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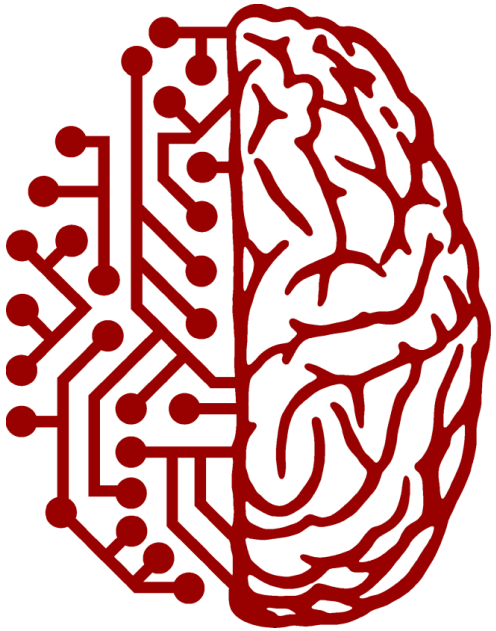

Fast inference with spiking networks

Mihai A. Petrovici

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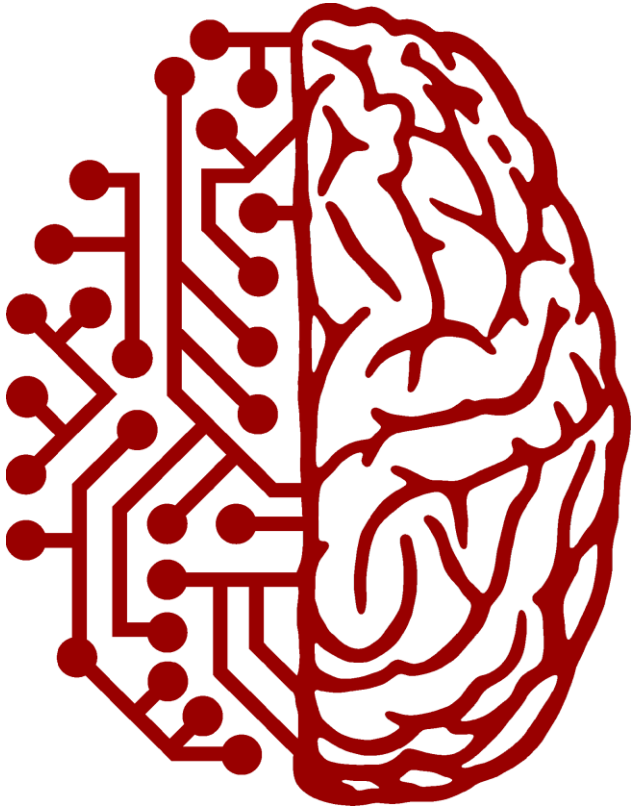
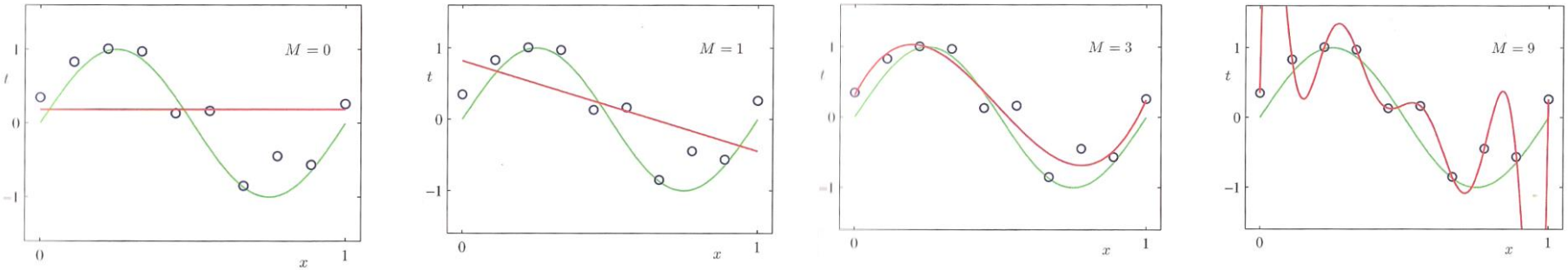
Electronic Vision(s)
Kirchhoff Institute for Physics
University of Heidelberg

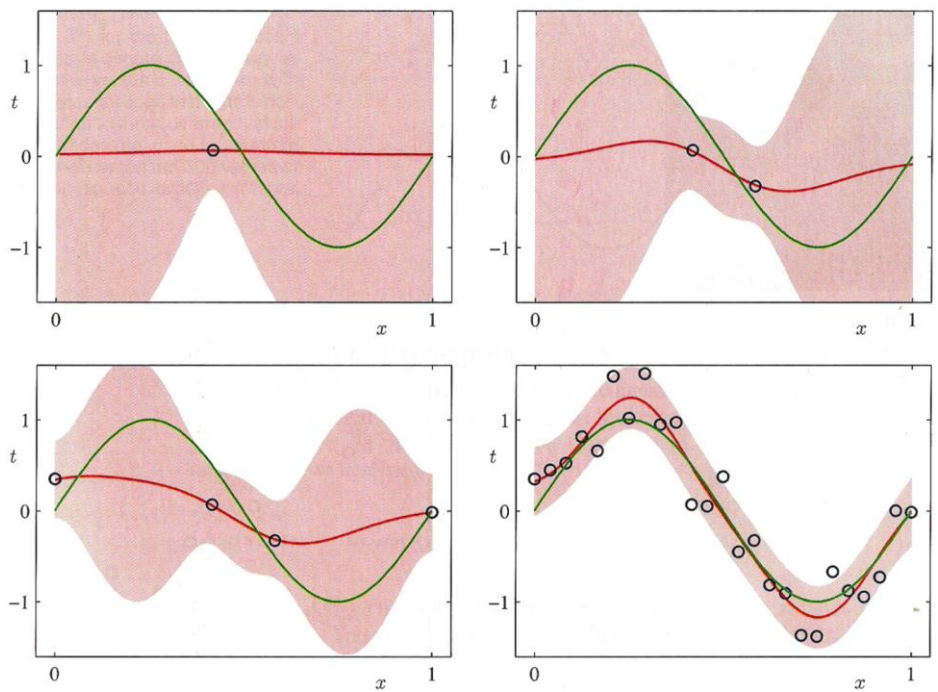


Probabilistic computation with spikes

Probabilistic (Bayesian) computing: motivation

get an estimate of the <u>distribution</u> of the sought function (expected value <u>and</u> uncertainty)

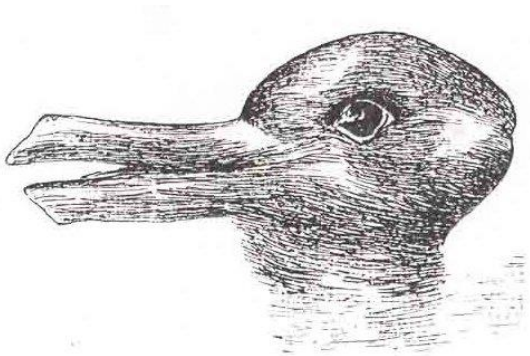


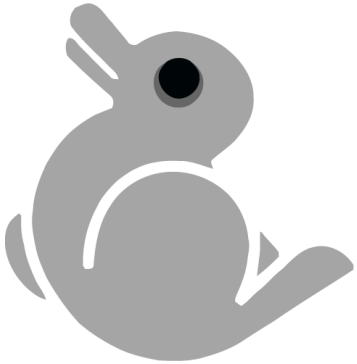


Bishop (2006)

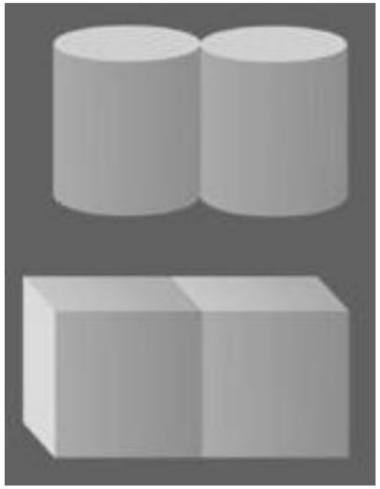
Probabilistic (Bayesian) computing: experimental evidence

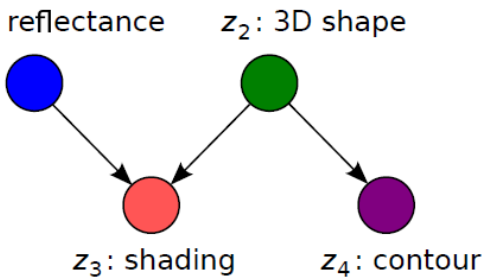
Rabbit/duck ambiguity



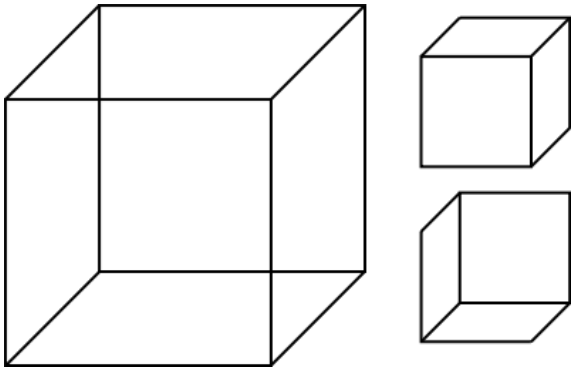


Knill-Kersten illusion



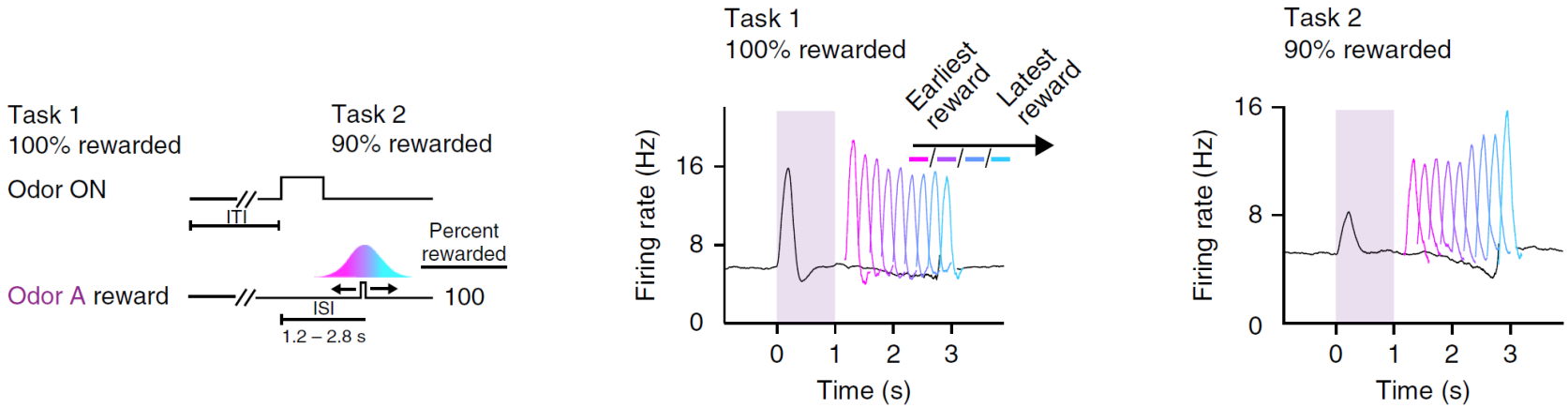


Necker cube

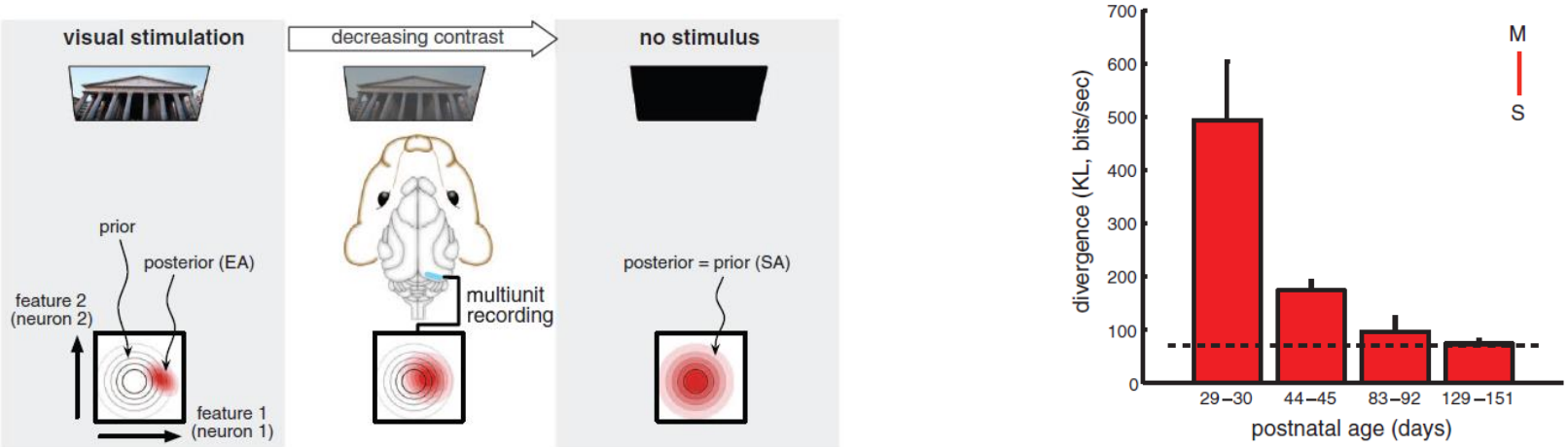


Probabilistic (Bayesian) computing: experimental evidence

Starkweather et al. (2017)



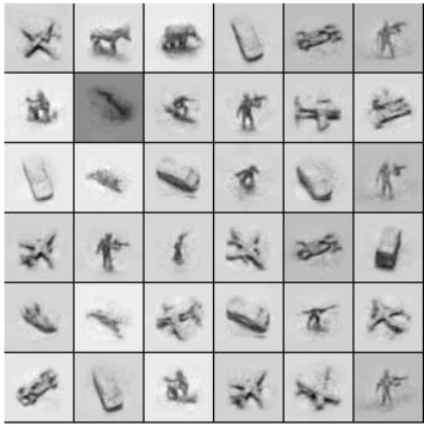
Berkes et al. (2011)



Probabilistic (Bayesian) computing in machine learning

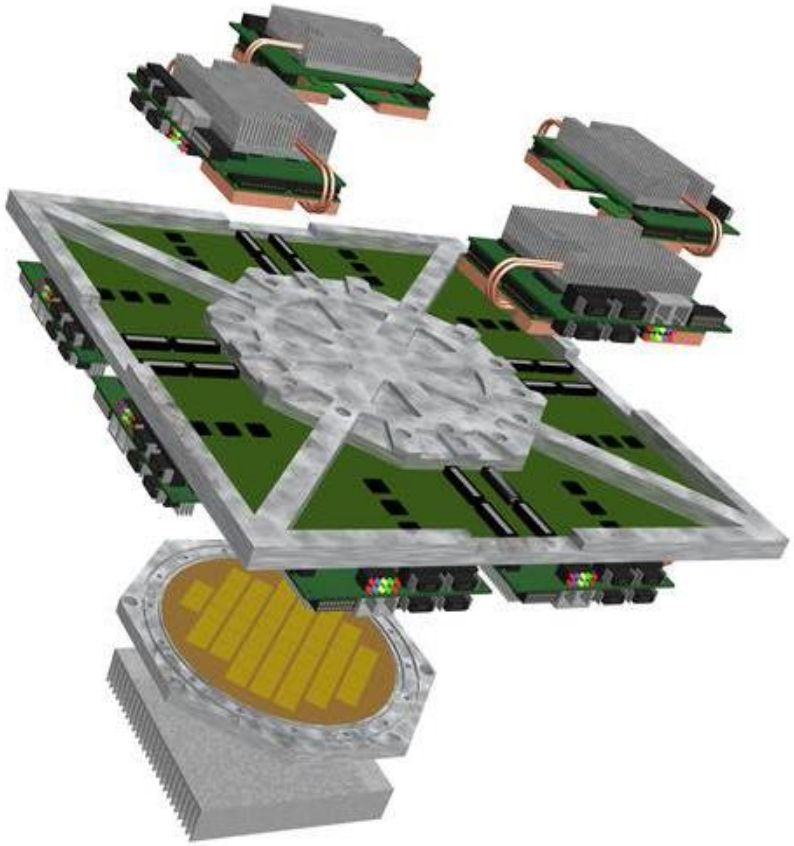
Deep generative models

Salakhutdinov & Hinton (2009)



