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Online summer school on optimization, interpolation and modular forms

— Problem Sheet 1: Conic Optimization —

Problem 1.1 Use the spectral theorem to show that the cone of positive semidefinite matrices is self dual.

Problem 1.2

a) Show that the set

$$\mathcal{D}^{n+1} = \{(X, s) \in \mathcal{S}^n_{\succeq 0} \times \mathbb{R}_{\geq 0} : (\det X)^{1/n} \geq s\}$$

is a proper convex cone.

b) Use the inequality of arithmetic and geometric means to determine its dual cone $(\mathcal{D}^{n+1})^*$.

Problem 1.3 Let $f \in \mathbb{R}[x_1, \ldots, x_n]_d$ be a homogeneous polynomial of degree d and let $\alpha \in \mathbb{R}^n$ be a vector. Compute $\langle f, (\alpha^{\mathsf{T}} x)^d \rangle$ with $x = (x_1, \ldots, x_n)$.

Problem 1.4 Show that the PARTITION problem has a solution if and only if

$$\sup \left\{ t : (c^{\mathsf{T}} x)^4 + n \sum_{i=1}^n x_i^4 - \left(\sum_{i=1}^n x_i^2 \right)^2 - t \in P_{n,4} \right\}$$

has optimal solution $t = 0$.