

MSCI

A Clear View of
Risk and Return

Supply–Demand Symmetry of Market Impact Models

November 2012

Carlo Acerbi

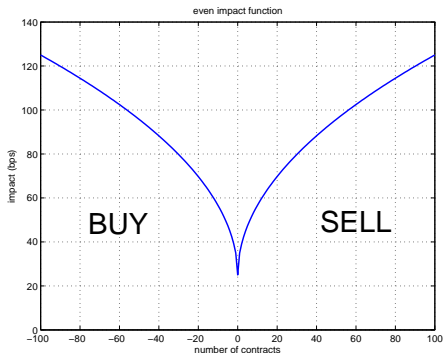
Outline

- 1 A Question Awaiting Formalization
- 2 Basic Formalism
 - Forex: Basic Facts
- 3 Supply–Demand Symmetry for Forex
- 4 But Then: Why Only Forex?
 - Geometrical Interpretation of Supply–Demand Symmetry
- 5 Some Results
- 6 Conclusions

A Question Awaiting Formalization

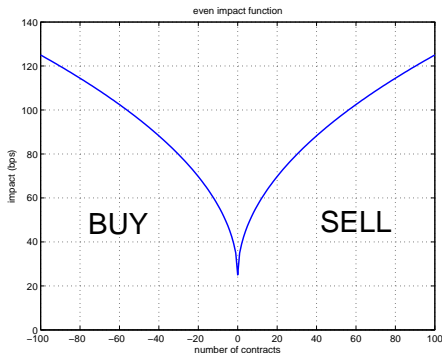
What is a Bid-Ask Symmetrical Market Impact?

- Is it an **even function** ?



What is a Bid–Ask Symmetrical Market Impact?

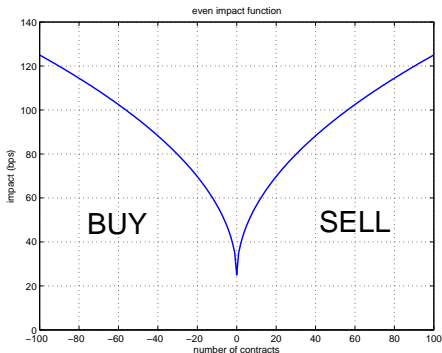
- Is it an even function ?



- Wow, nice plot! Great idea!

What is a Bid-Ask Symmetrical Market Impact?

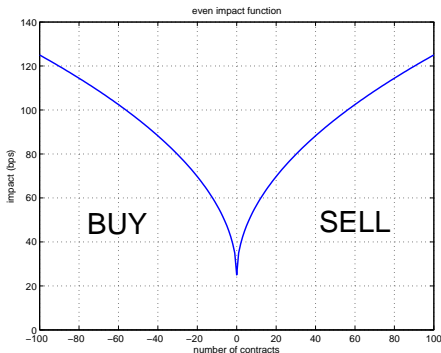
- Is it an even function ?



- Wow, nice plot! Great idea!
- But why even?

What is a Bid-Ask Symmetrical Market Impact?

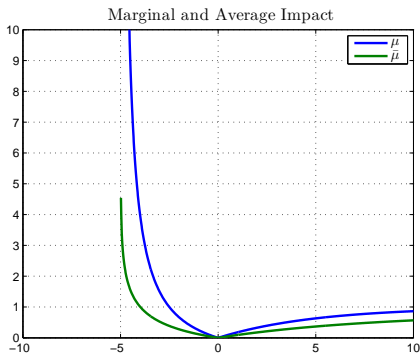
- Is it an even function ?



- Wow, nice plot! Great idea!
- But why even?
- Does it represent any financial symmetry?

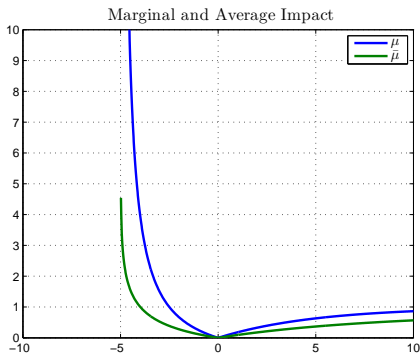
The Large Scale Picture Cannot Be Even

- Bid prices are floored at zero. Ask prices are not capped
- Bid impact is capped. Ask impact may be unbounded.



The Large Scale Picture Cannot Be Even

- Bid prices are floored at zero. Ask prices are not capped
- Bid impact is capped. Ask impact may be unbounded.



- Parity can't be a global fundamental symmetry

How to Formalize the Symmetry?

- What is the symmetry behind equivalence of supply and demand ?

How to Formalize the Symmetry?

- What is the symmetry behind equivalence of supply and demand ?
- Not so obvious for – say – equities or bonds

How to Formalize the Symmetry?

- What is the symmetry behind equivalence of supply and demand ?
- Not so obvious for – say – equities or bonds
- But **what about forex?** Buying one currency is selling another one

How to Formalize the Symmetry?

- What is the symmetry behind equivalence of supply and demand ?
- Not so obvious for – say – equities or bonds
- But **what about forex?** Buying one currency is selling another one
- **Buy/Sell symmetry \Leftrightarrow relativity under change of base currency**

How to Formalize the Symmetry?

- What is the symmetry behind equivalence of supply and demand ?
 - Not so obvious for – say – equities or bonds
 - But **what about forex?** Buying one currency is selling another one
 - Buy/Sell symmetry \Leftrightarrow relativity under change of base currency
-
- First we need an appropriate **formalism**

Basic Formalism

Price Impact of a General Security

Definition (Average price impact)

Expected price impact $\bar{\mu}(s, T)$ of an order of s contracts, executed through a time horizon T . Best execution assumed.

Price Impact of a General Security

Definition (**Average** price impact)

Expected price impact $\overline{\mu}(s, T)$ of an order of s contracts, executed through a time horizon T . Best execution assumed.

Definition (**Marginal** price impact)

Expected price impact $\mu(s, T)$ of additional ds contracts traded.

Price Impact of a General Security

Definition (Average price impact)

Expected price impact $\overline{\mu}(s, T)$ of an order of s contracts, executed through a time horizon T . Best execution assumed.

Definition (Marginal price impact)

Expected price impact $\mu(s, T)$ of additional ds contracts traded.

$$\overline{\mu}(s, T) = \frac{1}{s} \int_0^s \mu(z, T) dz$$

Price Impact of a General Security

Definition (Average price impact)

Expected price impact $\bar{\mu}(s, T)$ of an order of s contracts, executed through a time horizon T . Best execution assumed.

