MSCI A Clear View of Risk and Return

Supply–Demand Symmetry of Market Impact Models

November 2012

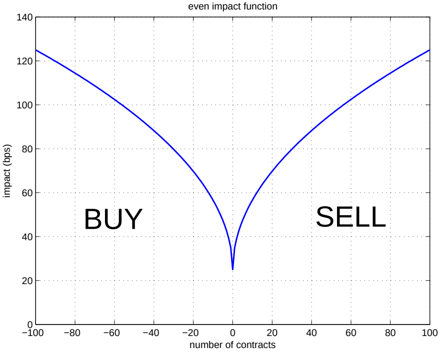
Carlo Acerbi

Outline

A Question Awaiting Formalization

What is a Bid–Ask Symmetrical Market Impact?

- Is it an even function ?



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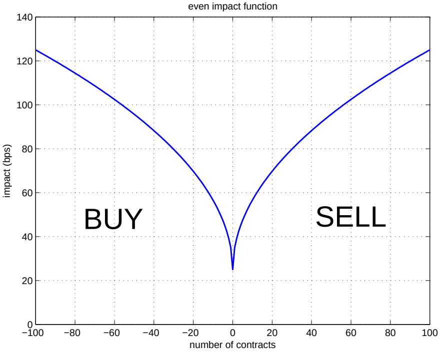

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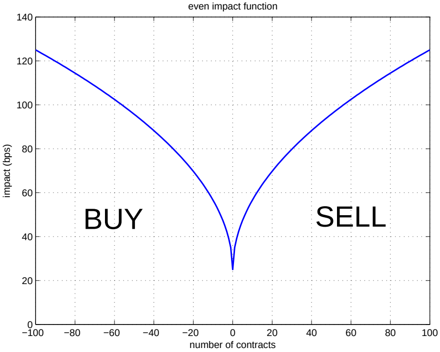

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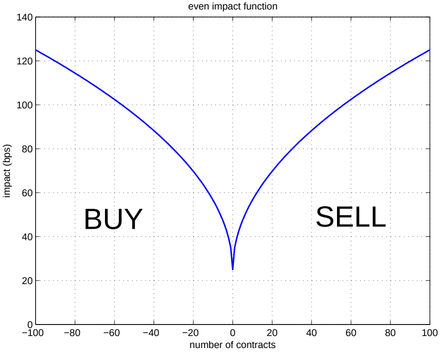

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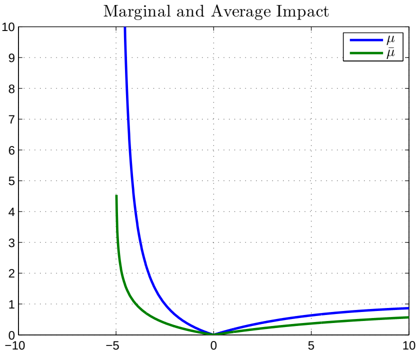
- But why even?
- Does it represent any financial symmetry?

The Large Scale Picture Cannot Be Even

- Bid prices are floored at zero. Ask prices are not capped
- Bid impact is capped. Ask impact may be unbounded.



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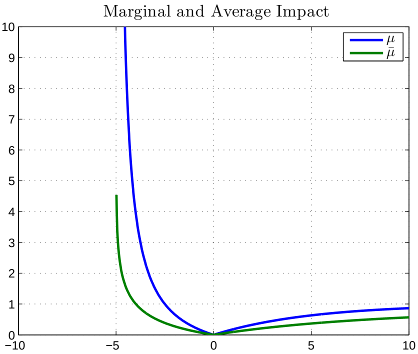

- Parity can't be a global fundamental symmetry

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- First we need an appropriate formalism

Basic Formalism

Definition (Average price impact)

Expected price impact $\overline{\mu}(s, T)$ of an order of s contracts, executed through a time horizon T. Best execution assumed.

Price Impact of a General Security

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Expected price impact $\mu(s, T)$ of additional ds contracts traded.

