

# FINANCE RESEARCH SEMINAR SUPPORTED BY UNIGESTION

## "Investment Horizons and Asset Prices under Asymmetric Information"

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### Abstract

I study a fully dynamic rational expectations economy with asymmetric information, where agents have finite investment horizons;  $T$ . Horizons affects asset prices through two key mechanisms: as  $T$  increases, 1) the age-adjusted risk aversion of the average investor falls, and 2) the risk transfer from forced liquidators into voluntary buyers drops. There are typically two equilibria: a stable equilibrium in which higher  $T$  lowers price volatility, and an unstable one with the opposite properties. Moreover, equilibria that fail to exist for low  $T$  can be recovered for high enough lifespan. Along the stable equilibrium, increasing  $T$  lowers price volatility and alleviates the uncertainty of uninformed investors. The low risk environment induces aggressive trading by the informed, which impound their knowledge into prices. Expected returns and return volatility are similar to an economy with full-information about fundamentals, even if the informed are relatively few. For short horizons, cautious trading disaggregates information from prices and the economy approaches one with no private information. Consistent with evidence of increased fund liquidations during episodes of financial distress, the results suggest that heightened volatility and uncertainty can be explained by the shortening of investor horizons in a rational economy with asymmetric information.

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# Investment Horizons and Asset Prices under Asymmetric Information \*

Elias Albagli

(Preliminary, comment welcome)

September 26, 2012

## Abstract

I study a fully dynamic rational expectations economy with asymmetric information, where agents have finite investment horizons;  $T$ . Horizons affects asset prices through two key mechanisms: as  $T$  increases, 1) the age-adjusted risk aversion of the average investor falls, and 2) the risk transfer from forced liquidators into voluntary buyers drops. There are typically two equilibria: a stable equilibrium in which higher  $T$  lowers price volatility, and an unstable one with the opposite properties. Moreover, equilibria that fail to exist for low  $T$  can be recovered for high enough lifespan. Along the stable equilibrium, increasing  $T$  lowers price volatility and alleviates the uncertainty of uninformed investors. The low risk environment induces aggressive trading by the informed, which impound their knowledge into prices. Expected returns and return volatility are similar to an economy with full-information about fundamentals, even if the informed are relatively few. For short horizons, cautious trading disaggregates information from prices and the economy approaches one with no private information. Consistent with evidence of increased fund liquidations during episodes of financial distress, the results suggest that heightened volatility and uncertainty can be explained by the shortening of investor horizons in a rational economy with asymmetric information.

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# 1 Introduction

It has long been recognized that short investment horizons can be a destabilizing force in asset markets. Vast theoretical and empirical research in the limits of arbitrage underscores how sophisticated traders investing other people’s money have good reasons to be concerned about short term performance, perhaps even more so than about the long-run prospect of their investments. Although this view has come a long way in the past 20 years, much less understood is the role played by short-term trading during episodes of financial distress. In particular, what role does short-term speculation have in jointly explaining the following stylized facts observed during financial crises: i) a tightening of funding constraints of financial intermediaries in the form of capital outflows, and the consequent increase in fund liquidations; ii) sharp declines in asset prices and increases in volatility and expected returns; and iii) spikes in market- and survey-based measures of economic uncertainty?

This paper contributes to filling this gap by providing a model where investors trade an infinitely lived asset, but have (arbitrary) finite investment horizons,  $T$ . The model studies how variations in these horizons affect the main properties of asset returns in equilibrium. In other words, the exercise performed here captures fact i) above –increased fund outflows during crises– by reducing the effective investing horizons of traders, and studies how it relates to facts ii) and iii).

The model is based on the dynamic rational expectations equilibrium analysis pioneered by Wang (1994). The economy is made up of a unit measure of competitive investors who trade an infinitely-lived asset to maximize utility of lifetime consumption under CARA preferences. There are two types of investors: those who observe private information about the persistent component of the dividend process (informed investors), and those who must infer it from dividends and prices (uninformed investors). Investors also differ in another important dimension: age. At any point in time, there are  $T$  generations of investors coexisting.  $T-1$  groups of equal mass (aged 1, 2, ...,  $T-1$ ) are still active in the market and can take voluntary positions, while the oldest generation (aged  $T$ ) is exiting the economy and must unwind its positions at prevailing prices. The net supply of the asset is itself random, akin to a noise trader’s supply. This causes prices to fluctuate for reasons orthogonal to fundamentals, and prevents them from fully revealing the information observed by informed investors.

This paper makes three main contributions. First, I describe two novel mechanisms through which investment horizons matter for the properties of assets returns, as well as for the uncertainty regarding the dividend process, conditional on public information. The first mechanism relates to the pricing of risk, which I label the *age-adjusted risk aversion* effect. As investors live for more periods, they are willing to absorb the liquidations of the dying generation at lower expected returns, since they can smooth their consumption over a longer lifespan and are less exposed to temporary price deviations caused by the random asset supply. To get a better understanding, consider a hypothetical economy where all investors are infinitely lived. In this economy, the marginal propensity to consume out of wealth is the ratio  $r/R$ —a number much smaller than one.<sup>1</sup> This coefficient is precisely how agents price the uncertainty regarding wealth fluctuations: the “age-adjusted” risk aversion parameter of all agents corresponds to  $\gamma \cdot r/R$ , where  $\gamma$  is the coefficient of absolute risk aversion. Take now the other extreme case in which agents live only 2

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<sup>1</sup>This is the economy considered by Wang (1994).

periods (the same argument holds for a static, one-shot game). In this economy, the marginal propensity to consume wealth is one, and therefore the effective risk aversion coincides with the age-adjusted one. In the present paper, the effective pricing of risk now depends on the age of the investor. Importantly, as investors live for longer horizons, risk aversion falls and increases the willingness to bear risk by the average investor in the economy. Although intuitive, this mechanism has, to the best of my knowledge, not been addressed formally in previous work.

The second mechanism relates to the quantity of risk that active investors must bear in equilibrium, which I label the *risk transfer* effect. As investors live longer, the relative size of the oldest generation (forced liquidators) shrinks in relation to the overall mass of investors (voluntary traders) implied by the steady state of the economy. To understand this second mechanism, take the standard OLG economy where there are only 2 vintages of rational, risk-averse investors (i.e., young/old). This case represents a rather extreme situation in which the dying generation (in mass 1/2) must unload all its positions into a single younger generation (also in mass 1/2) which must accommodate it in equilibrium. In other words, the whole risk in the economy must exchange hands every period! As studied by Spiegel (1998) and Watanabe (2008), this situation leads to 2 equilibria that are sustained by the extrapolative nature of expectations: a stable, low volatility equilibrium where innovation in the asset supply have small price impact, and an unstable equilibrium where they cause high volatility in returns.<sup>2</sup> Take now the polar opposite: an economy with infinite horizon that has only one vintage of investors with age given by the time elapsed in the economy (as in Wang, 1994). Because in this environment investors *always* trade voluntarily, one should expect the sustainable levels of price volatility to be quite different. To the best of my knowledge, the importance of this second mechanism has not been addressed before in formal asset market models.

The second contribution of the paper is a characterization result about existence and uniqueness of linear equilibria in generalized overlapping generations models. By building a parsimonious environment able to encompass arbitrary investing horizons and heterogeneous information structures, the model is able to nest a wide variety of economies whose equilibrium properties have not been addressed in prior work. In terms of existence, the results show that for any given set of parameters describing investor preferences and stochastic processes, economies which fail to exhibit equilibria for low investment horizons will always have equilibria for large enough  $T$ . The reverse interpretation of this result is that market equilibrium can break down as investors' horizons shorten. Regarding multiplicity, a finite investment horizon economy generically exhibits the two aforementioned equilibria (whenever equilibria exists). Along the stable, low volatility equilibrium, increases in investors' horizons lowers the price impact of asset supply innovations, decreasing price volatility. As  $T \rightarrow \infty$ , the low volatility equilibrium converges smoothly to the case studied by Wang (1994). Along the unstable equilibrium however, longer investment horizons leads to unbounded increases in price volatility. In the limit, this second equilibrium vanishes as  $T \rightarrow \infty$ . Intuitively, as investors live for more periods, both the increased willingness to take risks and the larger proportion of voluntary trades with respect to forced liquidations makes the high volatility equilibrium increasingly "difficult" to sustain.

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<sup>2</sup>In the single asset case, as studied here. In the  $N$ -risky asset case, there exists  $2^N$  equilibria.

The third contribution is the characterization of asset price informativeness and uncertainty as a function of investors' horizons. In particular, I study the behavior of asset prices along the stable equilibrium for three generic economies: a full-information benchmark where all investors observe the persistent component of dividend contemporaneously; a no-information economy in which all investors observe only the history of dividends and prices; and an asymmetric-information economy where only a relatively small mass of investors has access to persistent payoff information. I then show the following results: a) For long investment horizons, the intermediate economy behaves similarly to the full information benchmark. Indeed, the low risk environment implied by large  $T$  leads informed agents to react aggressively to private information, which then gets impounded into prices. Uninformed agents therefore extract precise information from prices, which reduces the level of uncertainty of the average investor. In this economy, price movements are largely driven by fundamental volatility, closely mimicking the full-information case. b) For short investment horizons, the asymmetric information economy approaches the no-information benchmark: the high risk environment implied by small  $T$  leads informed agents to trade more cautiously, disaggregating information from prices and increasing the uncertainty of the average investor about current and future fundamental asset values. In this economy, price movements are largely driven by asset supply innovations. c) The transition between the full- and no-information economy is explained by the higher sensitivity of the asymmetric information economy to shortening horizons, relative to the symmetric information benchmarks. This extra sensitivity is explained by an uncertainty-volatility spiral: as  $T$  falls, the higher uncertainty faced by the average trader lowers her willingness to absorb risk, increasing the price response to supply shocks. This increased volatility further deters informed investors' reaction to private information, which further reduces the informational content of prices, and so on.

The model presented here is most closely related to the literature studying trading in OLG environments. De Long, Shleifer and Waldman (1990), as well as Spiegel (1998), study economies with 2-period lived investors. Spiegel (1998) is closest to the present paper as in his model all investors are rational, and the random component of returns comes from (rather small) random innovations asset supply. Watanabe (2008) extends Spiegel's model to introduce asymmetric information about forthcoming dividends. In all these models however, investor horizons are fixed. The discussion on how the economy can transition between episodes of high and low price volatility therefore remains, by construction, relegated to the equilibrium switching argument *only*. He and Wang (1995), and Cvitanic et al. (2006) study dynamic, finite horizon economies with incomplete information. Since agents derive utility only from terminal wealth, the age-adjusted risk aversion coincides with the CARA parameter in both papers. Moreover, in these papers all investors grow old simultaneously, so there is no risk transfer from dying to active generations. The two main forces at work in the present paper are therefore quite different.

Other related papers study the impact of short-term investors in the context of 3-period models. In Froot, Scharfstein and Stein (1992), investors might choose to study information unrelated to fundamentals to the extent it can predict short-term price movements. Kondor (forthcoming) studies an economy with short-term traders, focusing on how public disclosures can simultaneously increase divergence of (rational) beliefs while lowering the conditional uncertainty about fundamentals. Cespa and Vives (2012) build a similar setup but focus on how persistent noise trading can generate two equilibria even in a finite

horizon economy. Also, in an early version of this paper, I study the impact of increased fund liquidations during downturns in effectively lowering investors' horizon, and its implications for price informativeness and expected returns (Albagli, 2009). It is of course difficult to compare the results obtained in a fully dynamic model from those derived from finite horizon environments. The key difference with these papers remains in that the present analysis allows for varying investment horizons –indeed, such variation constitutes the baseline of the results discussed– whereas 3-period models have a rigid lifespan structure.<sup>3</sup>

Finally, the work by Chien et al. (forthcoming) is also related. They build an economy where some investors are intermittent portfolio re-balancers. When equity prices drop following bad shocks, this forces a relatively small mass of sophisticated investors to bear aggregate risk disproportionately, inducing further price declines and an increase in expected returns. The economy they consider has symmetric information, infinitely lived agents and CRRA preferences, so the forces at work are quite different. Nevertheless, a varying mass of investors who must absorb aggregate risk is a common theme in both papers, and so the findings presented here are complementary to their work.