Definition (Marginal price impact)

Expected price impact $\mu(s, T)$ of additional ds contracts traded.

$$\bar{\mu}(s, T) = \frac{1}{s} \int_0^s \mu(z, T) dz$$

Convention

Buy trade if $s < 0$; sell trade if $s > 0$

Supply–Demand Curves

Let m be the fair price

Definition (Supply–Demand Curve (SDC))

Expected price of the entire order s

$$\overline{m}(s, T) = m - \text{sgn}(s) \overline{\mu}(s, T)$$

Supply–Demand Curves

Let m be the fair price

Definition (Supply–Demand Curve (SDC))

Expected price of the entire order s

$$\overline{m}(s, T) = m - \text{sgn}(s) \overline{\mu}(s, T)$$

Definition (Marginal Supply–Demand Curve (MSDC))

Expected price of additional ds contracts

$$m(s, T) = m - \text{sgn}(s) \mu(s, T)$$

Supply–Demand Curves

Let m be the fair price

Definition (Supply–Demand Curve (SDC))

Expected price of the entire order s

$$\overline{m}(s, T) = m - \text{sgn}(s) \overline{\mu}(s, T)$$

Definition (Marginal Supply–Demand Curve (MSDC))

Expected price of additional ds contracts

$$m(s, T) = m - \text{sgn}(s) \mu(s, T)$$

'Effective Order Book' Interpretation

We interpret a couple $\{ds, m(s, T)\}$ as a *quote* available within T

Supply–Demand Curves

Let m be the fair price

Definition (Supply–Demand Curve (SDC))

Expected price of the entire order s

$$\overline{m}(s, T) = m - \text{sgn}(s) \overline{\mu}(s, T)$$

Definition (Marginal Supply–Demand Curve (MSDC))

Expected price of additional ds contracts

$$m(s, T) = m - \text{sgn}(s) \mu(s, T)$$

'Effective Order Book' Interpretation

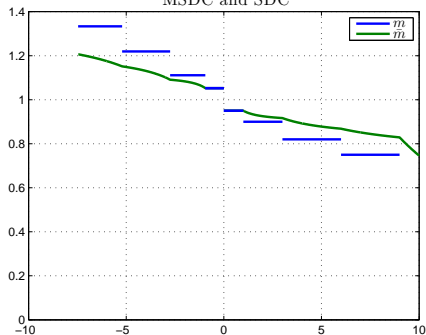
We interpret a couple $\{ds, m(s, T)\}$ as a *quote* available within T

Definition (Bid and Ask price)

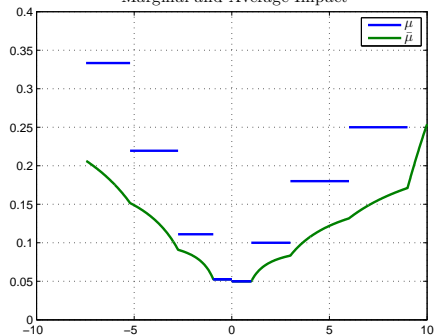
$$m^{\pm} \equiv m(0^{\pm}, \forall T) = \overline{m}(0^{\pm}, \forall T)$$

Example: Piecewise Constant MSDC

MSDC and SDC

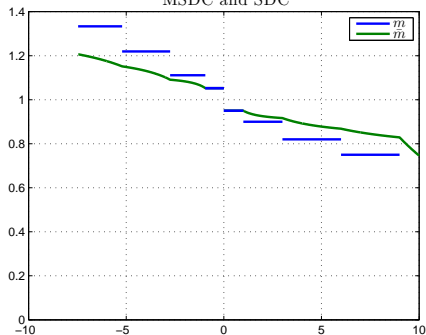


Marginal and Average Impact

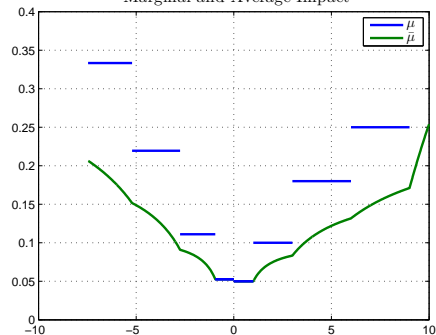


Example: Piecewise Constant MSDC

MSDC and SDC



Marginal and Average Impact



Is it symmetrical?

Liquidation Operator

Definition (Liquidation operator)

Expected order proceedings

$$L(s, T) = \overline{m}(s, T) s = \int_0^s m(z, T) dz$$

Cash in if $L > 0$, cash out if $L < 0$

Regular Market Hypothesis

The only fundamental hypotheses we make

Regular Market Hypothesis

The MSDC $m(s, T)$ is non-increasing in s , for all T .

Regular Market Hypothesis

The only fundamental hypotheses we make

Regular Market Hypothesis

The MSDC $m(s, T)$ is non-increasing in s , for all T .

- It amounts to impose that any further contract sold/bought meets worse and worse prices

Regular Market Hypothesis

The only fundamental hypotheses we make

Regular Market Hypothesis

The MSDC $m(s, T)$ is non-increasing in s , for all T .

- It amounts to impose that any further contract sold/bought meets worse and worse prices
- Or equivalently, that every quote in the market can be filled partially, for arbitrarily small sizes (no block quotes)

Regular Market Hypothesis

The only fundamental hypotheses we make

Regular Market Hypothesis

The MSDC $m(s, T)$ is non-increasing in s , for all T .

- It amounts to impose that any further contract sold/bought meets worse and worse prices
- Or equivalently, that every quote in the market can be filled partially, for arbitrarily small sizes (no block quotes)

Corollary

The liquidation operator L is concave

Forex: Just a Special Case of Security

- *One unit of foreign currency is just one particular security*

Forex: Just a Special Case of Security

- *One unit of foreign currency is just one particular security*
- *Fair exchange rate: X_f^d*
 - d : 'domestic' currency CCY_d
 - f : 'foreign' currency CCY_f
 - X_f^d expressed in CCY_d per unit CCY_f traded

Forex: Just a Special Case of Security

- *One unit of foreign currency is just one particular security*
- *Fair exchange rate: X_f^d*
 - d : 'domestic' currency CCY_d
 - f : 'foreign' currency CCY_f
 - X_f^d expressed in CCY_d per unit CCY_f traded
- MSDC: $X_f^d(s, T)$
 - s : number of 'foreign' currency units traded

Forex: Just a Special Case of Security

- *One unit of foreign currency is just one particular security*
- *Fair exchange rate: X_f^d*
 - d : 'domestic' currency CCY_d
 - f : 'foreign' currency CCY_f
 - X_f^d expressed in CCY_d per unit CCY_f traded
- MSDC: $X_f^d(s, T)$
 - s : number of 'foreign' currency units traded
- Similar convention for all other equivalent functions

Forex: Just a Special Case of Security

- *One unit of foreign currency is just one particular security*
- *Fair exchange rate: X_f^d*
 - d : 'domestic' currency CCY_d
 - f : 'foreign' currency CCY_f
 - X_f^d expressed in CCY_d per unit CCY_f traded
- MSDC: $X_f^d(s, T)$
 - s : number of 'foreign' currency units traded
- Similar convention for all other equivalent functions
- All the introduced functions admit a **dual representation** in the two currencies

