

## Invited Speakers - Titles & Abstracts

### Workshop on "Stochastic Dynamics and Stochastic Equations"

March 25-27, 2024, Bernoulli Center – Lausanne

#### Alexandre Richard

Title: Densities of SDEs driven by fractional Brownian motion, and application to McKean-Vlasov equations

Abstract:

In this talk, we consider first the SDE  $dX_t = b(t, X_t) + dB_t$ , where  $b$  is singular (e.g. a distribution) and  $B$  is a fractional Brownian motion. We present and review well-posedness results about this equation, giving criteria that relate the regularity of  $b$  and the parameter  $H$ . Then we study the time-space regularity of the density of the solution. Exploiting this regularity, we prove the existence of solutions for McKean-Vlasov equations of the form  $dY_t = \mu_t \ast b(t, Y_t) + dB_t$ , where  $\mu_t$  is the law of the solution  $Y_t$ , for a drift  $b$  which is allowed to be more singular than in the linear case. Joint work with L. Anzeletti, L. Galeati and E. Tanré.

#### Anastasia Papavasiliou

Title: Statistical inference for the diffusion coefficient of homogenisation limit of the fractional multiscale Ornstein-Uhlenbeck process.

Abstract:

The problem of inferring the diffusion coefficient of the homogenisation limit given observations from the multiscale SDE has been studied extensively in the last couple of decades. One of the first results was to derive an optimal subsampling rate for the discretised quadratic variation of the subsampled data to converge to the correct estimate (A. Stuart and G. Pavliotis, 2007). Following up from the recent progress in deriving the homogenisation limits for fractional multiscale SDEs, it is natural to ask the same question in the fractional setting. I will focus on the simplest such case, the fractional Ornstein-Uhlenbeck process with scaled fractional Brownian motion as its homogenisation limit (Gehring and Li, 2022) and present some first results on the subsampling rates that lead to unbiased estimates. The proofs are based on controlling the eigenvalues of the covariance matrices. This is joint work with Horatio Boedihardjo and Pablo Ramses Alonso Martin.

#### Antoine Jacquier

Title: Transportation-cost inequalities for non-linear Gaussian functionals

Abstract:

We study concentration properties for laws of non-linear Gaussian functionals on metric spaces. Our focus lies on measures with non-Gaussian tail behaviour which are beyond the reach of Talagrand's classical Transportation-Cost Inequalities (TCIs). Motivated by solutions of Rough Differential Equations

and relying on a suitable contraction principle, we prove generalised TCIs for functionals that arise in the theory of regularity structures and, in particular, in the cases of rough volatility and the two-dimensional Parabolic Anderson Model. In doing so, we also extend existing results on TCIs for diffusions driven by Gaussian processes.

**Arnaud Debussche** TBA

### **Arnaud Guillin**

Title : Uniform in time propagation of chaos for mean field models with singular interactions.

Abstract:

We will consider two mean field models with singular interactions: vortex 2D and Dyson-Ornstein-Uhlenbeck process. For this two models we will show the propagation of chaos uniformly in time. For the first one we will use an analytic approach and for the second one a coupling technique.

### **Aurélien Deya**

Title: Renormalization of a stochastic NLS model

Abstract:

We will start with a brief introduction about the main two lines of research

associated with random NLS (nonlinear Schrödinger) equations. Then we will focus on the interpretation and wellposedness of the model

$$(i\partial_t - \Delta)u = |u|^2 + B, u \cdot$$

$$2 + B, u \cdot$$

$$0 = 0, t \in \mathbb{R}^+, x \in T,$$

where  $B \cdot$  stands for a space-time fractional noise of indexes  $(H_0, H_1) \in (0, 1)^2$ . In the rough situation where  $2H_0 + H_1 \leq 2$ , this equation cannot be treated in a classical functional sense, making the interpretation of the nonlinearity  $|u|^2$  a major issue.

We will discuss about a possible renormalization procedure allowing to restore local wellposedness of the problem, provided the noise is not too rough. The talk is based on a joint work with Reika Fukuizumi and Laurent Thomann."

### **Ettienne Pardoux**

Title: Non-Markovian epidemic models and their large population limit

Abstract:

Almost 100 years ago, Kermack and McKendrick proposed epidemic models taking into account the fact that the infectivity of infectious individuals varies with their age of infection, and the susceptibility of

individuals losing their immunity increases continuously from 0 to a maximum which can be 1 or less. We show that the Kermack - Mc Kendrick models (or more general models) are Law of Large Numbers limits of realistic individual based non Markovian stochastic models, as the population size tends to infinity. We also consider infinite dimensional Markovian versions of our models, where we consider the age of infection as a new variable. We shall describe the PDE LLN limit, and the SPDE CLT limit in this framework. Finally, we shall point out some of the biases which the use of simplified Markov/ODE models introduces.