The rest of the paper is organized as follows. Section 2 lays out the main elements of the model and describes the equilibrium concept. Section 3 presents the characterization of existence and uniqueness, for different investment horizons, across economies with symmetric information. Section 4 then focuses on how investment horizons affect the stable equilibrium of the asymmetric information economy, discussing its implications for expected returns, price volatility and uncertainty. Section 5 concludes.

## 2 A Generalized OLG model with Asymmetric Information

### 2.1 Basic Setup

#### 2.1.1 Securities and Payoffs

Time is discrete:  $t = 1, 2, \dots, \infty$ . There is a risk-free asset in perfectly elastic supply yielding a gross return of  $R = 1 + r$ , and one risky asset -paying an infinite stream of dividends  $\{D_\tau\}_{\tau=1}^\infty$ . The dividend stream follows a mean-reverting process with unconditional mean  $\bar{F}$  and mean-reverting speed of  $0 \leq \rho_F \leq 1$ :

$$D_t = F_t + \varepsilon_t^D, \text{ with} \tag{1}$$

$$F_t = (1 - \rho_F)\bar{F} + \rho_F F_{t-1} + \varepsilon_t^F. \tag{2}$$

$F_t$  is the persistent payoff component. Due to the disturbances  $\varepsilon_t^D$  and  $\varepsilon_t^F$  however, the value of  $F_t$  is not revealed by the observation of dividends.

The risky asset is net supply of  $\theta_t$ , whose stochastic process is described by

$$\theta_t = (1 - \rho_\theta)\bar{\theta} + \rho_\theta \theta_{t-1} + \varepsilon_t^\theta, \tag{3}$$

where  $\bar{\theta} \geq 0$  is its unconditional mean,  $0 \leq \rho_\theta \leq 1$  is the mean-reverting speed of net supply to its steady

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<sup>3</sup>In Albagli (2009), changes in investing horizons are partly captured through comparative statics in the mass of investors that are forced to liquidate due to households' early withdrawals.

state average, and  $\varepsilon_t^\theta$  is a white noise disturbance. The error vector  $\varepsilon_t \equiv [\varepsilon_t^D \ \varepsilon_t^F \ \varepsilon_t^\theta]'$  is assumed to follow a joint normal distribution with mean zero, and variance-covariance matrix  $\Sigma = \text{diag}(\sigma_D^2, \sigma_F^2, \sigma_\theta^2)$ .

### 2.1.2 Investors

The mass of investors in the economy is normalized to unity. A fraction  $\mu$  of these, labeled uninformed investors, have access to publicly available information only. Letting  $\underline{h}_t \equiv \{h_{t-s}\}_{s=0}^{+\infty}$  denote the complete history of a variable  $h$  up to time  $t$ , public information corresponds to the whole history of dividends and prices of the risky asset, represented by the filtration  $\Omega_t^U = \{\underline{D}_t, \underline{P}_t\}$ . The complement share of investors (in mass  $1 - \mu$ ) are informed: in addition to public information  $\Omega_t^U$ , they observe the current realization of the persistent dividend component,  $F_t$ , directly.

Another crucial layer of heterogeneity in the model is age. Agents are born and live for  $T$  periods. At any point in time  $t$ , the economy then has  $T$  different generations of investors coexisting, aged  $j = 1, 2, \dots, T$  years. All investors born in period  $t$  maximize utility of lifetime consumption:  $\sum_{s=1}^T \beta^s U(C_{t+s})$ , where period-utility is given by negative exponential preferences:  $U(C) = -e^{-\gamma C}$ . The CARA coefficient  $\gamma$  is assumed to be the same across generations/investor types. All investors are born with exogenous (and innocuous) wealth  $w_0$ .

All generations have equal an mass of  $1/T$ , and the mix between informed and uninformed investors is the same in each generation. The economy then displays a constant age/information distribution of investors. For example, in an economy with lifespan  $T = 4$  and fraction of uninformed investors  $\mu = 0.8$ , there is a steady fraction equal to 20% of the agents that corresponds to 3-year old, uninformed investors.

### 2.1.3 Asset Markets

The risky asset can be traded by all investors alive in period  $t$ . Investors can take long or short positions at any time during their active trading years:  $j = \{1, 2, \dots, T-1\}$ , for which they can borrow (or save) unlimited amounts in the risk-free asset. The dying generation aged  $T$ , however, must liquidate the positions accumulated throughout its active trading years, as it may not leave the economy with a net debt to any other investor. Denoting  $X_{j,t}^U$  the demand of uninformed investors aged  $j$  in period  $t$ , and  $X_{j,t}^I$  the corresponding demand of the informed investors, aggregate demand for the asset is given by

$$AD : X_t \equiv \frac{1}{T} (\mu \cdot \sum_{j=1}^{T-1} X_{j,t}^U + (1 - \mu) \cdot \sum_{j=1}^{T-1} X_{j,t}^I). \quad (4)$$

Investors are price-takers and submit price-contingent demand orders (generalized limit orders) to a “Walrasian auctioneer”, who then sets a price  $P_t$  for the risky asset such that all orders are satisfied. Defining the dollar net excess return of investment in the risky asset as  $Q_{t+1} \equiv D_{t+1} + P_{t+1} - RP_t$ , the wealth of trader aged  $j$  consuming  $C_{j,t}^i$  and demanding  $X_{j,t}^i$  (for  $i = \{U, I\}$ ) evolves according to

$$W_{j+1,t+1}^i = (W_{j,t}^i - C_{j,t}^i)R + X_{j,t}^i Q_{t+1}. \quad (5)$$

## 2.2 Equilibrium Characterization

### 2.2.1 Recursive Representation

The key object to solve is the risky asset price,  $P_t$ . I conjecture (and later confirm) that the setup described above leads to the following price equation:

$$P_t = p_0 + \hat{p}_F F_t^U + p_F F_t + p_\theta \theta_t, \quad (6)$$

where  $F_t^U \equiv \mathbb{E}[F_t | \Omega_t^U]$  is defined as the uninformed investors' forecast of the persistent dividend component  $F_t$ , conditional on publicly available information. To characterize the equilibrium, it is useful to begin by writing the evolution of the main state variables in a recursive form. Let  $\Psi_{t+1} \equiv [1 \ F_{t+1} \ \theta_{t+1}]'$ . Given equations (1), (2), and (3), the evolution of  $\Psi_{t+1}$  can be written as

$$\Psi_{t+1} = A_\psi \Psi_t + B_\psi \epsilon_{t+1}^U, \quad (7)$$

where  $A_\psi$  and  $B_\psi$  are matrices of proper order, and the vector  $\epsilon_{t+1}^U \equiv [\epsilon_{t+1}^D \ \epsilon_{t+1}^F \ \epsilon_{t+1}^\theta \ \tilde{F}_t^U]'$  is the expanded error vector faced by the uninformed investors, who in addition to the exogenous shocks face uncertainty coming from their own forecast errors about  $F_t$ ;  $\tilde{F}_t^U \equiv \hat{F}_t^U - F_t$  (see the Appendix). Most of the equations of interest, including the evolution of beliefs, optimal demands, and prices, can be expressed in terms of this recursive representation.

The solution approach builds on the standard technique used in CARA-normal REE setups (see Vives (2008) for a textbook discussion). These models apply a guess-verify procedure which consists of 3 steps. First, conjecture that prices are linear in the underlying shocks. Based on this conjecture, update beliefs of investors (posterior means and variance) about future dividends and prices. Second, derive the optimal demands of investors. Third, impose market clearing and solve for the conjectured price coefficients in terms of the underlying model parameters.

More formally, for any filtration  $\Omega$ , let  $H(x|\Omega) : R \rightarrow [0, 1]$  denote the conditional posterior cdf of a random variable  $x$ . Let  $(j, i)$  denote the age/information type of each investor in the economy, with  $j = \{1, 2, \dots, T\}$ , and  $i = \{U, I\}$ , and let the filtration  $\Omega_t^U = \{\underline{D}_t, \underline{P}_t\}$  and  $\Omega_t^I = \{\underline{D}_t, \underline{P}_t, F_t\}$  represent the information available at time  $t$  to uninformed and informed investors, respectively.<sup>4</sup> The equilibrium concept is as follows:

*A competitive rational expectations equilibrium is defined by: 1. A price function given by (6), 2. Demand of the risky asset  $X_{j,t}^i = x(P_t, \Omega_t^i, j)$  by investor  $(j, i)$ , 3. Posterior beliefs  $H(\Psi_t | \Omega_t^U)$  and  $H(\Psi_t | \Omega_t^I)$  for uninformed and informed investors, respectively, such that  $\forall (j, i)$ : (i) Asset demands are optimal given prices and posterior beliefs; (ii) The asset markets clear at all times; and (iii) Posterior beliefs satisfy Bayes law.*

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<sup>4</sup>Whether we allow informed investors to observe the complete history  $\underline{F}_t$ , or just the current value  $F_t$ , is irrelevant for the results since  $\{\theta_t, F_t\}$  are sufficient statistics for predicting future returns.

### 2.2.2 Traders' Problem

For an investor aged  $j$  in period  $t$ , with information given by the filtration  $\Omega_t^i$ , the problem is given by

$$\max_{x,c} \mathbb{E}\left[-\sum_{s=0}^{T-j} \beta^s e^{-\gamma C_{j+s,t+s}} \mid \Omega_t^i\right], \text{ s.t. } W_{j+1,t+1}^i = (W_{j,t}^i - C_{j,t})R + X_{j,t}^i Q_{t+1}, \quad W_{1,t}^i = w_0. \quad (8)$$

This optimization remains analytically tractable as long as the evolution of future wealth, conditional on information, is normally distributed. The value function then takes a known form in terms of its dependence on the first and second conditional moments of investors' beliefs about the state variables driving future returns. With this tractable value function representation, asset demands and consumption/savings policies can be determined in closed form (see the discussion in Wang (1994) for more details).

We now check whether future excess returns  $Q_{t+1}$  are indeed conditionally normally distributed. For informed investors, this is immediate. Because the informed also observe the public information available to the uninformed, they know the value of the current forecast  $F_t^U$ . Since they also observe  $F_t$  privately, the price reveals the current realization of the aggregate supply,  $\theta_t$ . It is then straightforward to show that  $Q_{t+1}$  is conditionally gaussian for the informed investors.

For the uninformed, beliefs must be characterized with dynamic filtering methods. Note that from the price equation in (6), uninformed investors can back out a noisy signal about  $F_t$ , after subtracting the constant as well as the contribution of their own forecasts to the price. Let's label this signal the *informational content of price*, given by  $p_t \equiv F_t + \lambda \cdot \theta_t$ , with  $\lambda \equiv p_\theta/p_F$ . Together with the dividend in equation (1), these signals constitute the vector of public information about the state vector  $\Psi_t$ . Using the recursive characterization, the signal vector can be written as

$$S_t \equiv [D_t \ p_t]' = A_s \Psi_t + B_s \epsilon_t^U. \quad (9)$$

The next theorem describes the evolution of uninformed investors beliefs, showing that forecast errors indeed follow a conditionally normal distribution. Specifically, let  $\mathbb{O} \equiv \mathbb{E}[(\Psi_t - \mathbb{E}[\Psi_t \mid \Omega_t^U])(\Psi_t - \mathbb{E}[\Psi_t \mid \Omega_t^U])' \mid \Omega_t^U]$  denote the variance of the state vector, conditional on public information. Then,

**Theorem 1 (Beliefs with public information):** *The distribution of the state vector  $\Psi_t$ , conditional on the filtration  $\Omega_t^U = \{\underline{D}_t, \underline{P}_t\}$ , is normal with mean  $\mathbb{E}[\Psi_t \mid \Omega_t^U]$  and variance  $\mathbb{O}$ , where*

$$\mathbb{E}[\Psi_t \mid \Omega_t^U] = A_\psi \mathbb{E}[\Psi_{t-1} \mid \Omega_{t-1}^U] + K(S_t - \mathbb{E}[S_t \mid \Omega_{t-1}^U]), \quad (10)$$

and the conditional variance and projection matrix  $K$  jointly solve

$$\mathbb{O} = (I_3 - K A_s)(A_\psi \mathbb{O} A_\psi' + B_\psi \Delta B_\psi'), \quad (11)$$

$$K = (A_\psi \mathbb{O} A_\psi' + B_\psi \Delta B_\psi') A_s' (A_s (A_\psi \mathbb{O} A_\psi' + B_\psi \Delta B_\psi') A_s' + B_s \Delta B_s')^{-1}, \quad (12)$$

$$\Delta = \text{diag}(\sigma_D^2, \sigma_F^2, \sigma_\phi^2, \mathbb{O}(2, 2)).$$

*Proof: In the Appendix.*

Once we have checked that uninformed investors' beliefs follow a conditional gaussian distribution, we can state the results characterizing the value functions and the optimal consumption and investment policies chosen by different types of investors.