Notation: Dropping Time Dependence

- In what follows, we fix some horizon T and we stop indicating it

Forex: Basic Facts

Forex–Duality of the Framework

- The order sizes s_a and s_b of a trade in the two currencies are related by

$$s_a = -L_b^a(s_b)$$

Forex–Duality of the Framework

- The order sizes s_a and s_b of a trade in the two currencies are related by

$$s_a = -L_b^a(s_b) \quad \text{but also} \quad s_b = -L_a^b(s_a)$$

Forex–Duality of the Framework

- The order sizes s_a and s_b of a trade in the two currencies are related by

$$s_a = -L_b^a(s_b) \quad \text{but also} \quad s_b = -L_a^b(s_a)$$

Proposition

Dual liquidation operators are related by

$$-L_a^b = (-L_b^a)^{[-1]}$$

Forex–Duality of the Framework

- The order sizes s_a and s_b of a trade in the two currencies are related by

$$s_a = -L_b^a(s_b) \quad \text{but also} \quad s_b = -L_a^b(s_a)$$

Proposition

Dual liquidation operators are related by

$$-L_a^b = (-L_b^a)^{[-1]}$$

Proposition

Dual MSDCs are related by

$$X_a^b(s_a)X_b^a(s_b) = 1$$

Dual SDCs are related by

$$\overline{X}_a^b(s_a)\overline{X}_b^a(s_b) = 1$$

Forex–Duality of the Framework

- The order sizes s_a and s_b of a trade in the two currencies are related by

$$s_a = -L_b^a(s_b) \quad \text{but also} \quad s_b = -L_a^b(s_a)$$

Proposition

Dual liquidation operators are related by

$$-L_a^b = (-L_b^a)^{[-1]}$$

Proposition

Dual MSDCs are related by

$$X_a^b(s_a)X_b^a(s_b) = 1$$

*Dual **SDCs** are related by*

$$\overline{X}_a^b(s_a)\overline{X}_b^a(s_b) = 1$$

far less obvious!

Proofs

Proof.

By definition of inverse

$$[-L_a^b] \circ [-L_b^a](s_b) = s_b$$

Differentiating both sides by s_b we obtain

$$X_a^b(-L_b^a(s_b)) X_b^a(s_b) = 1$$



Proofs

Proof.

By definition of inverse

$$[-L_a^b] \circ [-L_b^a](s_b) = s_b$$

Differentiating both sides by s_b we obtain

$$X_a^b(-L_b^a(s_b)) X_b^a(s_b) = 1$$



Proof.

Applying twice the definition of SDC: $L(s) = \overline{X}(s) s$

$$[-L_a^b] \circ [-L_b^a](s_b) = \overline{X}_a^b(-L_b^a(s_b)) \cdot L_b^a(s_b) = \overline{X}_a^b(-L_b^a(s_b)) \cdot \overline{X}_b^a(s_b) \cdot s_b = s_b$$



Supply–Demand Symmetry for Forex

Invariance Under Change of Base Currency

- To impose supply–demand symmetry, we require that the two dual forex impact functions look identical to two investors with opposite base currency.

Invariance Under Change of Base Currency

- To impose supply–demand symmetry, we require that the two dual forex impact functions look identical to two investors with opposite base currency.
- ... up to a constant rescaling to account for notional disparity of the two currency units

Invariance Under Change of Base Currency

- To impose supply–demand symmetry, we require that the two dual forex impact functions look identical to two investors with opposite base currency.
- ... up to a constant rescaling to account for notional disparity of the two currency units

Example

Suppose the ¥/€ rate is 100 ¥/€. In a symmetrical market, we expect that the relative impact of liquidating €100 and the relative impact of liquidating ¥10'000 should be the same

Imposing Supply–Demand Symmetry for Small Forex Trades

- Impose that the dual relative bid–offer spreads are identical

$$\frac{X^- - X^+}{X} = \frac{1/X^+ - 1/X^-}{1/X}$$

Solving for X yields

Imposing Supply–Demand Symmetry for Small Forex Trades

- Impose that the dual relative bid–offer spreads are identical

$$\frac{X^- - X^+}{X} = \frac{1/X^+ - 1/X^-}{1/X}$$

Solving for X yields

Proposition

In a symmetrical forex LS, the fair rate is the geometric average of the bid rate and the offer rate.

$$X_b^a = \sqrt{X_b^{a+} X_b^{a-}}$$

Imposing Supply–Demand Symmetry for Forex, in General

Definition (Forex Supply–Demand Symmetry)

We say that a forex market is **symmetrical**, if there exists a constant $\alpha > 0$ such that the mapping $s_a/\alpha \leftrightarrow s_b$

$$\frac{s_a}{\alpha} = -\frac{1}{\alpha} L_b^a(s_b)$$

is an **involution**

$$-\frac{1}{\alpha} L_b^a = \left(-\frac{1}{\alpha} L_b^a \right)^{[-1]}$$

Imposing Supply–Demand Symmetry for Forex, in General

Definition (Forex Supply–Demand Symmetry)

We say that a forex market is **symmetrical**, if there exists a constant $\alpha > 0$ such that the mapping $s_a/\alpha \leftrightarrow s_b$

$$\frac{s_a}{\alpha} = -\frac{1}{\alpha} L_b^a(s_b)$$

is an involution

$$-\frac{1}{\alpha} L_b^a = \left(-\frac{1}{\alpha} L_b^a \right)^{[-1]}$$

Proposition

If such α exists, it's the fair rate

$$\alpha = X_b^a = \sqrt{X_b^{a+} X_b^{a-}}$$

Imposing Supply–Demand Symmetry for Forex, in General

Definition (Forex Supply–Demand Symmetry)

We say that a forex market is **symmetrical**, if there exists a constant $\alpha > 0$ such that the mapping $s_a/\alpha \leftrightarrow s_b$

$$\frac{s_a}{\alpha} = -\frac{1}{\alpha} L_b^a(s_b)$$

is an involution

$$-\frac{1}{\alpha} L_b^a = \left(-\frac{1}{\alpha} L_b^a \right)^{[-1]}$$

Proposition

If such α exists, it's the fair rate ... as you may have guessed

$$\alpha = X_b^a = \sqrt{X_b^{a+} X_b^{a-}}$$

Classification of Forex Symmetrical Markets

Theorem

A forex market displays supply–demand symmetry if and only if the liquidation operator $s_b \mapsto L_b^a(s_b)$ can be expressed as

$$L_b^a(s_b) = -X_b^a \phi(s_b)$$

where the function $\phi : \mathcal{D}_b^a \rightarrow \mathcal{D}_b^a$

- 1** *is an involution $\phi = \phi^{[-1]}$*
- 2** *is convex and strictly decreasing*
- 3** *$\phi(0) = 0$*

Classification of Forex Symmetrical Markets

Theorem

A forex market displays supply–demand symmetry if and only if the liquidation operator $s_b \mapsto L_b^a(s_b)$ can be expressed as

$$L_b^a(s_b) = -X_b^a \phi(s_b)$$

where the function $\phi : \mathcal{D}_b^a \rightarrow \mathcal{D}_b^a$

- 1** *is an involution $\phi = \phi^{[-1]}$*
- 2** *is convex and strictly decreasing*
- 3** *$\phi(0) = 0$*

Corollary

In a symmetrical forex market the MSDC and the SDC satisfy

$$X_b^a(s)X_b^a(\tilde{s}) = (X_b^a)^2$$

$$\overline{X}_b^a(s)\overline{X}_b^a(\tilde{s}) = (X_b^a)^2$$

at conjugated points s and $\tilde{s} = \phi(s)$.