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Convention

Buy trade if $s < 0$; sell trade if $s > 0$

Supply–Demand Curves

Let m be the fair price

Definition (Supply–Demand Curve (SDC))

Expected price of the entire order s

$$\overline{m}(s, T) = m - \mathrm{sgn}(s)\,\overline{\mu}(s, T)$$

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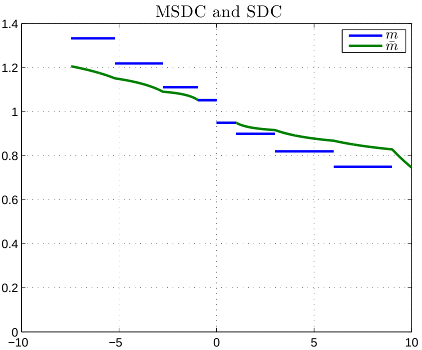
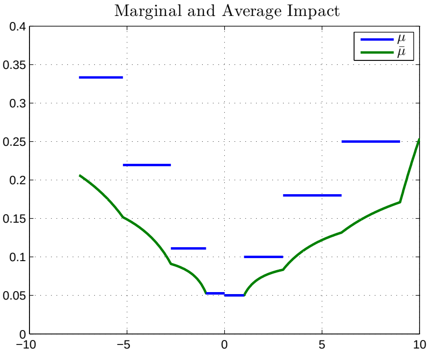
Definition (Bid and Ask price)

$$m^{\pm} \equiv m(0^{\pm}, \forall T) = \overline{m}(0^{\pm}, \forall T)$$

Example: Piecewise Constant MSDC





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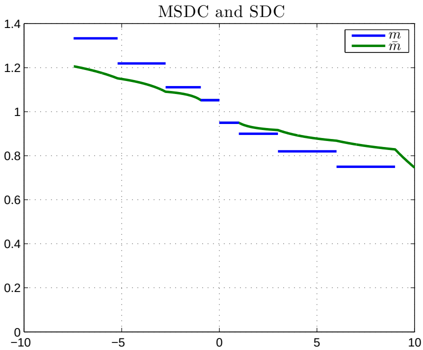

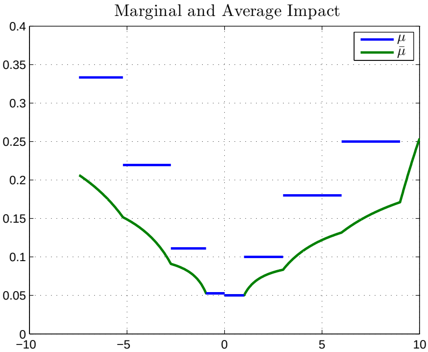

Is it symmetrical?

Liquidation Operator

Definition (Liquidation operator)

Expected order proceedings

$$L(s, T) = \overline{m}(s, T)\, s = \int_0^s m(z, T)\, dz$$

Cash in if $L > 0$, cash out if $L < 0$

Regular Market Hypothesis

The only fundamental hypotheses we make

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Corollary

The liquidation operator L is concave

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- MSDC: $X_f^d(s, T)$
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- Similar convention for all other equivalent functions
- All the introduced functions admit a dual representation in the two currencies

Notation: Dropping Time Dependence

- In what follows, we fix some horizon T and we stop indicating it

Forex: Basic Facts

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Forex–Duality of the Framework

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$$s_a = - L_b^a(s_b)$$

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Proofs

Proof.

By definition of inverse

$$[-L_a^b] \circ [-L_b^a](s_b) = s_b$$

Differentiating both sides by s_b we obtain

$$X_a^b(-L_b^a(s_b))\, X_b^a(s_b) = 1$$

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Proof.

