Problem 3.1 Define the Cayley-Menger determinant of $n$ points $x_1, \ldots, x_n$, where the pairwise distances $d(x_i, x_j) = \|x_i - x_j\|$ are given, by

$$\text{CM}(x_1, \ldots, x_n) = \left| \begin{array}{cccc} 0 & 1 & \cdots & 1 \\ 1 & d(x_1, x_1)^2 & \cdots & d(x_1, x_n)^2 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & d(x_n, x_1)^2 & \cdots & d(x_n, x_n)^2 \end{array} \right|.$$ 

a) The Cayley-Menger determinant of $n + 2$ points in $\mathbb{R}^n$ vanishes.

b) Let $L = \text{conv}\{v_0, \ldots, v_n\}$ be an $n$-dimensional simplex. Then the circumsphere of $L$ has the squared radius

$$R^2 = -\frac{1}{2} \frac{\det (d(v_i, v_j)^2)_{0 \leq i,j \leq n}}{\text{CM}(v_0, \ldots, v_n)}.$$ 

Problem 3.2 Consider $Q[x] = n \sum_{i=1}^{n} x_i^2 - \sum_{i \neq j} x_i x_j$. Show that $Q$ is locally optimal for the lattice sphere covering problem.