But Then: Why Only Forex?

A Currency is Just One Security Among All Others

- If you say

*"the euro for a yen based investor is as liquid an asset
as the yen is for a euro based investor"*

it might seem you're speaking of a forex symmetry only

A Currency is Just One Security Among All Others

- If you say

*“the euro for a yen based investor is as liquid an asset
as the yen is for a euro based investor”*

it might seem you're speaking of a forex symmetry only

- But if you equivalently say

“for a euro based investor selling yens is as liquid as buying yens”

you realize that the yen is just one security among all others

A Currency is Just One Security Among All Others

- If you say

*“the euro for a yen based investor is as liquid an asset
as the yen is for a euro based investor”*

it might seem you're speaking of a forex symmetry only

- But if you equivalently say

*“for a euro based investor selling **yens** is as liquid as buying **yens**”*

you realize that the yen is just one security among all others

- you could have been speaking of a stock, a gold bullion, an oil gallon, ...

A Currency is Just One Security Among All Others

- If you say

*“the euro for a yen based investor is as liquid an asset
as the yen is for a euro based investor”*

it might seem you're speaking of a forex symmetry only

- But if you equivalently say

*“for a euro based investor selling **yens** is as liquid as buying **yens**”*

you realize that the yen is just one security among all others

- you could have been speaking of a stock, a gold bullion, an oil gallon, ...
- similarities with “change of numeraire” type of symmetry

A Currency is Just One Security Among All Others

- If you say

*“the euro for a yen based investor is as liquid an asset
as the yen is for a euro based investor”*

it might seem you're speaking of a forex symmetry only

- But if you equivalently say

*“for a euro based investor selling **yens** is as liquid as buying **yens**”*

you realize that the yen is just one security among all others

- you could have been speaking of a stock, a gold bullion, an oil gallon, ...
- similarities with “change of numeraire” type of symmetry

Definition (Supply–Demand Symmetry for General Securities)

A security's market is symmetrical if it has the same properties of a forex symmetrical market

Classification of Symmetrical Markets for General Securities

Theorem

A security's market displays supply–demand symmetry if and only if the liquidation operator $s \mapsto L(s)$ can be expressed as

$$L(s) = -m\phi(s)$$

where the function $\phi : \mathcal{D} \rightarrow \mathcal{D}$

- 1** *is an involution $\phi = \phi^{[-1]}$*
- 2** *is convex and strictly decreasing*
- 3** *$\phi(0) = 0$*

Classification of Symmetrical Markets for General Securities

Theorem

A security's market displays supply–demand symmetry if and only if the liquidation operator $s \mapsto L(s)$ can be expressed as

$$L(s) = -m \phi(s)$$

where the function $\phi : \mathcal{D} \rightarrow \mathcal{D}$

- 1** *is an involution $\phi = \phi^{[-1]}$*
- 2** *is convex and strictly decreasing*
- 3** *$\phi(0) = 0$*

Corollary

In a security's symmetrical market the MSDC and the SDC satisfy

$$m(s) m(\tilde{s}) = m^2$$

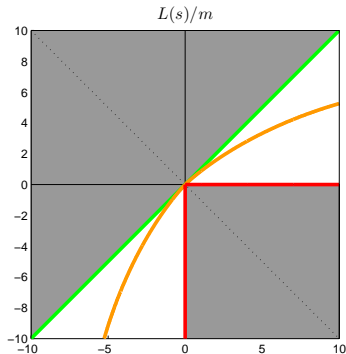
$$\overline{m}(s) \overline{m}(\tilde{s}) = m^2$$

at conjugated points s and $\tilde{s} = \phi(s)$.

Geometrical Interpretation of Supply–Demand Symmetry

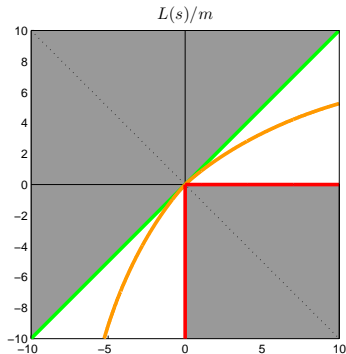
Geometrical Interpretation of Supply–Demand Symmetry

- $L(x)/m$: concave, symmetrical wrt $y = -x$, increasing, zero in zero



Geometrical Interpretation of Supply–Demand Symmetry

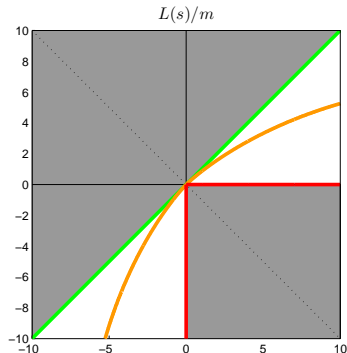
- $L(x)/m$: concave, symmetrical wrt $y = -x$, increasing, zero in zero



- the curve is forced to live in the white area of the plane

Geometrical Interpretation of Supply–Demand Symmetry

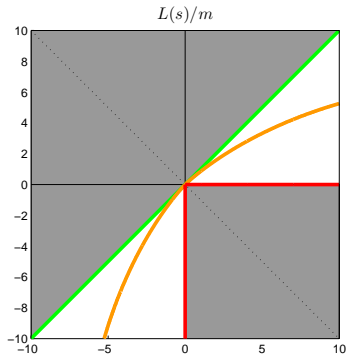
- $L(x)/m$: concave, symmetrical wrt $y = -x$, increasing, zero in zero



- the curve is forced to live in the white area of the plane
- two extremes: **perfectly liquid** and **perfectly illiquid** market

Geometrical Interpretation of Supply–Demand Symmetry

- $L(x)/m$: concave, symmetrical wrt $y = -x$, increasing, zero in zero



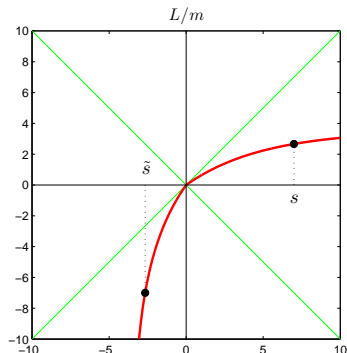
- the curve is forced to live in the white area of the plane
- two extremes: perfectly liquid and perfectly illiquid market
- natural notion of partial ordering of liquidity among different L 's

