Stationarity and risk premia in power markets

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Overview

1. Motivation and background for the talk
2. A measure change/ pricing measure in power markets
3. Forward pricing and analysis of the risk premium
4. Some notes on co-integration

Motivation and background
– Two stylized cases from power markets –

Case 1: Stationarity

- Empirical analysis shows stationary spot price dynamics in the EEX power market
  - *De-seasonalized* spot prices!
  - Barndorff-Nielsen, B., Veraart 2013
- In accordance with Paul Samuelson's view on stationarity of commodity prices.

Is the price of wheat or bread a mere random Walk? If we wait long enough, should we feel no surprise if a pound of bread sells for a penny or a billion dollars? For a trillion Cadillacs or 0.01 of a Cadillac?

- A possible stationary dynamics for power:

$$S(t) = \Lambda(t) + X(t) + Y(t)$$

- $\Lambda(t)$ deterministic seasonality function, X long-term variations, Y short-term (spike) factor

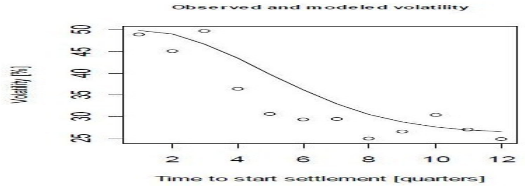
$$dX(t) = (\mu_X - \alpha_X X(t))\,dt + \sigma_X\,dB(t)$$
$$dY(t) = (\mu_Y - \alpha_Y Y(t))\,dt + dL(t)$$

- L a Lévy (jump) process, B a Brownian motion

- Problem: Forward prices become constants in the long end of the market
  - Pricing measure Q changes the mean-reversion level μ in the standard approach
  - Stationary spot also under the pricing measure Q

$$F(t,T) \sim \text{const.}, \; T >> t$$

- Not the case empirically (Andresen et al. 2010, NordPool forward prices)



- Popular power spot price model: Let long-term factor be non-stationary
  - two-factor dynamics of Lucia-Schwartz (2002)

$$dX(t) = \mu_X \, dt + \sigma_X \, dB(t)$$

- Forward is proportional to the long-term factor in the long end of the market

$$F(t, T) \sim X(t) \; T >> t$$

- ...but the spot is stationary?

Case 2: The risk premium in power markets

- Risk premium: difference between observed forward prices and predicted spot prices
    - Predicted at time of delivery

$$\mathrm{RP}(t, T) = F(t, T) - \mathbb{E}[S(T) \,|\, \mathcal{F}_t]$$

- Negative premium when far from maturity
    - Producers are hedging, accepting a price discount
- Possibly positive premium in the short end
    - Consumers are hedging the spike risk
- Empirical evidence for this in the NordPool and EEX
    - With emerging importance of wind and solar, short term premium negative in EEX, nowadays

Agenda

- Question 1 How to introduce a pricing measure Q allowing for stationary P-dynamics of the spot and non-constant forward prices in the long end?
- Question 2 How to introduce a pricing measure Q allowing for a stochastic risk premium, with a possibility for a sign change?

- We present an "all in one" pricing measure Q
  - Theory for this in B. and Ortiz-Latorre (2013)
  - Empirics in B., Cartea and Pedraz (2013)

A pricing measure Q

Brownian case

- Consider

$$dX(t) = (\mu_X - \alpha_X X(t))\, dt + \sigma_X\, dB(t)$$

- Define, with: $\theta_X \in \mathbb{R}, \beta_X \in [0,1]$

$$d\widetilde{B} = \boxed{-\frac{\theta_X + \alpha_X \beta_X X(t)}{\sigma_X}}\, dt + dB(t)$$

- Market price of risk depending stochastically on X
- By Girsanov's theorem, $\widetilde{B}(t)$ is a Brownian motion with respect to a $Q_X \sim P$
  - Note: Novikov's condition holds only up to some finite time
  - Must use "uniform pasting" of Novikov criteria to show measure change for arbitrary time!

- Q_X-dynamics of X

$$dX(t) = \left(\boxed{\mu_X + \theta_X} - \boxed{\alpha_X(1-\beta_X)} X(t) \right) dt + \sigma_X \, d\widetilde{B}(t)$$

- θ_X changes the *level* of mean reversion
  - The usual *market price of risk* chosen
- $\beta_X \in (0,1)$ yields a slow-down of the mean-reversion speed
  - $\beta_X = 1$: goes from stationary (P) to non-stationary (Q_X) dynamics!

