction, the Hamiltonian is linear in (l, δ) implying Mangasarian's sufficiency conditions are satisfied. Thus, the proposed δ^* is an optimal control and l^* maximizes $\mathcal{L}(l, m)$ on the space of piecewise continuously differentiable functions in \mathcal{A} . It follows from the Stone-Weierstrass Theorem this space is dense in \mathcal{A} endowed with the sup norm. And we know $\mathcal{L}(l, m)$ is continuous in l in this topology. Thus, the proposed state variable l^* maximizes $\mathcal{L}(l, m)$ amongst all $a \in \mathcal{A}$.

We next claim the proposed control policy implies the optimal liquid security is debt. Consider that $\delta^* = 1$ (and 0 otherwise) for all $x \in \mathcal{X}$ such that:

$$(\bar{\kappa} - m)\bar{F}(x) - (\underline{\kappa} - m)\underline{F}(x) + 2c(\psi\bar{\gamma} + (1 - \psi)\underline{\gamma}) \geq 0 \quad (96)$$

or

$$\frac{1 - \overline{F}(x)}{1 - \underline{F}(x)} \leq \frac{\underline{\kappa} - m}{\overline{\kappa} - m}. \quad (97)$$

Since MLR implies MHR, the left side of the preceding equation is non-decreasing so the optimal control switches from 1 to 0 at θ^* solving:

$$\frac{1 - \overline{F}(\theta^*)}{1 - \underline{F}(\theta^*)} = \frac{\underline{\kappa} - m}{\overline{\kappa} - m}. \blacksquare \quad (98)$$

Proposition 4: Optimal Face Value with Speculator Effort (High Carrying Costs)

It is claimed that the possibility of endogenous increases in σ result in strictly lower senior debt face value than when signal precision is fixed at $\underline{\sigma}$, provided the NSI constraint can be satisfied. To establish this, first note that with endogenous effort, the optimal face value is either θ_{int}^{**} or $\widehat{\theta}(\sigma^{\min})$. In the former case, the claim follows from the fact that any solution to the extended model's first-order condition (equation (50)) is strictly less than with σ fixed at $\underline{\sigma}$. In the latter case, the result is necessarily true if $\widehat{\theta}(\sigma^{\min}) = \underline{x}$ since the optimal debt face value with σ fixed at $\underline{\sigma}$ is strictly greater than \underline{x} . If $\widehat{\theta}(\sigma^{\min}) > \underline{x}$, the claim follows from the fact that it must then be the case that $\sigma^{\min} > \underline{\sigma}$ and the senior debt face value at which the NSI constraint is just satisfied is strictly less than the point at which the NSI constraint is satisfied when σ is fixed at $\underline{\sigma}$. ■

Proposition 5: Optimal Face Value with Speculator Effort (Low Carrying Costs)

It is claimed that the possibility of endogenous increases in σ result in strictly lower senior debt face value than when signal precision is fixed at $\underline{\sigma}$. To establish this, first note that with endogenous effort the optimal face value is either θ_{int}^{**} or $\widehat{\theta}(\sigma^{\max})$. In the former case, the result follows from the fact that any solution to the extended model's first-order condition (equation (50)) is strictly less than with σ fixed at $\underline{\sigma}$. In the latter case, the result follows from the fact that if the solution is at the corner $\widehat{\theta}(\sigma^{\max})$, the value function must be increasing in θ at the corner. But then the value function with σ fixed at $\underline{\sigma}$ must also be increasing in θ at this point as well. So with σ fixed at $\underline{\sigma}$ the optimal face value must exceed $\widehat{\theta}(\sigma^{\max})$. ■