Supply–Demand Symmetry for Stocks, in Words

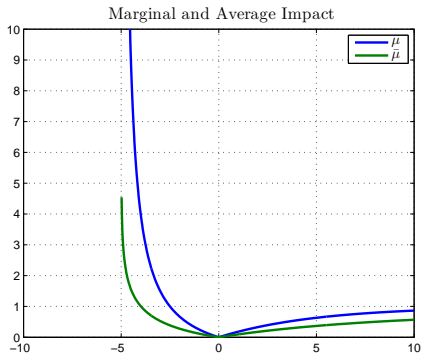
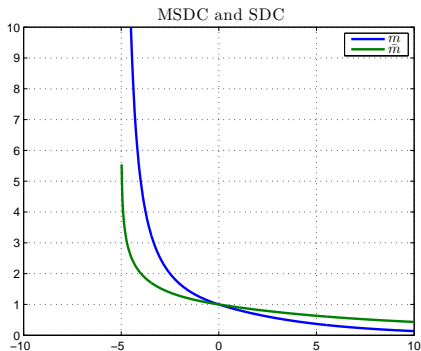
$$\tilde{s} = -L(s)/m \quad \text{and} \quad s = -L(\tilde{s})/m$$

Proposition (A Market is Symmetrical iff)

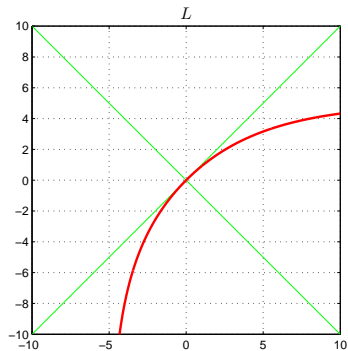
If $|\tilde{s}|$ stocks correspond in fair value to the liquidation of s stocks, then s stocks correspond in fair value to the cost of buying $|\tilde{s}|$ stocks, $\forall s$



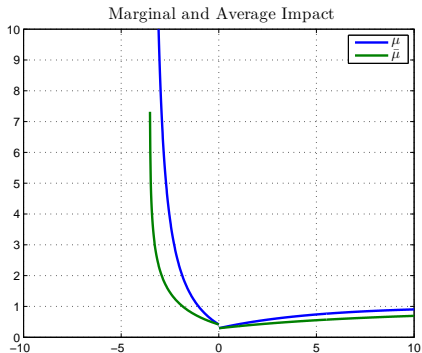
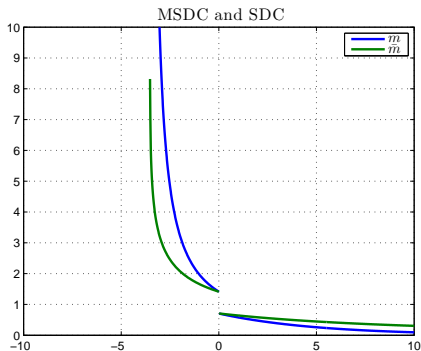
Example: Exponentially Decaying MSDC



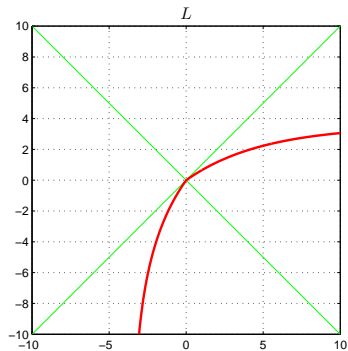
Example: Exponentially Decaying MSDC



Example: Exponentially Decaying MSDC with Spread

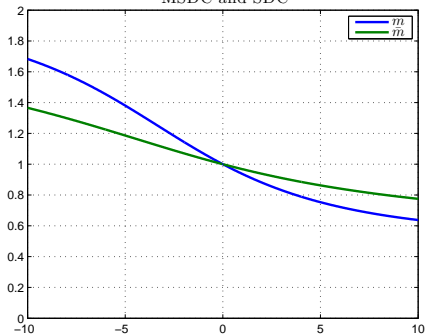


Example: Exponentially Decaying MSDC with Spread

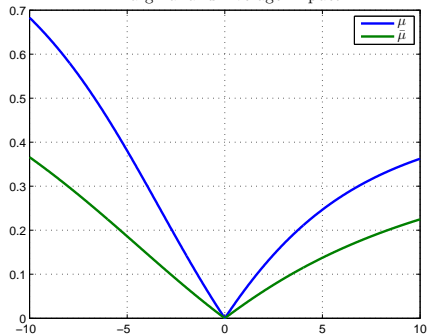


Example: Asymptotically Finite MSDC

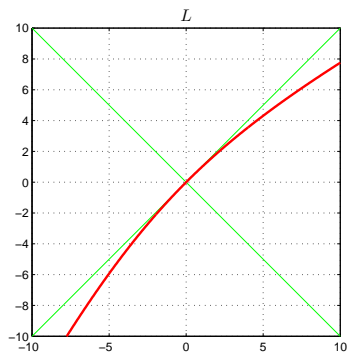
MSDC and SDC



Marginal and Average Impact

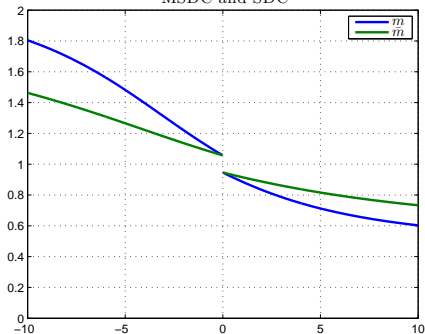


Example: Asymptotically Finite MSDC

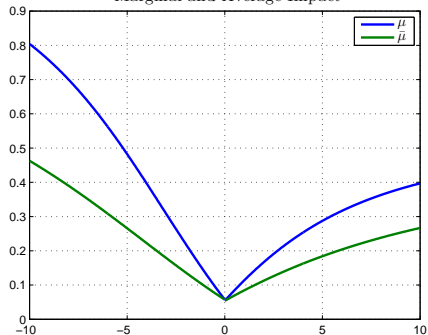


Example: Asymptotically Finite MSDC with Spread

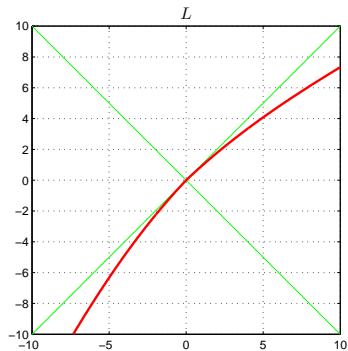
MSDC and SDC



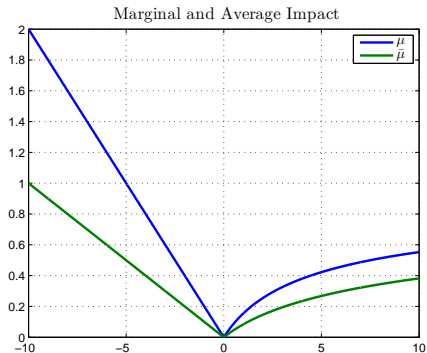
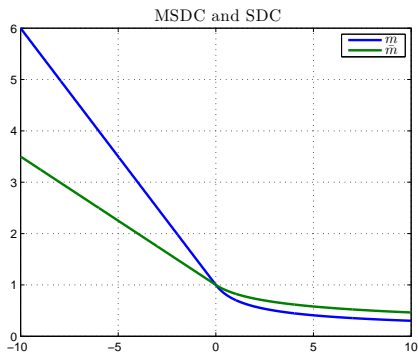
Marginal and Average Impact