Lévy case

- Consider

$$dY(t) = (\mu_Y - \alpha_Y Y(t))\,dt + dL(t)$$

- Assume L has only *positive* jumps (and no drift)
  - A so-called subordinator
- For $\theta_Y \in \mathbb{R}, \beta_Y \in [0,1]$, define

$$H(t,z) = e^{\theta_Y z}\left(1 + \frac{\alpha_Y \beta_Y}{\kappa_L''(\theta_Y)} z Y(t-)\right)$$

- $\kappa_L(\theta)$ is the cumulant of L
  - The log-moment generating function of $L(1)$
  - Exponential integrability condition assumed on L

- $Q_Y \sim P$ have Radon-Nikodym density

$$\left.\frac{dQ_Y}{dP}\right|_{\mathcal{F}_t} = \mathcal{E}\left(\int_0^{\cdot}\int_0^{\infty}(H(s,z)-1)\widetilde{N}(ds,dz)\right)(t)$$

- Here, $\widetilde{N}$ the compensated Poisson random measure of L
- The compensator measure (jump measure) of L is

$$\ell_Q(dt,dz) = H(t,z)\ell(dz)\,dt$$

- Here, $\ell(dz)$ is the Lévy measure of L (wrt P)
  - Note that ℓ_Q becomes stochastically dependent on Y
  - Jump size and intensity scaled by the state of Y
  - L looses its Lévy property under Q_Y, but remains a semimartingale

- Note that $\beta_Y = 0$ gives

$$\frac{dQ_Y}{dP}\Big|_{\mathcal{F}_t} = \mathcal{E}(\theta_Y L(t)) = \exp\left(\theta_Y L(t) - \kappa_L(\theta_Y)t\right)$$

- Esscher transform, with parameter θ_Y
  - Preserves the Lévy property of L under Q_Y
- Frequently used measure change for commodity models with jumps
- For $\beta_Y \in (0,1)$ we slow down the speed of mean-reversion
  - $\beta_Y = 1$ is it killed completely, to give a non-stationary dynamics

- Q_Y-dynamics of Y

$$dY(t) = \left(\boxed{\mu_Y + \kappa_L'(\theta_Y)} - \boxed{\alpha_Y(1-\beta_Y)} Y(t) \right) dt + d\widetilde{L}_Q(t)$$

- $\widetilde{L}_Q(t)$ Q_Y-martingale
  - It is L subtracted its Q_Y-mean value
- Let Q be the product measure of Q_X and Q_Y
- Proof of measure change from P to Q is the mathematical core of the paper
  - Must show that density process is a martingale, and not only a *local* martingale
  - Jump component is the challenging part

Forward pricing and risk premium

- Forward price, fixed time of maturity $T \geq t$

$$F(t,T) = \mathbb{E}_Q[S(T)\,|\,\mathcal{F}_t]$$

- Analytical price:

$$F(t,T) = \Lambda(T) + H_Q(T-t) + X(t)\mathrm{e}^{-\alpha_X(1-\beta_X)(T-t)} + Y(t)\mathrm{e}^{-\alpha_Y(1-\beta_Y)(T-t)}$$

- Here, $H_Q(x)$ deterministic function of the parameters of X, Y, and Q

$$H_Q(x) = \frac{\mu_X + \theta_X}{\alpha_X(1-\beta_X)}(1-\mathrm{e}^{-\alpha_X(1-\beta_X)x}) + \frac{\mu_Y + \kappa_L'(\theta_Y)}{\alpha_Y(1-\beta_Y)}(1-\mathrm{e}^{-\alpha_Y(1-\beta_Y)x})$$

Question 1: Stationary spot under P vs. non-constant forward prices....

- If $\beta_X, \beta_Y < 1$,

$$F(t,T) \sim \Lambda(T) + \text{const.}\,,\; T >> t$$

- If $\beta_X = 1$ (with $\beta_Y < 1$),

$$F(t,T) \sim \Lambda(T) + \text{const.} + X(t)\,,\; T >> t$$

- Forward price depends on X, the long-term factor
  - Exactly as in the Lucia-Schwartz model...
  - .. but now the spot is stationary under P, and non-stationary under Q!