Lemma 7: The Optimality of Tranching under a Continuum of Carrying Costs

For each posited pair $(\lambda_l^0, \lambda_i^0)$, we wish to find

$$\max_{l \in \mathcal{A}} - \int_{\underline{x}}^{\overline{x}} [\underline{f}(x) + \overline{f}(x)] l(x) dx$$

subject to LL; MN; and the constraints that the targeted information sensitivities are achieved. For brevity, let

$$\Omega \equiv \frac{(2\sigma - 1)\psi(1 - \psi)(\overline{\gamma} - \underline{\gamma})}{\psi\overline{\gamma} + (1 - \psi)\underline{\gamma}}.$$

In order to achieve the targeted information sensitivities, it must be that:

$$\begin{aligned}\lambda_l^0 &= \frac{\Omega(\bar{\mu}_l - \underline{\mu}_l)}{\bar{\mu}_l + \underline{\mu}_l} \Rightarrow \bar{\mu}_l = \left[\frac{\Omega + \lambda_l^0}{\Omega - \lambda_l^0} \right] \underline{\mu}_l \\ \lambda_i^0 &= \frac{\Omega[\bar{\mu}_x - \bar{\mu}_l - \underline{\mu}_x + \underline{\mu}_l]}{\bar{\mu}_x - \bar{\mu}_l + \underline{\mu}_x - \underline{\mu}_l}.\end{aligned}\tag{99}$$

Substituting the first constraint into the second, we impose the following inequality constraint to account for the information-sensitivity constraints.

$$\underline{\mu}_l \geq \frac{(\Omega - \lambda_l^0)[(\Omega + \lambda_i^0)\underline{\mu}_x - (\Omega - \lambda_i^0)\bar{\mu}_x]}{2\Omega(\lambda_i^0 - \lambda_l^0)}.\tag{100}$$

Given the maximand, it is apparent that the prior inequality constraint will indeed bind at an optimum.

We form the following Lagrangian, with the multiplier m here corresponding to the information-sensitivity constraint:

$$\mathcal{L}(l, m) = - \int_{\underline{x}}^{\bar{x}} [\underline{f}(x) + \bar{f}(x)] l(x) dx + m \left[\int_{\underline{x}}^{\bar{x}} \underline{f}(x) l(x) dx - \frac{(\Omega - \lambda_l^0)[(\Omega + \lambda_i^0)\underline{\mu}_x - (\Omega - \lambda_i^0)\bar{\mu}_x]}{2\Omega(\lambda_i^0 - \lambda_l^0)} \right].\tag{101}$$

From the LL and MN constraints it follows any $l \in \mathcal{A}$ is absolutely continuous. Further, the derivative $l' \equiv \delta$ is well-defined, with $\delta \in [0, 1]$, except on a subset of \mathcal{X} with Lebesgue measure zero. Using integration by parts, the Lagrangian can be rewritten as:

$$\mathcal{L}(l, m) = \int_{\underline{x}}^{\bar{x}} [\bar{F}(x) + (1 - m)\underline{F}(x)] \delta(x) dx + (m - 2)l(\bar{x}) - \frac{m(\Omega - \lambda_l^0)[(\Omega + \lambda_i^0)\underline{\mu}_x - (\Omega - \lambda_i^0)\bar{\mu}_x]}{2\Omega(\lambda_i^0 - \lambda_l^0)}.\tag{102}$$

The Hamiltonian is:

$$H(x, l, \delta, \pi, m) = [\bar{F}(x) + (1 - m)\underline{F}(x)]\delta(x) + \pi(x)\delta(x).\tag{103}$$

From Pontryagin's Maximum Principle, an optimal control policy δ^* maximizes the Hamiltonian, with the co-state variable π^* being a piecewise continuously differentiable function satisfying the following co-state equation at all points where the control δ^* is continuous:

$$\pi_l' = -H_l = 0.\tag{104}$$

Thus π^* is a constant, with the transversality condition demanding

$$\pi^* = m - 2.\tag{105}$$

Substituting this back into the Hamiltonian we obtain

$$H(x, l, \delta, \pi^*, m) = [\overline{F}(x) + (1 - m)\underline{F}(x) + m - 2]\delta(x). \quad (106)$$