Neuromorphic hardware

Schemmel et al. (2010)



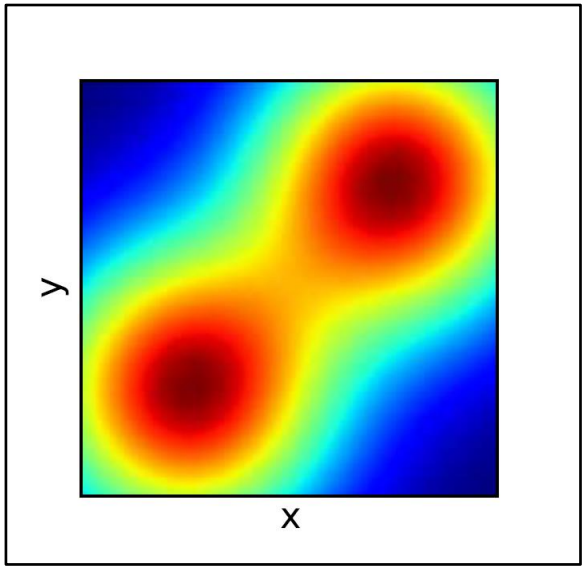
Requirements for probabilistic inference

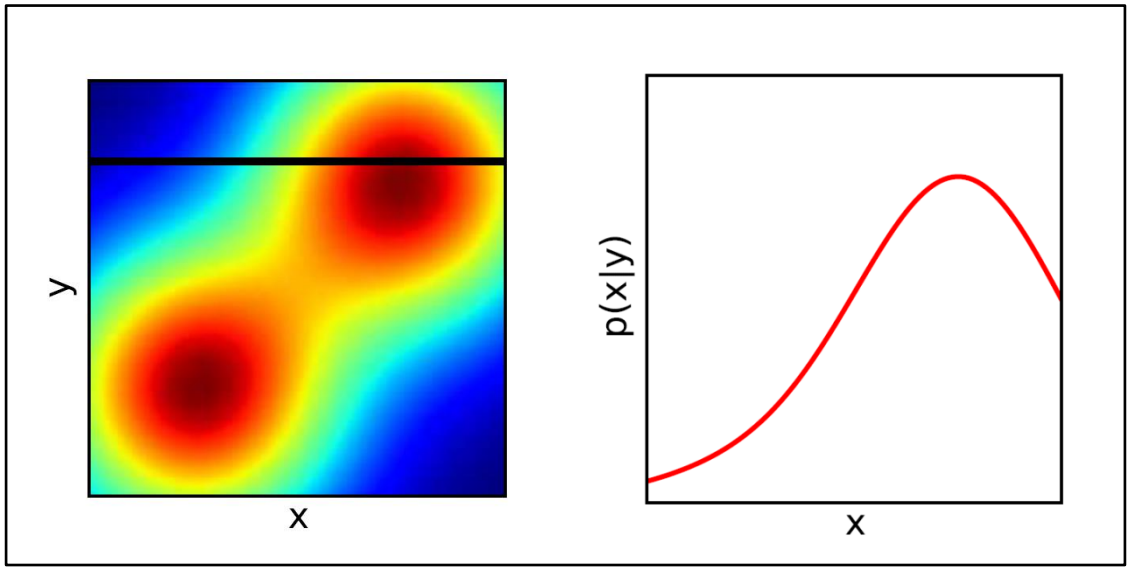
A system that performs probabilistic inference has to

→ represent probability distributions $p(z_1, z_2, \dots)$

→ calculate posterior (conditional) distributions $p(z_1, z_2, \dots | z_k, z_{k+1}, \dots)$

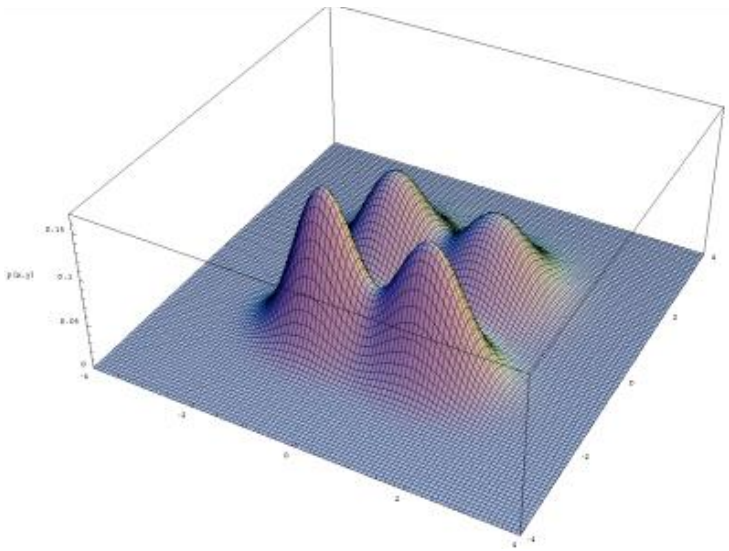
→ evaluate marginal distributions $p(z_1, z_2) = \sum_{z_3, z_4, \dots} p(z_1, z_2, z_3, \dots)$





Representation of probability distributions

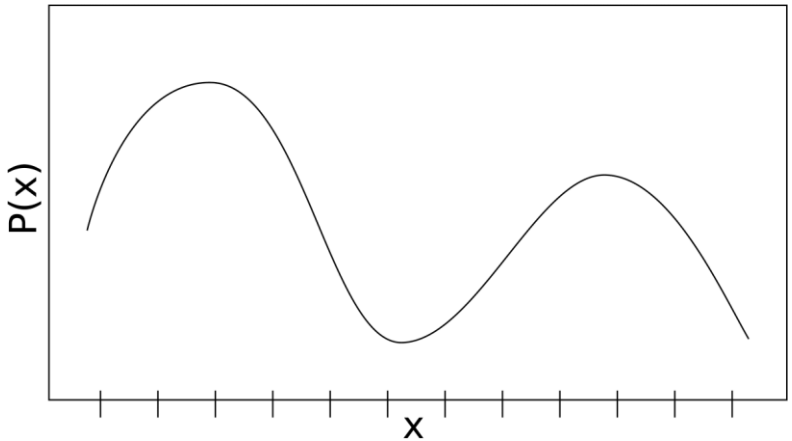
full state space



analytic / parametric

$$p(\boldsymbol{z}) = \frac{1}{Z}\exp\left[\frac{1}{2}\boldsymbol{z}^T\boldsymbol{W}\boldsymbol{z} + \boldsymbol{z}^T\boldsymbol{b}\right]$$

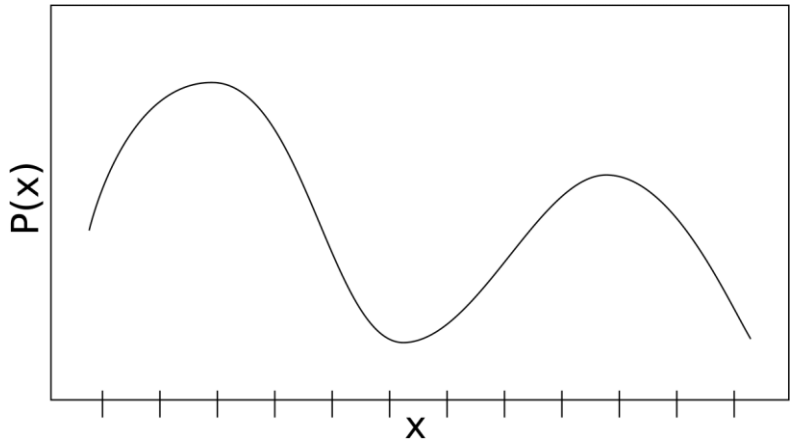
sampling



Sampling vs. parametric representation

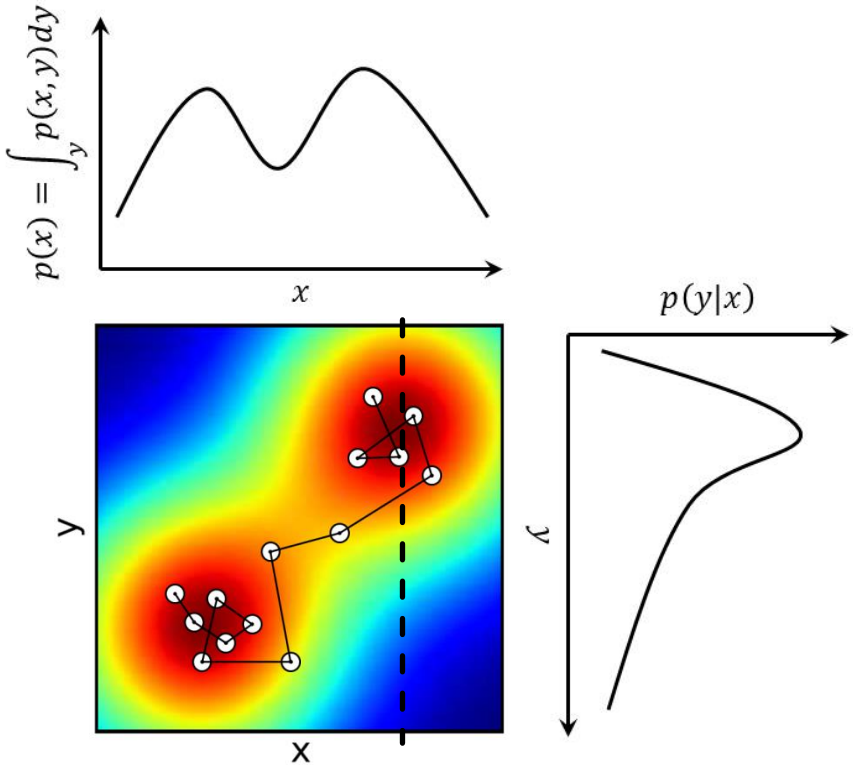
temporal aspects:

- increasingly correct representation
- anytime computing



computational complexity aspects:

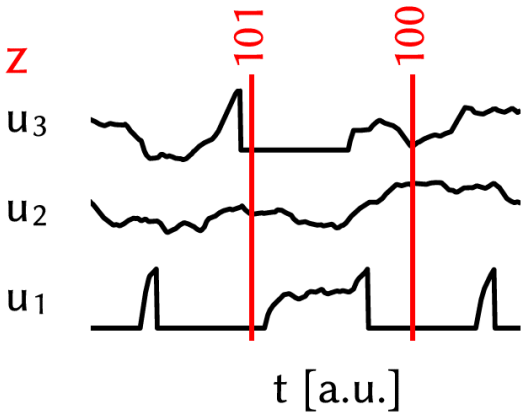
- computation of contitionals is simple
- marginalization is free



Spike-based encoding of an ensemble state

$z_k = 1 \;\Leftrightarrow\;$ neuron has spiked in $[t-\tau, t)$

$\rightarrow$ spike pattern encodes states $\boldsymbol{z}^{(t)}$

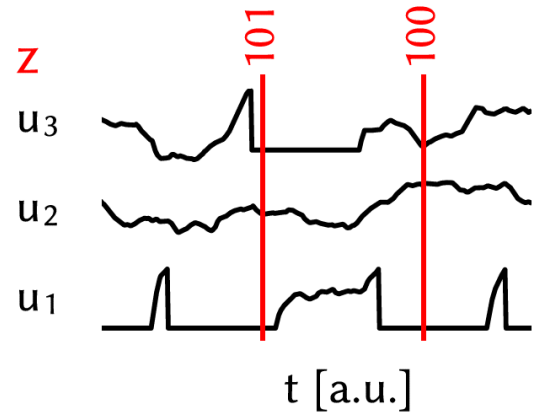


Büsing et al. (2011), Petrovici & Bill et al. (2016)

Emulation of Boltzmann machines

$z_k = 1 \;\Leftrightarrow\;$ neuron has spiked in $[t-\tau, t)$

$\rightarrow$ spike pattern encodes states $\boldsymbol{z}^{(t)}$



Neural computability condition

$$u_k = \log \frac{p(z_k = 1 \,|\boldsymbol{z}_{\setminus k})}{p(z_k = 0 \,|\boldsymbol{z}_{\setminus k})}$$

(which is equivalent to a logistic activation function $p(z_k = 1 \,|\boldsymbol{z}_{\setminus k}) = \frac{1}{1+\exp(-u_k)}$).

mediated by synaptic weights:

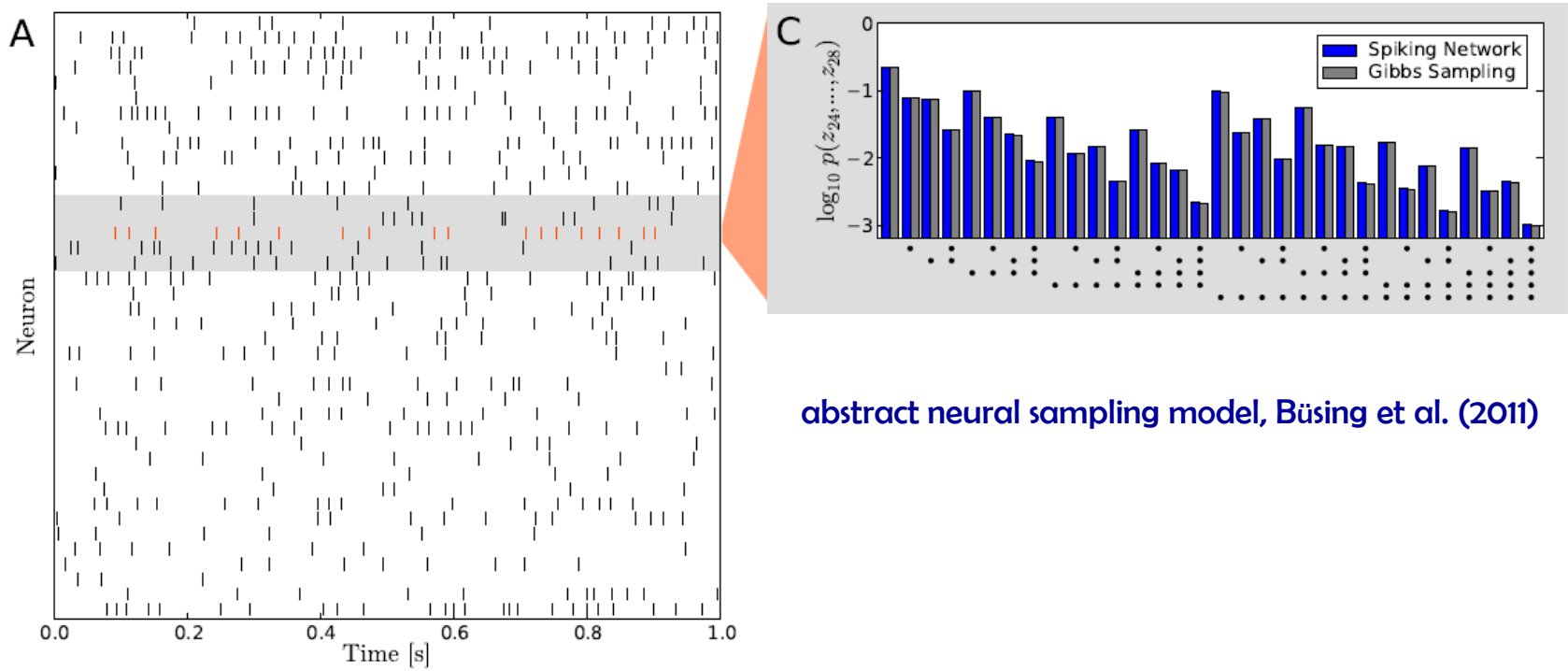
$$u_k = \sum_{i=1}^{K} W_{ki} z_i + b_k$$

