FEDERAL RESERVE BANK *of* NEW YORK

Intermediary Leverage Cycles and Financial Stability

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Outline

Questions about Financial Stability Policy

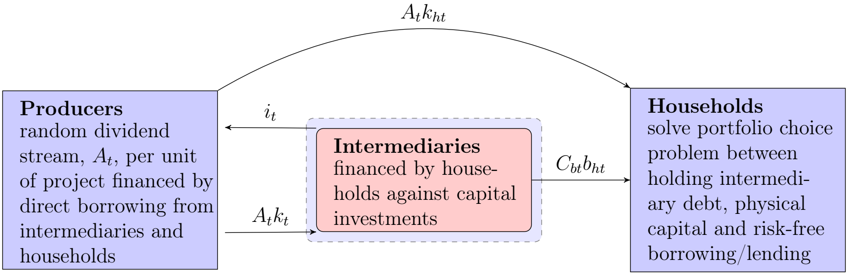
- Systemic distress of financial intermediaries raises questions about financial stability policies:
  - How does capital regulation affect the tradeoff between the pricing of credit and the amount of systemic risk?
  - How does macroprudential policy function in equilibrium?
  - What are the welfare implications of capital regulation?
- We develop a theoretical framework to address these questions

Our Approach

- We use a standard macro model with a financial sector and add one key assumption:
  - Funding constraints of financial intermediaries are risk based, so intermediaries have to hold more capital when the riskiness of their assets increases
- This assumption is empirically motivated and it allows us to capture stylized facts about:
  - Procyclical leverage of intermediary balance sheets
  - Procyclical share of intermediated credit
  - Relationship between asset risk premia and intermediary leverage

Outline

Economy Structure



Production

- Aggregate amount of capital K_t evolves as

$$dK_t = (I_t - \lambda_k)K_t dt$$

- Total output evolves as

$$Y_t = A_t K_t$$

- Stochastic productivity of capital $\{A_t = e^{a_t}\}_{t \geq 0}$

$$da_t = \bar{a}dt + \sigma_a dZ_{at}$$

- $p_{kt}A_t$ denotes the price of one unit of capital in terms of the consumption good

Households

- Household preferences are:

$$\mathbb{E}\left[\int_0^{+\infty} e^{-(\xi_t+\rho_h t)} \log c_t dt\right]$$

- Liquidity preference shocks (as in Allen and Gale (1994) and Diamond and Dybvig (1983)) are $\exp(-\xi_t)$

$$d\xi_t = \sigma_\xi \rho_{\xi,a} dZ_{at} + \sigma_\xi \sqrt{1-\rho_{\xi,a}^2} dZ_{\xi t}$$

- Households do not have access to the investment technology

$$dk_{ht} = -\lambda_k k_{ht} dt$$

Households' Optimization

$$\max_{\{c_t, k_{ht}, b_{ht}\}} \mathbb{E}\left[\int_0^{+\infty} e^{-(\xi_t+\rho_h t)} \log c_t dt\right]$$

subject to

$$dw_{ht} = r_{ft} w_{ht} dt + p_{kt} A_t k_{ht} \left(dR_{kt} - r_{ft} dt\right) + p_{bt} A_t b_{ht} \left(dR_{bt} - r_{ft} dt\right) - c_t dt$$

and no-shorting constraints

$$k_{ht} \geq 0$$
$$b_{ht} \geq 0$$

Intermediaries

- Financial intermediaries create new capital

$$dk_t = (\Phi(i_t) - \lambda_k)\, k_t dt$$

- Investment carries quadratic adjustment costs (Brunnermeier and Sannikov (2012))

$$\Phi(i_t) = \phi_0 \left(\sqrt{1 + \phi_1 i_t} - 1 \right)$$

- Intermediaries finance investment projects through inside equity and outside risky debt giving the budget constraint

$$p_{kt} A_t k_t = p_{bt} A_t b_t + w_t$$

Intermediaries' Risk Based Capital Constraint

- Risk based capital constraint (Danielsson, Shin, and Zigrand (2011))

$$\alpha\sqrt{\frac{1}{dt}\left\langle k_t d\left(p_{kt}A_t\right)\right\rangle^2} = w_t$$

- Implies a time-varying leverage constraint

$$\theta_t = \frac{p_{kt}A_t k_t}{w_t} = \frac{1}{\alpha\sqrt{\frac{1}{dt}\left\langle \frac{d(p_{kt}A_t)}{p_{kt}A_t}\right\rangle^2}}$$

- Note that the constraint is such that intermediary equity is proportional to the Value-at-Risk of total assets
- This will imply that default probabilities vary over time

Risk-based Capital Constraints

VaR is the potential loss in value of inventory positions due to adverse market movements over a defined time horizon with a specified confidence level. We typically employ a one-day time horizon with a 95% confidence level.