Applying twice the definition of SDC: $L(s) = \overline{X}(s)\, s$

$$[-L_a^b] \circ [-L_b^a](s_b) = \overline{X}_a^b(-L_b^a(s_b)) \cdot L_b^a(s_b) = \overline{X}_a^b(-L_b^a(s_b)) \cdot \overline{X}_b^a(s_b) \cdot s_b = s_b$$

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Supply–Demand Symmetry for Forex

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Example

Suppose the ¥/€ rate is 100 ¥/€. In a symmetrical market, we expect that the relative impact of liquidating €100 and the relative impact of liquidating ¥10'000 should be the same

Imposing Supply–Demand Symmetry for Small Forex Trades

- Impose that the dual relative bid–offer spreads are identical

$$\frac{X^- - X^+}{X} = \frac{1/X^+ - 1/X^-}{1/X}$$

Solving for X yields

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Proposition

In a symmetrical forex LS, the fair rate is the geometric average of the bid rate and the offer rate.

$$X_b^a = \sqrt{X_b^{a+} X_b^{a-}}$$

Imposing Supply–Demand Symmetry for Forex, in General

Definition (Forex Supply–Demand Symmetry)

We say that a forex market is symmetrical, if there exists a constant $\alpha > 0$ such that the mapping $s_a/\alpha \leftrightarrow s_b$

$$\frac{s_a}{\alpha} = -\frac{1}{\alpha} L_b^a(s_b)$$

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If such α exists, it's the fair rate

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Proposition

If such α exists, it's the fair rate ... as you may have guessed

$$\alpha = X_b^a = \sqrt{X_b^{a+} X_b^{a-}}$$

Classification of Forex Symmetrical Markets

Theorem

A forex market displays supply–demand symmetry if and only if the liquidation operator $s_b \mapsto L_b^a(s_b)$ *can be expressed as*

$$L_b^a(s_b) = -X_b^a\, \phi(s_b)$$

where the function $\phi : \mathcal{D}_b^a \to \mathcal{D}_b^a$

1. *is an involution* $\phi = \phi^{[-1]}$
2. *is convex and strictly decreasing*
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Corollary

In a symmetrical forex market the MSDC and the SDC satisfy

$$X_b^a(s) X_b^a(\tilde{s}) = (X_b^a)^2$$

$$\overline{X}_b^a(s) \overline{X}_b^a(\tilde{s}) = (X_b^a)^2$$

at conjugated points s *and* $\tilde{s} = \phi(s)$.

But Then: Why Only Forex?

A Currency is Just One Security Among All Others

- If you say

  "the euro for a yen based investor is as liquid an asset as the yen is for a euro based investor"

  it might seem you're speaking of a forex symmetry only

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Definition (Supply–Demand Symmetry for General Securities)

A security's market is symmetrical if it has the same properties of a forex symmetrical market

Classification of Symmetrical Markets for General Securities

Theorem

A security's market displays supply–demand symmetry if and only if the liquidation operator $s \mapsto L(s)$ can be expressed as

$$L(s) = -m\,\phi(s)$$

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Corollary

In a security's symmetrical market the MSDC and the SDC satisfy

$$m(s)\,m(\tilde{s}) = m^2$$

$$\overline{m}(s)\,\overline{m}(\tilde{s}) = m^2$$

at conjugated points s and $\tilde{s} = \phi(s)$.

Geometrical Interpretation of Supply–Demand Symmetry

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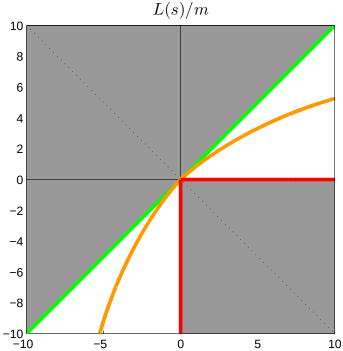
- $L(x)/m$: concave, symmetrical wrt $y = -x$, increasing, zero in zero



