Latent liquidity in limit order driven markets

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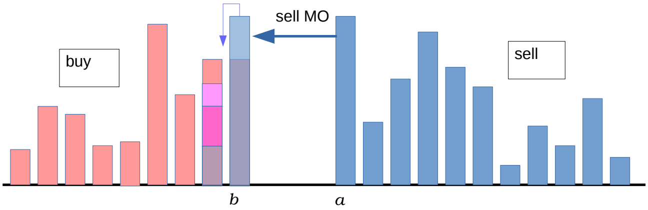
Limit order books

More than half of world's exchanges now operate LOBs as the market clearing mechanism :



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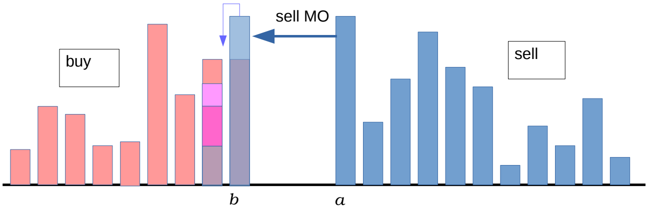

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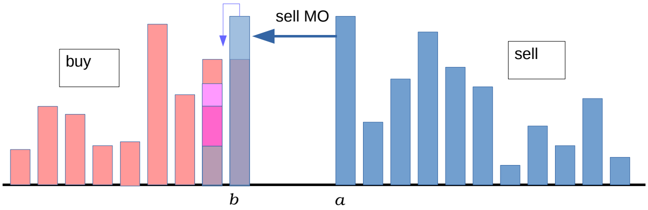


- Other market clearing mechanisms are imaginable (for instance : Centralized market maker).
- Are markets neutral as to the way market clearing is implemented ?
- I will argue here that they are not. LOBs create specific inventives for market participants, in particular with respect to the liquidity provision.

What is liquidity ?

Recall : Liquidity in LOBs

Liquidity taking = submission of market orders
Liquidity provision = submission of limit orders

Three criteria for a liquid market :

Tightness – Small bid-ask spread, i.e. market orders incur small instantaneous costs (instant liquidity taking)

Depth – Available volume at the best quotes are large.

Latency – A sequence of market orders can be executed within a short time horizon, i.e. executed volume is quickly refilled by liquidity providers.

What is liquidity II

Are "liquid" securities liquid ?

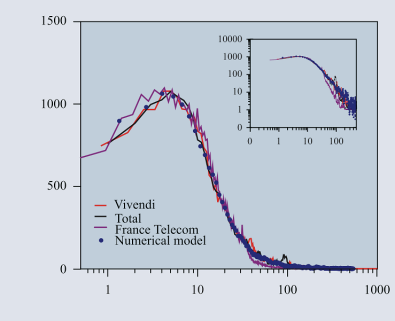
Daily traded volume of Apple Inc. ≈ 6.000.000.000$.

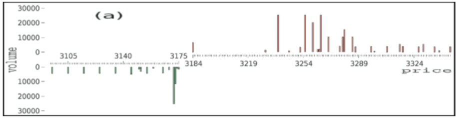
Market capitalization of Apple Inc. ≈ 600.000.000.000$ ≈ $100 \times V_D$. Imagine a large investment wishes to acquire 1% of Apple ; it takes *certainly several days* to execute this large order !

Therefore "liquid" stocks are in fact not that liquid at all. High-volume traders need to split a large order into small child orders and execute them incrementally.

This is a very widespread behaviour (optimal execution problem), so why do markets not provide sufficient liquidity for these traders ?

Empirical evidence for liquidity rationing : Average and typical shapes of the LOB





Conclusion : Liquidity in LOBs bends down and LOBs are sparse (for small-tick stocks). Aggregate demand and supply curves should increase ! [1]

1. J-P Bouchaud, J.D. Farmer et F Lillo. *How markets slowly digest changes in supply and demand.* http://arxiv.org/abs/0809.0822. 2008.