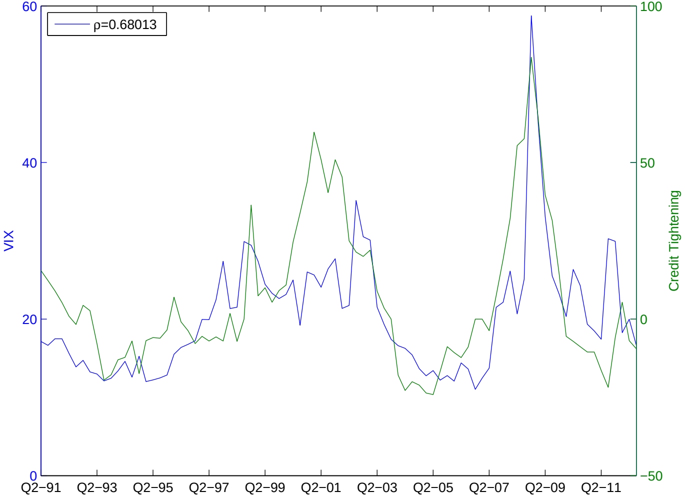
Average Daily VaR

in millions	Year Ended December		
Risk Categories	**2011**	2010	2009
Interest rates	**$ 94**	$ 93	$176
Equity prices	**33**	68	66
Currency rates	**20**	32	36
Commodity prices	**32**	33	36
Diversification effect [1]	**(66)**	(92)	(96)
Total	**$113**	$134	$218

1. Equals the difference between total VaR and the sum of the VaRs for the four risk categories. This effect arises because the four market risk categories are not perfectly correlated.

Source: Goldman Sachs 2011 Annual Report

Commercial Bank Lending Standards



Systemic Distress

- Solvency risk defined by

$$\tau_D = \inf_{t \geq 0} \{ w_t \leq \bar{\omega} p_{kt} A_t K_t \}$$

- Term structure of systemic distress

$$\delta_t(T) = \mathbb{P}(\tau_D \leq T | (w_t, \theta_t))$$

In distress

- Management changes
- Intermediary leverage reduced to $\underline{\theta} \approx 1$ by defaulting on debt
- Intermediary instantaneously restarts with wealth

$$w_{\tau_D^+} = \frac{\theta_{\tau_D}}{\underline{\theta}} w_{\tau_D}$$

Intermediaries' Optimization

- Intermediary maximizes equity holder value to solve

$$\max_{\{k_t,\beta_t,i_t\}} \quad \mathbb{E}\left[\int_0^{\tau_D} e^{-\rho t} w_t dt\right]$$

subject to the dynamic intermediary budget constraint

$$dw_t = k_t p_{kt} A_t \left(dR_{kt} + \left(\Phi\left(i_t\right) - i_t/p_{kt}\right) dt\right) - b_t p_{bt} A_t dR_{bt}$$

and the risk based capital constraint

$$\alpha \sqrt{\frac{1}{dt} \left\langle k_t d\left(p_{kt} A_t\right)\right\rangle^2} = w_t$$

Equilibrium

An equilibrium in this economy is a set of price processes $\{p_{kt}, p_{bt}, C_{bt}\}_{t\geq 0}$, a set of household decisions $\{k_{ht}, b_{ht}, c_t\}_{t\geq 0}$, and a set of intermediary decisions $\{k_t, \beta_t, i_t, \theta_t\}_{t\geq 0}$ such that:

1. Taking the price processes and the intermediary decisions as given, the household's choices solve the household's optimization problem, subject to the household budget constraint.
2. Taking the price processes and the household decisions as given, the intermediary's choices solve the intermediary optimization problem, subject to the intermediary wealth evolution and the risk based capital constraint.
3. The capital market clears:
$$K_t = k_t + k_{ht}.$$
4. The risky bond market clears:
$$b_t = b_{ht}.$$
5. The risk-free debt market clears:
$$w_{ht} = p_{kt} A_t k_{ht} + p_{bt} A_t b_{ht}.$$
6. The goods market clears:
$$c_t = A_t \left(K_t - i_t k_t\right).$$