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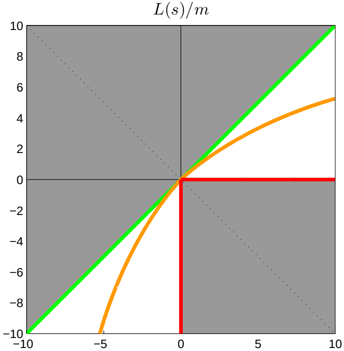

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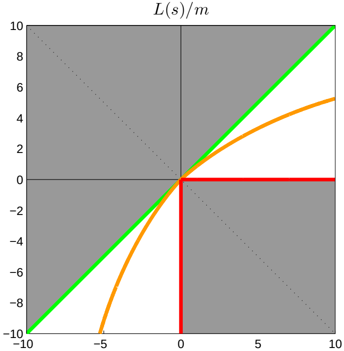

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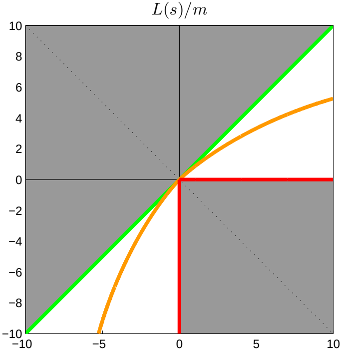


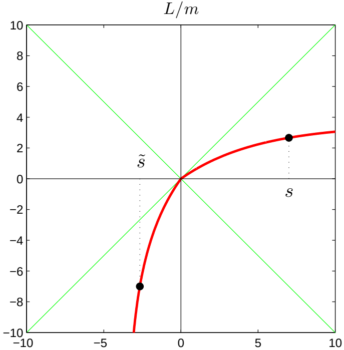
- the curve is forced to live in the white area of the plane
- two extremes: perfectly liquid and perfectly illiquid market
- natural notion of partial ordering of liquidity among different L's

Supply–Demand Symmetry for Stocks, in Words

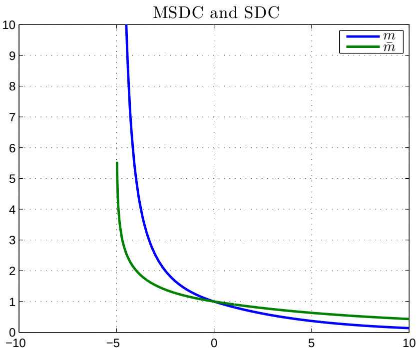
$$\tilde{s} = -L(s)/m \qquad \text{and} \qquad s = -L(\tilde{s})/m$$

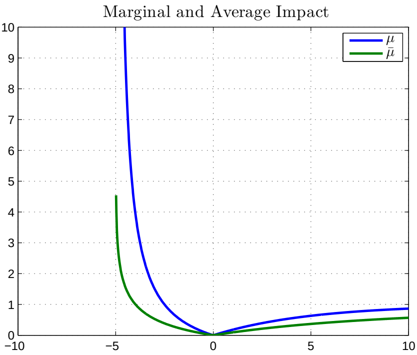
Proposition (A Market is Symmetrical iff)

If $|\tilde{s}|$ stocks correspond in fair value to the liquidation of s stocks, then s stocks correspond in fair value to the cost of buying $|\tilde{s}|$ stocks, $\forall s$



Example: Exponentially Decaying MSDC





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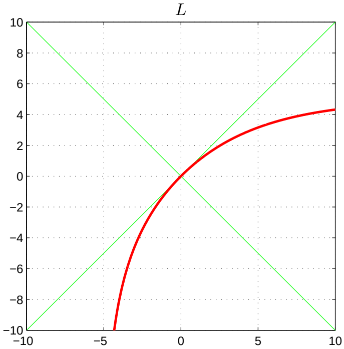
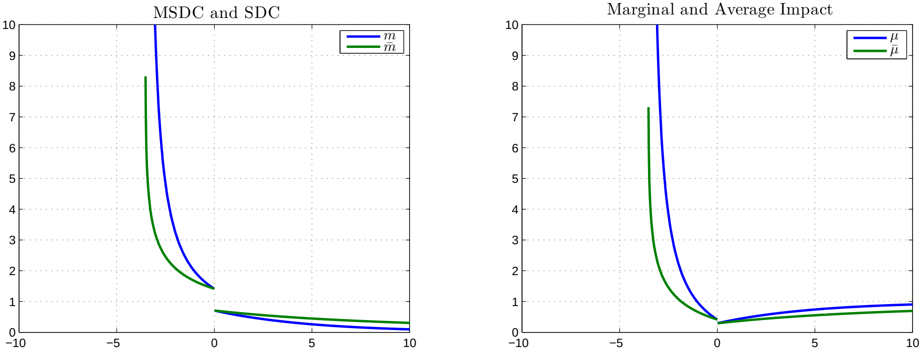


