

Research Report (EPFL)

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1 Overview

During my time at EPFL, my research focused on stochastic analysis, nonlinear PDEs, and interacting particle systems, with applications ranging from multi-species dynamics and multicomponent transport to modern mathematical finance. A central theme of my work at EPFL was the rigorous analysis of **highly degenerate nonlinear SPDEs**, especially those arising as hydrodynamic or fluctuation limits of microscopic stochastic systems. This period allowed me to significantly expand and deepen my research program, leading to substantial progress in four primary directions:

- stochastic cross-diffusion systems and hydrodynamic fluctuation theory,
- interacting particle systems and nonlinear mean-field SPDEs,
- stochastic Volterra equations, Markovian lifts, and rough volatility,
- algebraic and probabilistic structures related to signatures and polynomial processes.

2 Stochastic Cross-Diffusion Systems

At EPFL, I advanced my earlier work on cross-diffusion systems by developing a fluctuation-corrected stochastic version of the Shigesada–Kawasaki–Teramoto (SKT) system, derived from an underlying n -species particle model. In [5], I showed that this microscopic interpretation naturally leads to a system of nonlinear, degenerate SPDEs that incorporate *hydrodynamic fluctuation noise*. Using the entropy structure of the SKT system and a new entropy-based regularization method (introduced in earlier joint work [1, 7]), I established the existence of global nonnegative weak martingale solutions for these fluctuation-driven systems.

This project clarified which noise structures are physically and mathematically meaningful in degenerate multi-species systems. It also sets the stage for stochastic Maxwell–Stefan systems, which I began to investigate at EPFL as a natural next step.

3 Interacting Particle Systems and Nonlinear Mean-Field SPDEs

A second major research direction concerned interacting particle systems (IPS) arising from market capitalization models and stochastic portfolio theory. Together with C. Cuchiero, I studied the evolution of empirical measures

$$\rho_t^N = \frac{1}{N} \sum_{i=1}^N \delta_{X_t^i},$$

where each X_t^i models the market capitalization of a firm and evolves under stochastic dynamics with common noise and mean-field effects.

In [2, 3], we proved that, as $N \rightarrow \infty$, the empirical distribution converges to a nonlinear, fully degenerate SPDE, and we gave a complete analytic characterisation of the macroscopic dynamics. We established:

- a representation of the limiting SPDE as the conditional law of a McKean–Vlasov SDE,
- weighted Sobolev regularity of its marginal densities,
- a conditional propagation-of-chaos result.

I extended this analysis to *fluctuations* around the mean-field limit. In [6], I proved that the fluctuations of the empirical measure converge to a parabolic SPDE whose coefficients are determined by the hydrodynamic limit itself. For volatility-stabilized models, I further analysed the relation between the SPDE with common noise and its deterministic counterpart, identifying a fluctuation structure consistent with observed capital distribution curves.

These results form one of the first rigorous fluctuation theories for interacting financial systems in the area of so-called volatility-stabilised market models with common noise.

A final project in this direction is concerned with regularity estimates for cluster points of the limit of the empirical measure, without requiring any knowledge of the dynamics of the limiting measure-valued flow in the setting of general coefficients and degenerate/hypoelliptic particle dynamics.

4 Stochastic Volterra Equations and Markovian Lifts

Volterra-type stochastic equations are central in rough volatility modelling. At EPFL, I completed a general solution theory for stochastic Volterra equations with completely monotone kernels by lifting them to nonlocal stochastic evolution equations in weighted Sobolev spaces [4]. The main issue being that solutions of Volterra equations are typically *not Markov or semimartingales*, necessitating infinite-dimensional Markovian lifts. This work provides:

- existence and uniqueness in a broad setting,
- invariant measures for the lifted process and the original stochastic Volterra equation,
- an Itô-type formula adapted to the nonlocal memory structure.

This project creates a unified framework capable of treating classical models (such as rough Heston) as well as more general, non-Markovian volatility dynamics.

5 Signatures, Polynomial Structures, and Non-Markovian Effects

A fourth line of research I started at EPFL involved algebraic structures underlying rough-path signatures and their use in modelling. I worked on generalising polynomial processes to the Hopf-algebraic settings where coefficients depend on signature-type quantities, with implications for rough volatility, non-Markovian time series, and models with memory.

6 Summary

My research at EPFL substantially advanced several interconnected areas of modern stochastic analysis, particularly:

- the rigorous theory of fluctuation-corrected cross-diffusion SPDEs,
- mean-field and fluctuation limits of interacting financial systems,
- the analytic foundations of non-Markovian volatility models via stochastic Volterra equations,
- and algebraic-probabilistic tools for modelling rough and memory-dependent systems.

These achievements significantly broadened the analytic toolkit available for equations with degeneracies, common noise, and memory effects, and strengthened the conceptual link between stochastic analysis, nonlinear PDEs, and applications in physics, biology, and finance.

References

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