Related Literature

- **Leverage Cycles:** Geanakoplos (2003), Fostel and Geanakoplos (2008), Brunnermeier and Pedersen (2009)
- **Amplification in Macroeconomy:** Bernanke and Gertler (1989), Kiyotaki and Moore (1997)
- **Financial Intermediaries and the Macroeconomy:** Gertler and Kiyotaki (2012), Gertler, Kiyotaki, and Queralto (2011), He and Krishnamurthy (2012a,b), Brunnermeier and Sannikov (2011, 2012)

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Solution Strategy

- Equilibrium is characterized by two state variables, leverage θ_t and relative intermediary net worth ω_t

$$\omega_t = \frac{w_t}{w_t + w_{ht}} = \frac{w_t}{p_{kt} A_t K_t}$$

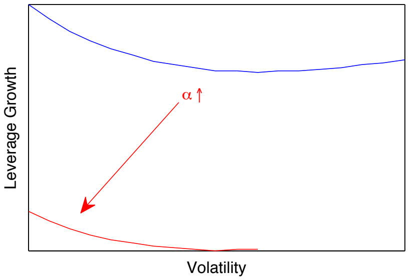
- Represent state dynamics as

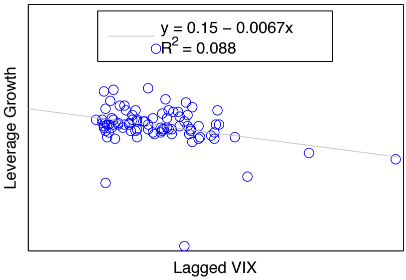
$$\frac{d\omega_t}{\omega_t} = \mu_{\omega t} dt + \sigma_{\omega a,t} dZ_{at} + \sigma_{\omega \xi,t} dZ_{\xi t}$$
$$\frac{d\theta_t}{\theta_t} = \mu_{\theta t} dt + \sigma_{\theta a,t} dZ_{at} + \sigma_{\theta \xi,t} dZ_{\xi t}$$

- Risk-based capital constraint implies

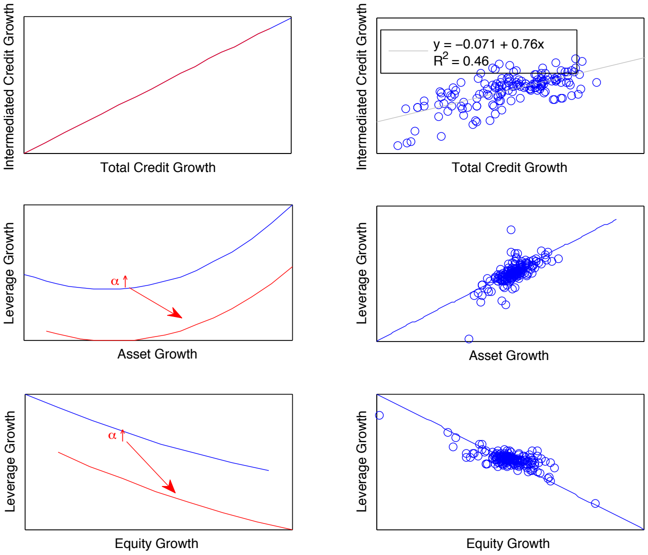
$$\alpha^{-2} \theta_t^{-2} = \sigma_{ka,t}^2 + \sigma_{k\xi,t}^2$$

Volatility Risk





Intermediary Balance Sheets



Optimal Household Choices

Denote by $\pi_{kt} = \left(p_{kt} A_t k_{ht}\right) / w_{ht}$ and $\pi_{bt} = \left(p_{bt} A_t b_{ht}\right) / w_{ht}$

Lemma 3.1

The household's optimal consumption choice satisfies:

$$c_t = \left(\rho_h - \frac{\sigma_\xi^2}{2}\right) w_{ht}.$$

In the unconstrained region, the household's optimal portfolio choice is given by:

$$\begin{bmatrix} \pi_{kt} \\ \pi_{bt} \end{bmatrix} = \left(\begin{bmatrix} \sigma_{ka,t} & \sigma_{k\xi,t} \\ \sigma_{ba,t} & \sigma_{b\xi,t} \end{bmatrix} \begin{bmatrix} \sigma_{ka,t} & \sigma_{ba,t} \\ \sigma_{k\xi,t} & \sigma_{b\xi,t} \end{bmatrix} \right)^{-1} \begin{bmatrix} \mu_{Rk,t} - r_{ft} \\ \mu_{Rb,t} - r_{ft} \end{bmatrix}$$
$$- \sigma_\xi \begin{bmatrix} \sigma_{ka,t} & \sigma_{ba,t} \\ \sigma_{k\xi,t} & \sigma_{b\xi,t} \end{bmatrix}^{-1} \begin{bmatrix} \rho_{\xi,a} \\ \sqrt{1 - \rho_{\xi,a}^2} \end{bmatrix}.$$