Example: Exponentially Decaying MSDC with Spread



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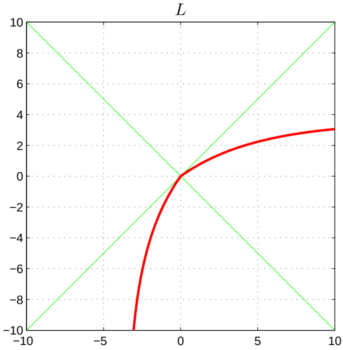
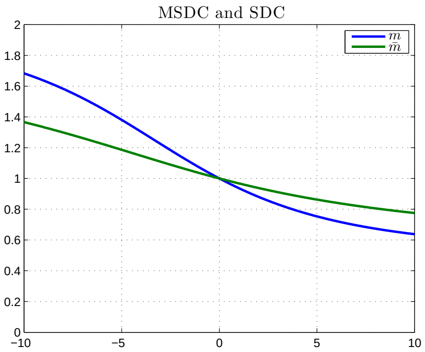
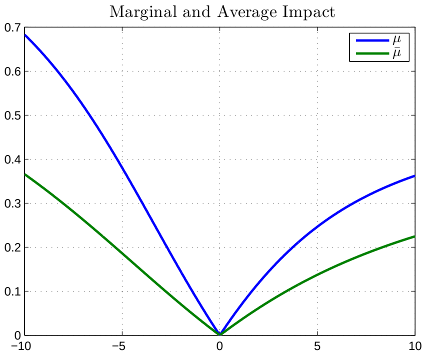


Example: Asymptotically Finite MSDC





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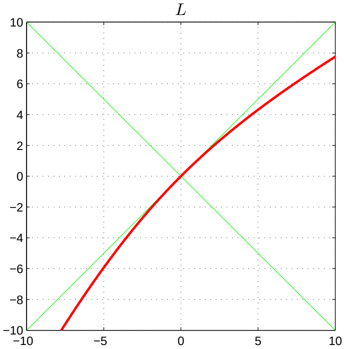
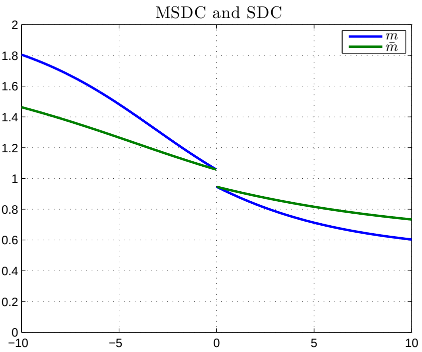
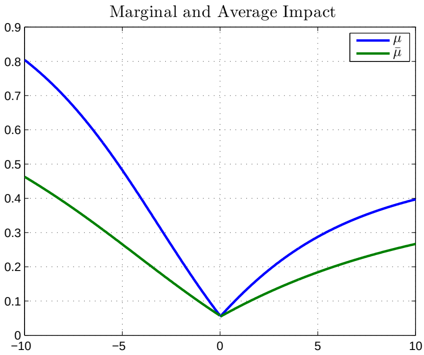