Maximizing H over admissible δ , a candidate optimal control policy is:

$$\delta^* = 1_{\{x \in \mathcal{X}: \overline{F}(x) + (1-m)\underline{F}(x) + m - 2 \geq 0\}}. \quad (107)$$

Further, from the transversality condition it follows LL binds at \underline{x} so the implied state variable is:

$$l^*(x) = \underline{x} + \int_{\underline{x}}^x \delta^*(\tilde{x}) d\tilde{x}. \quad (108)$$

By construction, the Hamiltonian is linear in (l, δ) implying Mangasarian's sufficiency conditions are satisfied. Thus, the proposed δ^* is an optimal control and l^* maximizes $\mathcal{L}(l, m)$ on the space of piecewise continuously differentiable functions in \mathcal{A} . It follows from the Stone-Weierstrass Theorem this space is dense in \mathcal{A} endowed with the sup norm. And we know $\mathcal{L}(l, m)$ is continuous in l in this topology. Thus, the proposed state variable l^* maximizes $\mathcal{L}(l, m)$ amongst all $a \in \mathcal{A}$.

We next claim the proposed control policy implies the optimal liquid security is debt. Consider that $\delta^* = 1$ (and 0 otherwise) for all $x \in \mathcal{X}$ such that:

$$\frac{1 - \overline{F}(x)}{1 - \underline{F}(x)} \leq m - 1. \quad (109)$$

Since MLR implies MHR, the left side of the preceding equation is non-decreasing so the optimal control switches from 1 to 0 at θ solving:

$$\frac{1 - \overline{F}(\theta)}{1 - \underline{F}(\theta)} = m - 1. \blacksquare \quad (110)$$

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Table 1: Order Flow Possibilities

State	Signal	S Order	UI Order	Total Order	Probability	LP Belief
$\bar{\omega}$	$\bar{\omega}$	0	$-\underline{u}$	$-\underline{u}$	$\frac{\sigma(1-\psi)}{2}$	σ
$\bar{\omega}$	$\bar{\omega}$	0	$-\bar{u}$	$-\bar{u}$	$\frac{\sigma\psi}{2}$	$1 - \sigma - \psi + 2\sigma\psi$
$\bar{\omega}$	$\underline{\omega}$	$-(\bar{u} - \underline{u})$	$-\underline{u}$	$-\bar{u}$	$\frac{(1-\sigma)(1-\psi)}{2}$	$1 - \sigma - \psi + 2\sigma\psi$
$\bar{\omega}$	$\underline{\omega}$	$-(\bar{u} - \underline{u})$	$-\bar{u}$	$-(2\bar{u} - \underline{u})$	$\frac{(1-\sigma)\psi}{2}$	$1 - \sigma$
$\underline{\omega}$	$\underline{\omega}$	$-(\bar{u} - \underline{u})$	$-\underline{u}$	$-\bar{u}$	$\frac{\sigma(1-\psi)}{2}$	$1 - \sigma - \psi + 2\sigma\psi$
$\underline{\omega}$	$\underline{\omega}$	$-(\bar{u} - \underline{u})$	$-\bar{u}$	$-(2\bar{u} - \underline{u})$	$\frac{\sigma\psi}{2}$	$1 - \sigma$
$\underline{\omega}$	$\bar{\omega}$	0	$-\underline{u}$	$-\underline{u}$	$\frac{(1-\sigma)(1-\psi)}{2}$	σ
$\underline{\omega}$	$\bar{\omega}$	0	$-\bar{u}$	$-\bar{u}$	$\frac{(1-\sigma)\psi}{2}$	$1 - \sigma - \psi + 2\sigma\psi$

Table 2: Optimal Face Values on Senior Tranche

	Normal Risk Level	Amplified Risk Level
Fixed σ : High c	155	109
Variable σ : High c	148	108
More Noise: High c	144	106
Fixed σ : Low c	115	102
Variable σ : Low c	112	102
More Noise: Low c	111	102