**Theorem 2 (consumption/investment policies):** *Let  $W_{j,t}^I$  and  $W_{j,t}^U$  denote the current wealth of a  $j$ -aged informed and uninformed investor, respectively. Let  $M_t \equiv [1 \ F_t \ \theta_t \ \tilde{F}_t^U]'$  denote the current projection of the informed investors about the expanded state variable driving future returns, and let  $M_t^U \equiv [1 \ \hat{F}_t^U \ \hat{\theta}_t^U]'$  denote the current projection of the uninformed investors about the current state variables. Then,*

1. *The value function and optimal rules of the informed investor correspond to*

$$J^I(W_{j,t}^I; M_t; j; t) = -\beta^t e^{-\alpha_j W_{j,t}^I - V_j^I(M_t)}, \quad (13)$$

$$X_{j,t}^I = \left( \frac{A_Q}{\alpha_{j+1} \Gamma_{j+1}^I} - \frac{h_{j+1}^I}{\alpha_{j+1} \Gamma_{j+1}^I} \right) \cdot M_t, \quad (14)$$

$$C_{j,t}^I = c_{j+1}^I + \left( \frac{\alpha_{j+1} R}{\alpha_{j+1} R + \gamma} \right) W_{j,t}^I + \frac{M_t' m_{j+1}^I M_t}{2(\alpha_{j+1} R + \gamma)}. \quad (15)$$

2. *The value function and optimal rules of the uninformed investor correspond to*

$$J^U(W_{j,t}^U; M_t^U; j; t) = -\beta^t e^{-\alpha_j W_{j,t}^U - V_j^U(M_t^U)}, \quad (16)$$

$$X_{j,t}^U = \left( \frac{A_Q^U}{\alpha_{j+1} \Gamma_{j+1}^U} - \frac{h_{j+1}^U}{\alpha_{j+1} \Gamma_{j+1}^U} \right) \cdot M_t^U, \quad (17)$$

$$C_{j,t}^U = c_{j+1}^U + \left( \frac{\alpha_{j+1} R}{\alpha_{j+1} R + \gamma} \right) W_{j,t}^U + \frac{M_t^{U'} m_{j+1}^U M_t^U}{2(\alpha_{j+1} R + \gamma)}. \quad (18)$$

*Proof: In the Appendix.*

Future returns  $Q_{t+1}$  depend on the actual state variables  $F_t$  and  $\theta_t$ , but also on the uninformed investors' projection about these variables. This can be conveniently reduced to a dependence on the expanded state vector  $M_t \equiv [1 \ F_t \ \theta_t \ \tilde{F}_t^U]'$ , which includes uninformed investors' forecast error about current state variables.<sup>5</sup> While this vector is perfectly observed by the informed investors, it is observed with noise by the uninformed investors. Conditional on their information however, their forecast error is a zero-mean, normally distributed random variable. Hence, for both investor types future returns are linear in these projections ( $M_t$  for the informed,  $M_t^U$  for the uninformed), plus additional white noise error with gaussian distribution. The problem then remains tractable and value functions and optimal

<sup>5</sup>This is because the forecast error of the uninformed about  $F_t$  is perfectly colinear with her forecast error about  $\theta_t$ .

policies have the closed-form expression stated above.

Optimal portfolios take the standard form found in other dynamic CARA-gaussian models. Take the case for informed investors. The term  $A_Q/(\alpha_{j+1}\Gamma_{j+1}^I)$  is a mean-variance efficient portfolio capturing the tradeoff between expected returns (numerator) and risk (denominator), where  $\alpha_{j+1}$  is the age-dependent risk aversion coefficient, and  $\Gamma_{j+1}^I$  is the renormalized covariance matrix of returns. In simple terms, this ratio reflects the response in investors demand coming from an increase in expected returns. The second term is a demand hedging component, which arises from the fact that innovations in returns also affect expected returns further into the future. More precisely, the error innovation  $\epsilon_{t+1}$  not only affects returns  $Q_{t+1}$ , but also the value function at  $t+1$ , giving rise to an additional source of risk (see Wang (1994) for more details). What makes this particular problem different from other dynamic REE environments is of course the dependence of these components on the age of the investor.

The solution method, barring some special cases commented below, relies on numerical procedures. Beginning with a known terminal value function for the dying generation, one can iteratively compute the value functions at earlier ages for each investor to find the optimal consumption and investment rules for all the different ages actively interacting in the asset market. Equilibrium prices can then be solved by imposing the market clearing condition:

$$\frac{1}{T}(\mu \cdot \sum_{j=1}^{T-1} X_{j,t}^U + (1 - \mu) \cdot \sum_{j=1}^{T-1} X_{j,t}^I) = \theta_t. \quad (19)$$

In the next two sections I analyze the properties of the equilibrium and discuss the implications of varying investment horizons;  $T$ , for particular classes of economies.

### 3 Symmetric Information Economies

In this section, I characterize the equilibrium of the model and discuss the implications of varying the investment horizon  $T$ , for economies with symmetric information. I study both the cases where the mass of uninformed agents is  $\mu = 1$  (no-information economy) and  $\mu = 0$  (full information economy). Apart from being more tractable and allowing for the derivation of some analytical results, these economies go a long way in conveying the intuition for the main mechanisms that are triggered from variations in horizons, and so provide a natural starting point of the analysis.

#### 3.1 Existence and multiplicity of equilibria

Table 1 introduces the baseline parameters that I will use throughout (unless otherwise stated). Although the purpose of the paper is not to provide a detailed calibration exercise for fitting asset pricing data, some features of the parameter choice are relevant to discuss. I have chosen the variance of the persistent dividend process as a normalization (equal to one) and made it relatively persistent ( $\rho_F = 0.95$ ). In

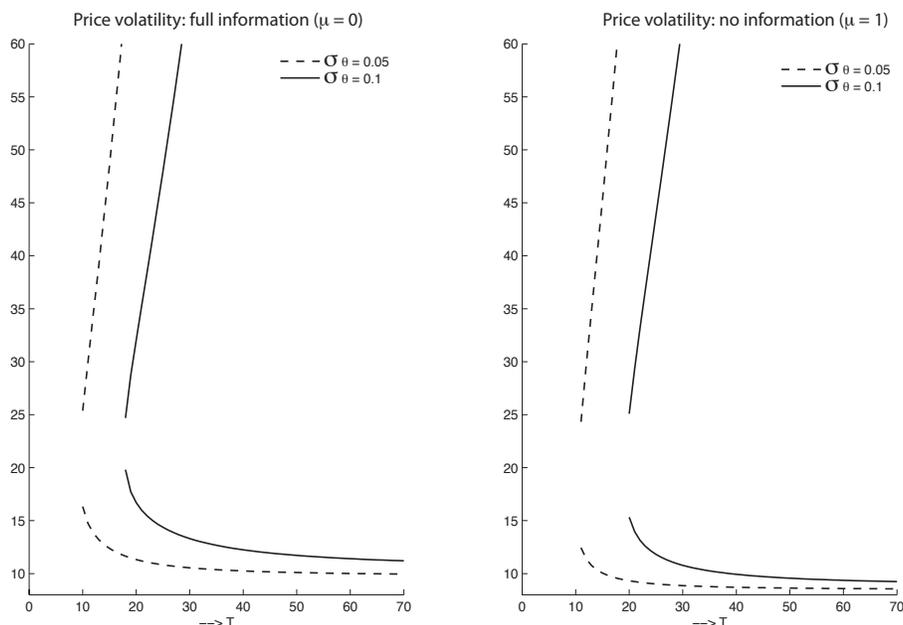
comparison, the temporary dividend component is relatively volatile (i.e,  $\sigma_D = 3$  vs.  $\sigma_F = 1$ ).<sup>6</sup> The average net supply  $\bar{\theta}$  of the risky asset is also normalized to one, in accordance to the measure of agents in the economy. Its standard deviation  $\sigma_\theta$  is set to 10% of its unconditional value.

Table 1: Baseline parameters

$\sigma_D$	$\sigma_F$	$\sigma_\theta$	$\rho_F$	$\rho_\theta$	$\gamma$	$r$	$\beta$	$F$	$\theta$
3	1	0.1	0.95	0.6	1	0.05	1/(1.05)	20	1

Figure 1 plots the relation between the volatility of prices  $P_{t+1}$ , given an information set which conditions on public information  $\{\underline{D}_t, \underline{P}_t\}$ , and the investment horizon,  $T$ . The first thing to note from the figure is that an equilibrium does not exist for all investment horizons. Under the baseline parameters, the full-information economy exhibits equilibria starting from the critical horizon of  $T^* = 18$  onwards, while the no-information economy requires  $T \geq 20$  (dashed lines). The fact that overlapping generations models for the stock market can fail to exhibit equilibria is discussed by Spiegel (1998), Watanabe (2008), and Banerjee (2011). As the figure suggest, for the traditional OLG setup in which  $T = 2$ , conditions for existence are indeed quite stringent and one must adjust some of the key parameters related to risk. Spiegel (1998), for instance, assumes a very small volatility of the random supply;  $\sigma_\theta$ . The dotted lines show equilibria for  $\sigma_\theta = 0.05$ , for which the economy reaches the existence region at lower values of  $T^*$ .

Figure 1: Existence and multiplicity



<sup>6</sup>Although the transitory dividend component volatility does not enter the price equation (6) directly, it makes inference of the persistent dividend component  $F_t$  more difficult for the uninformed investors.

The second apparent feature of the equilibria is multiplicity. For each investment horizon equal or larger than the minimum required for existence, there are generally 2 equilibria.<sup>7</sup> Across these equilibria, only the price coefficient associated with innovations in the persistent dividend component ( $\hat{p}_F$  for  $\mu = 1$ , and  $p_F$  for  $\mu = 0$ ) remains the same. The rest of the parameters vary reflecting two self-fulfilling equilibria. In one equilibrium, the price impact of the random supply is relatively small ( $p_\theta$  is small, in absolute value). If investors trading in the current period believe this equilibrium will prevail in the future, they require a modest compensation for taking on the other side of random supply innovations, which then have minor effects on prices and returns. As a result of this low risk environment, the average price of the security is high, reflected by a large coefficient  $p_0$ . But the situation might well be the converse. If investors expect future supply innovations to have large effect on future prices, they will become reluctant to absorb the current (stochastic) supply of the stock, whose price will then respond to innovations in  $\theta$  with a large, negative coefficient. The high risk faced by investors in this environment requires compensation, reflected in a large average premium demanded on the security (a low, or even negative value of  $p_0$  in the price equation).

Although the existence and multiplicity results have already been addressed in prior work, little is known about how *variations* in investor horizons can affect the properties of the equilibria. Figure 1 shows that, along the low volatility equilibrium, increasing the lifespan of investors reduces price volatility. In other words, the low volatility equilibrium becomes increasingly “easy” to attain as investors live for longer horizons. As mentioned earlier, this is due to two main mechanisms at work: an *age-adjusted risk aversion* effect, and a *risk transfer* effect, which I will explain in more detail momentarily. Moreover, the low volatility equilibrium is stable in the best-response sense. That is, if the coefficients in the price equation are perturbed slightly (for instance, a more negative  $p_\theta$ ), the best-response of investors will make the economy converge back at the low volatility equilibrium after a few iterations (in this case, investors would take the opportunity to buy the relatively cheap supply, reverting the perturbation of  $p_\theta$  to a less negative value).