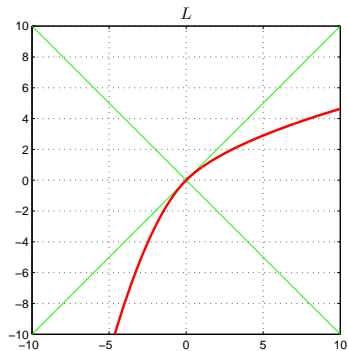
Example: Asymptotically Finite MSDC with Spread



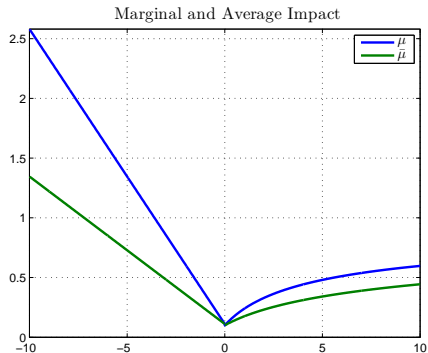
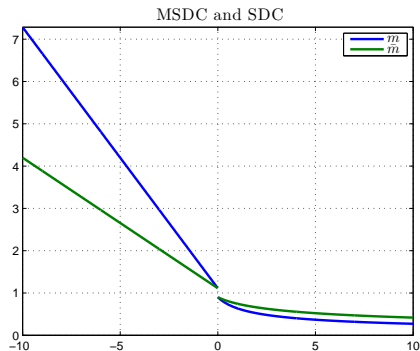
Example: Linear Ask MSDC



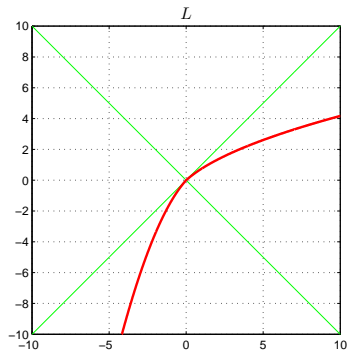
Example: Linear Ask MSDC



Example: Linear Ask MSDC with Spread

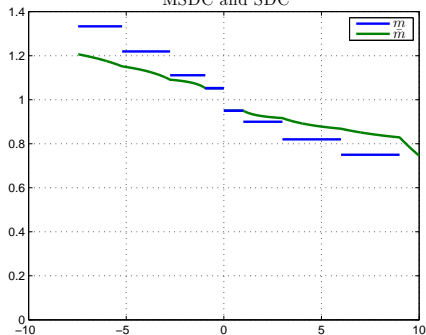


Example: Linear Ask MSDC with Spread

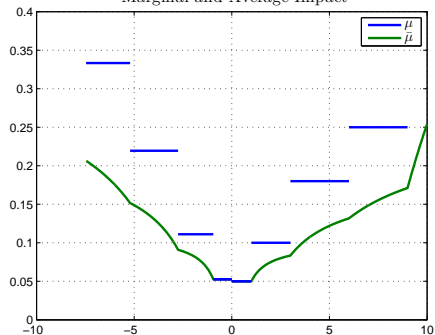


Example: Piecewise Constant MSDC

MSDC and SDC

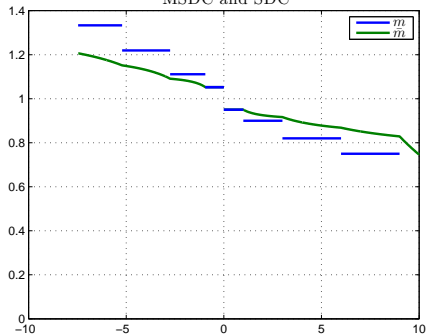


Marginal and Average Impact

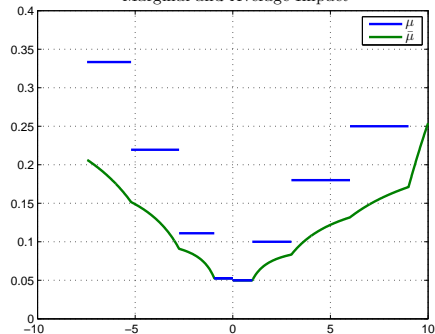


Example: Piecewise Constant MSDC

MSDC and SDC

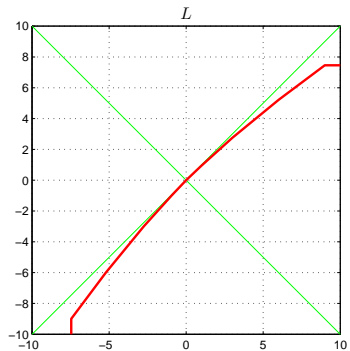


Marginal and Average Impact



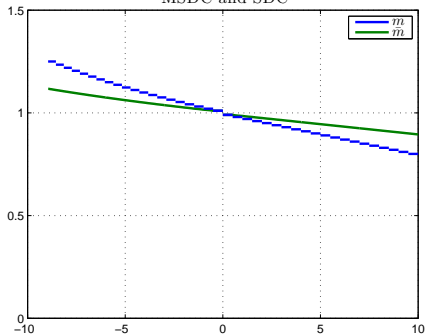
It was symmetrical!

Example: Piecewise Constant MSDC

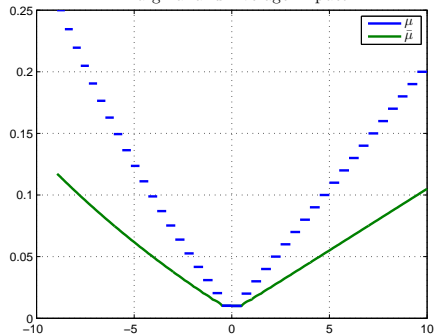


Example: Another Piecewise Constant MSDC

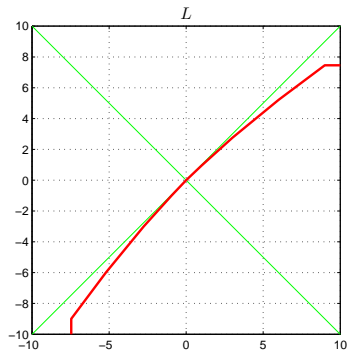
MSDC and SDC



Marginal and Average Impact



Example: Another Piecewise Constant MSDC



Some Results

An Even Impact Always Corresponds to Excess of Supply

- Given any market 'wing', there exists one and only one symmetrical wing that completes a symmetrical market

An Even Impact Always Corresponds to Excess of Supply

- Given any market 'wing', there exists one and only one symmetrical wing that completes a symmetrical market
- Therefore, we can build sound notions of
 - 'more liquid market'
 - 'more liquid wing'