Example: Asymptotically Finite MSDC with Spread





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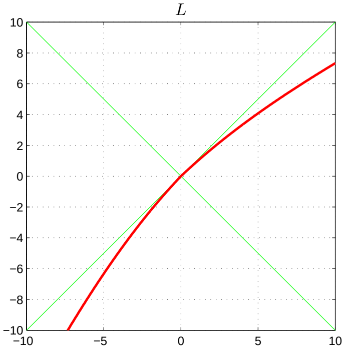
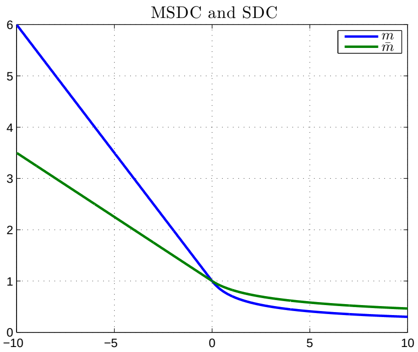
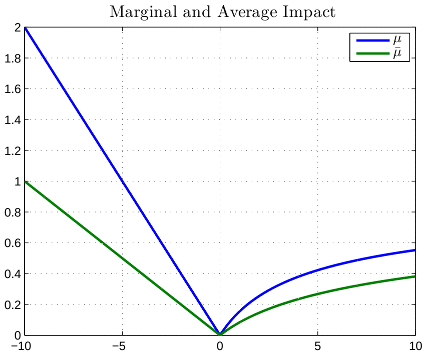


Example: Linear Ask MSDC





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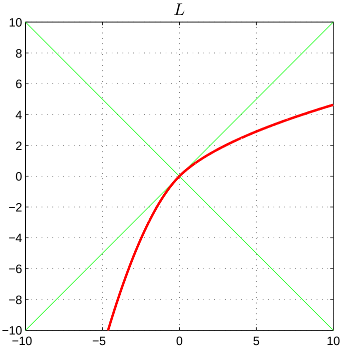

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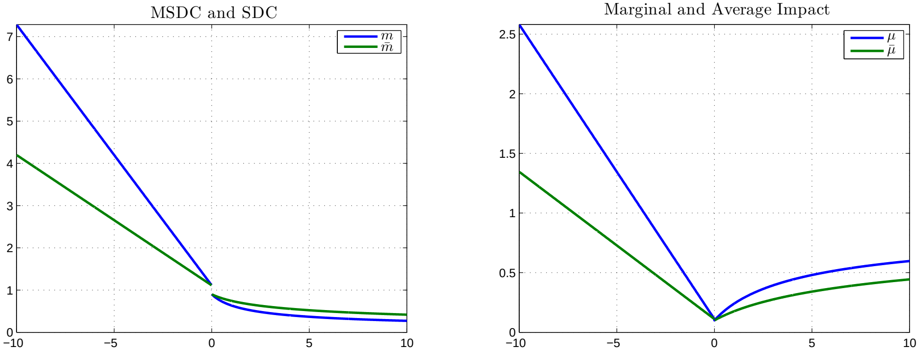

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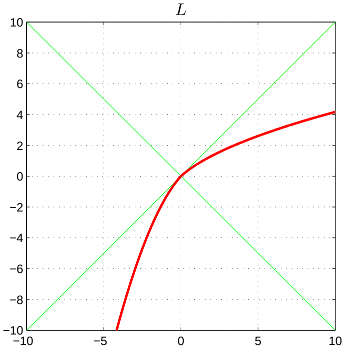

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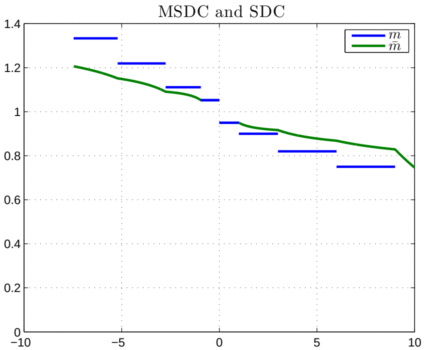