The high volatility equilibria, in contrast, exhibits the exact opposite features. As investors live for longer horizons, this equilibrium becomes increasingly “difficult” to attain: it takes an increasingly volatile security to induce investors to demand the levels of compensations that are consistent with such equilibrium.

There are additional features of the high volatility equilibrium that make it a less suitable candidate for equilibrium selection. First, as noted by Spiegel (1998) and made apparent in Figure 1, the volatility along the high-volatility equilibrium is *decreasing* in the variability of the random supply shock;  $\sigma_\theta$  (compare the solid vs. dashed lines in the figure). Second, it is unstable in the best-response sense. As can be learned from the numerical simulations, slight perturbations in the price coefficients lead to investor behavior that push the economy further away from the high-volatility equilibrium.<sup>8</sup> Third, the extreme levels of asset price volatility for longer lifespans make it increasingly hard to find positive average prices. Fourth, while the low-volatility equilibrium converges smoothly to the infinite horizon economy, the high volatility

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<sup>7</sup>Although one could adjust parameters such that for the critical horizon  $T^*$  the two equilibria coincide.

<sup>8</sup>For instance, if the price coefficient  $p_\theta$  is set to a slightly higher value (i.e., less negative), the economy converges to the stable, low-volatility equilibrium. If  $p_\theta$  is set to a slightly more negative value, the economy diverges.

equilibrium vanishes as  $T \rightarrow \infty$ . Actually, it is possible to show this result analytically for a simplified version of the symmetric information economy (see the Appendix for the proof).

**Proposition 1 (infinite horizon limit):** *Set  $\sigma_D = 0$  and  $\rho_\theta = \rho_F = 1$ . As  $T \rightarrow \infty$ , and for all values of the remainder parameters:*

- a. An equilibrium exists.*
- b. The equilibrium is unique.*

The result in proposition 1 is of theoretical interest on its own. To my knowledge, a formal proof of the existence and uniqueness of equilibria in dynamic REE settings when  $T \rightarrow \infty$  has not been stated in prior work. Wang (1994), for instance, states that an equilibrium price equation similar to expression (6) can be solved for numerically. Whether this is the case for all possible parameters, or whether the solution is unique, is left an open question.

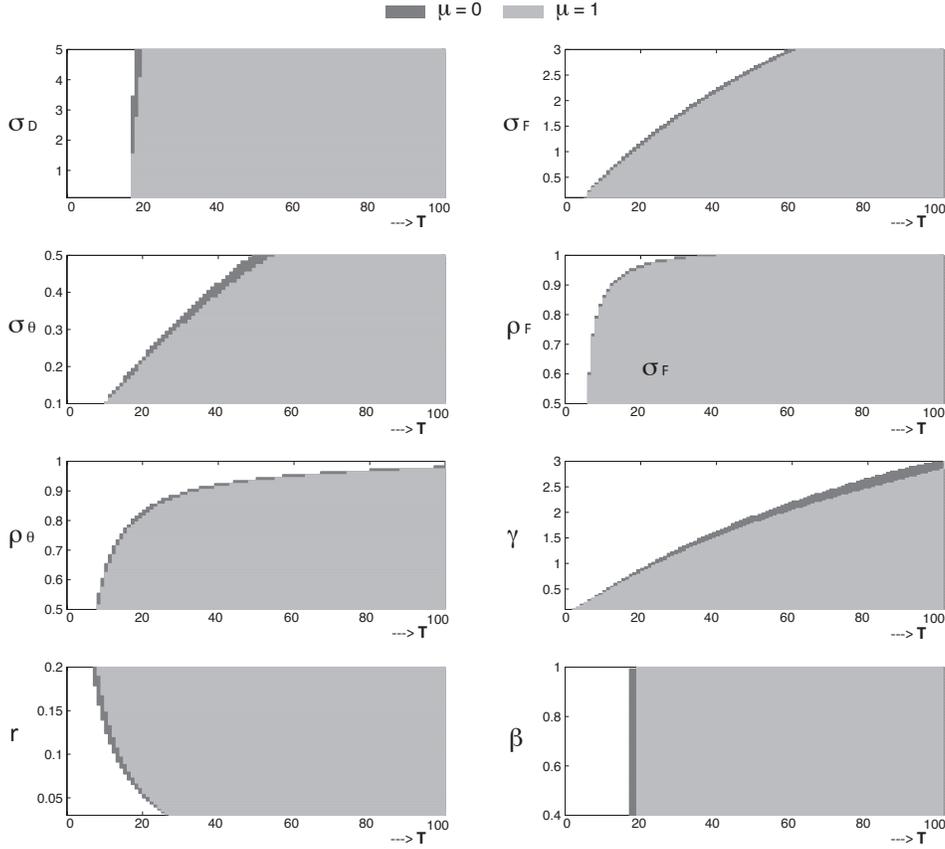
The economics behind this result –the survival of the low volatility equilibrium only, in the limit– seem intuitive. When investors are infinitely lived, they can always accommodate a highly volatile asset price by voluntarily buying when it is underpriced, and selling when it is overpriced. The attractiveness of the highly volatile asset in the infinite horizon economy is, of course, not an equilibrium, as competitive agents will bid the price up when it is underpriced, and down when it is overpriced. In equilibrium, only a moderate level of price volatility is sustainable. As Figure 1 illustrates, this volatility corresponds to the limit of the stable equilibrium in the finite horizon economy. For this and the other reasons cited above, I will focus the attention on the low-volatility equilibrium in the remainder of paper.

Figure 2 gives a more general picture of the equilibrium existence regions. Starting from the benchmark parameters, each panel shows the critical investment horizon  $T^*$  that sustains equilibria, for both the full- and no-information economy. The parameter dependence of  $T^*$  are intuitive. Parameters that increase the volatility of the dividend process (higher  $\sigma_D$ ,  $\sigma_F$ , or an increase in the persistence  $\rho_F$ ) increase the fundamental risk of the security, and require increasing critical horizons  $T^*$  for existence. Similarly, increases in non-fundamental risk related to the random supply shock (higher  $\sigma_\theta$ , or an increase in the persistence  $\rho_\theta$ ) also shift up the minimal required horizon. Naturally, risk aversion  $\gamma$  also increases  $T^*$ , while the converse is true for the risk-free rate, since prices respond less to fundamental innovations when investors discount future flows at a higher rate. Finally, although  $\beta$  matters for the consumption path chosen by investors, it has no effect in their trading decisions (as trading choices are wealth-independent), and hence no impact on prices or the critical horizon  $T^*$ . Moreover, for all parameter values, the full-information information economy reaches an equilibrium at a weakly lower value of  $T^*$  than the no-information economy, since in the former investors have less uncertainty about the fundamental dividend process.

### 3.2 Variations in investment horizons: two key mechanisms

I now analyze in more detail the two main mechanisms driving the relation between price volatility and investment horizons, in the case of economies with symmetric information.

Figure 2: Existence regions



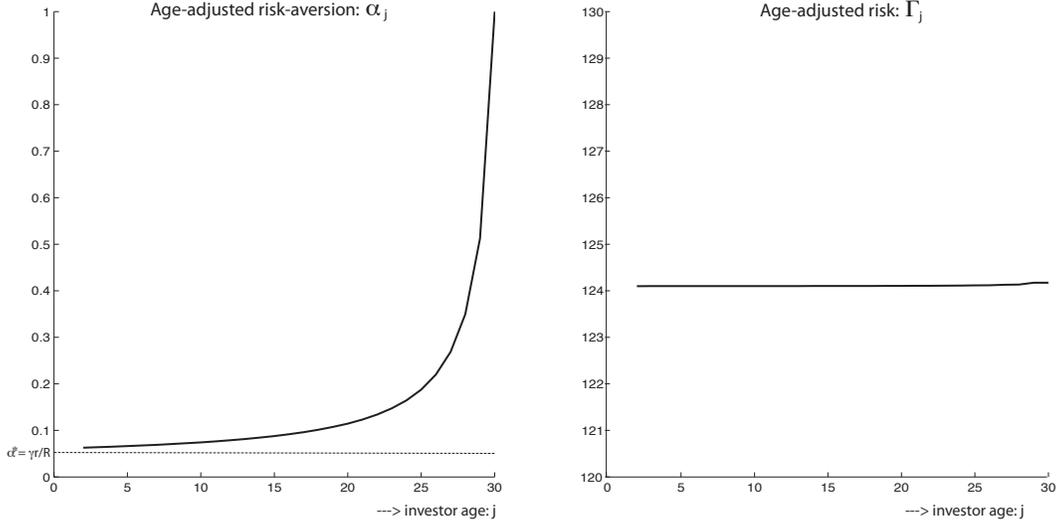
### 3.2.1 Age-adjusted risk aversion effect

The first mechanism is related to the changes in the *pricing* of risk induced by changes in  $T$ , which I will refer to as the *age-adjusted risk aversion* effect. A key driver of the action in dynamic trading environment is not the concern of investors about future dividends, but about future *price changes*. Obviously, in the traditional OLG setting in which investors live two periods, these concerns are maximized: if in the future period the realization of the random supply drives prices down, investors exiting the economy have no further periods to recover.

When investors live for more period however, they are less affected by price fluctuations since they are not forced to unwind their portfolio at adverse prices –unless they have reached the terminal date  $T$ . In the words of De long et al. (1990), this mechanisms is akin to receiving “dividend insurance”: investors who purchase assets with high fundamental value diminish their total risk exposure to non-fundamental shocks, the more so the longer they live and consume dividends.

To understand this mechanism more formally, consider the optimal portfolio decisions of the informed investors in the full-information economy by setting  $\mu = 0$  (an equivalent argument holds for other information structures). Figure 3 plots the age-dependent denominator term  $\alpha_{j+1}\Gamma_{j+1}^I$  by its separate

Figure 3: Age-adjusted risk aversion effect



components, for an investment horizon  $T = 30$ . The key source of variation is the  $\alpha_{j+1}$  term, which represents an age-adjusted, risk-aversion parameter. As shown in the appendix,  $\alpha_{j+1}$  can be solved recursively through the equation:

$$\alpha_j = \frac{\gamma \alpha_{j+1} R}{\alpha_{j+1} R + \gamma} \quad (20)$$

The economics behind this expression can be understood as follows. Take the case of an investor aged  $j = 30$ . This investor is now retiring and hence her value function is just determined by the utility of terminal wealth. In other words, the value of  $\alpha_{30} = \gamma$ . When this investor was aged  $j = 29$  one period ago, this is the value of  $\alpha$  that applies for her portfolio decisions, just as in a static CARA investment problem. At the other extreme, for a newly born investor aged  $j = 1$ , the current risk of her transaction  $\Gamma_{j+1}^I$  is not accounted one-for-one in terms of its utility (value-function) impact, since she still has many periods to recover from fluctuations in returns. This does not mean trading is riskless: changes in wealth resulting from the risky portfolio choice will affect her wealth, current consumption and value function. But as the consumption policy expression (15) shows, current consumption choices are only affected by changes in current wealth through

$$\frac{\partial C_{j,t}^I}{\partial W_{j,t}^I} = \frac{\gamma \alpha_{j+1} R}{\alpha_{j+1} R + \gamma} = \alpha_j,$$

which is *precisely* the age-adjusted risk aversion coefficient used to price risk in the demand equation (14).