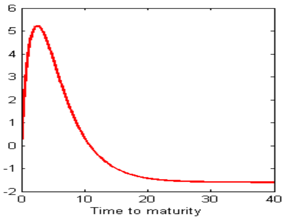
Question 2: Stochastic risk premium with sign change...

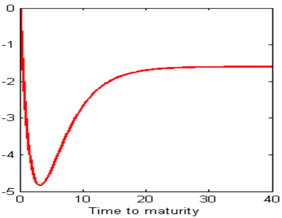
- Risk premium is

$$\begin{aligned} \mathrm{RP}(t,T) = H_Q(T-t) - H_P(T-t) \\ + X(t)\mathrm{e}^{-\alpha_X(T-t)}(\mathrm{e}^{\alpha_X\beta_X(T-t)} - 1) \\ + Y(t)\mathrm{e}^{-\alpha_Y(T-t)}(\mathrm{e}^{\alpha_Y\beta_Y(T-t)} - 1) \end{aligned}$$

- For $T-t$ small, X and Y will be influential in the risk premium
  - Y is positive, as the jumps are positive
  - Hence, may lead to a positive risk premium if Y is sufficiently big (e.g. a spike)
- If $\beta_X, \beta_Y < 1$, $\mathrm{RP}(t,T) \sim \text{const.}$, $T >> t$
  - 'const.' can be either negative or positive depending on parameter choices of Q

- Risk premium term structure for a positive spike (left)
  - Arbitrary, but illustrative parameter values
  - Positive short term premium from retailer's hedging
- To mimic inflow of wind power, use factor $-Y$
  - Negative spikes (and even possibly negative prices)
  - Gives a negative contribution to the risk premium in the short end (right)
  - Intensified hedging from coal/gas producers on the short term





Emphasis of risk in measure change

- Note: P-dynamics of X has stationary (limiting) distribution

$$X(t) \sim \mathcal{N}\left(\frac{\mu_X}{\alpha_X}, \frac{\sigma_X^2}{2\alpha_X}\right)$$

- Under Q, the stationary distribution becomes (when $\beta_X < 1$)

$$X(t) \sim \mathcal{N}\left(\frac{\mu_X + \theta_X}{\alpha_X(1-\beta_X)}, \frac{\sigma_X^2}{2\alpha_X(1-\beta_X)}\right)$$

- The volatility of X becomes bigger under Q!
  - We emphasise the variations of X *more* under Q than under P
  - For Y we let "spikes last longer" under Q

Some words on co-integration

- Co-integrated spot price model (log-prices, under P!)

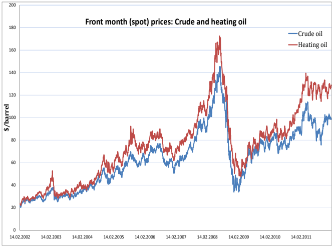
$$\ln S_i(t) = X(t) + Y_i(t)\,, i = 1, 2$$

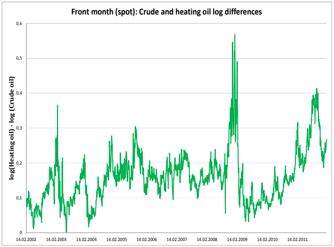
$$dX(t) = \mu_X\,dt + \sigma_X\,dB(t)$$
$$dY_i(t) = (\mu_i - \alpha_i Y_i(t))\,dt + \sigma_i\,dW_i(t)\,, i = 1, 2$$

- B, and W_i correlated Brownian motions
  - Short-term stationary, long-term non-stationary
  - Classical commodity spot price model (Lucia & Schwartz 2002) (!)
- Stationary difference

$$\ln S_1(t) - \ln S_2(t) = Y_1(t) - Y_2(t)$$

- Example: Crude oil and heating oil at NYMEX
  - Both series look non-stationary
  - and highly dependent





