Taming the Leverage Cycle

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An anecdote about a leverage cycle

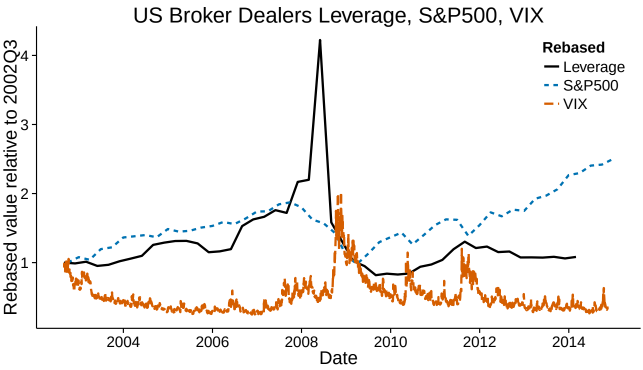

Figure: Leverage of US Broker-Dealers (solid black line), S&P500 index (dashed blue line), VIX S&P500 (red dash-dotted line.

Intuition for leverage cycle

Basic idea:

In good times, when perceived risk is low, leverage goes up. In bad times, when risk is high, leverage goes down. Prices respond to leverage and risk responds to prices.

$$
\begin{aligned}
\text{Leverage} &= F\left(\text{Perceived risk}\right), \\
\text{Prices} &= G\left(\text{Leverage}\right), \\
\text{Perceived risk} &= H\left(\text{Prices}\right).
\end{aligned}
$$

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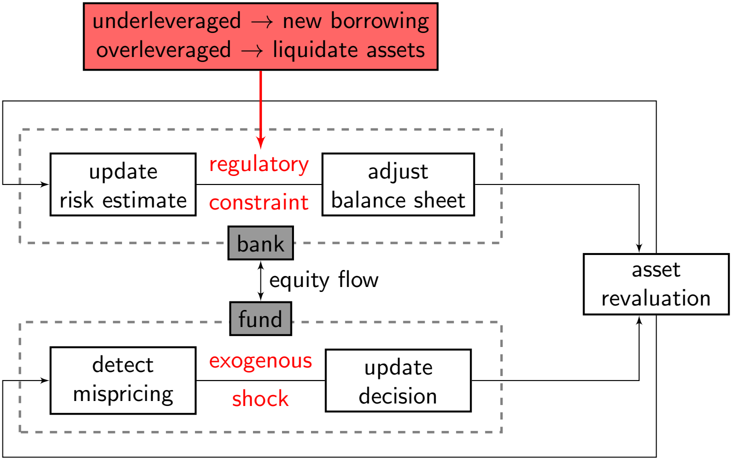
What do we need to model this?

- Leveraged investor (bank).
- An unleveraged fundamentalist investor.
- A risky and a risk free asset.

Stochastic discrete time model of leverage cycles



Outline for the remainder of this talk

1. A model of a leveraged bank and a fund investor.

2. Emergence of endogenous risk $\rightarrow$ leverage cycles.

3. Optimal leverage policy in the presence of both exogenous and endogenous risk.

Bank leverage regulation

Motivation: VaR constraint with normal returns

$$\lambda(t) \leq \bar{\lambda}(t) = F_{\mathsf{VaR}}(\sigma^2(t)) = \frac{1}{\sigma(t)\Phi^{-1}(a)} \propto \frac{1}{\sigma(t)}.$$

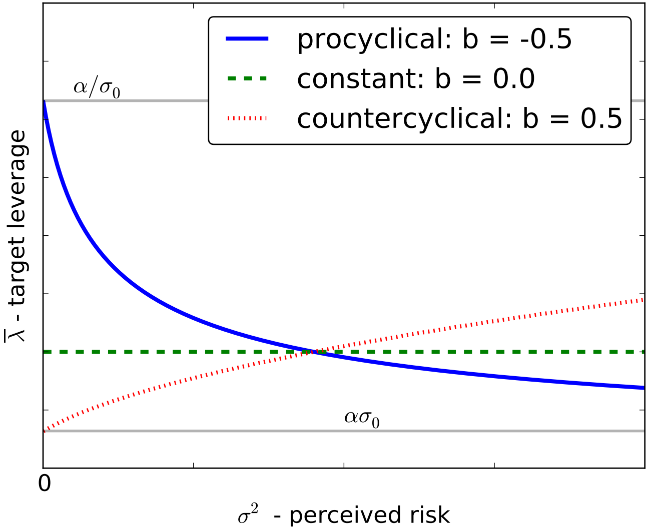
Our model: 3 parameter leverage constraint

$$\lambda(t) \leq \bar{\lambda}(t) = F_{(\alpha,\sigma_0^2,b)}(\sigma(t)) := \alpha(\sigma^2(t) + \sigma_0^2)^b.$$

Note:

- Due to profit maximization: $\lambda(t) \approx \bar{\lambda}(t) :=$ target leverage,
- α: bank risk level (leverage at a given level of risk),
- $b < 0$: procyclical w.r.t $\sigma(t)$,
- $b > 0$: countercyclical w.r.t $\sigma(t)$,
- σ_0: lower/upper bound on leverage.