As an illustrative reference for this mechanism, consider the extreme case of a hypothetical economy in which investors live for infinite periods (as analyzed by Wang (1994)). In this case, the value of  $\alpha$  can be found as the stationary solution to (20), corresponding to  $\alpha^* = \gamma r / R$ . As shown in figure 3, the age-adjusted risk-aversion of a newly born investor in the finite horizon economy converges towards the infinitely-lived investor (age-adjusted) risk aversion coefficient,  $\alpha^*$ . For an investor with an intermediate

age, say  $j = 20$ , the situation is somewhat in between the static benchmark and the infinite horizon economy. It follows that as the investment horizon grows larger, the elasticity of the average investor's demand to changes in expected returns increases, through a fall in the demand denominator in expression (14). To my knowledge, this mechanism has not been studied formally in overlapping generations models of the financial market.<sup>9</sup>

### 3.2.2 Risk transfer effect

The second mechanism is related to changes in the *amount* of risk that must be absorbed in equilibrium by all active generations, induced by changes in  $T$ . I label this mechanism the *risk transfer* effect. In an economy in which agents live for  $T$  periods, each period there are  $T - 1$  generations of voluntary investors, compared to a single generation of retirees. In consequence, the relative risk transfer between these generations is the ratio  $1/(T - 1)$  (when all generations have equal mass, which is a condition for the steady-state analyzed here). This mechanism can be illustrated by a convenient decomposition of the market-clearing condition in (19). Continuing with the full-information economy as an example, this gives

$$\begin{aligned} \frac{1}{T}(X_{1,t}^I + X_{2,t}^I + \dots + X_{T-1,t}^I) &= \underbrace{X_{1,t-1}^I + X_{2,t-1}^I + \dots + X_{T-1,t-1}^I}_{\theta_{t-1}} + \underbrace{(1 - \rho\theta)(\bar{\theta} - \theta_{t-1}) + \varepsilon_t^\theta}_{\Delta\theta_t} \quad (21) \\ \Rightarrow \frac{1}{T}(X_{1,t}^I + \Delta X_{2,t}^I + \dots + \Delta X_{T-1,t}^I) &= X_{T-1,t-1}^I + \Delta\theta_t \end{aligned}$$

The demand in the left-hand side of (21) is composed of all current active investors. Of these, investors aged  $2, 3 \dots, T - 1$  were also present in the previous period, and hence their *net demands* correspond to  $\Delta X_{2,t}^I = X_{2,t}^I - X_{1,t-1}^I$  for the investor currently aged 2,  $\Delta X_{3,t}^I$  for the investor aged 3, and so on. For all these investors, the change in net positions is voluntary. Only for the investor aged  $T - 1$  in the previous period the net demand is exogenously set at  $-X_{T-1,t-1}^I$ . In equilibrium, the negative of this amount, plus the supply innovation  $(1 - \rho\theta)(\bar{\theta} - \theta_{t-1}) + \varepsilon_t^\theta$ , must be absorbed by changes in the net positions of all active investors. Hence, a relative risk transfer from a mass of  $1/T$  retirees to a mass of  $(T - 1)/T$  voluntary investors: a risk transfer ratio of  $1/(T - 1)$ .

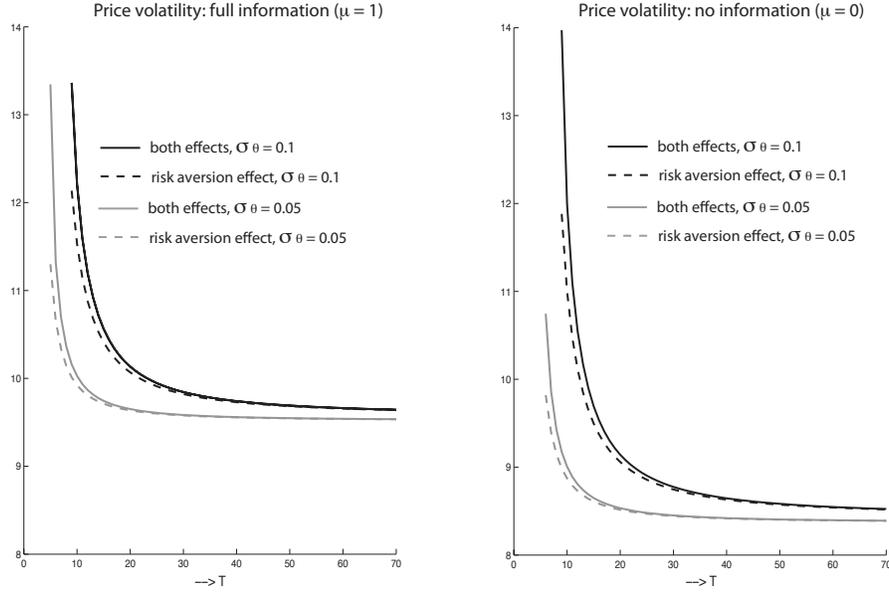
It is clear in this context why the results of OLG models with  $T = 2$  are so extreme. On the one hand, current participants in the market must purchase an asset which they must completely unwind in a single future trading round (the *age-adjusted risk aversion* effect). But moreover, when they do liquidate in the next period, their entire positions must be purchased by a generation with equal mass as them (the *risk transfer* effect). Simply put, the whole risk of the economy must forcefully exchange hands every period! To my knowledge, this second mechanism has not been studied formally in overlapping generations models of the financial market.

To gauge the relative importance of this two channels, I perform a decomposition of the change in

<sup>9</sup>The models in He and Wang (1995) and Cvitanic et al. (2006) also considers a finite investment horizon. However, since agents derive utility only from their terminal consumption of wealth, they price changes in current wealth one-to-one (i.e., the marginal propensity to consume wealth changes is 1). Therefore, the age-adjusted risk aversion coefficient coincides with the CARA parameter.

price volatility through the following exercise. I fix the mass of active (voluntary) investors in the market to one (which corresponds to the infinite horizon economy). I then vary the investment horizon  $T$ , letting only the average demands of investors change as a result of the *age-adjusted risk aversion* effect—hence, shutting off the *risk transfer* mechanism. Figure 4 shows the volatility of prices (conditional on public

Figure 4: Relative contributions of the effects



information) that emerges in these economies. As before, I include both the baseline parameters and a case with lower volatility of the random supply,  $\sigma_\theta = 0.05$ . As the figure makes clear, both effects are important in delivering the change in price volatility due to changing investment horizons. For the baseline parameters (black lines), the contribution of the *risk transfer* effect is about 33% of the total price volatility increase produced by shrinking  $T$  from 70 to 9, in the case of the full-information economy. For the no-information economy, the contribution of the *risk transfer effect* reaches 39% over the same range. For economies that admit equilibria at lower investment horizons—which is the case when  $\sigma_\theta = 0.05$  (gray lines)—the importance of the *risk transfer* effect grows larger, since changes in the relative risk transfer ratio,  $1/(T - 1)$ , become more significant as  $T$  approaches the origin. For the full- and no-information economy, the *risk transfer* effect account for 66% and 42% of total price volatility changes, respectively.

## 4 Asymmetric Information Economies

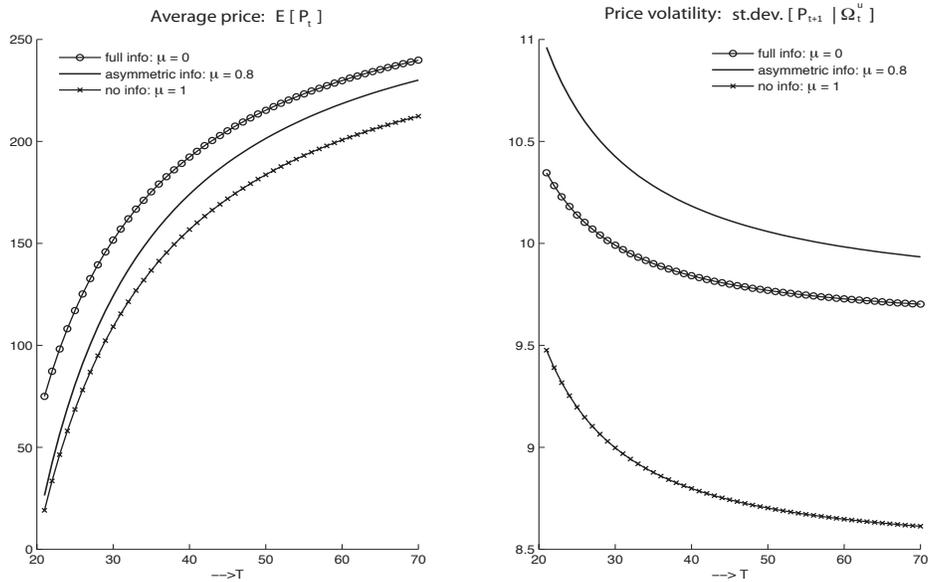
This section analyzes the effects of changes in investment horizons for the more general asymmetric information economy. From the discussion in section 2, it is clear that the endogenous price signal,  $p_t \equiv F_t + \lambda \cdot \theta_t$ , will be relatively more informative about the persistent dividend component when the ratio  $\lambda \equiv p_\theta/p_F$  is small (in absolute magnitude). From the discussion in section 3, we learned that as

horizons shrink, the volatility of prices spikes due to the increased response of prices to random supply innovations. It follows that in the asymmetric information economy, the informational role of markets will be diminished as the lifespan of investors is reduced. I now formally describe how the main pricing moments in the asymmetric information economy behave as a function of investment horizons, contrasting them with respect to the symmetric information benchmarks of section 3. I then explore how this variation affects the informational efficiency of the market.

#### 4.1 Asset pricing moments

The left hand side of figure 5 plots the unconditional average of prices for three different economies: the full information benchmark ( $\mu = 0$ , circled line), the no-information benchmark ( $\mu = 1$ , crossed line), and the asymmetric information economy when only 20% of investors are informed ( $\mu = 0.8$ , plain line). The ordering of mean prices across these economies follow a one-to-one rank with the amount of fundamental information available to agents: prices are the highest for the full-information economy, intermediate for the asymmetric information economy, and the lowest for the no-information case.

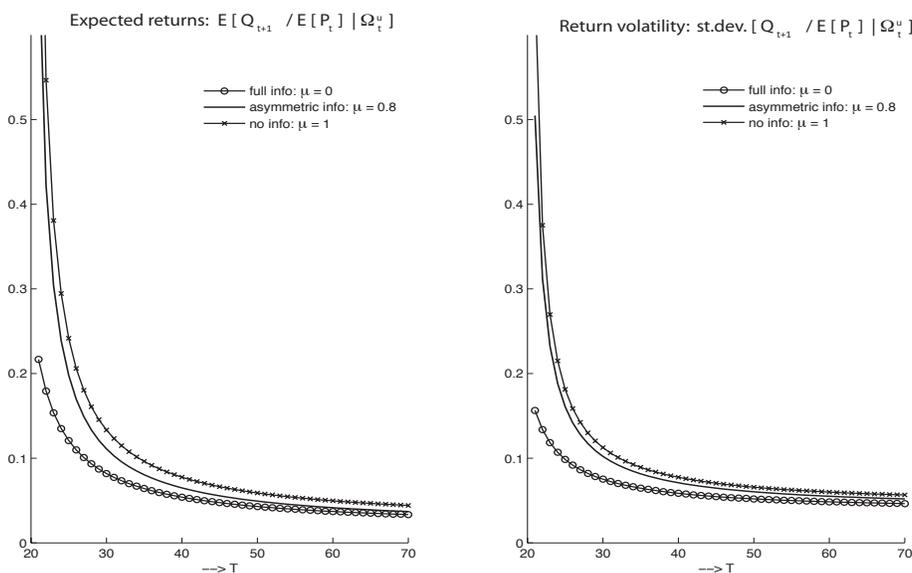
Figure 5: Average prices and volatility



Interestingly, the right panel of the figure reveals that price volatility (the standard deviation of future prices, conditional on current public information) might not follow the same ordering. In particular, the no-information economy displays the least amount of price volatility, whereas the asymmetric-information economy is the most volatile. The reason behind this result is intuitive. For the no-information economy, prices convey less information about fundamentals and hence vary less to reflect changes in the underlying persistent dividend component. As the investment horizon shrinks, volatility spikes, but mostly to reflect an increased role of supply shocks in driving price movements. In the full-information economy, volatility

is larger for all investment horizons due to the high impact that fundamentals have on prices when they can be observed without noise. Once again, as horizons grow short, the increased importance of supply shocks raise price volatility. The asymmetric information economy, on the other hand, inherits the behavior of the benchmark economies to a different degree depending on the investment horizon under consideration. For relatively long investment horizons, volatility is higher than in the no-information economy due to the better capacity of prices to reflect changes in fundamentals. For the parameters chosen, it is also above the volatility displayed by the full-information economy, as prices react more to innovations in supply when fundamental uncertainty is larger. Moreover, as  $T$  decreases, the volatility of prices in the asymmetric information case increases with a sensitivity similar to that of the no-information economy, reflecting the increased uncertainty of uninformed investors and the increased reaction of prices to non-fundamental innovations.