Equilibrium Expected Returns

Expected return to capital

$$\mu_{Rk,t} - r_{ft} = \underbrace{\left(\sigma^2_{ka,t} + \sigma^2_{k\xi,t}\right)\frac{1-\theta_t\omega_t}{1-\omega_t}}_{\text{compensation for own risk}} + \underbrace{\left(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t}\right)\frac{\omega_t\left(\theta_t-1\right)}{1-\omega_t}}_{\text{compensation for risk of correlated asset}}$$

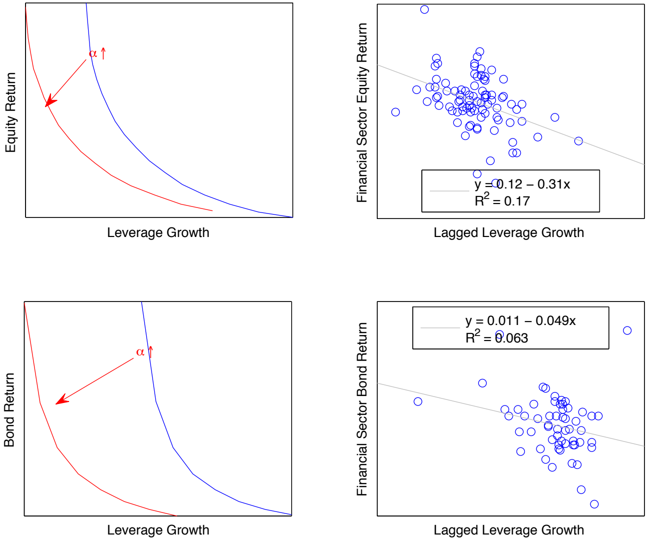
$$+ \underbrace{\sigma_\xi\left(\sigma_{ka,t}\rho_{\xi,a} + \sigma_{k\xi,t}\sqrt{1-\rho^2_{\xi,a}}\right)}_{\text{compensation for liquidity risk}}$$

Expected return to intermediary debt

$$\mu_{Rb,t} - r_{ft} = \underbrace{\left(\sigma^2_{ba,t} + \sigma^2_{b\xi,t}\right)\frac{\omega_t\left(\theta_t-1\right)}{1-\omega_t}}_{\text{compensation for own risk}} + \underbrace{\left(\sigma_{ka,t}\sigma_{ba,t} + \sigma_{k\xi,t}\sigma_{b\xi,t}\right)\frac{1-\theta_t\omega_t}{1-\omega_t}}_{\text{compensation for risk of correlated asset}}$$

$$+ \underbrace{\sigma_\xi\left(\sigma_{ba,t}\rho_{\xi,a} + \sigma_{b\xi,t}\sqrt{1-\rho^2_{\xi,a}}\right)}_{\text{compensation for liquidity risk}}$$

Excess Returns



Equilibrium Prices of Risk I

Shocks

$$
\begin{aligned}
d\hat{y}_t &= \sigma_a^{-1}\left(d\log Y_t - \mathbb{E}_t\left[d\log Y_t\right]\right) = dZ_{at} \\
d\hat{\theta}_t &= \left(\sigma^2_{\theta a,t} + \sigma^2_{\theta\xi,t}\right)^{-\frac{1}{2}}\left(\frac{d\theta_t}{\theta_t} - \mathbb{E}_t\left[\frac{d\theta_t}{\theta_t}\right]\right) \\
&= \frac{\sigma_{\theta a,t}}{\sqrt{\sigma^2_{\theta a,t} + \sigma^2_{\theta\xi,t}}} dZ_{at} + \frac{\sigma_{\theta\xi,t}}{\sqrt{\sigma^2_{\theta a,t} + \sigma^2_{\theta\xi,t}}} dZ_{\xi t}.
\end{aligned}
$$