Boltzmann distribution over $z_k \in \{0, 1\}$

$$p(\boldsymbol{z}) = \frac{1}{Z} \exp\left[\frac{1}{2}\boldsymbol{z}^T \boldsymbol{W} \boldsymbol{z} + \boldsymbol{z}^T \boldsymbol{b}\right]$$

Büsing et al. (2011), Petrovici & Bill et al. (2016)

Emulation of Boltzmann machines



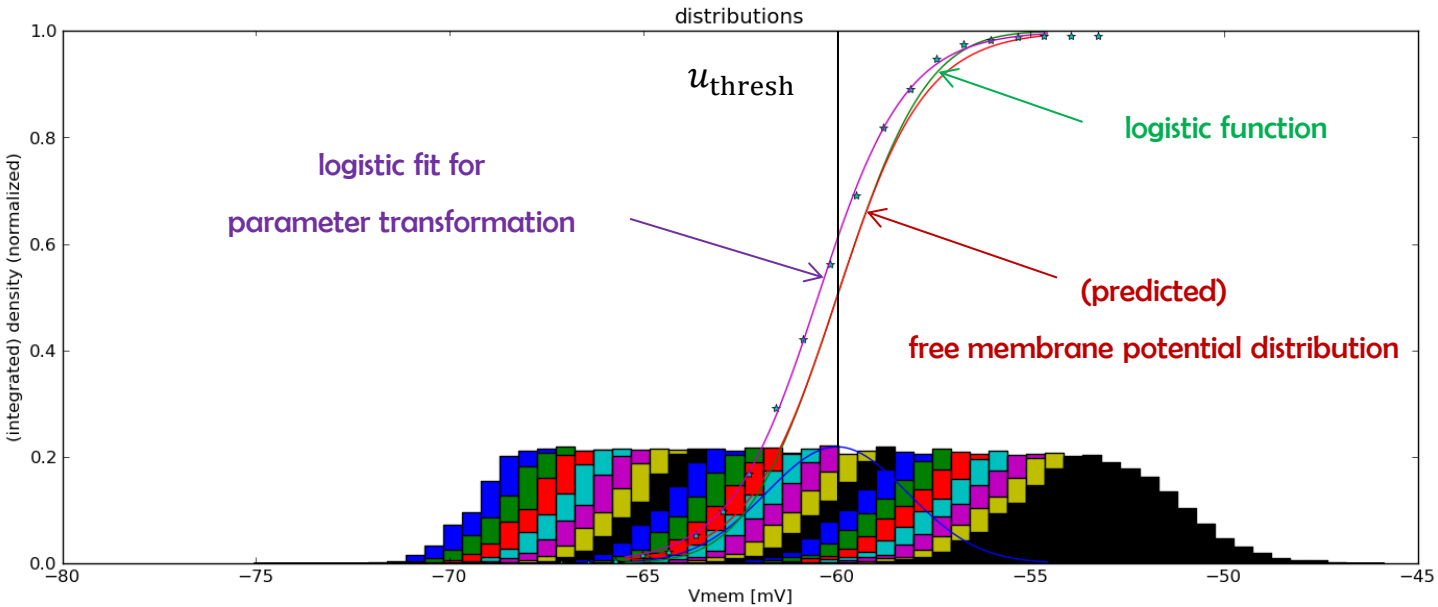
abstract neural sampling model, Büsing et al. (2011)

Neural computability condition

$$u_k = \log \frac{p(z_k = 1 \,|\boldsymbol{z}_{\setminus k})}{p(z_k = 0 \,|\boldsymbol{z}_{\setminus k})}$$

(which is equivalent to a logistic activation function $p(z_k = 1 \,|\boldsymbol{z}_{\setminus k}) = \frac{1}{1+\exp(-u_k)}$).

Idea: Stochasticity by Poisson background



$$\Rightarrow \quad p_{\text{spike}} \approx \text{erf}[\alpha \cdot (u_{\text{eff}} - \langle u_0 \rangle)]$$

$$\approx \sigma[\alpha \cdot (u_{\text{eff}} - \langle u_0 \rangle)]$$

unfortunately, neurons are a bit more complicated...

The diffusion approximation

noise source: Poisson spike trains
high background firing rates
relatively low synaptic weights

$\Rightarrow$ membrane as Ornstein-Uhlenbeck process

$$du(t) = \Theta \cdot [\mu - u(t)]dt + \sigma\, dW(t)$$

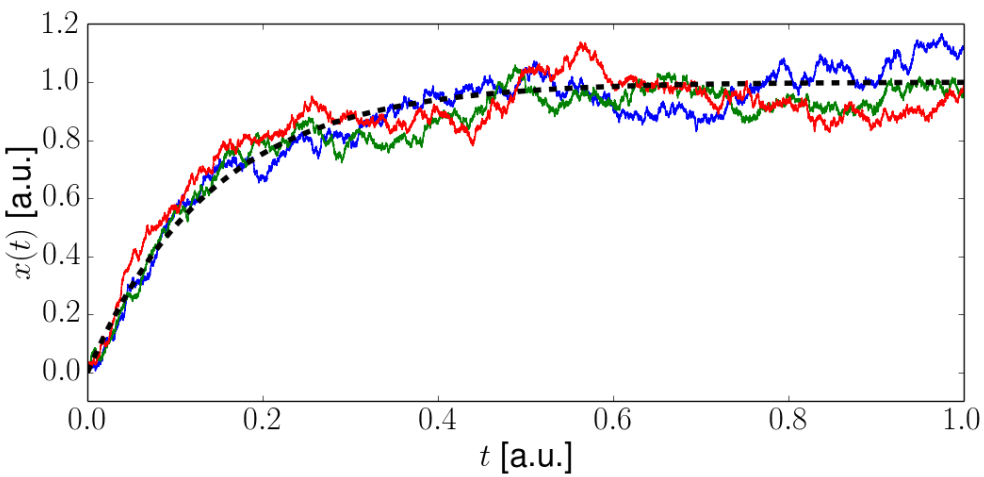
Ricciardi & Sacerdote (1979)

for COBA LIF:

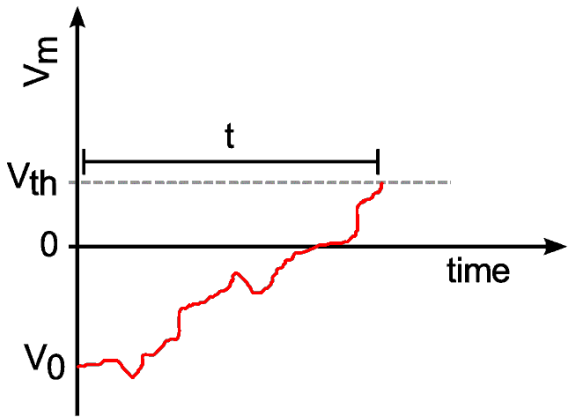
$$\Theta = \frac{1}{\tau_{\text{syn}}}$$

$$\mu = \frac{I^{\text{ext}} + g_{\text{l}} E_{\text{l}} + \sum_{i=1}^{n} \nu_i w_i E_i^{\text{rev}} \tau_{\text{syn}}}{\langle g^{\text{tot}} \rangle}$$

$$\sigma^2 = \frac{\sum_{i=1}^{n} \nu_i [w_i (E_i^{\text{rev}} - \mu)]^2 \tau_{\text{syn}}}{2 \langle g^{\text{tot}} \rangle^2}$$



First-passage-time calculations



$$\text{FPT}(a,0) \;:=\; \langle \inf\{t \geq 0 : V_t = a | V_0 = 0\}\rangle$$
$$= \sqrt{\frac{\pi}{\rho\sigma^2}} \int_0^{|a|} \left[1 + \operatorname{erf}\left(\sqrt{\frac{\rho}{\sigma^2}}\, x\right)\right] \exp\left(\frac{\rho x^2}{\sigma^2}\right) dx$$

Thomas (1975)

however, remember that $\tau_{\text{ref}} \approx \tau_{\text{syn}}$!

$\Rightarrow$ membrane does not forget !

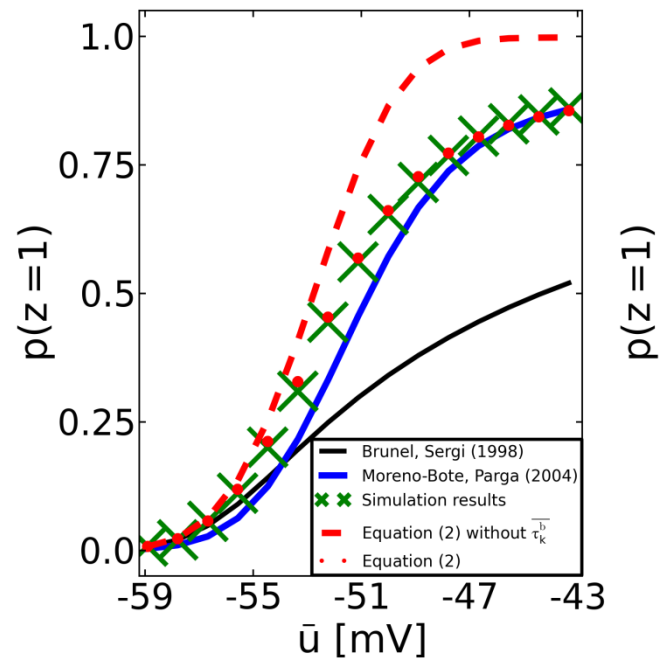
Brunel & Sergi (1998)

assumption: $\tau_{\text{syn}} \ll \tau_{\text{m}}$

this allows expansion in $\sqrt{\frac{\tau_{\text{syn}}}{\tau_{\text{m}}}}$:

$$\langle T \rangle = \tau\sqrt{\pi} \int_{\rho-\mu/\sigma}^{\vartheta_{\text{eff}}-\mu/\sigma} [1 + \operatorname{erf} x] \exp(x^2)\, dx$$

$$\nu_k = \frac{1}{\tau_{\text{ref}} + \text{FPT}(V_{th}, V_0)}$$

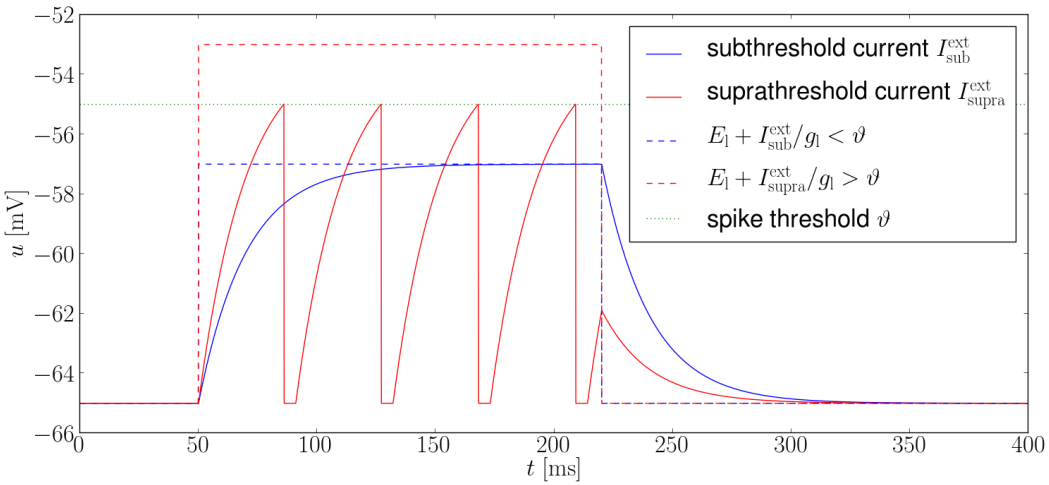


The adiabatic approximation

Moreno-Bote & Parga (2004)

assumption: $\tau_{\mathrm{syn}} \gg \tau_{\mathrm{m}}$

then, the synaptic input appears quasistatic to the membrane

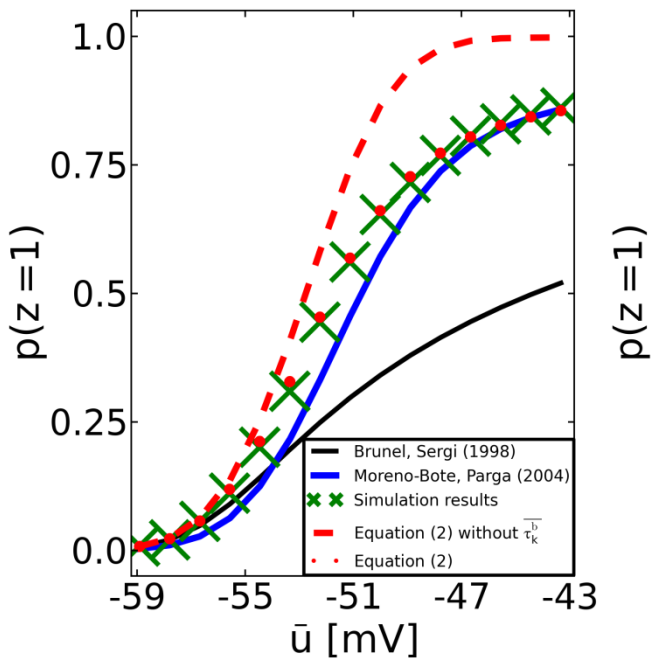


$$\tilde{\nu} = \left(\tau_{\mathrm{m}} \frac{\varrho - \tilde{u}}{\vartheta - \tilde{u}}\right)^{-1}$$

$$\nu = \int_{\vartheta}^{\infty} \tilde{\nu}(\tilde{u})\, p(\tilde{u})\, d\tilde{u}$$

however, remember that $\tau_{\mathrm{ref}} \approx \tau_{\mathrm{syn}}$!

$\Rightarrow$ adiabatic approximation does not hold !