Figure 6: Risk premium and return volatility



Price volatility is, however, not the best metric to account for the risk faced by investors, since what matters for market participants is the the moments of returns. Figure 6 plots the expected return (risk premium) and the volatility of returns across the different economies.<sup>10</sup> The following pattern emerges in this figure which is independent of the parameters chosen. First, both expected returns and return volatility are the lowest for the full information economy. Indeed, although prices are relatively volatile in this economy, prices are also the highest, so the volatility of returns is diminished. Regarding the risk premium, it is intuitive that since investors have the most information about fundamentals in this case, their willingness to participate in the market is the highest, which is reflected by lower required returns.

At the other extreme, the risk premium and return volatility is the highest for the no-information

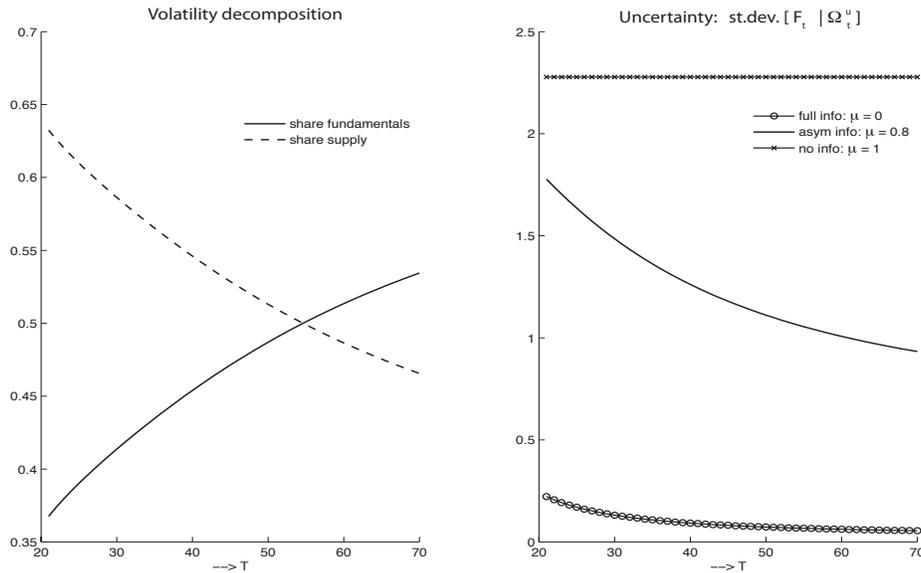
<sup>10</sup>Since prices can be negative or arbitrarily close to zero in linear price models, I define the expected return as the ratio between the expected dollar return conditional on public information,  $\mathbb{E}[Q_{t+1} | \Omega_t^u]$ , and the unconditional mean price,  $\mathbb{E}[P_t]$ .

economy, since agents face the largest amount of uncertainty regarding the fundamental value of the asset. Interestingly, the asymmetric information economy lies strictly in between the symmetric information benchmarks. I will now explain in more detail the economic forces behind this result.

## 4.2 Market efficiency

Figure 7 plots the decomposition of price volatility (left panel) and the resulting uncertainty of the uninformed investors (right panel), as a function of investment horizons. The share of price fluctuations due to fundamentals is calculated as the fraction of the price variance explained by innovations in  $F_t$ , while the non-fundamental share includes the fraction of price variance coming from both supply shocks, and the forecast errors incurred by the uninformed investors (both shares sum to one). Clearly, fundamental volatility dominates for relatively large values of the investment horizon, while the converse is true for short lifespans. In consequence, the informational content of prices is diminished as investment horizons shorten, and the uncertainty of the uninformed investors increases. As the right panel of the figure shows, the uncertainty of the uninformed (the standard deviation of  $F_t$ , conditional on the history of public information) nearly doubles as horizons fall from  $T = 70$  to  $T = 20$ .

Figure 7: Variance decomposition and uncertainty



Indeed, this figure provides the central intuition for the behavior of investors' required returns in the asymmetric information economy plotted in figure 6. In particular, for long investment horizons, the risk premium under asymmetric information comes very close to the returns required in the full-information economy, whereas the no-information economy stands out with a larger compensation for risk. However, as horizons shorten, the asymmetric information economy approaches the no-information economy, both demanding significantly higher compensation for purchasing the security than the full-

information economy.

What explains the apparent switching of the asymmetric information economy between these different benchmark regimes? Precisely the endogenous nature of information in rational expectations environments. In a competitive risk-averse framework, the central force determining how aggressively informed investors react to their private knowledge is risk. For long investment horizons, both the *age-adjusted risk aversion* and the *risk transfer* effect lead to investor demands which are relatively elastic to expected returns. In consequence, the market is deep and supply innovations have modest price impacts. In response to this low risk environment, competitive CARA-investors who possess superior knowledge will trade on this information aggressively, and information that is originally received privately by, say, 20% of the trader population will mostly find its way into the price. Conversely, for short investment horizons, the aforementioned effects incite cautious trading: demands are inelastic to private information about future returns. In consequence, information that is private to a group of investors *remains private* to such group. This explains why asset prices in the asymmetric information economy approach the behavior exhibited by the full-information economy for long horizons, since in this case the economy resembles one which a large group of investors are, in effect, pretty well informed in equilibrium. On the contrary, shrinking investment horizons set in motion a process of information *disaggregation* from prices, and the knowledge of the population becomes in effect much closer to the no-information benchmark –with asset pricing moments following suit.

The above fact can also be explained in terms of the excess sensitivity of the risk premium to changes in  $T$  in the asymmetric information economy, relative to the symmetric knowledge benchmarks. This excess sensitivity is explained by the interdependence between the uninformed investors' level of uncertainty and the risk faced by the informed. To understand this link, let's describe in more detail the consequences of a reduction in the demand elasticity of informed investors to private information. If these agents respond less to information, the equilibrium price reveals less information about the fundamental  $F_t$ , and uncertainty about fundamentals is higher for the uninformed. In response, the willingness to make the market is reduced for these agents, which require a larger compensation for bearing a large amount of risk. In consequence, supply innovations are met with wider price fluctuations. The increased volatility of prices, in turn, increases the risk for the informed, inducing them to react even less strongly to private information. This last observations closes the interdependence, or "spiral", between the risk faced by the informed investors, and uncertainty of the uninformed.

However, one should note that there are limits to the strength of this spiral effect as well. For one, the uncertainty of the uninformed in the asymmetric information economy is capped above by the knowledge of investors in the no-information benchmark, which do learn about the persistent component  $F_t$  from the observation of dividends. More interestingly however, when horizons shrink and the uncertainty of the fundamental process grows larger, at the same time the price is becoming relatively *more* informative about the noisy supply. Since this second source of price variation takes the center stage for relatively short horizons (Figure 7, left panel), knowing relatively more about this second component becomes increasingly important. In effect, the feedback loop between price volatility and investor uncertainty

grows weaker due to this counteracting force.<sup>11</sup>

I conclude this section with some remarks on quantitative issues. First and foremost, CARA-normal models are obviously not designed to capture realistic variations in asset pricing moments as the more traditional theoretical asset pricing literature. This paper is no exception, and the central results should be interpreted as an illustration of the qualitative mechanisms that are triggered by changes in effective investment horizons. That said, the proposed mechanism of varying investment horizons does seem to play an important role, at least in the confines of the model.

One could, of course, be suspicious about how big changes in investment horizons need to be for pricing moments to show interesting fluctuations. Is a reduction from 70 to 20 periods (say, years) a reasonable proxy for what happens at the business cycle frequency? In this respect, note that the *inverse* of the investment horizon corresponds exactly to the fraction of investors which are liquidating forcefully at any given time (the mass of the dying generation). In this light, an increase in liquidations from 1.42% (i.e., 1/70) to 5% (i.e., 1/20) does not seem exaggerated if one considers the empirical findings of fund liquidations during periods of financial distress. For instance, Ben-David et al. (2012) find that hedge funds reduced their exposure in equity markets in almost 30% during the 2008:Q3-Q4 contraction, which corresponded roughly to 1% of all outstanding equities. Importantly, their results indicate that most of this selling was actually *forced* by investors withdrawing financing. Carhart et al. (2002) study attrition rates in the mutual fund industry, and find that while 3.6% of funds disappear yearly on average over their sample, the standard deviation is quite high, at 2.4%. Chen et al. (2008) measure distress selling of troubled –but still alive– mutual funds, and report average distress-driven sales between 0.6-1% of mutual funds holdings at quarterly frequency (when using the asset-weighted measure of outflows). Moreover, this fraction spikes considerably (nearly doubles) during the mayor episode of financial turbulence covered in their sample: the demise of LTCM in 1998. Taken together, variations in forced liquidations between 1% and 5% per year seem to be in the ball park of the magnitudes reported in these studies for variation at the business cycle frequency.

The second observation relates more generally to the quantitative implications of endogenous information aggregation in explaining asset pricing moments. As figure 6 shows, although the asymmetric information economy reacts much more to changes in investment horizons than the full-information economy, one could argue that the no-information economy does a reasonable job in delivering comparable variations in risk premium and return volatility. What is then the value added of a (more complex) model that highlights asymmetric information?

The answer is that symmetric information benchmarks have no implications for market efficiency. For the no-information economy, prices contain no additional information about the persistent dividend process –over and above from what can be learned from dividends. In the full-information economy, on the other hand, the price system is irrelevant as a source of information since agents already know the value of fundamentals. It is in this respect that the analysis of asymmetric information in financial markets becomes crucial. On one hand, it stresses that while return volatility will be high when horizons shorten, it is the non-fundamental part of volatility that takes the center stage during these episodes,

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<sup>11</sup>For a more detailed description of this counteracting mechanism, see the discussion in Avdis (2011).

and that this matters for the allocative role of financial markets only to the extent that prices convey valuable information to investors. On the other, it gives a plausible explanation for why most measures of economic uncertainty used often in empirical work tend to spike during contractions and episodes of financial distress.

## 5 Conclusions

This paper analyzed the role played by finite investment horizons in financial markets. The main message of the paper is twofold. First, horizons matter for both the pricing of risk of the average investor (the *age-adjusted risk aversion* effect) and the amount of risk that must be held in equilibrium by the surviving agents in a generalized OLG framework (the *risk transfer* effect). Both mechanism have the potential of delivering interesting variations in expected returns and return volatilities when we interpret periods of fund withdrawals as an effective shortening of investors' horizons.

Second, investment horizons have important implications for the informational role of prices and market efficiency in an economy with asymmetric information. While long investment horizons incite informed investors to release their private information into prices through the trading process, the high risk environment triggered by short investment horizons can significantly reduce asset price informativeness. In particular, even if the fraction of informed investors is relatively low, the degree of information contained in prices can show ample variations across different horizon regimes, leading the economy to align closely to the full-information benchmark for long horizons, but much closer to the no-information framework for short investors' lifespans. This findings suggests information *disaggregation* from the price process can be an important mechanism for understanding variations in economic uncertainty more specifically, and market efficiency more generally.

There are several interesting avenues in which one can extend and complement the current analysis. First, although the focus of the present paper is studying the asset pricing effects of varying investment horizons, the analysis is limited to the comparison between steady states of different economies, rather than actual time variation in horizons within the same economy. While a very interesting question to study, this type of analysis is difficult to handle within rational expectations models as equilibrium prices lose their linear form. This makes the standard solution methods inapplicable and the analysis becomes intractable.

A related extension which *can* be analyzed under the current framework is the effects of deterministic changes in population demographics. The analysis suggests that a generation of "baby boomers" entering the economy could lead to higher asset prices and reduced uncertainty early in their life, but to declining prices, heightened volatility and poor informational efficiency as the generation approaches old age. Although the equilibrium in this economy would exhibit time-dependent price coefficients, the analysis remains tractable as long as these demographic changes are perfectly anticipated.

Finally, one could also study the incentives to acquire information for different investment horizons. While investors have more incentives to become informed when prices are less reliable, the analysis suggests these are also the times when the lifespan over which they plan to use such information is

shorter. This leads to non-trivial predictions about the effects of investment horizons on endogenous information acquisition. I leave these important questions for future research.