Risk-neutral dynamics

- In *perfect* spot markets, S_i will have r as drift under Q
  - Q is an equivalent martingale measure
  - Co-integration is removed under Q
  - S_i a bivariate geometric Brownian motion
  - Duan & Pliska (2004)
- In commodities, many frictions....
  - Storage, transportation, no-storage, convenience yield
- Apply our measure change on the Y_i-factors
  - Change speeds of mean reversion α_i
  - As well as level c_i
  - ...and drift μ of X
- Hence, Q dynamics of S_i remains co-integrated under Q

- Forward price $F_i(t,T)$ at time $t \leq T$ for a contract delivering S_i at time T

$$F_i(t,T) = H_i(T-t)\exp\left(X(t) + e^{-\alpha_i(1-\beta_i)(T-t)}Y_i(t)\right), i = 1,2$$

- H_i known deterministic functions
  - Given by the parameters of the spot
- Remark: F_1 and F_2 are co-integrated in the Musiela parametrization $x = T - t$

$$\ln F_1(t,t+x) - \ln F_2(t,t+x) = \ln H_1(x) - \ln H_2(x) + e^{-\alpha_1(1-\beta_1)x}Y_1(t) - e^{-\alpha_2(1-\beta_2)x}Y_2(t)$$

- Forward price dynamics

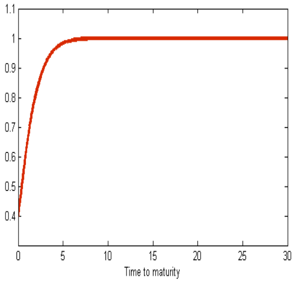
$$\frac{dF_i(t,T)}{F_i(t,T)} = \sigma_X \, d\widetilde{B}(t) + \sigma_i e^{-\alpha_i(1-\beta_i)(T-t)} \, d\widetilde{W}_i(t) \, , i = 1, 2$$

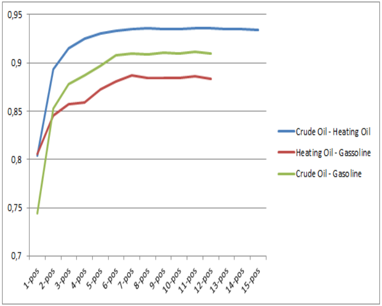
- In the long end of the market forward prices are perfectly correlated

$$\frac{dF_i(t,T)}{F_i(t,T)} \sim \sigma \, dB(t) \, , i = 1, 2$$

- For general delivery times T
  - Analytical expressions for the vol and correlation term structure
  - For logreturns of F_1 and F_2

- Empirical example:
  - Theoretical correlation term structure (left) vs. NYMEX empirical forward price correlations (right)
  - Arbitrary but reasonable parameters for the theoretical curve
  - 3 years of daily data up to Feb 1, 2012 from NYMEX





- "Margrabe-Black-76" formula for spread option on $F_1(t,T)$ and $F_2(t,T)$ with exercise time $\tau \leq T$

$$C(t,\tau,T) = F_1(t,T)\Phi(d_1) - F_2(t,T)\Phi(d_2)$$

where,

$$d_1 = d_2 + \sqrt{\int_t^\tau g_\rho^2(T-s)\,ds}\,,\; d_2 = \frac{\ln F_1(t,T) - \ln F_2(t,T) - \frac{1}{2}\int_t^\tau g_\rho^2(T-s)\,ds}{\sqrt{\int_t^\tau g_\rho^2(T-s)\,ds}}$$

$$g_\rho^2(x) = \sigma_1^2 e^{-2\alpha_1(1-\beta_1)x} - 2\rho\sigma_1\sigma_2 e^{-(\alpha_1(1-\beta_1)+\alpha_2(1-\beta_2))x} + \sigma_2^2 e^{-2\alpha_2(1-\beta_2)x}$$

- No dependence on long-term volatility σ_X!
  - But dependence on speed of mean-reversion...

Concluding remarks

- Proposed a measure change that slows down speed of mean reversion in Ornstein-Uhlenbeck models
- Provides a theoretical foundation for
  - Stationary spot prices, but non-constant forward prices in the long end of the curve
  - Stochastic risk premium, with possibly positive premium in the short end of the curve
- Focused on artihmetic Ornstein-Uhlenbeck models
  - In paper: geoemtric case analysed as well
  - Parameters in forward price given by Volterra equations due to affine structure
- Applied the measure change to co-integration
  - Co-integration is preserved under the pricing measure

Thank you for your attention!

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