The membrane autocorrelation propagation

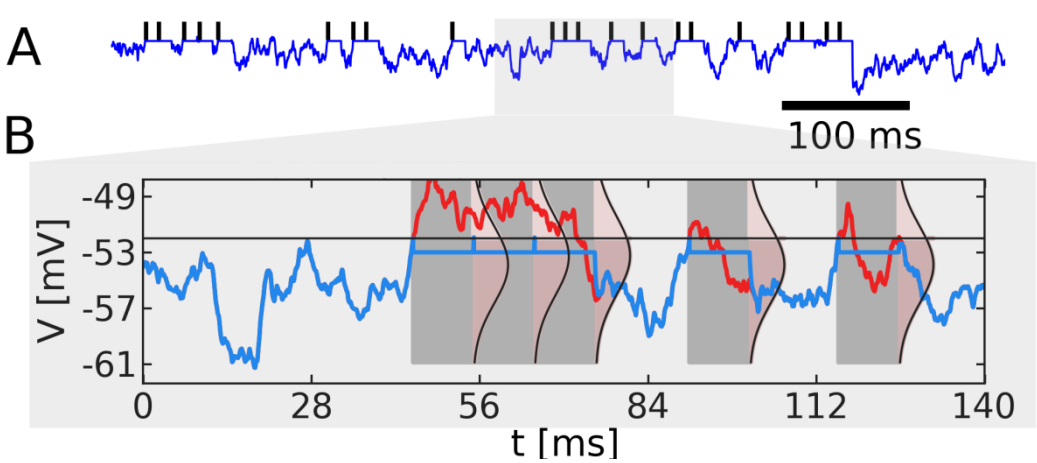
$$p(z_k = 1) = \frac{t_{k,\,\text{refractory}}}{t_{\text{total}}} = \frac{\sum_n P_n \, n \, \tau_{\text{ref}}}{\sum_n P_n \cdot \left(n \, \tau_{\text{ref}} + \sum_{k=1}^{n-1} \overline{\tau_k^b} + T_n \right)}$$

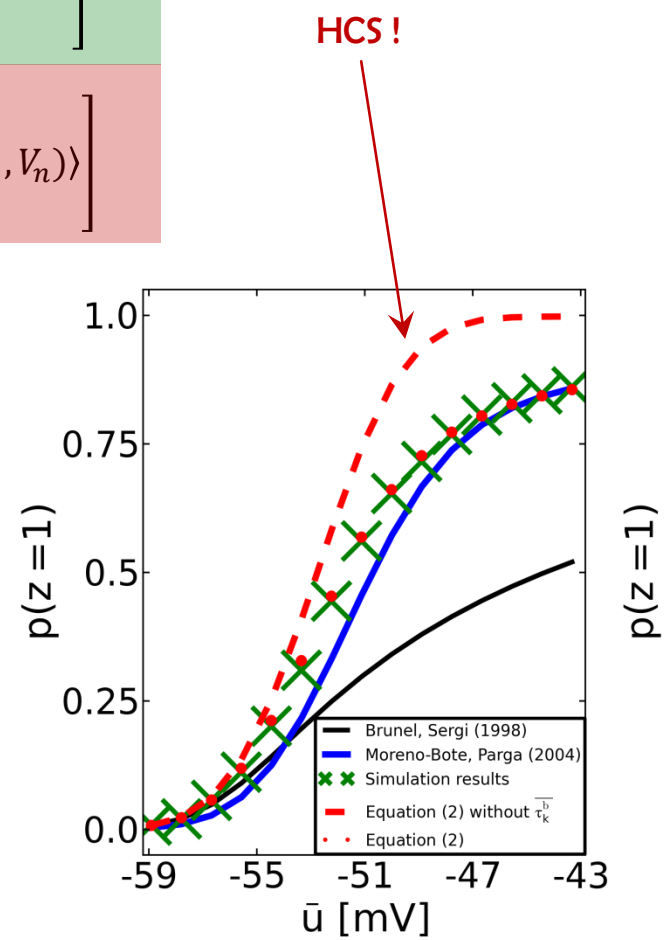
Petrovici & Bill et al. (2016)

$$P_n = \left(1 - \sum_{i=1}^{n-1} P_i \right) \cdot \int_{V_{\text{thr}}}^{\infty} dV_{n-1} \; p(V_{n-1} | V_{n-1} > V_{\text{thr}}) \left[\int_{-\infty}^{V_{\text{thr}}} dV_n \; p(V_n | V_{n-1}) \right]$$

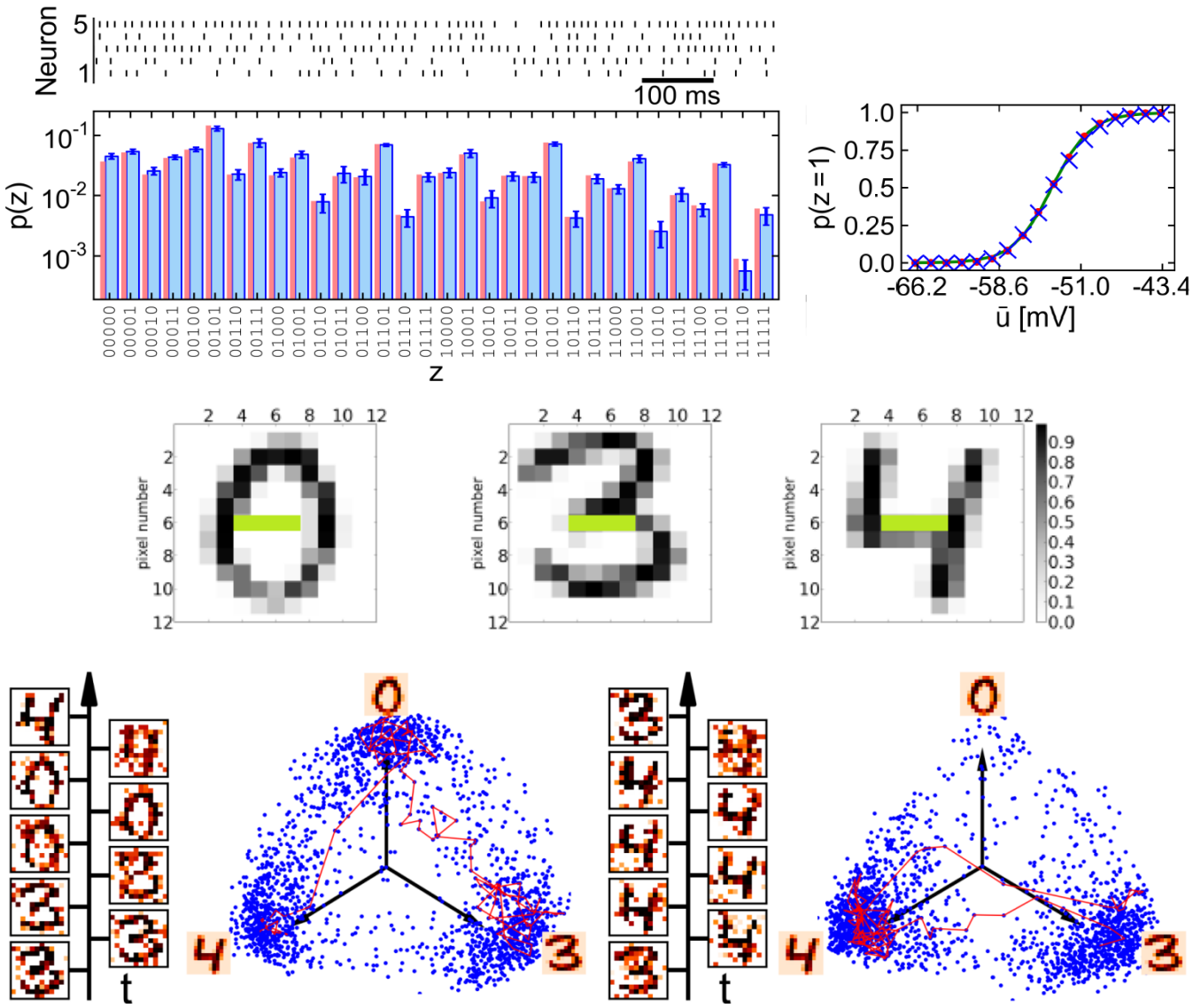
$$T_n = \int_{V_{\text{thr}}}^{\infty} dV_{n-1} \; p(V_{n-1} | V_{n-1} > V_{\text{thr}}) \left[\int_{-\infty}^{V_{\text{thr}}} dV_n \; p(V_n | V_{n-1}) \langle FPT(V_{\text{thr}}, V_n) \rangle \right]$$

$$\overline{\tau_k^b} = \int_{\vartheta}^{\infty} du_k \, \tau_{\text{eff}} \ln \left(\frac{\varrho - u_k}{\vartheta - u_k} \right) p(u_k | u_k > \vartheta, u_{k-1})$$



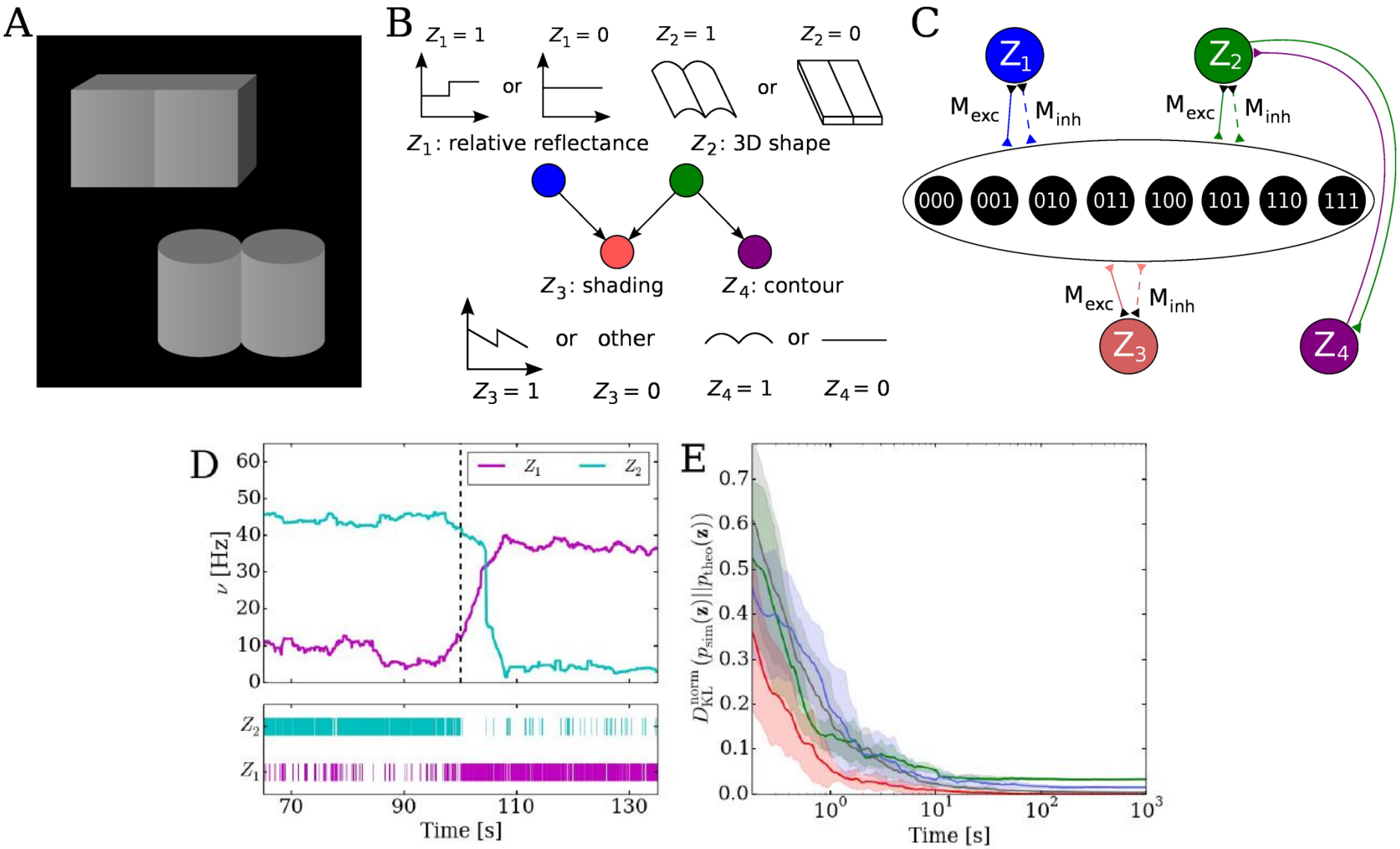


(Fully visible) LIF-based Boltzmann machines



Petrovici & Bill et al. (2016)

Beyond Boltzmann: Spiking Bayesian networks



Probst & Petrovici et al. (2015)

Deep spiking discriminative architectures

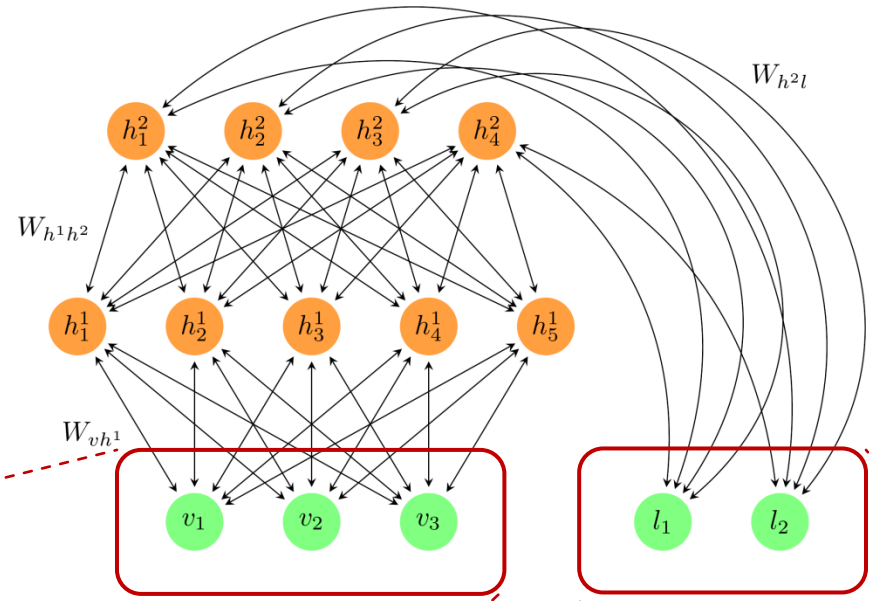
Training of RBMs/DBMs on MNIST:

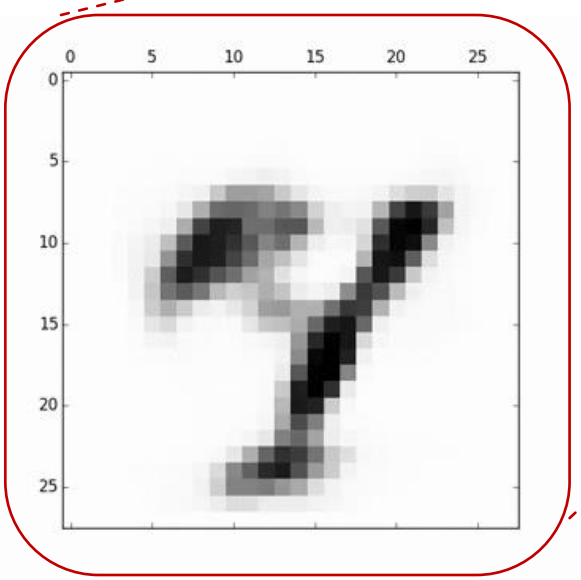
- maximum likelihood learning

$$\Delta w_{ij} \propto \langle z_i z_j \rangle_{\text{data}} - \langle z_i z_j \rangle_{\text{model}}$$

$$\Delta b_i \propto \langle z_i \rangle_{\text{data}} - \langle z_i \rangle_{\text{model}}$$