This is joint work with Guodong Pang (Rice University), and in part with also Raphaël Forien (INRAE) and Arsene Brice Zotsa Ngoufack (PhD student at AMU).

### **Eyal Neuman**

Title: The radius of a self-repelling star polymer

Abstract:

We study the effective radius of weakly self-avoiding star polymers in one, two, and three dimensions. Our model includes  $N$  Brownian motions up to time  $T$ , started at the origin and subject to exponential penalization based on the amount of time they spend close to each other, or close to themselves.

The effective radius measures the typical distance from the origin. Our main result gives estimates for the effective radius where in two and three dimensions we impose the restriction that  $T \leq N$ . One of the highlights of our results is that in two dimensions, we find that the radius is proportional to  $T^{3/4}$ , up to logarithmic corrections. Our result may shed light on the well-known conjecture that for a single self-avoiding random walk in two dimensions, the end-to-end distance up to time  $T$  is roughly  $T^{3/4}$ .

The talk is based on a joint work with Carl Mueller.

### **Greg Pavliotis**

Title: Parameter Estimation for McKean SDEs

Abstract:

In this talk we will present recent results on statistical inference for systems of weakly interacting diffusions and, in particular, for their mean field limit that is described by a nonlinear McKean SDE. We consider the case where paths of particles of the interacting particle system are observed. The goal is to infer parameters in the mean field SDE. Various methodologies are presented, including stochastic gradient descent in continuous time, spectral-theoretic techniques and the method of moments.

### **Ivan Nourdin**

Title: Limit theorems for  $p$ -domain functionals of stationary Gaussian fields

Abstract:

We will investigate central and non-central limit theorems for  $p$ -domain functionals of stationary Gaussian fields. Our main tool will be the Malliavin-Stein approach. Based on a joint work with Nikolai Leonenko, Leonardo Maini and Francesca Pistolato.

### **Laure Coutin**

Title : PDE for the density of a diffusion and its running supremum

Abstract :

In this talk, we prove that the joint law of a diffusion and the running supremum of its first component is absolutely continuous and that its density satisfies a weak partial differential equation with boundary condition. We also prove uniqueness of the solution of that PDE. This talk is based on a joint work with L. Huang and M. Pontier

### **Mate Gerencser**

Title: SHE and KPZ with rougher than white noise

Abstract:

We consider the KPZ equation (or after a Cole-Hopf transform, SHE, or nonlinear variants of the latter) in one dimension driven by noise that is rougher than white, e.g. it is derivative of white of order  $\gamma$ . It is known that a change in behavior takes place at  $\gamma=1/4$ , as opposed to the criticality threshold  $\gamma=1/2$ . We describe some recent contributions in both the  $\gamma<1/4$  and  $\gamma>1/4$  regimes, complementing works of [Hu-Huang-Le-Nualart-Tindel '17] and [Hairer '24]. Based partly on joint work with Fabio Toninelli.

### **Michele Coti Zelati**

Title: Exponential mixing for random flows

Abstract:

We consider random dynamical systems driven by noise that is absolutely continuous with respect to the Lebesgue measure, and exhibit sufficient conditions that imply exponential mixing. As a corollary, we show that the so-called Pierrehumbert model, consisting of alternating shear flows with randomized phases, is exponentially mixing.

### **Oleg Butkovskiy**

Title: Optimal weak uniqueness for SDEs driven by fractional Brownian motion and for stochastic heat equation with distributional drift.

Abstract:

Joint work with Leonid Mytnik (Technion, Israel). We consider the stochastic differential equation

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$$dX_t = b(X_t) dt + dB_t^H,$$

\$\$

Where the drift  $b$  is a Schwartz distribution lying in the space  $\mathcal{C}^\alpha$ ,  $\alpha < 0$ , and  $B^H$  is a fractional Brownian motion of Hurst index  $H \in (0, 1/2]$ . If  $H = 1/2$ , then both weak and strong uniqueness theories for this SDE have been developed. However, the situation is much more complicated if  $H \neq 1/2$ , as the main tool, Zvonkin transformation, becomes unavailable in this setting.