An Even Impact Always Corresponds to Excess of Supply

- Given any market 'wing', there exists one and only one symmetrical wing that completes a symmetrical market
- Therefore, we can build sound notions of
 - 'more liquid market'
 - 'more liquid wing'
- but also a notion of
 - 'excess of demand' (resp. 'of supply'): sell side more (resp. less) liquid than buy side

An Even Impact Always Corresponds to Excess of Supply

- Given any market 'wing', there exists one and only one symmetrical wing that completes a symmetrical market
- Therefore, we can build sound notions of
 - 'more liquid market'
 - 'more liquid wing'
- but also a notion of
 - 'excess of demand' (resp. 'of supply'): sell side more (resp. less) liquid than buy side

Proposition

If the average impact is an even function, the market has an excess of supply

An Even Impact Always Corresponds to Excess of Supply

- Given any market 'wing', there exists one and only one symmetrical wing that completes a symmetrical market
- Therefore, we can build sound notions of
 - 'more liquid market'
 - 'more liquid wing'
- but also a notion of
 - 'excess of demand' (resp. 'of supply'): sell side more (resp. less) liquid than buy side

Proposition

If the average impact is an even function, the market has an excess of supply

Proof.

Not so obvious



Even Impact Always Corresponds to Excess of Supply

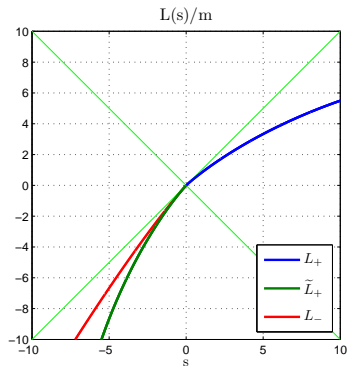


Figure : Illustration of the proposition. L_+ represents the bid wing of a LS. The plot compares the ask wing L_- obtained assuming that impact is even and the ask wing \tilde{L}_+ assuming a symmetrical market. The former, is always more liquid.

Even Impact as Small Size Limit of Supply–Demand Symmetry

Proposition

Consider a symmetrical market. Express the bid wing MSDC as

$$m(s) = m_+ - m_+ \psi(s) \quad s > 0$$

with $\lim_{s \rightarrow 0} \psi(s) = 0$. Then, the opposite ask wing MSDC can be approximated as an expansion in powers of ψ , to give

$$m(s) = m_- + m_- \psi(-sm_-/m) + \mathcal{O}(\psi^2) \quad s < 0$$

Even Impact as Small Size Limit of Supply–Demand Symmetry

Proposition

Consider a symmetrical market. Express the bid wing MSDC as

$$m(s) = m_+ - m_+ \psi(s) \quad s > 0$$

with $\lim_{s \rightarrow 0} \psi(s) = 0$. Then, the opposite ask wing MSDC can be approximated as an expansion in powers of ψ , to give

$$m(s) = m_- + m_- \psi(-sm_-/m) + \mathcal{O}(\psi^2) \quad s < 0$$

Proof.

An application of the Lagrange inversion theorem



Even Impact as Small Size Limit of Supply–Demand Symmetry

Proposition

Consider a symmetrical market. Express the bid wing MSDC as

$$m(s) = m_+ - m_+ \psi(s) \quad s > 0$$

with $\lim_{s \rightarrow 0} \psi(s) = 0$. Then, the opposite ask wing MSDC can be approximated as an expansion in powers of ψ , to give

$$m(s) = m_- + m_- \psi(-sm_-/m) + \mathcal{O}(\psi^2) \quad s < 0$$

Proof.

An application of the Lagrange inversion theorem



Corollary

At small impact regimes, a symmetrical market can be approximated by an even impact function iff the bid–offer spread is zero, in which case

$$\mu(s) = m\psi(|s|) + \mathcal{O}(\psi^2) \quad \forall s$$

Zooming at Low Impact Scale with No Spread

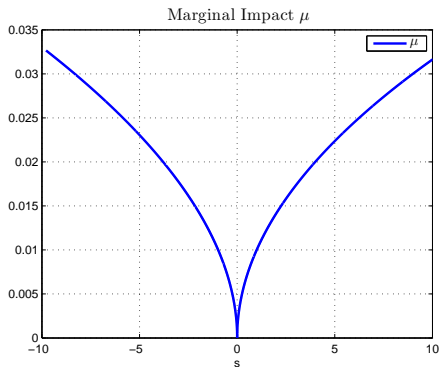


Figure : A symmetrical (power-law) marginal impact with no bid-ask spread, zoomed at small impact scale. The function is very close to an even one.

Zooming at Low Impact Scale with Spread

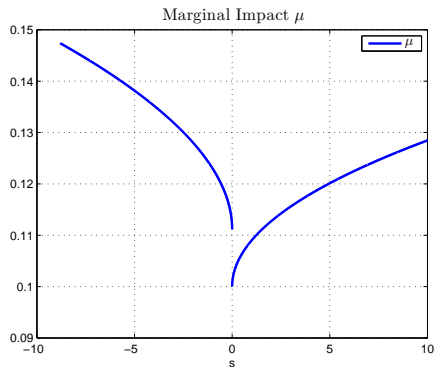


Figure : A symmetrical (power-law) marginal impact with finite bid-ask spread. The function can not be approximated by an even one at any scale.

Zooming at Low Impact Scale with Spread

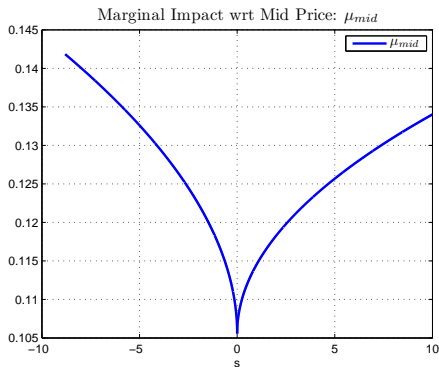


Figure : Even if we compute impact from mid price instead of fair price, to offset the central gap, the ask wing remains steeper.

Comparing with the Literature

- All models in the literature (with a massive amount of empirical evidence) assume even functions to describe supply–demand equilibrium.

Comparing with the Literature

- All models in the literature (with a massive amount of empirical evidence) assume even functions to describe supply–demand equilibrium. Are they all wrong ?

Comparing with the Literature

- All models in the literature (with a massive amount of empirical evidence) assume even functions to describe supply–demand equilibrium. Are they all wrong ?
- **No.** The last proposition tells us that they may be just looking into small impact regimes, neglecting bid–offer spread. Which is in fact the typical assumption in most models

Comparing with the Literature

- All models in the literature (with a massive amount of empirical evidence) assume even functions to describe supply–demand equilibrium. Are they all wrong ?
- No. The last proposition tells us that they may be just looking into small impact regimes, neglecting bid–offer spread. Which is in fact the typical assumption in most models
- Our notion of symmetry makes **testable predictions at all size scales and impact regimes**, that are supposed to extend previous findings.