- coupled adaptive tempering

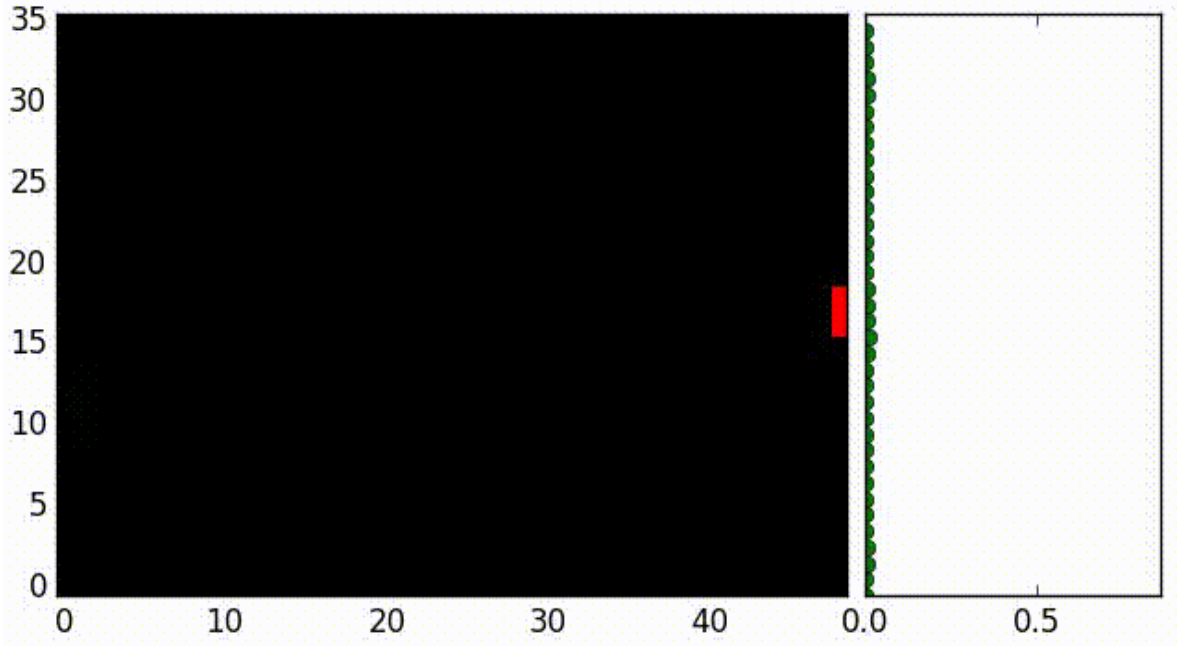


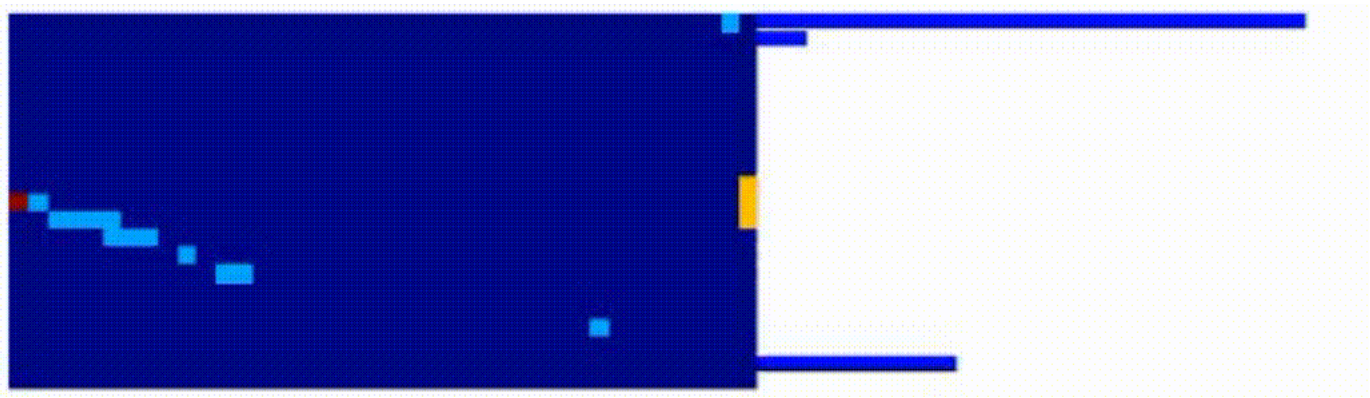


96.9 % correct classification with less than 2000 neurons

Leng & Petrovici et al. (2016)

Deep pong





Roth, Zenk (2017)

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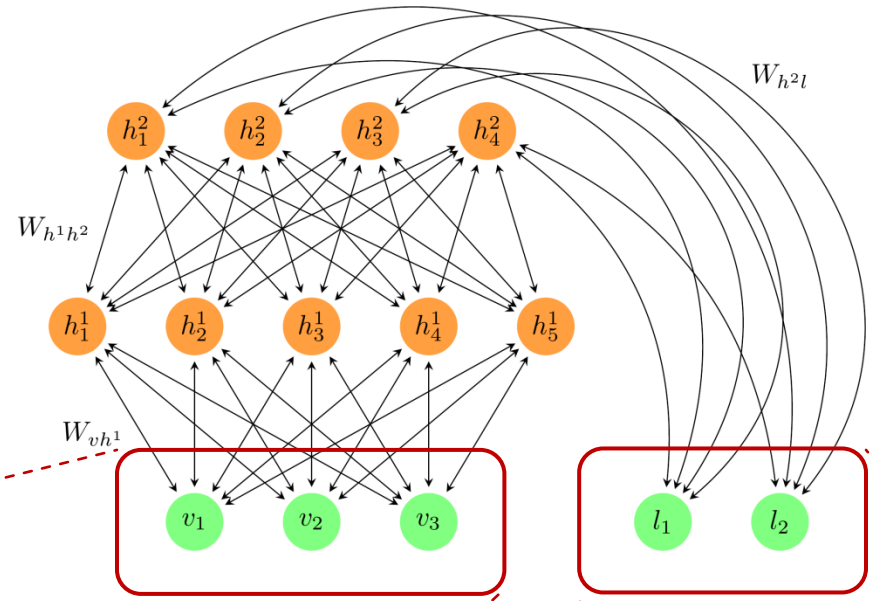
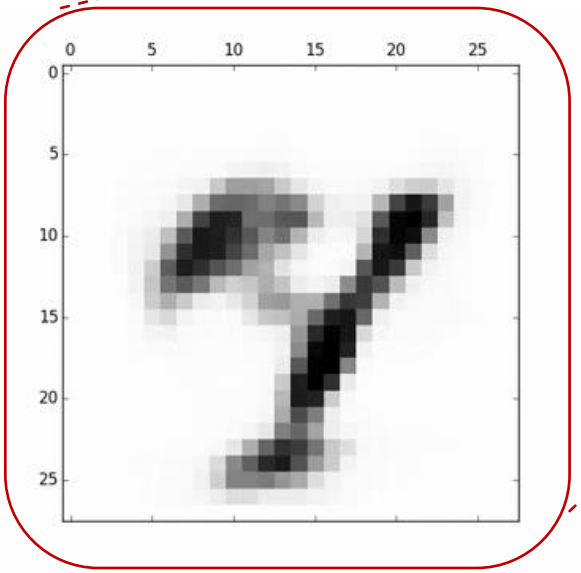
- coupled adaptive tempering



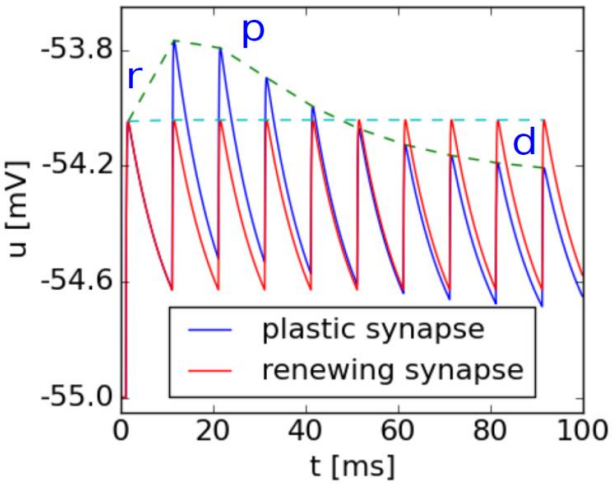


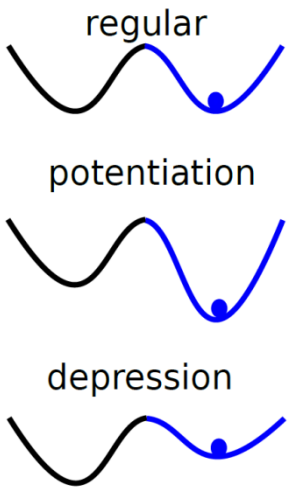
96.9 % correct classification with less than 2000 neurons

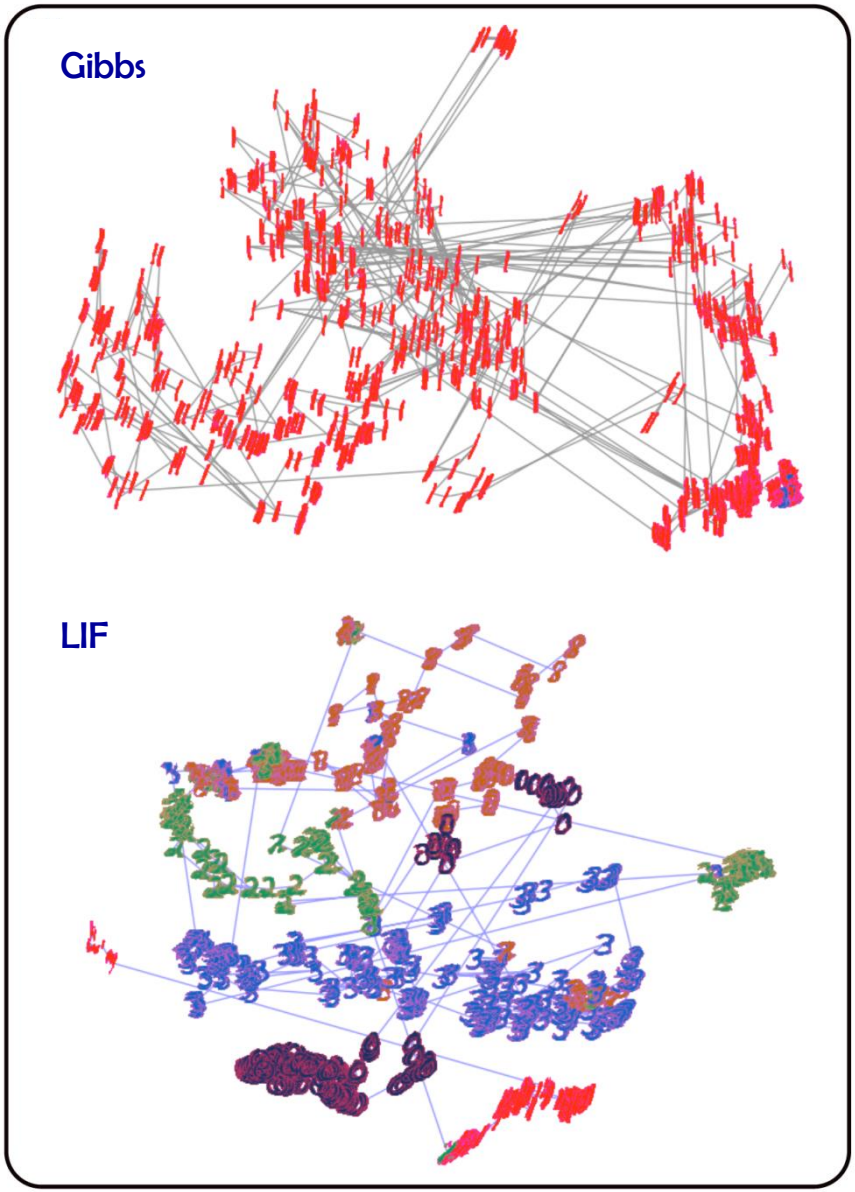
Leng & Petrovici et al. (2016)

Short-term plasticity enables superior mixing

STP model: Tsodyks & Markram (1997)



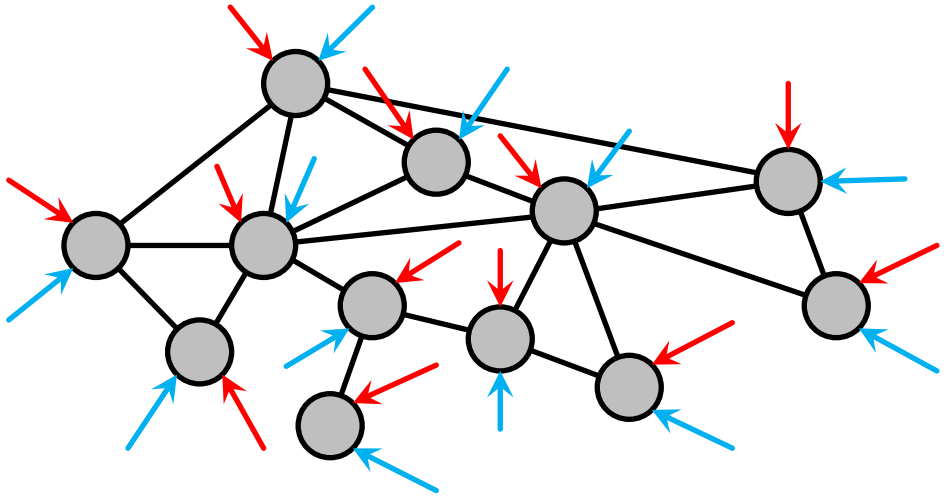




Leng & Petrovici et al. (2016)

... so where does the noise come from?

1st approximation: independent Poisson sources



unrealistic in both biological & artificial systems

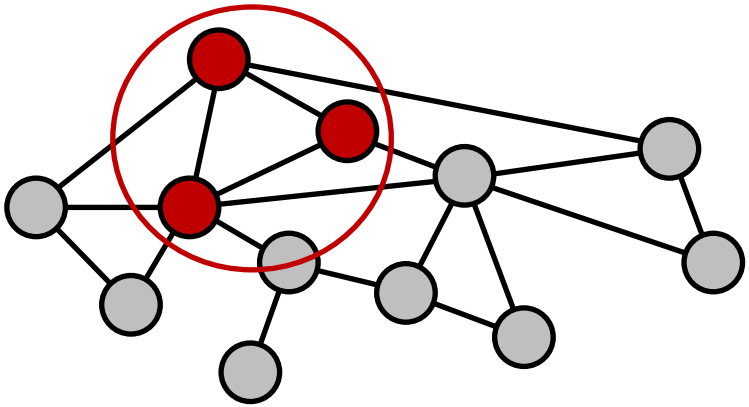
better: common pool of presynaptic partners

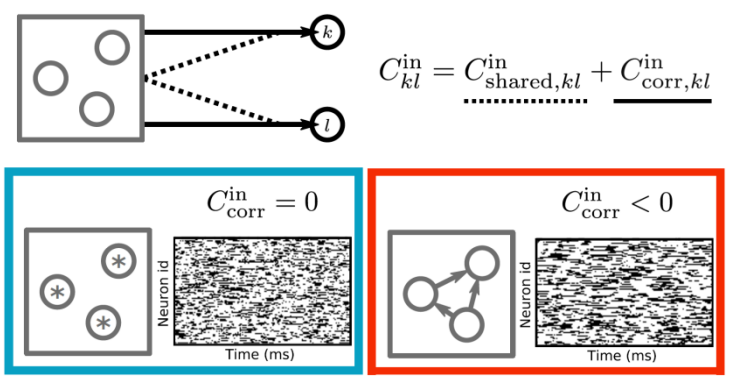
$\Rightarrow$ correlated inputs

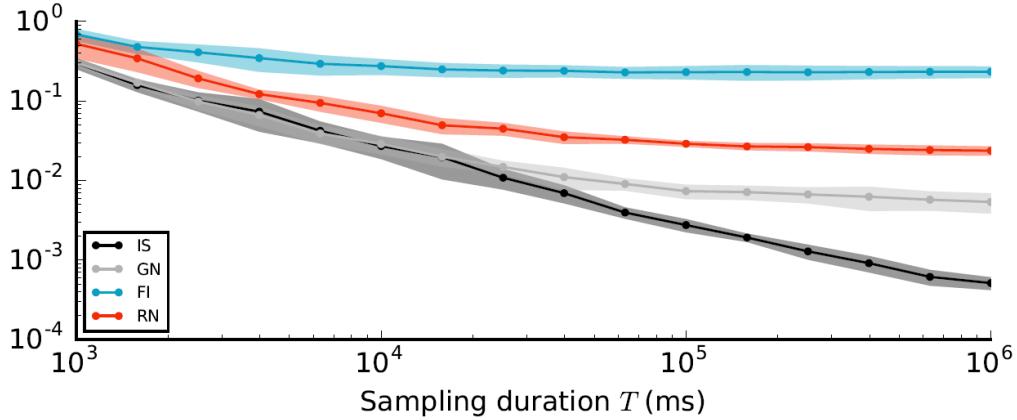
$\Rightarrow$ deviation from target distribution