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## 6 Appendix

### Proof of Theorems 1 and 2:

To characterize beliefs, optimal consumption and investment rules, and equilibrium prices, one must first find the matrices  $A_\psi$ ,  $B_\psi$  in expression (7). From equations (1), (2), and (3), the evolution of  $\Psi_{t+1}$  is given by (7) as long as

$$A_\psi = \begin{bmatrix} 1 & 0 & 0 \\ (1 - \rho_F)\bar{F} & \rho_F & 0 \\ (1 - \rho_\theta)\bar{\theta} & 0 & \rho_\theta \end{bmatrix}, \text{ and } B_\psi = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

I will prove Theorem's 1 and 2 simultaneously through the following 4 steps. 1) For given coefficients in the price equation, solve the Bayesian filtering problem of the uninformed, and find the autoregressive process of their forecast errors. 2) Find the recursive representation of the conditional state vectors  $M_t$  and  $M_t^U$ , and the conditional distributions of future excess returns,  $Q_{t+1}$ . 3) Solve optimal demands, and impose market clearing to find equilibrium prices. The resulting price function is then found by equating the market clearing price to the initial price conjecture.

**Step 1:** Together with the coefficients in the price equation (6), the recursive representation in (7) leads directly to the results in Theorem 1, which is just the Bayesian updating of beliefs described by the Kalman filter. A derivation of the Kalman filter can be found in most advanced statistics textbooks.<sup>12</sup> Writing  $\mathbb{E}[\Psi_t | \Omega_t^U] \equiv \Psi_t^U$  and  $\Psi_t^U - \Psi_t \equiv \tilde{\Psi}_t^U$  for notational convenience, the evolution of the uninformed investors' forecast error vector can be found from manipulation of (10):

$$\begin{aligned} \tilde{\Psi}_{t+1}^U &= A_U \cdot \tilde{\Psi}_t^U + B_U \cdot \epsilon_{t+1}^U, \text{ with} \\ A_U &\equiv (I_3 - K A_s) A_\Psi, \text{ and } B_U \equiv (K(A_s B_\Psi + B_s) - B_\Psi). \end{aligned}$$

But notice that the observation of the price implies forecast errors about  $F_t$  and  $\theta_t$  are perfectly colinear for the uninformed; i.e,  $F_t + \lambda \cdot \theta_t = F_t^U + \lambda \cdot \theta_t^U$ , or  $\tilde{\theta}_t^U = -\lambda^{-1} \cdot \tilde{F}_t^U$ . This allows to rewrite the vector  $\tilde{\Psi}_{t+1}^U$  as  $\tilde{\Psi}_{t+1}^U = [0 \ \tilde{F}_{t+1}^U \ \tilde{\theta}_{t+1}^U]'$  =  $A_U \cdot [0 \ \tilde{F}_t^U \ \tilde{\theta}_t^U]'$  +  $B_U \cdot \epsilon_{t+1}^U$ , or

$$\begin{aligned} \tilde{F}_{t+1}^U &= \rho_U \cdot \tilde{F}_t^U + b_U \cdot \epsilon_{t+1}^U, \\ \text{with } \rho_U &\equiv A_U(2,2) - \lambda^{-1} A_U(2,3), \text{ and } b_U \equiv B_U(2, :). \end{aligned} \tag{22}$$

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<sup>12</sup>See Ljungqvist and Sargent (2000) for applications in macroeconomics.

**Step 2:** Using (22), the evolution of the conditional state vectors  $M_t$  and  $M_t^U$  can now be found:

$$M_{t+1} = A_M \cdot M_t + B_M \cdot \epsilon_{t+1}, \quad (23)$$

$$M_{t+1}^U = A_M^U \cdot M_t^U + B_M^U \cdot \epsilon_{t+1}^U, \quad (24)$$

$$\text{with } A_M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ (1 - \rho_F)\bar{F} & \rho_F & 0 & 0 \\ (1 - \rho_\theta)\bar{\theta} & 0 & \rho_\theta & 0 \\ 0 & 0 & 0 & \rho_U \end{bmatrix}, \quad B_M = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ B_U(2,1) & B_U(2,2) & B_U(2,3) \end{bmatrix},$$

$$A_M^U = A_\Psi, \quad B_M^U = K(A_s A_\Psi l_0 v_0 + A_s B_\Psi + B_s), \quad l_0 \equiv [0 \quad -1 \quad -\lambda^{-1}]', \quad \text{and } v_0 \equiv [0 \quad 0 \quad 0 \quad 1].$$

Expressions (23) and (24) can now be used to express the conditional moments of future returns for informed and uninformed investors. Note that the forecast  $F_t^U$  by in the price equation (6) can be replaced to express prices as a function of current state variables and the forecast error of the uninformed,

$$P_t = p_0 + p_1 \cdot F_t + p_2 \cdot \theta_t + p_3 \cdot \tilde{F}_t^U \equiv P \cdot M_t, \quad (25)$$

where  $p_1 = \hat{p}_F + p_F$ ,  $p_2 = p_\theta$ ,  $p_3 = \hat{p}_F$ , and  $P \equiv [p_0 \quad p_1 \quad p_2 \quad p_3]$ . Moreover, writing the dividend in (1) as  $D_{t+1} = A_D \cdot \Psi_{t+1} + B_D \cdot \epsilon_{t+1}$ , with  $A_D \equiv [0 \quad 1 \quad 0]$ ,  $B_D \equiv [1 \quad 0 \quad 0]$ , the future return  $Q_{t+1}$  can now be written as:

$$Q_{t+1} = A_Q \cdot M_t + B_Q \cdot \epsilon_{t+1}, \quad (26)$$

$$\text{with } A_Q \equiv A_D A_M + P \cdot (A_M - I_4 R), \quad \text{and } B_Q \equiv (A_D + P) \cdot B_M + B_D.$$

For uninformed investors, the corresponding expression in terms of their conditional expectations and forecast errors is:

$$Q_{t+1} = A_Q^U \cdot M_t^U + B_Q^U \cdot \epsilon_{t+1}^U, \quad (27)$$

$$\text{with } A_Q^U \equiv A_Q m_0, \quad B_Q^U \equiv A_Q m_1 v_0 + B_Q m_0', \quad m_0 \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}', \quad \text{and } m_1 \equiv [0 \quad -1 \quad \lambda^{-1} \quad 1]'$$

Expression (26) and (27) confirm that future returns indeed follow a conditional gaussian distribution for both informed and uninformed investors.

**Step 3:** Optimal investment and consumption policies can now be found by applying a known result on value functions in gaussian-exponential environments (see Vives, 2008). In particular, the conjectured value function at  $t + 1$  for an informed investor aged  $j$  at time  $t$  takes the form

$$J^I(W_{j+1,t+1}^I; M_{t+1}; j + 1; t + 1) = -\beta^{t+1} \cdot \exp\{-\alpha_{j+1} W_{j+1,t}^I - \frac{1}{2} M_{t+1}' V_{j+1}^I M_{t+1}\}.$$

The value function at  $t$  then takes the form

$$\begin{aligned}
J^I(W_{j,t}^I; M_t; j; t) &= \max_{\{X,C\}} - \exp\{-\gamma C\} - \beta \delta_{j+1}^I \exp\{-\alpha_{j+1} R(W_{j,t}^I + C) - \alpha_{j+1} X A_Q M_t \quad (28) \\
&\quad - \frac{1}{2} M_t' v_{j+1}^{I,aa} M_t + \frac{1}{2} (\alpha_{j+1} X B_Q + M_t' v_{j+1}^{I,ab} M_t) (\Xi_{j+1}^I)^{-1} (\alpha_{j+1} X B_Q + M_t' v_{j+1}^{I,ab} M_t)'\}, \\
\text{with } v_{j+1}^{I,aa} &\equiv A_M' V_{j+1}^I A_M; \quad v_{j+1}^{I,bb} \equiv B_M' V_{j+1}^I B_M; \quad v_{j+1}^{I,ab} \equiv A_M' V_{j+1}^I B_M; \\
\Xi_{j+1}^I &\equiv \Sigma^{-1} + v_{j+1}^{I,bb}, \quad \text{and } \delta_{j+1}^I \equiv |\Sigma \cdot \Xi_{j+1}^I|^{-1/2}.
\end{aligned}$$

The optimal consumption/investment policies can now be found through differentiation. Letting  $\Gamma_{j+1}^I \equiv B_Q (\Xi_{j+1}^I)^{-1} B_Q'$ , we get

$$X_{j,t}^I = \frac{A_Q - h_{j+1}^I}{\alpha_{j+1} \Gamma_{j+1}^I} \cdot M_t, \quad (29)$$

$$C_{j,t}^I = c_{j+1}^I + \left( \frac{\alpha_{j+1} R}{\alpha_{j+1} R + \gamma} \right) \cdot W_{j,t}^I + \frac{M_t' m_{j+1}^I M_t}{2(\alpha_{j+1} R + \gamma)}, \quad (30)$$

$$\begin{aligned}
\text{where } c_{j+1}^I &\equiv \log(\gamma / (\beta \delta_{j+1}^I \alpha_{j+1} R)) / (\alpha_{j+1} R + \gamma); \quad h_{j+1}^I \equiv B_Q (\Xi_{j+1}^I)^{-1} v_{j+1}^{I,ab'}, \\
\text{and } m_{j+1}^I &\equiv g'_{j+1} (\Gamma_{j+1}^I)^{-1} g_{j+1} + v_{j+1}^{I,aa} - v_{j+1}^{I,ab} (\Xi_{j+1}^I)^{-1} v_{j+1}^{I,ab'}.
\end{aligned}$$

The matrices  $V_{j+1}^I$  and scalars  $\alpha_j$ , for  $j = 1, 2 \dots T$ , can be solved recursively by setting  $\alpha_T = \gamma$ ,  $V_T^I = 0_4$ , and using the recursive equations found in the f.o.c.s:

$$\alpha_j = \frac{\gamma \alpha_{j+1} R}{\alpha_{j+1} R + \gamma}, \quad \text{and } V_j^I = m_{t+1}^I \cdot \left( \frac{\alpha_j}{\alpha_{j+1} R} \right) + 2 \cdot i_{1,1} \cdot (\gamma c_{j+1}^I + \log \frac{\alpha_j}{\gamma}). \quad (31)$$

where  $i_{1,1}$  is a 4x4 matrix with all element equal to zero except for the first, which equals one. For the uninformed investors the procedure is similar, but one must replace  $M_t$ ,  $A_Q$ ,  $B_Q$ , and  $\Sigma$ , for  $M_t^U$ ,  $A_Q^U$ ,  $B_Q^U$ , and  $\Delta$ , respectively. This allows solving investment and consumption policies, using the appropriate superscript  $U$  instead of  $I$ . I can now imposing the market clearing condition (19). Note that although the conditional state vector has different dimensions for the informed and uninformed investors, the uninformed forecasts can be replaced by the actual values of the state variables plus the forecast noise. This leads to the following equation that must be satisfied by the price:

$$P = Y \cdot Z^{-1} \quad (32)$$

where

$$\begin{aligned}
Z &\equiv [(A_M - I_4 R) \cdot (m_0 \bar{w}_0^U - m_1 v_0 \bar{w}_1^U) \cdot (m'_0 + x_0) + I_4 \bar{w}_0^I - B_M \cdot (m'_0 \bar{w}_1^U \cdot (m'_0 + x_0) + I_4 \bar{w}_1^I)], \\
Y &\equiv [A_\theta - (A_D A_M m_0 \bar{w}_0^U - (A_D A_M m_1 v_0 + (A_D B_M + B_D) m'_0) \bar{w}_1^U) \cdot (m'_0 + x_0) \cdot \\
&\quad A_D A_M \bar{w}_0^I + (A_D B_M + B_D) \bar{w}_1^I], \quad \bar{w}_0^U \equiv \frac{1}{T} \sum_{s=2}^T (\alpha_s \Gamma_s^U)^{-1}, \quad \bar{w}_0^I \equiv \frac{1}{T} \sum_{s=2}^T (\alpha_s \Gamma_s^I)^{-1}, \\
\bar{w}_1^U &\equiv \frac{1}{T} \sum_{s=2}^T (\Xi_s^U)^{-1} v_s^{U,ab'} (\alpha_s \Gamma_s^U)^{-1}, \quad \bar{w}_1^I \equiv \frac{1}{T} \sum_{s=2}^T (\Xi_s^I)^{-1} v_s^{I,ab'} (\alpha_s \Gamma_s^I)^{-1}, \\
A_\theta &\equiv [\bar{\theta} \ 0 \ 1 \ 0], \quad \text{and } x_0 \equiv [0 \ 1 \ -\lambda^{-1}]' \cdot v_0.
\end{aligned}$$

### Proof of Proposition 1:

This proof consists of three steps. First, I derive the price equation that arises from the market clearing condition. This leads to a quadratic equation for the random supply coefficient  $p_\theta$  in the price equation (6). I will show here that both roots of the equation depend on a particular coefficient of the value function matrix  $V$  (the stationary value function matrix in the infinite horizon case). This coefficient is the ninth element of the matrix, associated with the utility (value function) impact of random supply innovations, which I label  $V_\theta$ . In the second step of the proof, I derive a second equation which describes the element  $V_\theta$  as a function of the coefficient  $p_\theta$ . An equilibrium is a pair  $\{p_\theta, V_\theta\}$  satisfying both equations. In the third step, I show that a) an intersection between these functions always exists (part a) of the proposition), and b) it is unique (part b) of the proposition).

