Discussion of "A preferred-Habitat Model of the Term structure of interest rates" by Dimitri Vayanos and Jean-Luc Vila

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Summary of the paper

- Propose a theoretical model of the term structure based on
  - Agents with preferred-habitat (i.e, exogenous demand for specific maturity bonds)
  - Risk-averse arbitrageurs
- That can explain many stylized facts:
  - Predictability in long-term bonds
  - Forward rates are biased predictors of expected future short rates
  - Two first principal components explain most of the variation in the yield curve
  - Relation between forward rates and investor demand.
- Ultimate goals:
  - 'Considering this mechanism leads to a new set of intuitions and predictions.'
  - 'Because model is structural rather than reduced-form, can suggest which specifications [...] are more economically plausible.'

The stylized facts

- Predictability in bond returns
  - Returns on long-term bonds are larger when the slope of the yield curve is steep
  - Average term premia are small.
  - Term premia are time varying (and even may change sign).

  (Fama and Bliss (1987), Fama-French (1993), Duffee (2002), Dai-Singleton (2002), Cochrane and Piazzesi (2005))

Table I
Regressions of excess returns to Treasury bonds
July 1961 through December 1998

Monthly excess returns to portfolios of Treasury coupon bonds are regressed on the previous month's term-structure slope and an estimate of the interest rate volatility during the previous month. The slope of the term structure is measured by the difference between five-year and three-month zero-coupon yields (interpolated from coupon bonds). Monthly volatility is measured by the square root of the sum of squared daily changes in the five-year zero-coupon bond yield. Asymptotic t-statistics, adjusted for generalized heteroskedasticity, are in parentheses. There are 449 monthly observations.

Maturity (years)	Mean excess return (%)	Coef on slope	Coef on volatility	Std. dev. of fitted excess rets
$0 < m \leq 1$	0.011	0.027 (1.76)	0.116 (0.96)	0.036
$1 < m \leq 2$	0.045	0.085 (1.85)	0.413 (1.27)	0.119
$2 < m \leq 3$	0.064	0.132 (1.88)	0.582 (1.20)	0.179
$3 < m \leq 4$	0.074	0.187 (2.38)	0.706 (1.35)	0.241
$4 < m \leq 5$	0.063	0.214 (2.37)	0.692 (1.16)	0.265
$5 < m \leq 10$	0.094	0.296 (2.69)	0.804 (1.08)	0.354

source: Duffee (2001)

- In contradiction with expectation's hypothesis.
- But consistent with evidence from equity and currency markets about predictability: 'high yields forecast high returns... presumably reflecting the fact that discount rates are high'

Absence of arbitrage in Short rate models

- Suppose the short rate model is exogenously given by:

$$dr_t = \kappa(\theta - r_t)dt + \sigma_r dz_t$$

- Assume zero-coupon bond $P(t, T_i) = P^i(t, r_t)$. Construct a portfolio $V(t) = n_1 P^1(t) + n_2 P^2(t)$, that is self financing:

$$\begin{aligned} dV_t &= n_1(t)dP^1(t) + n_2(t)dP^2(t) \\ &= \left[n_1 \left(\frac{1}{2} P^1_{rr} \sigma_r^2 + P^1_r \mu_r + P^1_t \right) + n_2 \left(\frac{1}{2} P^2_{rr} \sigma_r^2 + P^2_r \mu_r + P^2_t \right) \right] dt \\ &+ \left(n_1 P^1_r + n_2 P^2_r \right) \sigma_r dw_t \end{aligned}$$

If we choose $\{n_1, n_2\}$ such that the portfolio is locally risk-free

$$n_1 P^1_r + n_2 P^2_r = 0 \quad (1)$$

it should earn the risk-free rate:

$$n_1 \left(\frac{1}{2} P^1_{rr} \sigma_r^2 + P^1_r \mu_r + P^1_t \right) + n_2 \left(\frac{1}{2} P^2_{rr} \sigma_r^2 + P^2_r \mu_r + P^2_t \right) = r(n_1 P^1 + n_2 P^2) \quad (2)$$

Absence of arbitrage in Short rate models

- Combining equations (1) and (2), we find

$$\frac{1}{P_r^1}\left[\frac{1}{2}P_{rr}^1\sigma_r^2 + P_r^1\mu_r + P_t^1 - rP^1\right] = \frac{1}{P_r^2}\left[\frac{1}{2}P_{rr}^2\sigma_r^2 + P_r^2\mu_r + P_t^2 - rP^2\right]. \quad (3)$$

Therefore, there must exist a market price of risk process $\gamma(t)$ *independent of maturities* such that:

$$\gamma(t) = \frac{\frac{1}{2}P_{rr}\sigma_r^2 + P_r\mu_r + P_t - rP}{\sigma_r P_r} \equiv \frac{\mu_P(t,T) - r_t}{\sigma_P(t,T)}. \quad (4)$$

⇒ Absence of arbitrage implies that the instantaneous Sharpe ratio across all bonds is equalized (in a one-factor model).

Specification of risk-premia and predictability in bond returns

- How do we choose the instantaneous Sharpe ratio?
  - First generation: 'completely affine' models.
    - Vasicek: γ constant.
    - CIR: $\gamma(r) = \gamma\sqrt{r}$ proportional to short rate volatility.
  - Second Generation: 'essentially affine' (Duffee (2002), DS (2002)):
    - Vasicek: $\gamma(r) = \gamma_0 + \gamma_r r$.
    - CIR: $\gamma(r) = \gamma_0\sqrt{r} + \frac{\gamma_r}{\sqrt{r}}$.