Cyclicality parameter b: procyclical vs. countercyclical policies



Risk estimation and portfolio adjustment

Historical estimation of volatility

Let $p(t)$ be the price of the risky asset at time t. Then the bank's **perceived risk** evolves as

$$\sigma^2(t+\tau) = (1-\tau\delta)\sigma^2(t) + \tau\delta\left(\log\left[\frac{p(t)}{p(t-\tau)}\right]\frac{t_{\mathrm{VaR}}}{\tau}\right)^2.$$

Balance sheet

Adjust size of balance sheet to meet target leverage:

$$\Delta B(t) = \tau\theta\{\overline{\lambda}(t)(A_{\mathrm{B}}(t) - L_{\mathrm{B}}(t)) - A_{\mathrm{B}}(t)\}.$$

Adjust equity to meet equity target:

$$\kappa_{\mathrm{B}}(t) = \tau\eta\{\overline{E} - (A_{\mathrm{B}}(t) - L_{\mathrm{B}}(t))\}$$

The fund stabilizes the price dynamics of the risky asset

Fund characteristics:

- Not leveraged.
- Fund has a notion of a fundamental value μ of the risky asset.
- Dynamics of portfolio weight for risky asset:

$$\Delta w_F(t+\tau) \propto \rho(\mu - p(t)) + \sqrt{\tau} s(t) \xi(t),$$

where $\xi(t) \sim \mathcal{N}(0,1)$ and $s(t)$ follow GARCH(1,1).

Note:

- Fund stabilizes prices (buys if price below fundamental, sells above).
- For $s = 0$ we obtain deterministic system.
- Fund is source of "clustered" exogenous volatility.

Market mechanism for risky asset

1. Bank and fund demand function:

$$D_{\mathrm{B}}(t+\tau) = \frac{1}{p(t+\tau)} w_{\mathrm{B}}(n(t)p(t+\tau) + c_{\mathrm{B}}(t) + \Delta B(t)),$$
$$D_{\mathrm{F}}(t+\tau) = \frac{1}{p(t+\tau)} w_{\mathrm{F}}(t+\tau)((1-n(t))p(t+\tau) + c_{\mathrm{F}}(t)).$$

2. Compute $p(t+\tau)$ by market clearing:

$$1 = D_{\mathrm{B}}(t+\tau) + D_{\mathrm{F}}(t+\tau)$$

3. Compute new ownership of risky asset for bank $n(t+\tau)$ and fund $1-n(t+\tau)$

We can collect full model in 6D map

Map:

$$x(t+\tau) = g(x(t))$$

State vector:

$$x(t) = [p(t), \sigma^2(t), n(t), L_{\mathrm{B}}(t), w_F(t), p'(t)]^T,$$

where:

- p: Price of risky asset.
- σ^2: Perceived risk.
- n: Amount of asset owned by bank.
- L_{B}: Liabilities of bank.
- w_F: Investment into risky asset by fund.
- p': Past price of risky asset.

Guiding principles for choice of main parameters

1. Properties of the leverage cycle:
   - Peak-to-trough ratio ≈ 2,
   - Period of cycles ≈ 10 years,

   determines α (bank risk level), $\bar{E}$ (bank equity target).
2. Timescale for risk estimation:
   - $t_\delta = 1/\delta \approx 2$ years (based on RiskMetrics).