Empirical evidence for liquidity rationing II : Selective liquididy taking

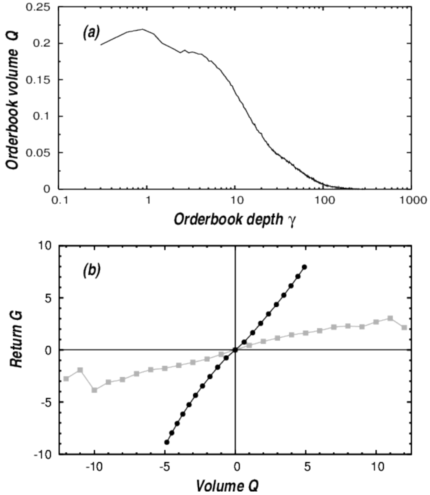
The probability that the volume of a market order exceeds the volume at the best quote is very small. A large fraction of market orders exactly match the available volume at the best quote thereby executing the whole limit order queue.

Stock	$\mathbb{P}_{>}$	$\mathbb{P}_{=}$	tick
APP	7.5%	48.7%	small
AMZ	6.6%	53.0%	small
GOO	14.4%	25.4%	small
MST	0.9%	16.3%	large
CIS	0.19%	7.8%	large
ORA	0.12%	28.3%	large

Conclusion : Liquidity takers condition the size of their order on the available volume. The large values of $\mathbb{P}_{=}$ suggest that traders frequently wish to execute more volume.

Empirical evidence for liquidity rationing III : Virtual/true market depth

Method of Rosenow&Weber :



Average shape of the order book.

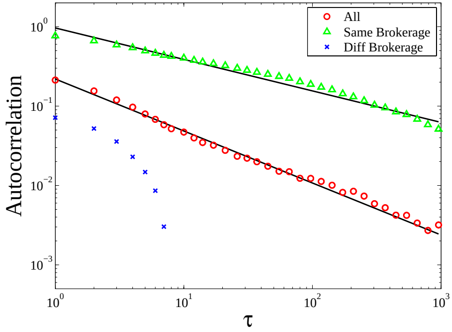
Virtual market depth = (inverse) impact if only the initial volume in the LOB would be executed.

True market depth = (inverse) measured impact

Empirical evidence for liquidity rationing IV : Order splitting

High-volume traders need to split large orders into small child orders to execute them incrementally (over hours/days). This creates a correlated market order flow.

$$\epsilon_t = +1 \text{ if MO at } t \text{ was a buy} \,;\; \epsilon_t = -1 \text{ if MO was a sell.}$$



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The order flow is highly predictable :

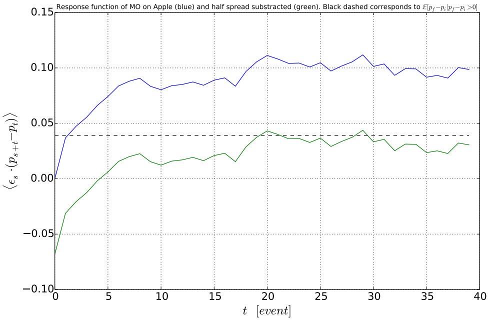
$$\mathbb{P}[\epsilon_{t'} = +1 | \epsilon_t = +1] > \frac{1}{2} \forall t' > t \; .$$

Order splitting and the bid-ask spread

Observed average Price impact (API) of a single market order :

$$\mathbb{P}[\epsilon_{t'} = +1 | \epsilon_t = +1] > \frac{1}{2} \forall t' > t \ .$$

Therefore, the measured API of a market order includes the *future* imbalance of the order flow :



API of market orders increases until reaching a plateau value $\in [s/2, s]$.
Price impact of one-shot market orders that penetrate the book is the same.

Reasons for liquidity rationing in LOBs

- A human marketplace is often conceptualized with the Walrasian paradigm : Repeated auctions lead to market clearing at the intersection between the supply and demand functions.
- LOBs implement an asymmetry between liquidity providers and takers : A transaction can only take place after the prior submission of a limit order, which is a commitment to trade without the guarantee of execution.
- Liquidity providers therefore face information leakage costs and adverse selection risks.
- Conclusion : The precise market clearing mechanism has a great effect on the behaviour of market participants. The LOB creates an incentive not to submit limit orders !

Liquidity rationing in LOBs

Market participants conceal their trading intentions. Therefore, there is a large pool of “latent liquidity” behind the visible submitted liquidity. This latent liquidity is gradually revealed and financial markets never truely clear.