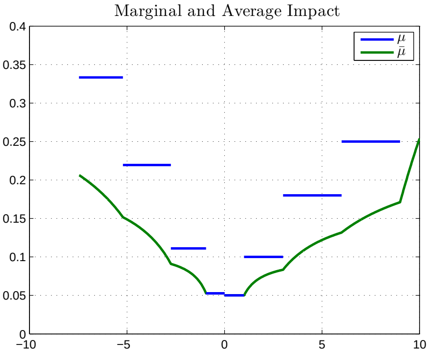

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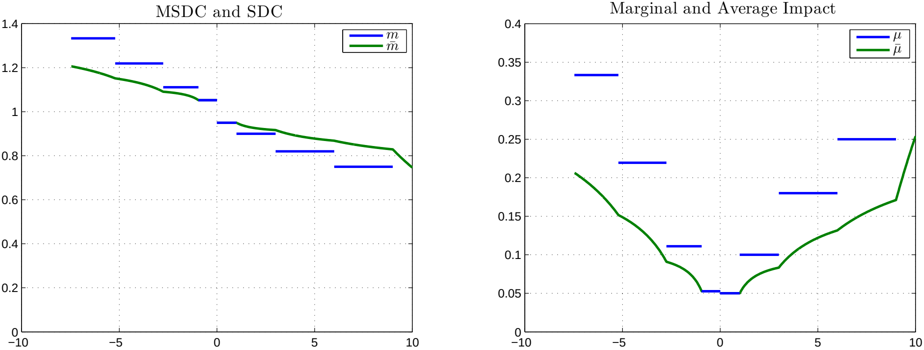

It was symmetrical!

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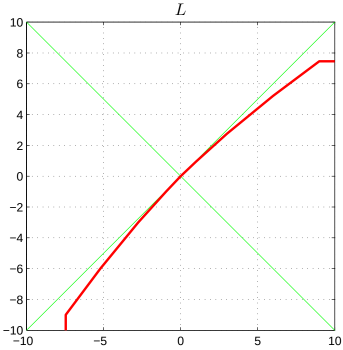

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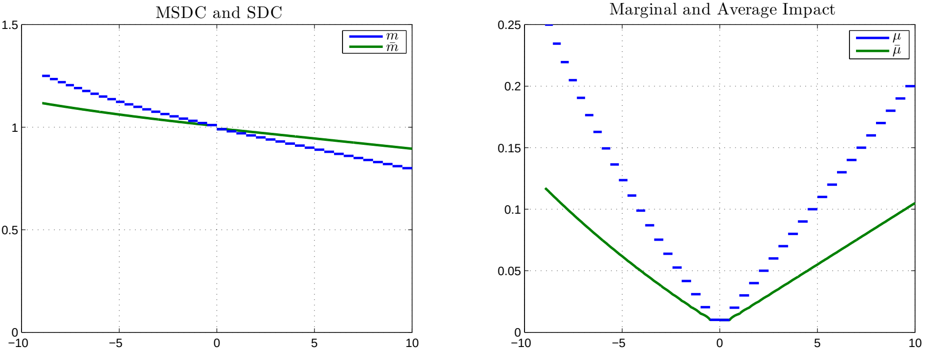

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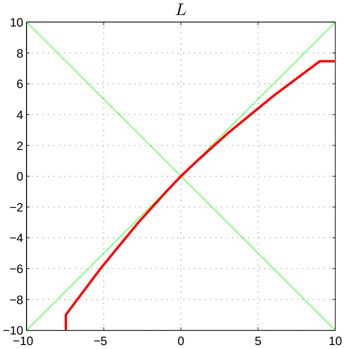

Some Results

An Even Impact Always Corresponds to Excess of Supply

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Proof.