**Step 1:** In the infinite horizon case with symmetric information, there exists only one type of investor whose asset demand (equation (29)) can be restated (dropping the age and information subscripts) as:

$$X_t = \frac{A_Q - B_Q \Xi^{-1} v^{ab'}}{\alpha \Gamma} \Psi_t,$$

where  $A_Q = [0 \ (1 - rp_f) \ -(R - \rho)p_\theta]$  and  $B_Q = [(1 + p_f) \ p_\theta]$  are the row vectors associated with the loadings of future returns on the vector of current state variables, and future disturbances, respectively. It is straightforward to show that for this economy,

$$\Xi = \begin{bmatrix} \sigma_F^{-2} & 0 \\ 0 & \sigma_\theta^{-2} + V_\theta \end{bmatrix}, \quad \Gamma = (1 + p_F)^2 \sigma_F^2 + p_\theta^2 \sigma_\theta^2, \quad \text{and } \alpha = \frac{\gamma r}{R}.$$

Moreover, from equation (28), it can be shown that the second term in the demand's numerator corresponds to  $B_Q \Xi^{-1} v^{ab'} = [0 \ 0 \ p_\theta \rho_\theta V_\theta / (\sigma_\theta^{-2} + V_\theta)]$ . However, in the infinite horizon economy there are no forced liquidations, so that the supply persistent coefficient is null,  $\rho_\theta = 0$ , which simplifies the demand equation considerably. The market clearing condition then simply reads

$$\frac{[0 \quad (1 - rp_F) \quad - (R - \rho)p_\theta]}{\gamma(r/R)((1 + p_F)^2\sigma_F^2 + p_\theta^2(\sigma_\theta^{-2} + V_\theta)^{-1})} \cdot [1 \quad F_t \quad \theta_t]' = [0 \quad 0 \quad 1] \cdot [1 \quad F_t \quad \theta_t]',$$

which gives rise to two equations determining the price coefficients  $p_F$  and  $p_\theta$ .

$$0 = 1 - rp_F, \quad (33)$$

$$0 = p_\theta^2 + p_\theta(R^2/\gamma r)(\sigma_\theta^{-2} + V_\theta) + (R/r)^2\sigma_F^2(\sigma_\theta^{-2} + V_\theta). \quad (34)$$

**Step 2:** to find the dependence of the value function term  $V_\theta$  on the model parameters and the price coefficient  $p_\theta$ , note that we can adapt equation (31) above to the infinite horizon case to write

$$V = \frac{m}{R} + 2 \cdot i_{1,1} \cdot (\gamma c + \log \frac{\alpha}{\gamma}), \quad (35)$$

but since  $i_{1,1}$  is zero for all terms besides the first, the expression for the ninth term in  $V$  reduces to

$$\begin{aligned} V_\theta &= \frac{m(3,3)}{R} = \frac{R}{p_\theta^2}(1 + p_F)^2\sigma_F^2 + p_\theta^2(\sigma_\theta^{-2} + V_\theta)^{-1}, \text{ or} \\ p_\theta &= -\frac{R\sigma_F}{r} \frac{V_\theta^{1/2}}{(R - V_\theta/(\sigma_\theta^{-2} + V_\theta))^{1/2}}, \end{aligned} \quad (36)$$

where I focus only on the negative root.<sup>13</sup>

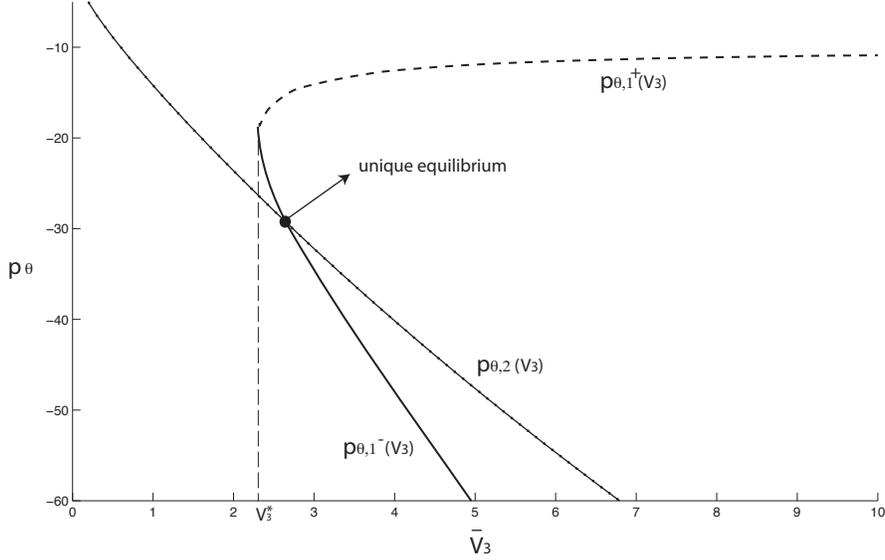
**Step 3:** I now study equations (34) and (36). Figure 8 provides the loci of these equations in the  $\{V_\theta, p_\theta\}$  space. I have set all parameters values in this figure to one, but the shape of the figure remains qualitatively similar for any parameter combination (aside from scaling effects). Note that to find existence, it suffices to show that, over the range of values of  $V_\theta$  for which (34) has a real solution, it intersects with equation (36) at least once. Uniqueness then relies, of course, in showing that this intersection is indeed the only possible one. It is convenient to label the function in (34) as  $p_{\theta,1}^+(V_\theta)$  and  $p_{\theta,1}^-(V_\theta)$ , denoting the positive and negative roots. Likewise, I label the function in (36) as  $p_{\theta,2}(V_\theta)$ . I will prove existence and uniqueness simultaneously by establishing the following facts:

- i)  $\partial p_{\theta,1}^+(\cdot)/\partial V_\theta > 0$ ;
- ii)  $\partial p_{\theta,1}^-(\cdot)/\partial V_\theta < 0$ ,  $\partial^2 p_{\theta,1}^-(\cdot)/\partial V_\theta^2 > 0$ , and  $\lim_{V_\theta \rightarrow \infty} \partial p_{\theta,1}^-(\cdot)/\partial V_\theta = \text{constant}$ ,
- iii)  $\partial p_{\theta,2}(\cdot)/\partial V_\theta < 0$ ,  $\partial^2 p_{\theta,2}(\cdot)/\partial V_\theta^2 > 0$ , and  $\lim_{V_\theta \rightarrow \infty} \partial p_{\theta,2}(\cdot)/\partial V_\theta = 0$ , and
- iv)  $p_{\theta,2}(V_\theta^*) < p_{\theta,1}^-(V_\theta^*)$ , where  $V_\theta^*$  is implicitly defined by the point where  $p_{\theta,1}^-(\cdot) = p_{\theta,1}^+(\cdot)$ .

Indeed, fact iv) implies that, at the lowest (positive) value of  $V_\theta$  for which (34) admits a real solution, equation  $p_{\theta,2}(\cdot)$  lies strictly below. Together with the convergence result of the equations' slopes stated in facts ii) and iii), this implies that the negative root  $p_{\theta,1}^-(\cdot)$  and  $p_{\theta,2}(\cdot)$  will intersect at least once, establishing existence. Moreover, the monotonicity of slopes established in facts ii) and iii) also implies

<sup>13</sup>It is easy to show that positives values of  $p_\theta$  are not sustainable as an equilibrium.

Figure 8: Infinite horizon economy with symmetric information



that this intersection is unique. Finally, the positive slope of the positive root  $p_{\theta,1}^+(\cdot)$  established in part i) implies that equation  $p_{\theta,2}(\cdot)$  can never intersect it, establishing global uniqueness of the solution.

Equation (34) can be rewritten as:

$$p_{\theta,1}^+ = \frac{-R^2(\sigma_\theta^{-2} + V_\theta)}{2\gamma r} + \frac{R^2(\sigma_\theta^{-2} + V_\theta)}{2\gamma r} \sqrt{1 - (2\gamma\sigma_F/R)^2(\sigma_\theta^{-2} + V_\theta)^{-1}}, \quad (37)$$

$$p_{\theta,1}^- = \frac{-R^2(\sigma_\theta^{-2} + V_\theta)}{2\gamma r} - \frac{R^2(\sigma_\theta^{-2} - V_\theta)}{2\gamma r} \sqrt{1 - (2\gamma\sigma_F/R)^2(\sigma_\theta^{-2} + V_\theta)^{-1}}. \quad (38)$$

To establish fact i), simply derive equation (37) w.r.t.  $V_\theta$ , which yields the result immediately. For fact ii), let  $K \equiv 2\gamma\sigma_F/R$ . Deriving the negative root in equation (38) yields

$$\partial p_{\theta,1}^-(\cdot)/\partial V_\theta = \frac{R^2}{2\gamma r} \left[ -1 - \frac{\sigma_\theta^{-2} + V_\theta - K^2/2}{((\sigma_\theta^{-2} + V_\theta - K^2)(\sigma_\theta^{-2} + V_\theta))^{1/2}} \right]$$

Note that because  $\sigma_\theta^{-2} + V_\theta \geq K^2$  for all values of  $V_\theta$  which admit a real solution to equation (34), it follows that the slope is unambiguously negative (and it can be trivially checked that its second derivative is strictly positive). Moreover, as  $V_\theta \rightarrow \infty$ , the above expression converges to  $-R^2/(\gamma r)$ . To establish fact (iii), let  $J \equiv R\sigma_F/r$ . The slope of equation (36) is then given by

$$\begin{aligned} \partial p_{\theta,2}(\cdot)/\partial V_\theta &= -\frac{L}{2} \left[ (V_\theta(R - V_\theta(\sigma_\theta^{-2} + V_\theta)^{-1}))^{-1/2} \right. \\ &\quad \left. + ((\sigma_\theta^{-2} + V_\theta)(R - V_\theta(\sigma_\theta^{-2} + V_\theta)^{-1}))^{-3/2} \sigma_\theta^{-2} (\sigma_\theta^{-2} + V_\theta)^{-1/2} \right], \end{aligned}$$

which is strictly negative. From inspection of the above expression, it is trivial to check that the second derivative  $\partial^2 p_{\theta,2}(\cdot)/\partial V_\theta^2$  is strictly positive. Moreover, as  $V_\theta \rightarrow \infty$ , the expression converges to zero. To establish fact iv), compute the value of  $p_\theta$  that attains for the lowest value of  $V_\theta = V^*$  admitting a real solution to equation (34). This yields  $p_{\theta,1}^-(V^*) = -2\gamma\sigma_F^2/r$ . If we now replace this value for  $p_\theta$  in equation 36, it can be easily established that the corresponding value of  $V_\theta$  is strictly lower than  $V^*$ . This implies fact iv) since  $\partial p_{\theta,2}(\cdot)/\partial V_\theta$ , which completes the proof.