The micro-macro model of financial markets

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- News feeds contain a few messages per day per stock. Also, hedging of portfolios occurs (max) twice a day. These are the fundamental time scales that change trading intentions of investors.
- Separation of time scales (high-frequency versus fundamental news) and separation of liquidity (submitted limit orders versus latent liquidity)

Definition of latent liquidity

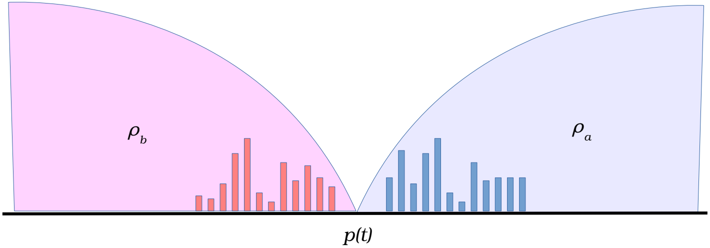
- If $p(t)$ decreases sufficiently, investor k starts buying (the opposite is true when the price increases) : Thresholds $x_{S/B}^{(k)}$.
- The latent order book collects the expected intended trading volumes $\mathbb{E}[Q_{S/B}^{(k)}|\mathcal{F}_t^k]$ at time t of all investors, once a trading intention is triggered.
- Consider now functions $\mathcal{L}_S$ and $\mathcal{L}_B$, the *latent buy and sell order books*, defined on a discrete grid $\tau\mathbb{Z}$ (with τ the tick size) and image in the positive real values, defined as

$$\mathcal{L}_B, \mathcal{L}_S : \tau\mathbb{Z} \times \mathbb{R}^+ \mapsto \mathbb{R}^+$$

$$\mathcal{L}_B(x,t) = \sum_{k \,:\, x_B^{(k)}(t)=x} \mathbb{E}[Q_B^{(k)}(t)|\mathcal{F}_t^k] \,,$$

$$\mathcal{L}_S(x,t) = \sum_{k \,:\, x_S^{(k)}(t)=x} \mathbb{E}[Q_S^{(k)}(t)|\mathcal{F}_t^k] \,.$$

“Dormant” versus “outstanding” latent liquidity



When $p(t)$ moves latent liqudity becomes “outstanding” and transforms into a trading intention.

Exogeneous sell order flow : Dynamics of outstanding latent liquidity

Outstanding latent buy liquidity :

$$n^B(p_t) = \int_{p_t}^{\infty} \mathrm{d}x \rho_b(x)$$

Clearing of outstanding latent liquidity :

- Executed limit orders are refilled.
- A fraction $\lambda_{\mathcal{L}}$ of outstanding latent liquidity is submitted via market orders (per unit time).
- All orders have fixed size ω.

Linear and constant latent liquidity (no exogeneous news) :

$$\rho_b(x) \approx L(p(0) - x)$$

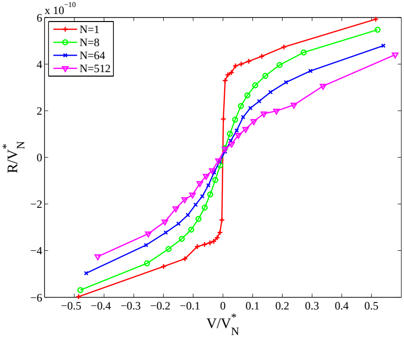
Conservation equation for outstanding buy liquidity :

$$\dot{n}^B(t) = L\dot{p}(t)p(t) - \lambda_{\mathcal{L}} n^B(t) - \lambda_S(t) \, .$$

The micro-macro model of financial markets. Price impact of an exogeneous sell order flow

Linear price impact ρ of the order flow which comprises of an **exogeneous** and an **endogeneous** component :

$$\dot{p}(t) = \underbrace{-\rho\lambda_S}_{\text{exogeneous}} + \underbrace{\rho\lambda_{\mathcal{L}} n^B(t)}_{\text{endogeneous}} \,.$$