Examples of leverage cycles: we consider four parameter scenarios

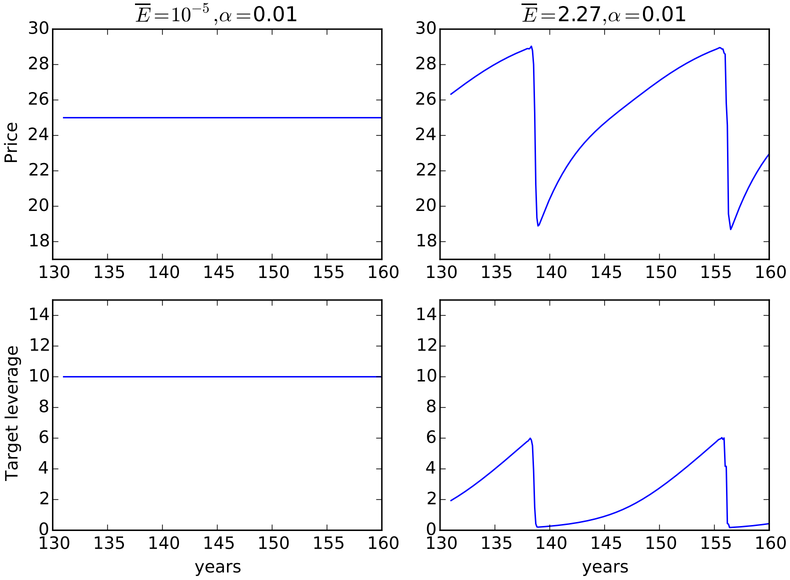
(i) Deterministic, small bank (weak endogenous risk): $\overline{E} = 10^{-5}$ and $s = 0$,

(ii) Deterministic, large bank (strong endogenous risk): $\overline{E} = 2.27$ and $s = 0$,

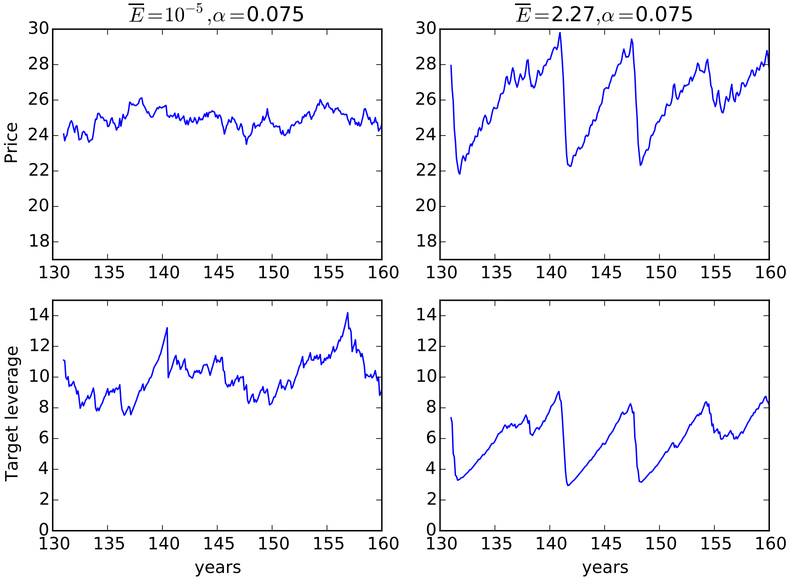
(iii) Stochastic, small bank (weak endogenous risk): $\overline{E} = 10^{-5}$ and $s > 0$.

(iv) Stochastic, large bank (strong endogenous risk): $\overline{E} = 2.27$ and $s > 0$,

Deterministic: (i) small bank vs. (ii) large bank



Stochastic: (iii) small bank vs. (iv) large bank



How do leverage cycles depend on the model parameters?

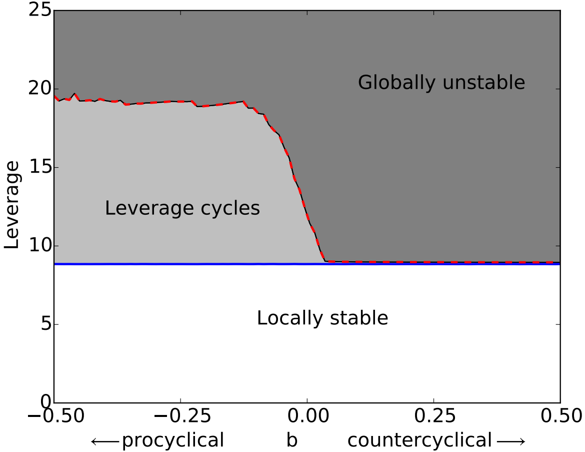


Figure: Deterministic model (eigenvalues)

How do leverage cycles depend on the model parameters?

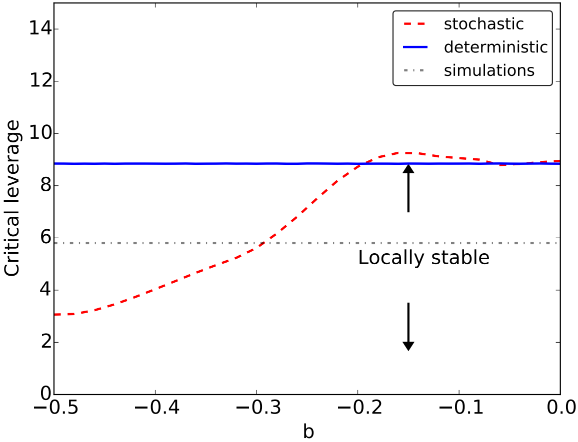


Figure: Critical leverage for emergence of leverage cycles: deterministic/stochastic (Lyapunov exponents)