Equilibrium Prices of Risk II

Price of risk of leverage

$$\eta_{\theta t} = \sqrt{1 + \frac{(\sigma_{ka,t} - \sigma_a)^2}{\sigma^2_{k\xi,t}}} \left(-\frac{2\theta_t \omega_t p_{kt}}{\beta (1 - \omega_t)} \sigma_{k\xi,t} + \sigma_\xi \sqrt{1 - \rho^2_{\xi,a}} \right)$$

- Price of risk of leverage is always positive (Adrian, Etula, and Muir (2011)), and depends on leverage growth in a nonmonotonic fashion (Adrian, Moench, and Shin (2010) find a negative relationship)

Equilibrium Prices of Risk III

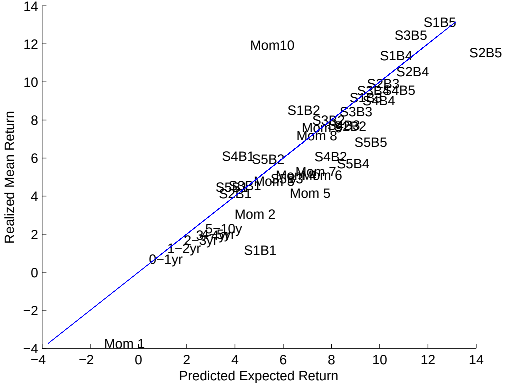


Figure: Source: Adrian, Etula, and Muir (2011)

Equilibrium Prices of Risk IV

Price of risk of output

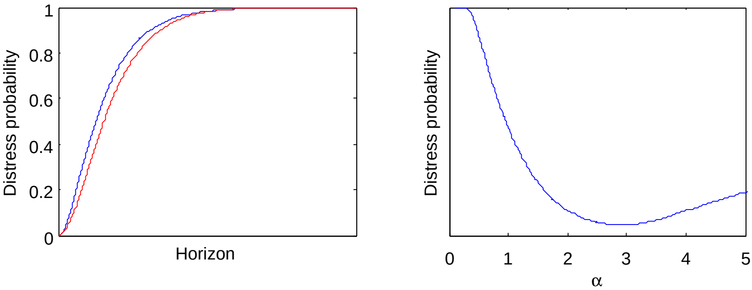
$$\eta_{yt} = \sigma_a + \sigma_\xi \left(\rho_{\xi,a} - \frac{\sigma_{ka,t} - \sigma_a}{\sigma_{k\xi,t}} \sqrt{1 - \rho_{\xi,a}^2} \right)$$

- Switches sign, consistent with insignificant estimates of price of output risk
- Usually becomes negative when exposure to liquidity shock is small