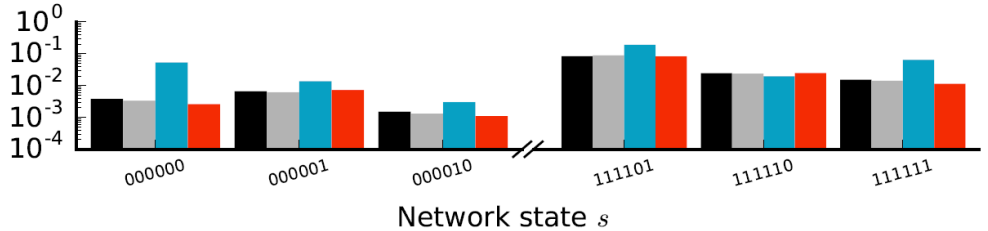
Embedded stochastic inference machines

more realistic: sea of noise



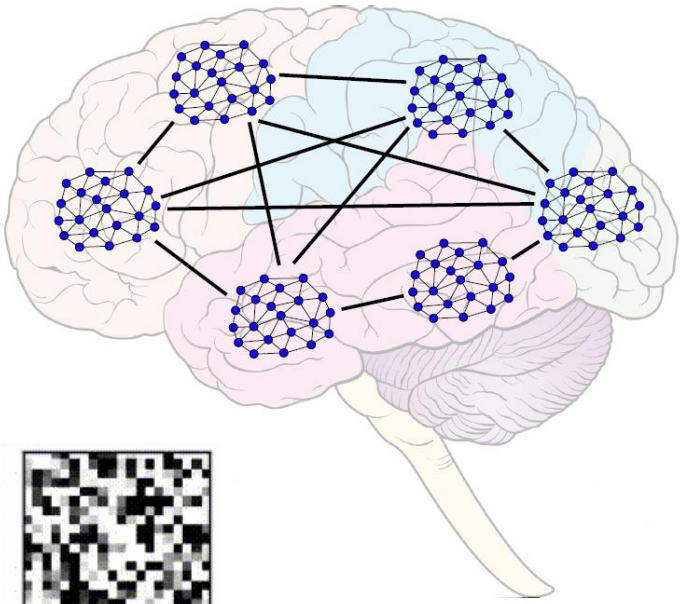


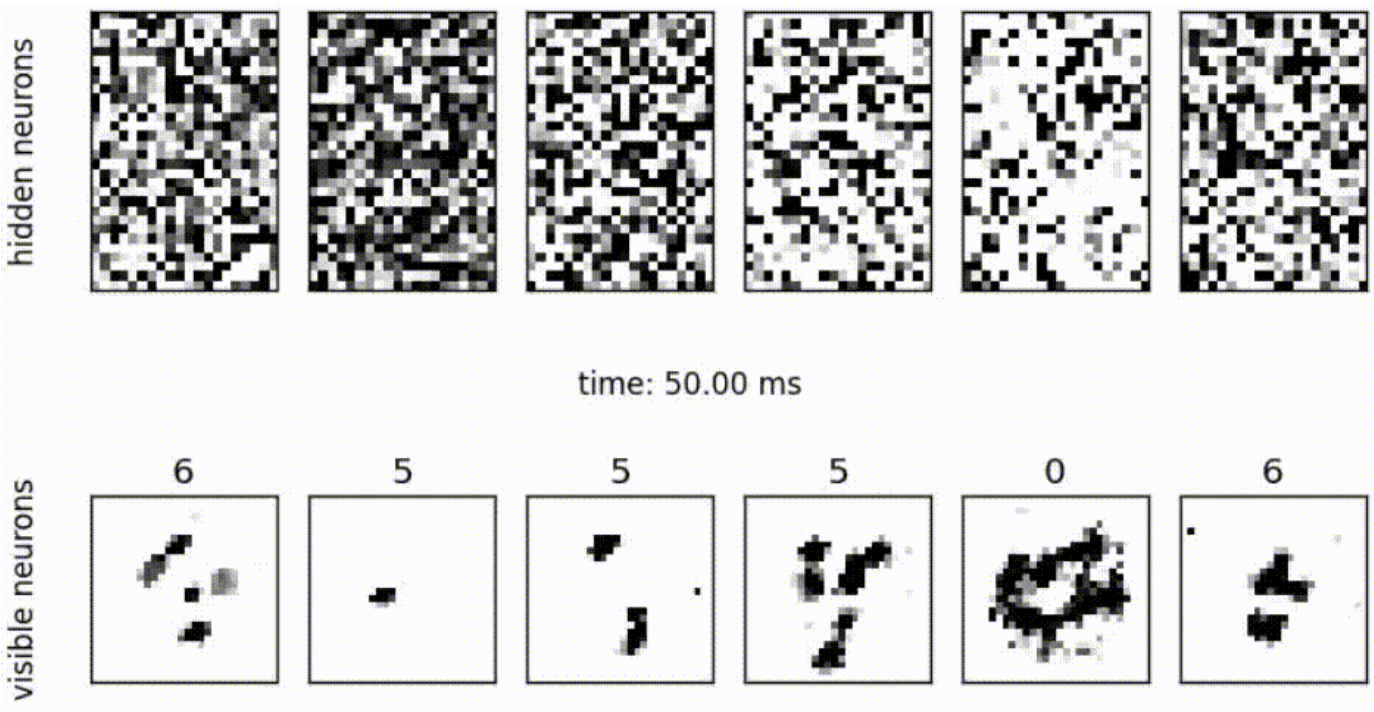




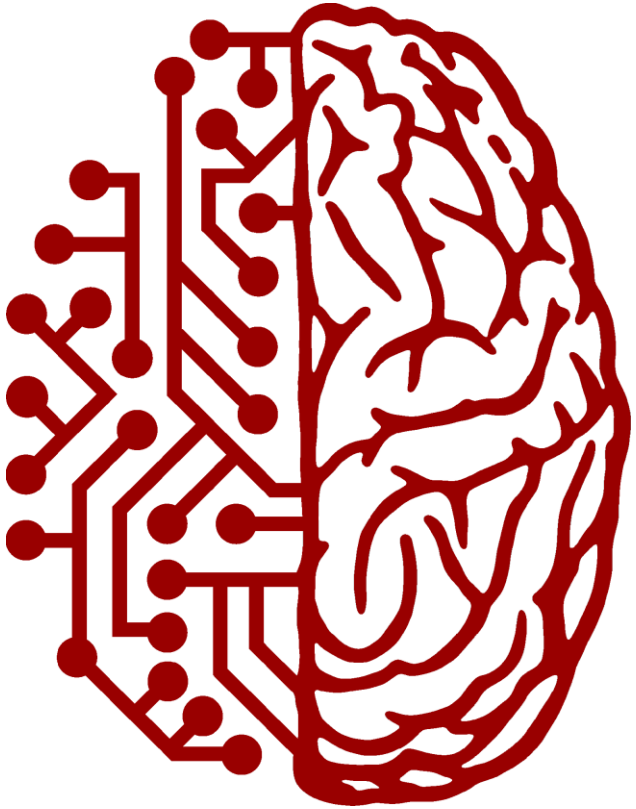
Jordan et al. (2015)

Noiseless stochastic computation



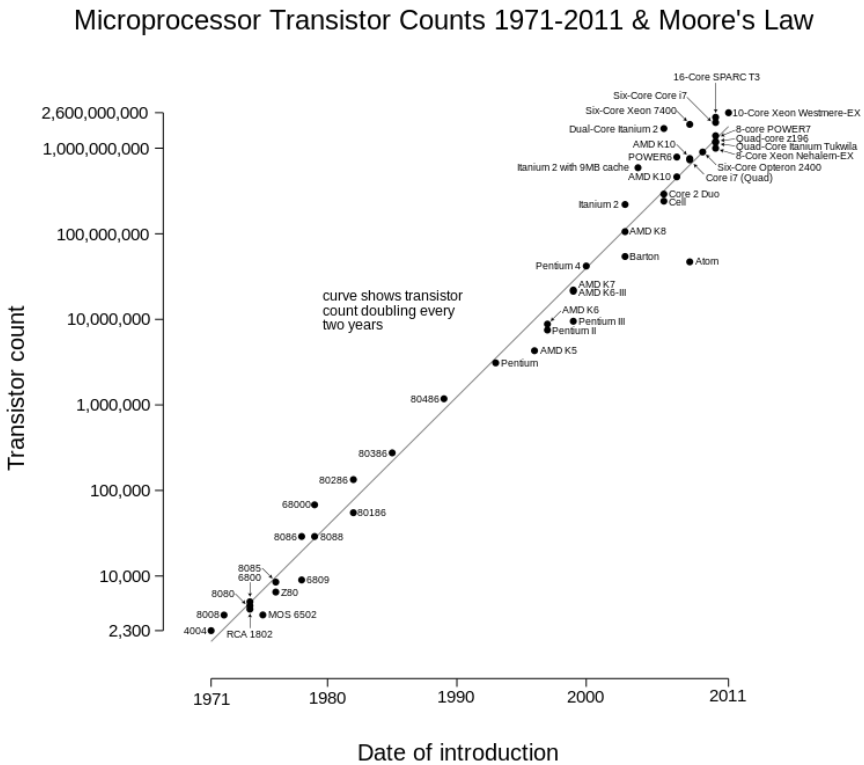


ongoing work with Dominik Dold and Ilja Bytschok



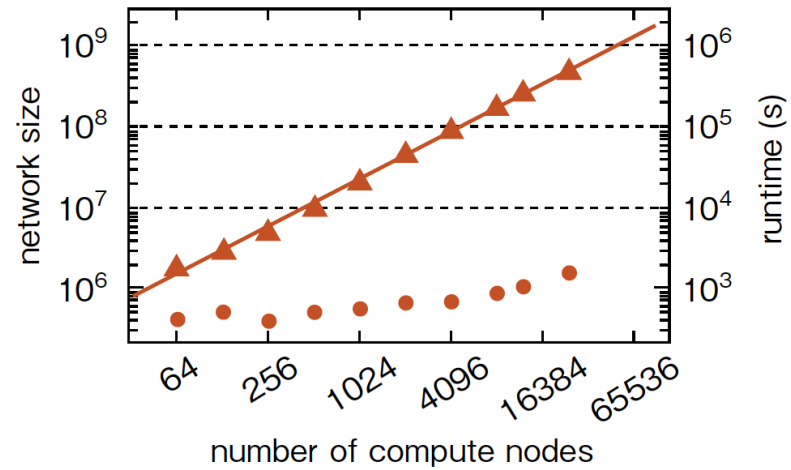
Physical emulation of spiking networks

Simulation: size & time



simulation speed 1520:1

compared to biological real-time



Diesmann (2012)

	nature	simulation
synaptic plasticity	seconds	hours
learning	days	years
development	years	millennia
evolution	> millennia	> millions of years

Simulation & emulation: energy scaling

Energy Scales

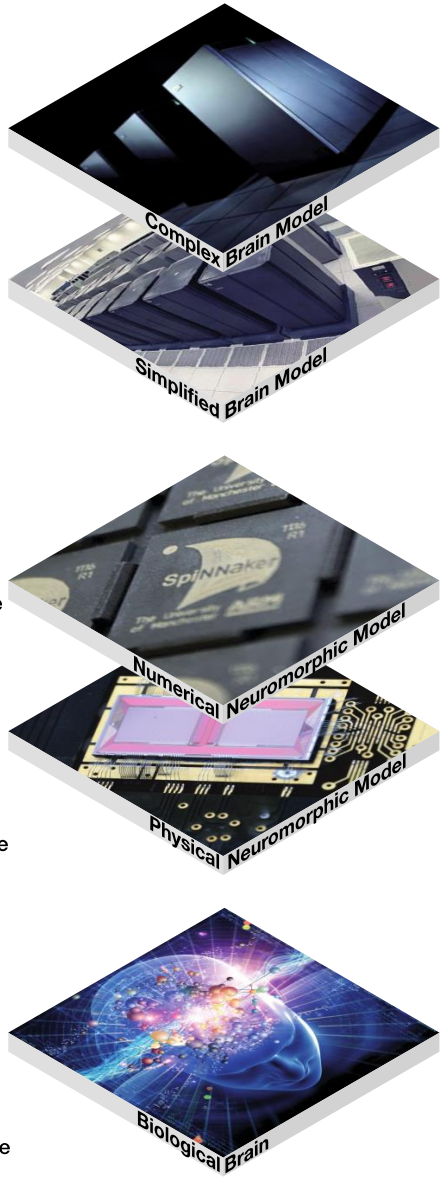
10^{0} J
1 Joule

10^{-4} J
0.1 milliJoule

10^{-8} J
10 nanoJoule

10^{-10} J
0.1 nanoJoule

10^{-14} J
10 femtoJoule

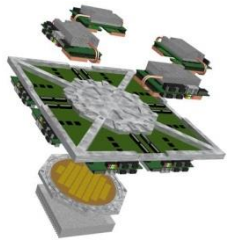


E F F I C I E N C Y

Blue Brain Project

nest:: initiative

SpiNNaker



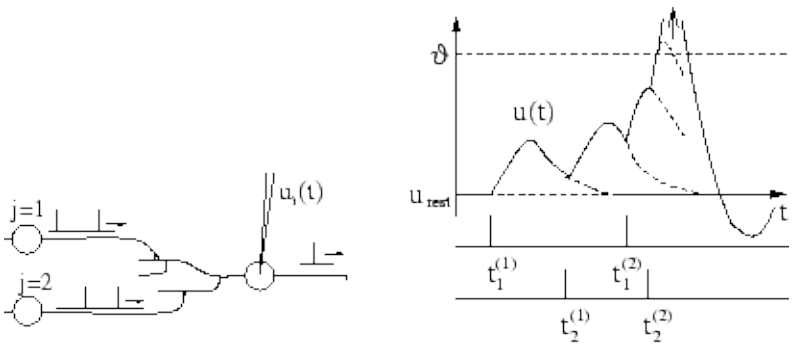
BrainScaleS

1.000.000 times more energy-efficient

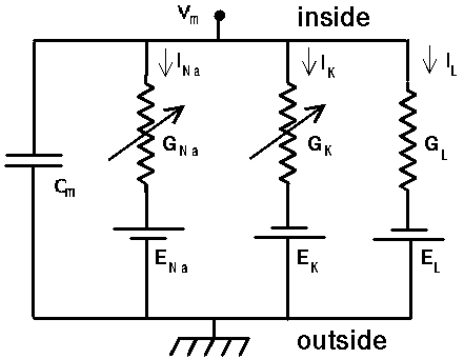
10.000 times less energy-efficient

Analog neuromorphic hardware

Electrophysiology



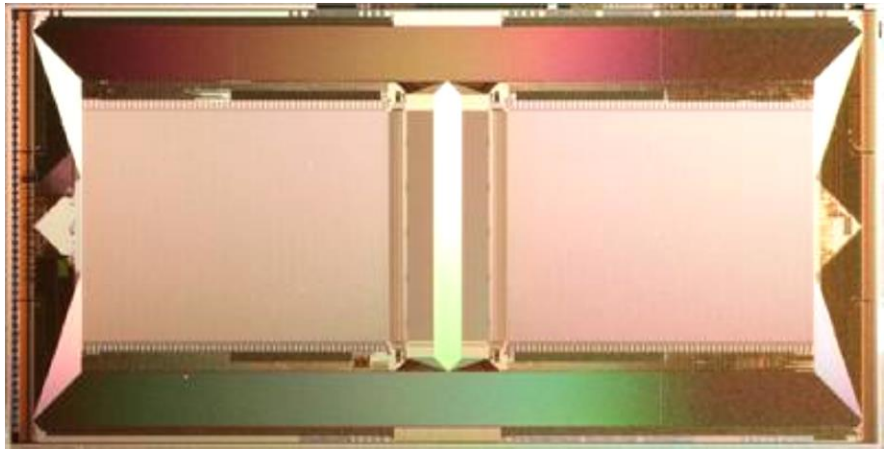
Hodgkin-Huxley-Model



$$C_m \dot{u} = g_L(u - E_L) + g_{\text{syn}}\,(u - E_{\text{syn}})$$

$$\cancel{g_{\text{K}}\, n^4 (u - E_{\text{K}}) + g_{\text{Na}}\, m^3 h (u - E_{\text{Na}})}$$