Which cyclicality parameter (b) minimizes bank losses for a given leverage: intuition

Case 1: Small bank, strong exogenous volatility clustering

1. No endogenous volatility due to leverage cycles.
2. Expect Value-at-Risk policy to be optimal ($b = -0.5$).

Case 2: Large bank, weak exogenous volatility clustering

1. Strong endogenous volatility due to leverage cycles.
2. Exogenous volatility is roughly constant.
3. Expect constant leverage policy to be optimal ($b = 0$).

Which cyclicality parameter (b) minimizes bank losses for a given leverage: results

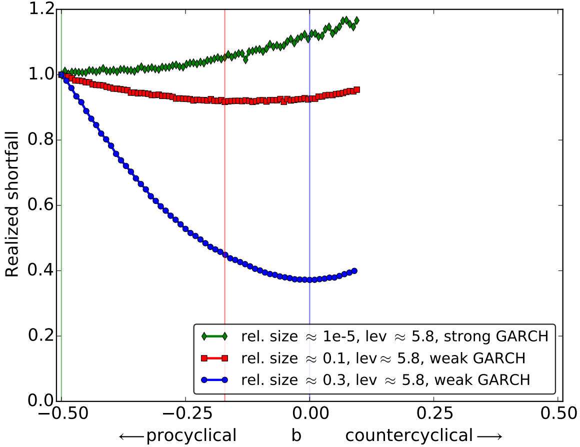


Figure: Realized shortfall at constant leverage.

Conclusions

1. Feedback between risk management, risk estimation and asset prices can lead to **endogenous volatility**.
2. Endogenous volatility increases with **bank leverage and size**.
3. **Bank losses can be reduced** by an appropriate leverage policy but choice of parameter depends crucially on levels of endogenous and exogenous volatility.

BACK UP

Full 6 D model (1/2)

Recall:

$$x(t) = [\sigma^2(t), w_F(t), p(t), n(t), L_B(t), p'(t)]^T, \tag{1}$$

Definitions:

$$\begin{aligned}
&\text{Bank assets} & A_B(t) &= p(t)n(t)/w_B, \\
&\text{Target leverage} & \bar{\lambda}(t) &= \alpha(\sigma^2(t) + \sigma_0^2)^b, \\
&\text{Balance sheet adjustment} & \Delta B(t) &= \tau\theta(\bar{\lambda}(t)(A_B(t) - L_B(t)) - A_B(t)), \\
&\text{Equity redistribution} & \kappa_B(t) &= -\kappa_F(t) = \tau\eta(\overline{E} - (A_B(t) - L_B(t))), \\
&\text{Bank cash} & c_B(t) &= (1 - w_B)n(t)p(t)/w_B + \kappa_B(t), \\
&\text{Fund cash} & c_F(t) &= (1 - w_F(t))(1 - n(t))p(t)/w_F(t) + \kappa_F(t).
\end{aligned} \tag{2}$$

Full 6 D model (2/2)

Dynamical system:

$$x(t+\tau) = g(x(t)) \tag{3}$$

where the function g is the following 6-dimensional map:

$$\sigma^2(t+\tau) = (1-\tau\delta)\sigma^2(t) + \tau\delta\left(\log\left[\frac{p(t)}{p'(t)}\right]\frac{t_{\mathrm{VaR}}}{\tau}\right)^2, \tag{4a}$$

$$w_{\mathrm{F}}(t+\tau) = w_{\mathrm{F}}(t) + \frac{w_{\mathrm{F}}(t)}{p(t)}\left[\tau\rho(\mu - p(t)) + \sqrt{\tau}s\xi(t)\right], \tag{4b}$$

$$p(t+\tau) = \frac{w_{\mathrm{B}}(c_{\mathrm{B}}(t) + \Delta B(t)) + w_{\mathrm{F}}(t+\tau)c_{\mathrm{F}}(t)}{1 - w_{\mathrm{B}}n(t) - (1-n(t))w_{\mathrm{F}}(t+\tau)}, \tag{4c}$$

$$n(t+\tau) = \frac{w_{\mathrm{B}}(n(t)p(t+\tau) + c_{\mathrm{B}}(t) + \Delta B(t))}{p(t+\tau)}, \tag{4d}$$

$$L_{\mathrm{B}}(t+\tau) = L_{\mathrm{B}}(t) + \Delta B(t), \tag{4e}$$

$$p'(t+\tau) = p(t). \tag{4f}$$

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