Impact formula

We find an inhomogeneous Riccati equation for the average impacted price :

$$\dot{p}(t) + \omega\lambda_{\mathcal{L}} p(t) - \frac{\rho L \lambda_{\mathcal{L}}}{2} p(t)^2 = 2\rho\lambda_{\mathcal{L}} Q_S(t) - \rho\lambda_S(t) ,$$

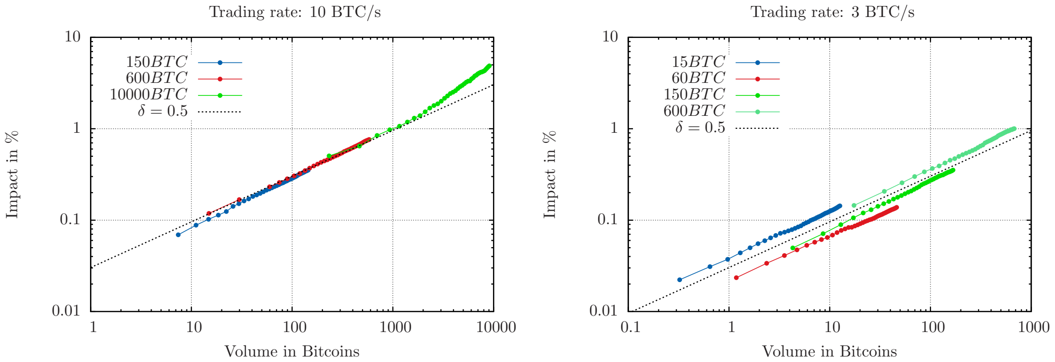
with $Q_S(t) = -\omega \int_0^t \mathrm{d}t' \lambda_S(t')$ the aggregate traded volume. When markets clear efficiently, i.e. $\lambda_{\mathcal{L}} \to \infty$, we obtain

$$p(t) \simeq \frac{\omega}{\rho L} - \sqrt{\frac{\omega^2}{\rho L^2} - \frac{4}{L} Q_S(t)} , \quad Q_S(t) < 0 .$$

For large Q_S we find square-root impact.

Empirical square-root law

Impact of large sequential orders on the Bitcoin/USD exchange market : [2]



2. J. Donier et J. Bonart. *A million metaorder analysis of market impact on the BitCoin*. http://papers.ssrn.com/sol3/Papers.cfm?abstract_id=2536001. 2014.

Non-Markovian optimal execution

Linearized price impact equation for $\lambda_{\mathcal{L}}$ large :

$$p(t) \simeq \frac{\omega}{L} - \sqrt{q_t} + \frac{1}{2\rho L \lambda_{\mathcal{L}}} \frac{\dot{q}_t}{q_t} - \frac{1}{4\rho\omega\lambda_{\mathcal{L}}} \frac{\dot{q}_t}{\sqrt{q_t}} \ ,$$

$$q_t = \frac{\omega^2}{\rho^2 L^2} - \frac{4 Q_S(t)}{L} \ .$$

Optimal execution problem :

$$\arg\max_{\lambda_S} \left[-\omega \int_0^T \mathrm{d}t' \ \lambda_S(t) p(t) \right] \ , \ \text{under the constraint}$$

$$Q = \omega \int_0^T \mathrm{d}t' \ \lambda_S(t) \ .$$

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$$Q = \omega \int_0^T \mathrm{d}t' \; \lambda_S(t) \; .$$

We find the Euler-Lagrange equations :

$$\frac{1}{L}\ddot{q}_t q_t - \frac{1}{2L}\dot{q}_t^2 - \frac{1}{2\omega}\ddot{q}_t q_t^{3/2} + \frac{1}{8\omega}\dot{q}_t^2\sqrt{q_t} = 0 \;,$$

with the boundary condition

$$q_0 = \frac{\omega^2}{\rho^2 L^2} \;, \quad q_T = \frac{4 Q_T}{L} + \frac{\omega^2}{\rho^2 L^2} \; .$$

Conclusion

- LOBs implement an asymmetry between liquidity providers and takers. This leads to liquidity rationing.
- We have developed a macroscopic approach to market clearing which takes into account the strategic behaviour of investors who are interested in trading (order splitting).
- An external order flow generates an endogeneous order flow in the opposite direction.
- The resulting price impact is non-linear and non-Markovian.