Not so obvious □

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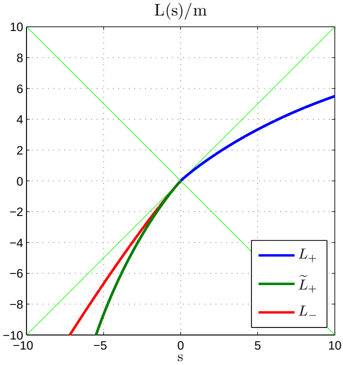

Figure : Illustration of the proposition. L_+ represents the bid wing of a LS. The plot compares the ask wing L_- obtained assuming that impact is even and the ask wing $\widetilde{L}_+$ assuming a symmetrical market. The former, is always more liquid.

Even Impact as Small Size Limit of Supply–Demand Symmetry

Proposition

Consider a symmetrical market. Express the bid wing MSDC as

$$m(s) = m_+ - m_+ \, \psi(s) \qquad\qquad s > 0$$

with $\lim_{s\to 0} \psi(s) = 0$. *Then, the opposite ask wing MSDC can be approximated as an expansion in powers of* ψ, *to give*

$$m(s) = m_- + m_- \, \psi(-sm_-/m) + \mathcal{O}(\psi^2) \qquad\qquad s < 0$$

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Corollary

At small impact regimes, a symmetrical market can be approximated by an even impact function iff the bid–offer spread is zero, in which case

$$\mu(s) = m\psi(|s|) + \mathcal{O}(\psi^2) \qquad\qquad \forall s$$

Zooming at Low Impact Scale with No Spread

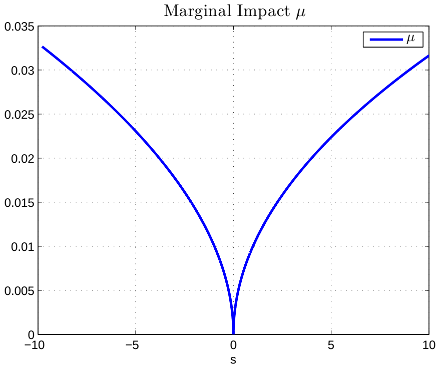

Figure : A symmetrical (power–law) marginal impact with no bid–ask spread, zoomed at small impact scale. The function is very close to an even one.

Zooming at Low Impact Scale with Spread

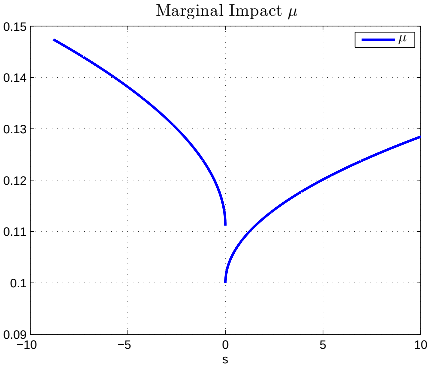

Figure : A symmetrical (power–law) marginal impact with finite bid–ask spread. The function can not be approximated by an even one at any scale.

Zooming at Low Impact Scale with Spread

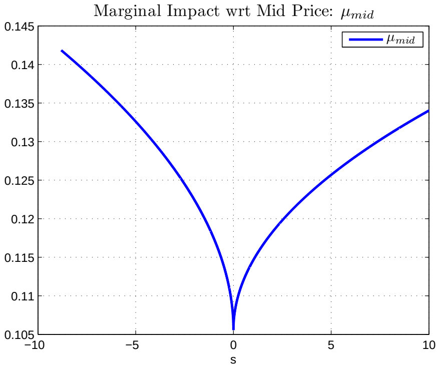

Figure : Even if we compute impact from mid price instead of fair price, to offset the central gap, the ask wing remains steeper.

Comparing with the Literature

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- Our notion of symmetry makes testable predictions at all size scales and impact regimes, that are supposed to extend previous findings.

Conclusions

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- Even market impact functions can describe a supply–demand equilibrium only in absence of bid/ask spread and for small trade sizes. They always express an excess of supply.
- Supply–demand symmetry should represent the equilibria points of no market imbalance in all market impact models

Thanks!

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