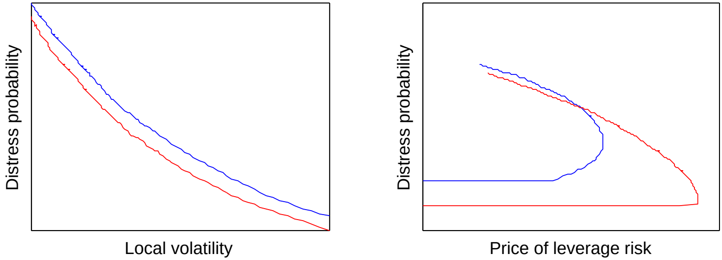
More

Outline

Term Structure of Systemic Risk



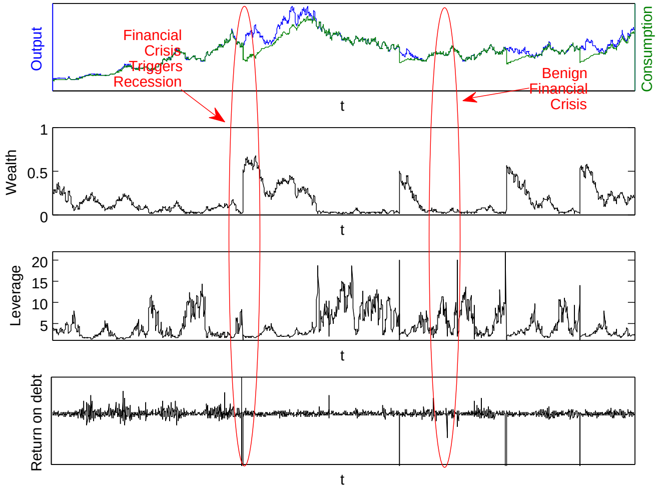
Volatility Paradox



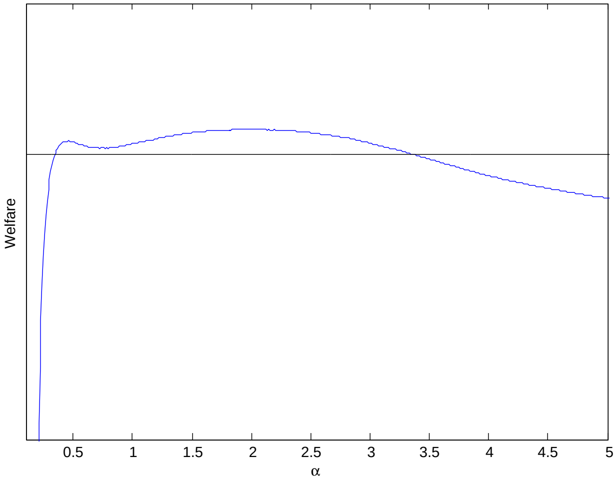
Constant Leverage Benchmark

- Constant expected output and consumption growth
- But lower level of output and consumption growth
- Constant investment and price of capital
- Liquidity shocks have no impact on real activity

A Sample Path of the Economy



Household Welfare



Conclusion

- Dynamic, general equilibrium theory of financial intermediaries' leverage cycle with endogenous amplification and endogenous systemic risk
- Conceptual basis for policies towards financial stability
- Systemic risk return tradeoff: tighter intermediary capital requirements tend to shift the term structure of systemic downward, at the cost of increased prices of risk today

- Model captures important stylized facts:
  1. Procyclical intermediary leverage
  2. Procyclicality of intermediated credit
  3. Financial sector equity return and intermediary leverage growth
  4. Exposure to intermediary leverage shocks as pricing factor

Tobias Adrian, Emanuel Moench, and Hyun Song Shin. Financial Intermediation, Asset Prices, and Macroeconomic Dynamics. Federal Reserve Bank of New York Staff Reports No. 442, 2010.

Tobias Adrian, Erkko Etula, and Tyler Muir. Financial Intermediaries and the Cross-Section of Asset Returns. Federal Reserve Bank of New York Staff Reports No. 464, 2011.

Franklin Allen and Douglas Gale. Limited market participation and volatility of asset prices. *American Economic Review*, 84:933–955, 1994.

Ben Bernanke and Mark Gertler. Agency Costs, Net Worth, and Business Fluctuations. *American Economic Review*, 79(1):14–31, 1989.

Markus K. Brunnermeier and Lasse Heje Pedersen. Market Liquidity and Funding Liquidity. *Review of Financial Studies*, 22(6):2201–2238, 2009.

Markus K. Brunnermeier and Yuliy Sannikov. The I Theory of Money. Unpublished working paper, Princeton University, 2011.

Markus K. Brunnermeier and Yuliy Sannikov. A Macroeconomic Model with a Financial Sector. Unpublished working paper, Princeton University, 2012.

Jon Danielsson, Hyun Song Shin, and Jean-Pierre Zigrand. Balance sheet capacity and endogenous risk. Working Paper, 2011.

Douglas W. Diamond and Philip H. Dybvig. Bank runs, deposit insurance and liquidity. *Journal of Political Economy*, 93(1):401–419, 1983.

Ana Fostel and John Geanakoplos. Leverage Cycles and the Anxious Economy. *American Economic Review*, 98(4):1211–1244, 2008.

John Geanakoplos. Liquidity, Default, and Crashes: Endogenous Contracts in General Equilibrium. In M. Dewatripont, L.P. Hansen, and S.J. Turnovsky, editors, *Advances in Economics and Econometrics II*, pages 107–205. Econometric Society, 2003.

Mark Gertler and Nobuhiro Kiyotaki. Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy. Unpublished working papers, Princeton University, 2012.

Mark Gertler, Nobuhiro Kiyotaki, and Albert Queralto. Financial Crises, Bank Risk Exposure, and Government Financial Policy. Unpublished working papers, Princeton University, 2011.

Zhiguo He and Arvind Krishnamurthy. A model of capital and crises. *Review of Economic Studies*, 79(2):735–777, 2012a.

Zhiguo He and Arvind Krishnamurthy. Intermediary asset pricing. *American Economic Review*, 2012b.

Nobuhiro Kiyotaki and John Moore. Credit Cycles. *Journal of Political Economy*, 105(2):211–248, 1997.