- Why are essentially affine models of the market price of risk useful?
  Can be seen from the definition of excess expected return:

$$\mu_P(t, T) - r_t = \gamma(r_t)\sigma_P(t, T)$$

⇒ First generation risk-premia impose that compensation for risk is a fixed multiple of short-rate volatility. In particular, risk-premia cannot switch sign over time.

⇒ More general structure of risk-premia breaks this link, which is necessary to capture evidence on predictability (Duffee (2002).

N.B.: $\sigma_P(t, T) = -\sigma_r \frac{P_r(t,T)}{P(t,T)} < 0$ so need $\gamma_r > 0$ to generate relation between slope and expected return.

Intuition for the Habitat-model

- Risk-averse arbitrageurs can freely trade bonds of all maturities.

⇒ No-arbitrage holds and Sharpe ratios ($\gamma(t)$) across all bonds are equalized.

⇒ Arbitrageurs are indifferent between all bonds. Can achieve same payoff with one bond and short rate.

- Consider portfolio choice with one bond (maturity T) and risk-free rate.

$$dW(t) = r(t)W(t) + X(t)\left(\frac{dP(t,T)}{P(t,T)} - r(t)\right)dt \quad (5)$$

$$= r(t)W(t) + X(t)\sigma_P(t,T)(\gamma(t)dt + dz(t)) \quad (6)$$

- Arbitrageurs are instantaneous mean-variance optimizers:

$$\max_{X(t)} \mathrm{E}[dW(t)] - \frac{a}{2}V[dW(t)]$$

So first order condition is

$$X(t) = \frac{\gamma(t)}{a\,\sigma_P(t,T)}$$

- Now, in equilibrium net demand is zero. So if $Y(t, T)$ is dollar demand for bond of maturity T by Habitat-agent we get:

$$X(t) = -Y(t, T)$$

- Using first order condition of arbitrageurs

$$\frac{\gamma(t)}{a\,\sigma_P(t, T)} = -Y(t, T)$$

- So, in particular, if we assume that demand of Habitat-agents is linear in short rate

$$Y(t, T) = -\frac{\beta(t) + \beta_r r(t)}{\sigma_P(t, T)}$$

  we get Duffee's essentially affine model in equilibrium, with mpr linear in short rate:

$$\gamma(t) = a\beta(t) + a\beta_r r(t)$$

  - Note: Can have stochastic demand shocks $\beta(t)$ which leads to a two-factor model of the term structure where one factor is a pure risk-premium factor (it only affects bond prices and not the dynamics of the short rate - Duffee (2002)).

Intuition

- To solve the predictability puzzles we need Sharpe ratio to go up when rates go down (and slope increases), i.e.,

$$\beta_r > 0$$

$\Rightarrow$ Demand of Habitat-agents must be increasing in the short rate (remember $\sigma_P(t, T) < 0!$).

- Intuition:
  - For Risk-averse arbitrageurs the risk-free benchmark is the short rate (their 'habitat').
  - When rates decrease, Habitat-agent demand decreases ($Y \downarrow$) and since arbitrageurs must take the other side ($X \uparrow$).
  - In equilibrium, Sharpe ratio on long-term bond has to increase to incite arbitrageurs to hold larger amount of risky bonds.
    (Arbitrageurs are instantaneous mean-variance optimizers so they only care about Sharpe ratio).

Comments

- In effect, the paper provides one possible justification for the essentially affine model of Duffee (2002).
- There are others, e.g., Lucas (1978) exchange economy with log C_t utility agent and aggregate consumption given by:

$$\frac{dC_t}{C_t} = (r_t + \gamma(r_t)^2)dt + \gamma(r_t)dz_t$$

- In particular, all the implications for prices are identical to those of Duffee:
  - predictability
  - forward vs. expected future spot
  - PCA
- How can we test this model's specific implications?
  - $\Rightarrow$ Can we identify the Habitat-agents and tie variation in risk-premia to their demand for long-term bonds.
- Who are the Habitat agents?
  - Pension funds in the UK during and post pension reform?
  - Wouldn't their demand for long term bonds go up when rates decrease (as the PV of liabilities increase)?

Comments about the model

- Arbitrageurs are also Habitat agents with a Habitat equal to instantaneous maturity.
- Would be interesting to model Habitat from Micro-economic foundations rather than ad-hoc demand function (e.g., Ingersoll (1987)).
- Problem with perfect market, continuous time set-up with finite factors is that portfolio problem is ill-defined. In a one factor model, only one bond (in addition of the risk-free rate) is needed to replicate any other bond perfectly. So notion of Habitat seems a bit awkward.
- Interesting to relax 'perfect-market' assumptions (restrictions to trading etc...) or finite factor assumption such as in a string or Brownian field setup (e.g., PCD and Goldstein (2004)).

Conclusion

- Interesting idea to propose microeconomic foundations for essentially affine term structure models.
- Can we go further in modeling microeconomic foundations of these Habitat agents?
- Can we identify their asset demand empirically to test the model's new predictions?
- Is habitat the most likely explanation for predictability in long-term bond returns?
- Can it explain similar phenomenon (high yield predicting high future returns) we see in equity and currency markets?