HICANN chip



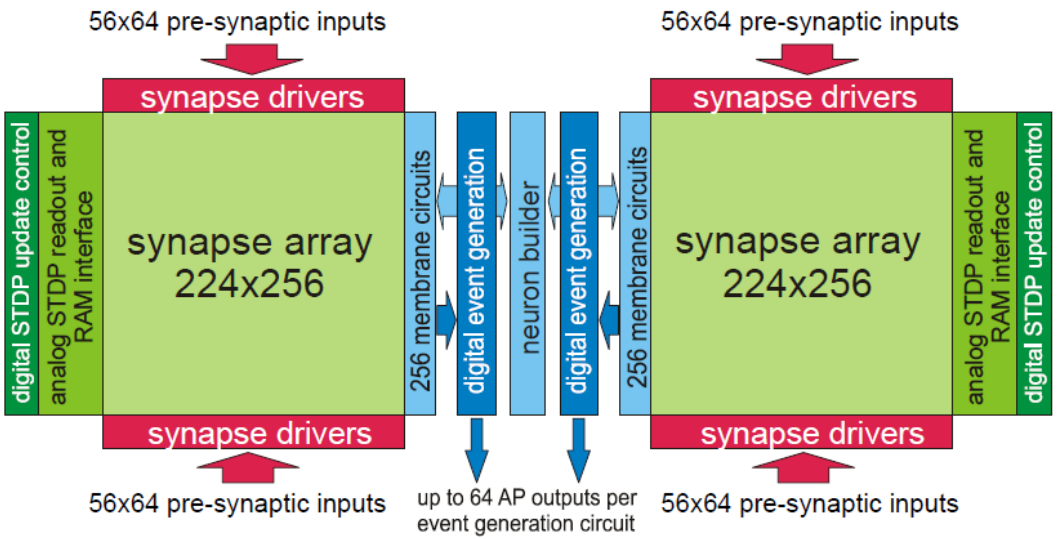
Schemmel et al. (2010)

Adaptive Exponential I&F Model

$$C_m \dot{u} = g_L(u - E_L) + g_{\text{syn}}\left(u - E_{\text{syn}}\right) + g_L \Delta_T \exp\left(\frac{V - V_T}{\Delta_T}\right) - w$$

$$\tau_w \dot{w} = a(V - E_L) - w$$

Analog neuromorphic hardware



HICANN chip

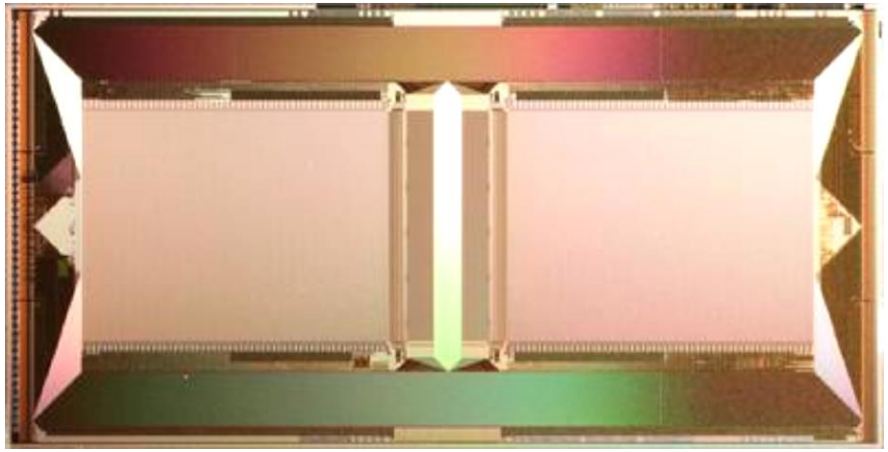
- mixed-signal VLSI:

  membrane → analog

  spikes → digital

- inherent speedup:

  $10^3 - 10^5$



Schemmel et al. (2010)

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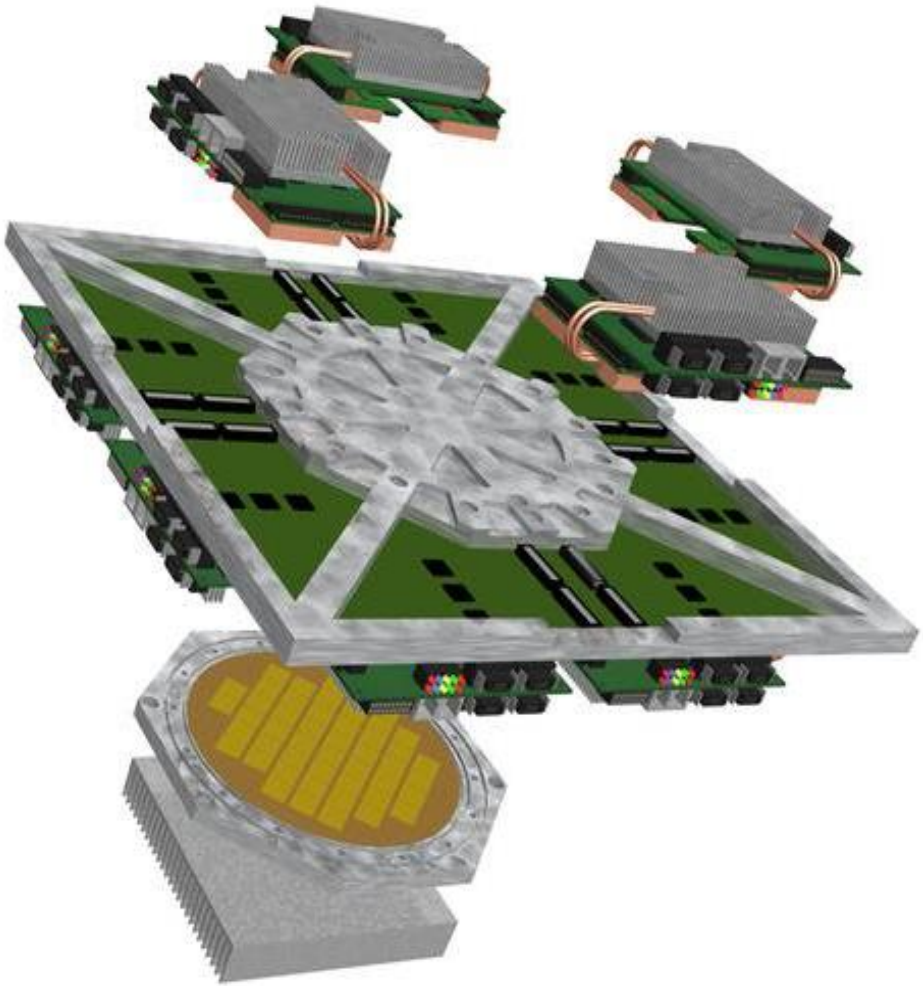
$$\tau_w \dot{w} = a(V - E_L) - w$$

Waferscale integration → BrainScaleS system

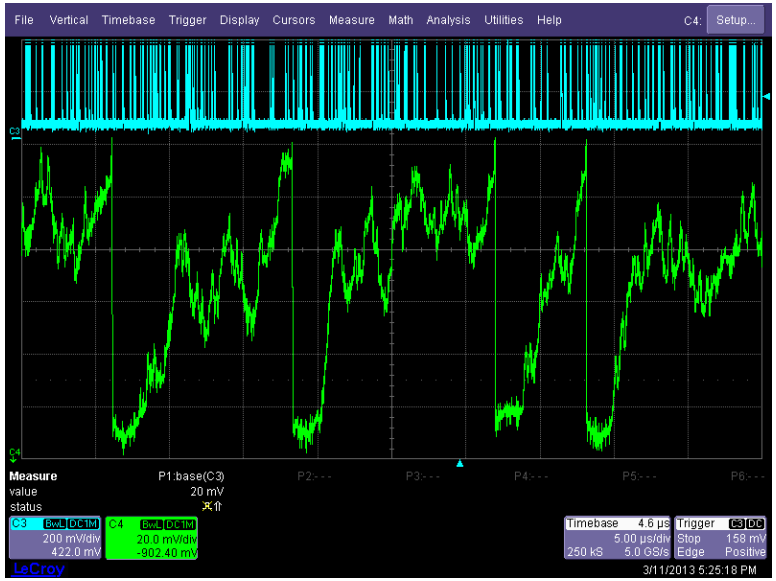


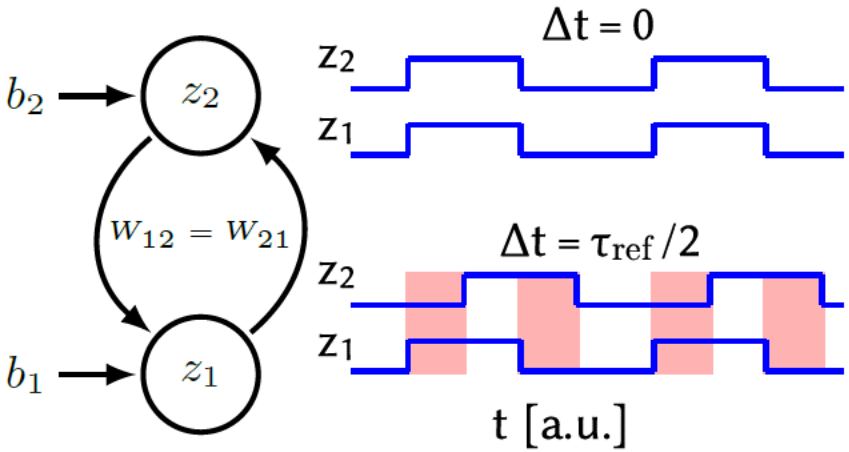
Schemmel et al. (2010)

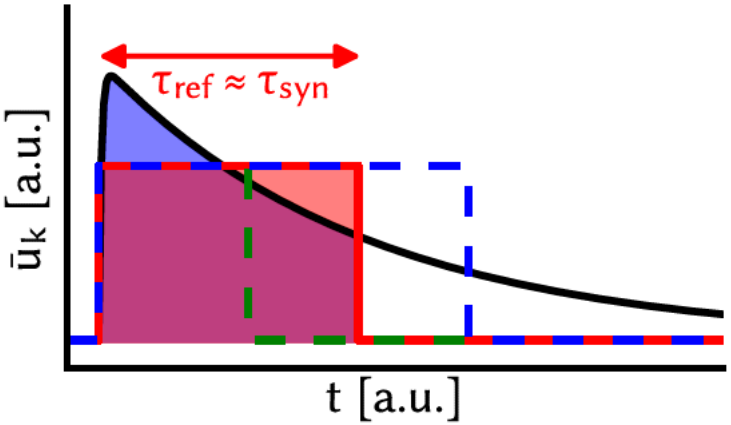
The Hybrid Modeling Facility in Heidelberg

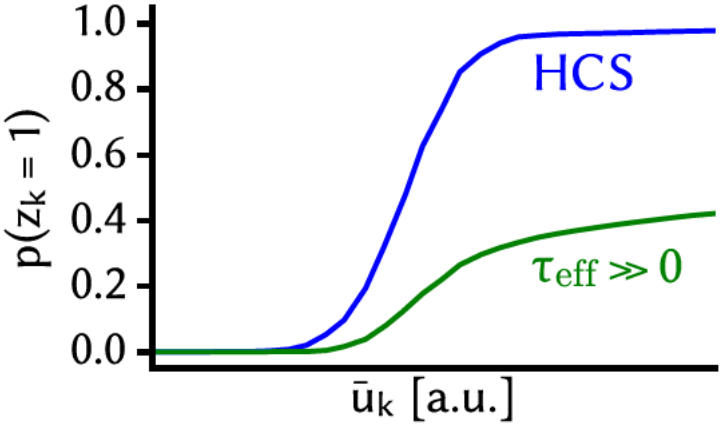
4 million AdEx neurons, 1 billion conductance-based synapses, under construction

Hardware is not software...