Conclusions

Summing Up

- We have formalized the conditions that describe equivalence of supply and demand for a general security

Summing Up

- We have formalized the conditions that describe equivalence of supply and demand for a general security
- We have characterized and classified all the possible solutions to the above conditions

Summing Up

- We have formalized the conditions that describe equivalence of supply and demand for a general security
- We have characterized and classified all the possible solutions to the above conditions
- The resulting symmetry generalizes the traditional idea of 'even' impact functions at all trade size scales

Summing Up

- We have formalized the conditions that describe equivalence of supply and demand for a general security
- We have characterized and classified all the possible solutions to the above conditions
- The resulting symmetry generalizes the traditional idea of 'even' impact functions at all trade size scales
- Even market impact functions can describe a supply–demand equilibrium only in absence of bid/ask spread and for small trade sizes. They always express an excess of supply.

Summing Up

- We have formalized the conditions that describe equivalence of supply and demand for a general security
- We have characterized and classified all the possible solutions to the above conditions
- The resulting symmetry generalizes the traditional idea of 'even' impact functions at all trade size scales
- Even market impact functions can describe a supply–demand equilibrium only in absence of bid/ask spread and for small trade sizes. They always express an excess of supply.
- Supply–demand symmetry should represent the equilibria points of no market imbalance in all market impact models

Thanks!

MSCI 24 Hour Global Client Service

Americas

| | |
|---------------|----------------------------|
| Americas | 1.888.588.4567 (toll free) |
| Atlanta | +1.404.551.3212 |
| Boston | +1.617.532.0920 |
| Chicago | +1.312.706.4999 |
| Monterrey | +52.81.1253.4020 |
| Montreal | +1.514.847.7506 |
| New York | +1.212.804.3901 |
| San Francisco | +1.415.836.8800 |
| Sao Paulo | +55.11.3706.1360 |
| Stamford | +1.203.325.5630 |
| Toronto | +1.416.628.1007 |

Europe, Middle East & Africa

| | |
|-----------|---------------------------|
| Cape Town | +27.21.673.0100 |
| Frankfurt | +49.69.133.859.00 |
| Geneva | +41.22.817.9777 |
| London | +44.20.7618.2222 |
| Milan | +39.02.5849.0415 |
| Paris | 0800.91.59.17 (toll free) |

Asia Pacific

| | |
|-------------|----------------------------|
| China North | 10800.852.1032 (toll free) |
| China South | 10800.152.1032 (toll free) |
| Hong Kong | +852.2844.9333 |
| Seoul | +798.8521.3392 (toll free) |
| Singapore | 800.852.3749 (toll free) |
| Sydney | +61.2.9033.9333 |
| Tokyo | +81.3.5226.8222 |

clientservice@msci.com
www.msci.com

Barra Knowledge Base – Online Answers to Barra Questions: www.barra.com/support

Notice and Disclaimer

- This document and all of the information contained in it, including without limitation all text, data, graphs, charts (collectively, the "Information") is the property of MSCI Inc. or its subsidiaries (collectively, "MSCI"), or MSCIs licensors, direct or indirect suppliers or any third party involved in making or compiling any Information (collectively, with MSCI, the "Information Providers"), and is provided for informational purposes only. The Information may not be reproduced or disseminated in whole or in part without prior written permission from MSCI.
- The Information may not be used to create derivative works or to verify or correct other data or information. For example (but without limitation), the Information may not be used to create indices, databases, risk models, analytics, software, or in connection with the issuing, offering, sponsoring, managing or marketing of any securities, portfolios, financial products or other investment vehicles utilizing or based on, linked to, tracking or otherwise derived from the Information or any other MSCI data, information, products or services.
- The user of the Information assumes the entire risk of any use it may make or permit to be made of the Information. NONE OF THE INFORMATION PROVIDERS MAKES ANY EXPRESS OR IMPLIED WARRANTIES OR REPRESENTATIONS WITH RESPECT TO THE INFORMATION (OR THE RESULTS TO BE OBTAINED BY THE USE THEREOF), AND TO THE MAXIMUM EXTENT PERMITTED BY APPLICABLE LAW, EACH INFORMATION PROVIDER EXPRESSLY DISCLAIMS ALL IMPLIED WARRANTIES (INCLUDING, WITHOUT LIMITATION, ANY IMPLIED WARRANTIES OF ORIGINALITY, ACCURACY, TIMELINESS, NON-INFRINGEMENT, COMPLETENESS, MERCHANTABILITY AND FITNESS FOR A PARTICULAR PURPOSE) WITH RESPECT TO ANY OF THE INFORMATION.
- Without limiting any of the foregoing and to the maximum extent permitted by applicable law, in no event shall any Information Provider have any liability regarding any of the Information for any direct, indirect, special, punitive, consequential (including lost profits) or any other damages even if notified of the possibility of such damages. The foregoing shall not exclude or limit any liability that may not by applicable law be excluded or limited, including without limitation (as applicable), any liability for death or personal injury to the extent that such injury results from the negligence or wilful default of itself, its servants, agents or sub-contractors.
- Information containing any historical information, data or analysis should not be taken as an indication or guarantee of any future performance, analysis, forecast or prediction. Past performance does not guarantee future results.
- None of the Information constitutes an offer to sell (or a solicitation of an offer to buy), any security, financial product or other investment vehicle or any trading strategy.
- MSCIs indirect wholly-owned subsidiary Institutional Shareholder Services, Inc. ("ISS") is a Registered Investment Adviser under the Investment Advisers Act of 1940. Except with respect to any applicable products or services from ISS (including applicable products or services from MSCI ESG Research Information, which are provided by ISS), none of MSCIs products or services recommends, endorses, approves or otherwise expresses any opinion regarding any issuer, securities, financial products or instruments or trading strategies and none of MSCIs products or services is intended to constitute investment advice or a recommendation to make (or refrain from making) any kind of investment decision and may not be relied on as such.
- The MSCI ESG Indices use ratings and other data, analysis and information from MSCI ESG Research. MSCI ESG Research is produced by ISS or its subsidiaries. Issuers mentioned or included in any MSCI ESG Research materials may be a client of MSCI, ISS, or another MSCI subsidiary, or the parent of, or affiliated with, a client of MSCI, ISS, or another MSCI subsidiary, including ISS Corporate Services, Inc., which provides tools and services to issuers. MSCI ESG Research materials, including materials utilized in any MSCI ESG Indices or other products, have not been submitted to, nor received approval from, the United States Securities and Exchange Commission or any other regulatory body.
- Any use of or access to products, services or information of MSCI requires a license from MSCI. MSCI, Barra, RiskMetrics, ISS, CFRA, FEA, and other MSCI brands and product names are the trademarks, service marks, or registered trademarks or service marks of MSCI or its subsidiaries in the United States and other jurisdictions. The Global Industry Classification Standard (GICS) was developed by and is the exclusive property of MSCI and Standard & Poors. "Global Industry Classification Standard (GICS)" is a service mark of MSCI and Standard & Poors.

©2012 MSCI Inc. All rights reserved.

RV Jan 2012