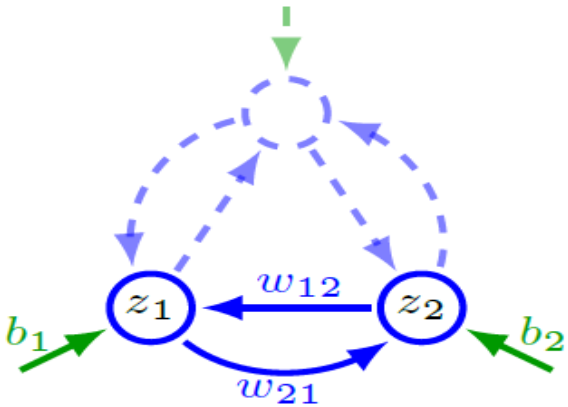


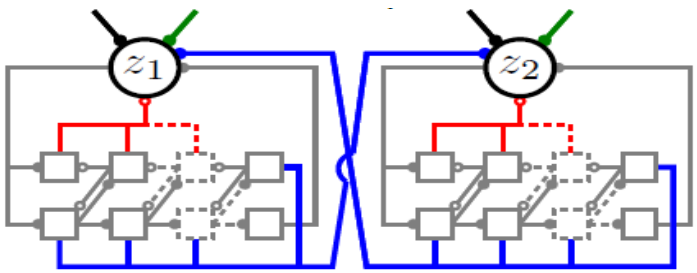


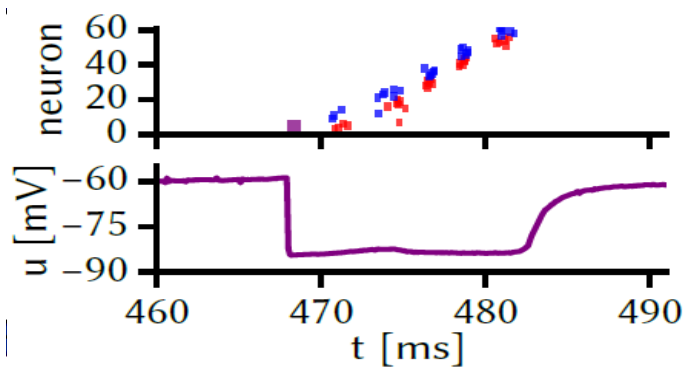


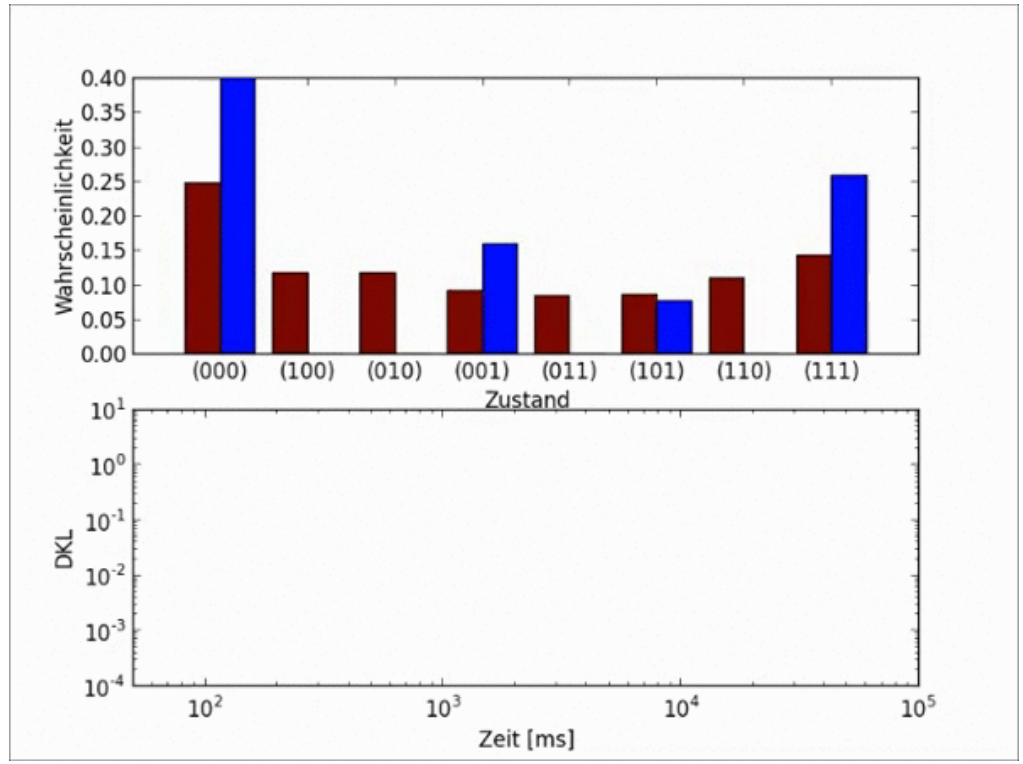


LIF sampling on accelerated hardware









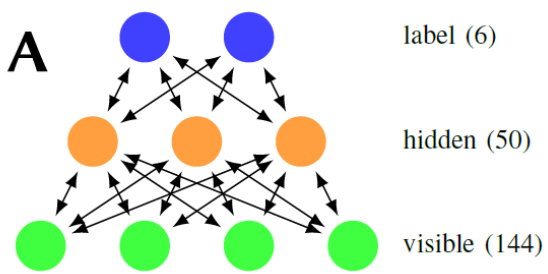
bio time: 100 s

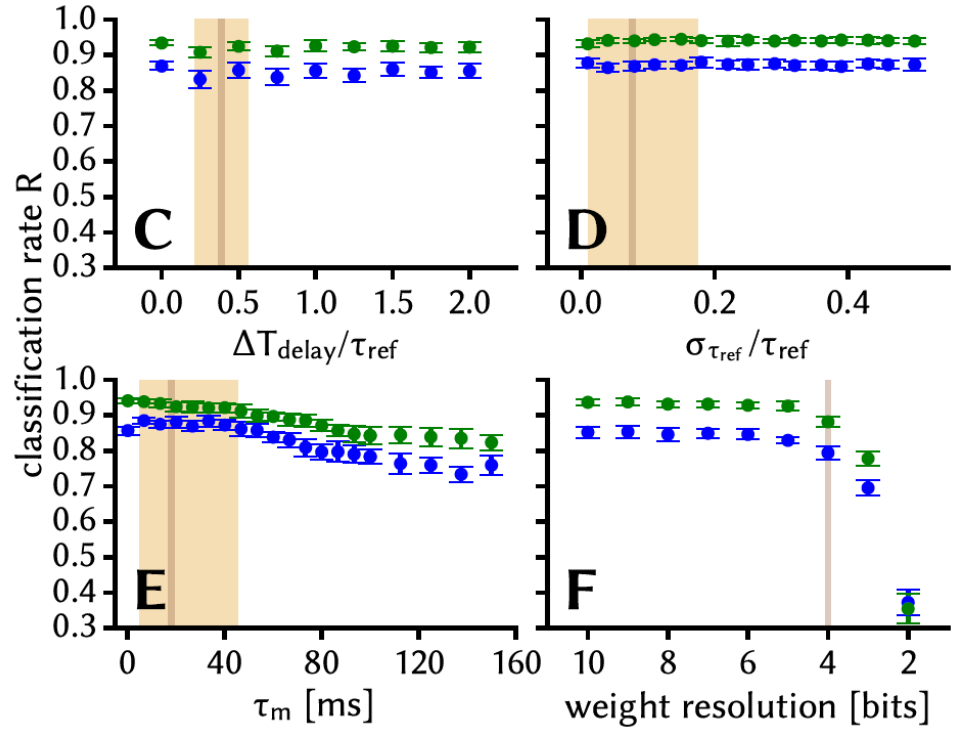
software simulation: 1 s

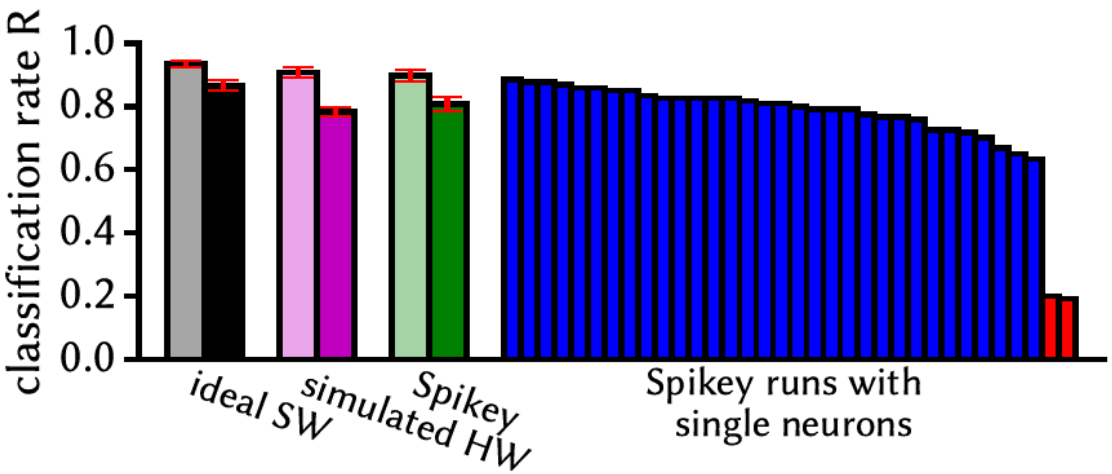
hardware emulation: 10 ms

Petrovici & Stöckel et al. (2015), Petrovici et al. (2017)

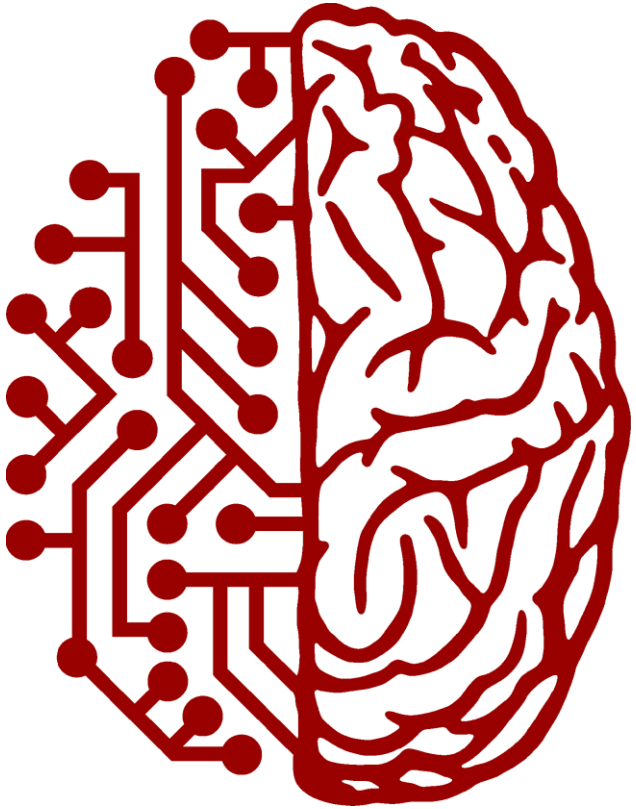
Robustness from structure





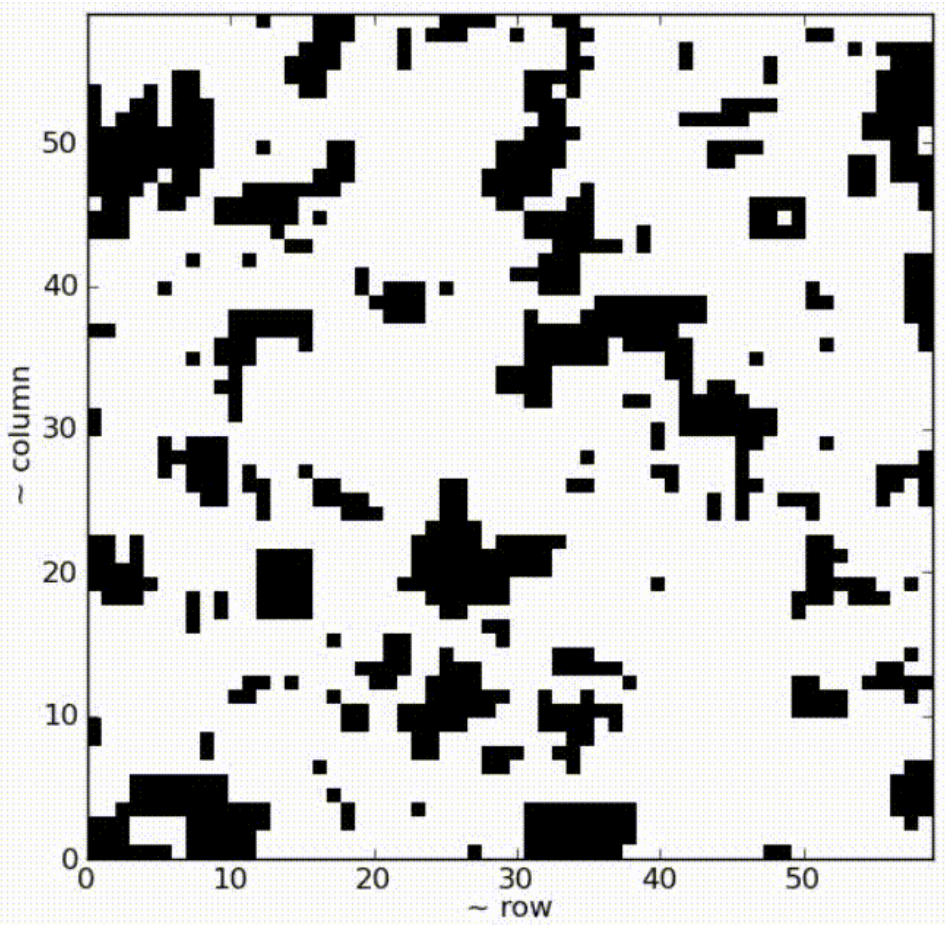


Petrovici & Schröder et al. (2017)



Outlook / Work in progress

Ensemble dynamics



$$p(\sigma) = \frac{1}{Z} \exp\left[\frac{1}{2}\sigma^T J\,\sigma + \sigma^T b\right]$$

spiking networks

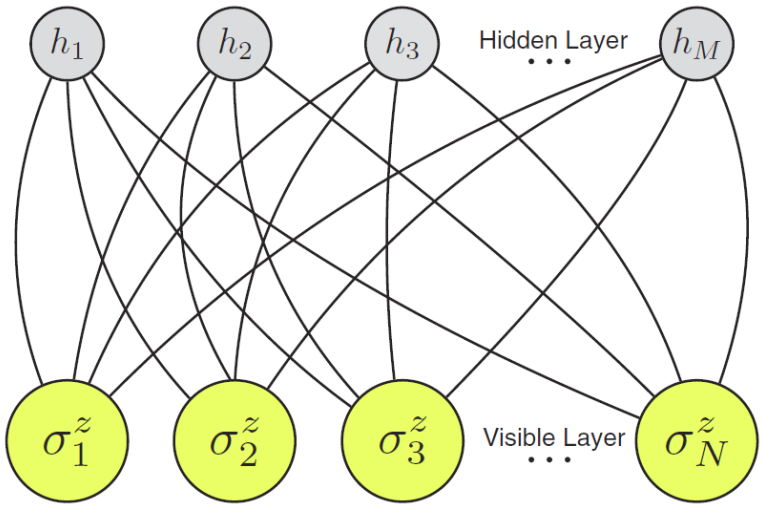
modeling

magnetic systems

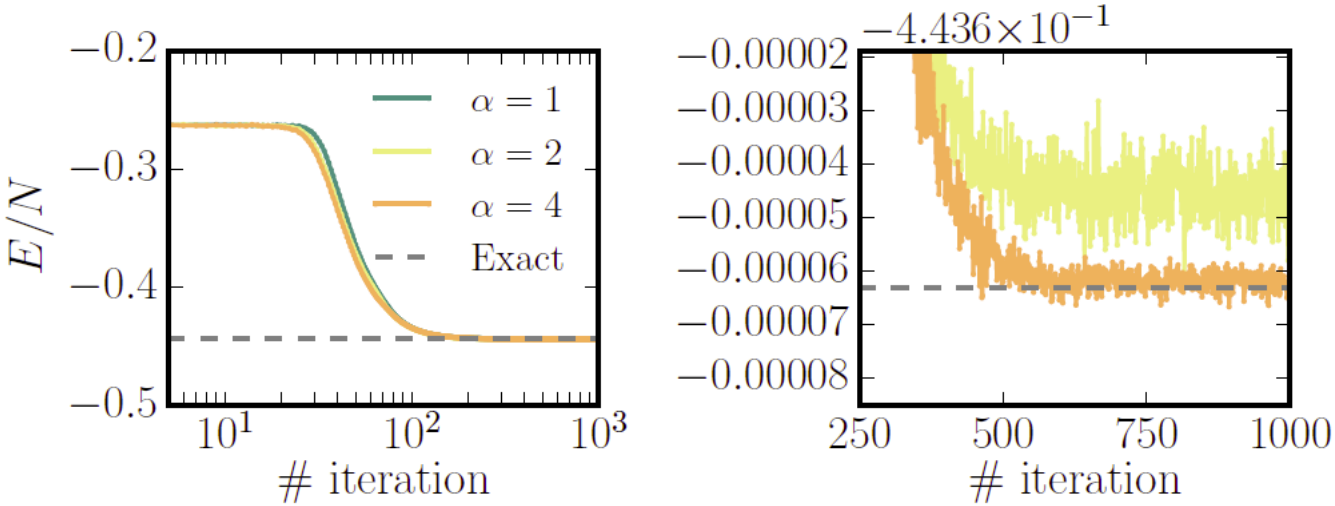
$$p(z) = \frac{1}{Z} \exp\left[\frac{1}{2} z^T W z + z^T b\right]$$

ongoing work with Andreas Baumbach

Quantum many-body problems



$$\Delta w \propto \frac{\partial}{\partial w} \frac{\langle \Psi_M | H | \Psi_M \rangle}{\langle \Psi_M | \Psi_M \rangle}$$



Carleo & Troyer (2017)

Learning rules

$$\dot{w}_k^a = \eta \left(\phi(U) - \phi(\alpha V^a)\right) \mathrm{PSP}_k$$

- **maximum likelihood learning**

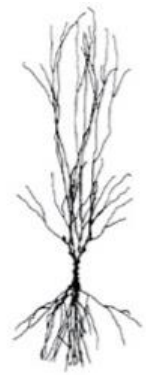
$$\Delta w_{ij} \propto \langle z_i z_j \rangle_{\text{data}} - \langle z_i z_j \rangle_{\text{model}}$$

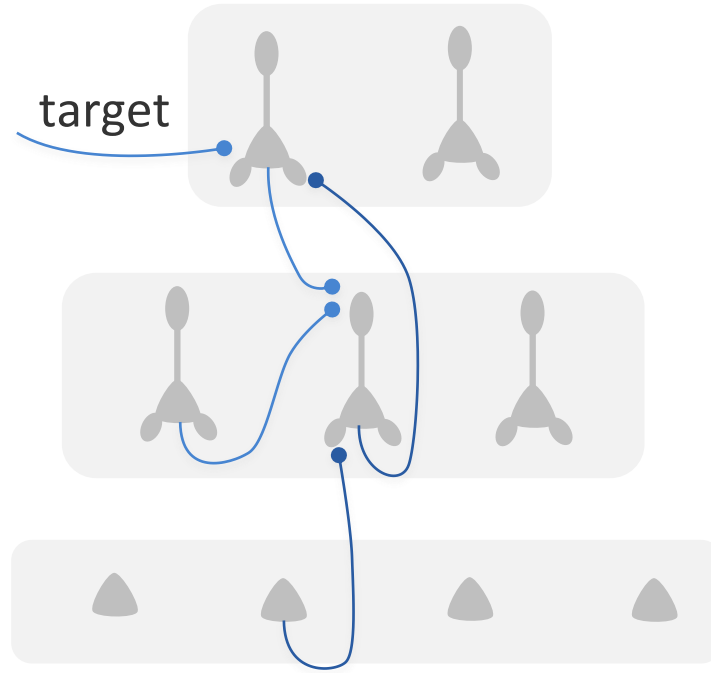
$$\Delta b_i \propto \langle z_i \rangle_{\text{data}} - \langle z_i \rangle_{\text{model}}$$

- **backprop**

$$\Delta w_{ij} \propto \frac{\partial E}{\partial o_j} \frac{\partial o_j}{\partial \text{net}_j} \frac{\partial \text{net}_j}{\partial w_{ij}}$$

vs.





ongoing work with Joao Sacramento and Walter Senn

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