The breakthroughs of Catellier and Gubinelli, and later by Le, allowed to establish strong well-posedness of this SDE via sewing/stochastic sewing arguments. However, weak uniqueness of this SDE remained a challenge for quite a while, since a direct application of stochastic sewing alone does not seem to be very fruitful. I will explain how a combination of stochastic sewing with certain arguments from ergodic theory allows to show weak uniqueness in the whole regime where weak existence is known, that is  $\alpha > 1/2 - 1/(2H)$ . If time permits, we will discuss how one can get a weak rate of convergence of the corresponding Euler scheme and see how a similar argument yields weak uniqueness of stochastic heat equation with distributional drift also in the whole regime where weak existence is known.

### **Peter Friz**

Title: On the analysis of some SPDEs via RSDEs

Abstract:

Many SPDEs arise from SDE dynamics under partial conditioning of the noise. My talk will circulate on three concrete examples, the Zakai equation from non-linear filtering, the pathwise control problem suggested by Lions-Souganidis, and last not least a rough PDE approach to pricing in non-Markovian stochastic volatility models. Underlying all these examples is the notion of rough stochastic differential equations, recently introduced (jointly with K. Lê and A. Hocquet).

### **Sebastian Kassing**

Title: Stochastic Modified Flows, Mean-Field Limits and Dynamics of Stochastic Gradient Descent

Abstract:

We propose new limiting dynamics for stochastic gradient descent in the small learning rate regime called stochastic modified flows. These SDEs are driven by a cylindrical Brownian motion and improve the so-called stochastic modified equations by having regular diffusion coefficients and by matching the multi-point statistics. As a second contribution, we introduce distribution dependent stochastic modified flows (based on McKean-Vlasov SDEs) which we prove to describe the fluctuating limiting dynamics of stochastic gradient descent in the small learning rate - infinite width scaling regime.

This talk is based on joint work with Benjamin Gess (University of Bielefeld) and Vitalii Konarovskyi (University of Hamburg).

## **Wilhelm Stannat**

Title: Peng's Maximum Principle for SPDE and applications

Abstract:

We present an extension of Peng's maximum principle for semilinear SPDEs in one space-dimension with non-convex control domains and control-dependent diffusion coefficients to the case of general cost functionals with Nemytskiitype coefficients. The analysis is based on a new approach to the representation of the second order adjoint state as the solution of a function-valued backward SPDE.

Using this representation, we calculate the viscosity super- and subdifferential of the value function evaluated along an optimal trajectory for controlled semilinear SPDEs. This establishes the well-known connection between Pontryagin's maximum principle and dynamic programming within the framework of viscosity solutions. As a corollary, we derive that the correction term in the stochastic Hamiltonian arising in non-smooth stochastic control problems is non-positive. These results directly lead us to a stochastic verification theorem for fully nonlinear Hamilton–Jacobi–Bellman equations in the framework of viscosity solutions.

The talk is based on joint work with L. Wessels. [1] W. Stannat, L. Wessels: Peng's Maximum Principle for Stochastic Partial Differential Equations, *SIAM J. Control Optim.*, 59 (2021), 3552-3573

[2] W. Stannat, L. Wessels: Necessary and sufficient conditions for optimal control of semilinear Stochastic Partial Differential Equations, arXiv:2112.09639, to appear in *Ann. Appl. Probab.*

## **Yuzhao Wang**

Title: Gibbs measures with genreal traps.

Abstract:

We will discuss some of the recent progress on the construction of Gibbs measures associated with the nonlinear focusing Schrödinger equation, specifically in the presence of trapping potentials. Our focus lies on identifying sharp conditions related to the normalizability of these measures. Notably, when dealing with subharmonic potentials, , we observe a novel critical nonlinearity (below the usual mass-critical exponent) for which the Gibbs measures exhibit a phase transition.

## **Zdzislaw Brzezniak**

Title: Reflection of Stochastic Evolution Equations in Infinite Dimensional Domains

Abstract:

In this paper, we establish the existence and the uniqueness of solutions of stochastic evolution equations (SEEs) with reflection in an infinite dimensional ball. Our framework is sufficiently general to include e.g. the stochastic Navier-Stokes equations. Based on a joint work with Tusheng Zhang (Manchester and Hefei) *Ann. Inst. Henri Poincaré Probab. Stat.* (2023).

**Short Talks & Posters - Titles & Abstracts**  
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**Amal Machtalay**

Title : Stochastic mean-field games

Abstract :

Mean Field Game (MFG in short) represents a cutting-edge mathematical framework used to model and analyze decision-making behaviors within a large population of interacting agents. This framework is particularly effective in fields like crowd dynamics, economics, and finance, among others. The stochastic aspect of MFGs incorporates elements of randomness into the system, either through the dynamics of the agents or via external factors, making the model more realistic for many applications.

**Dennis Chemnitz**

Title: Dynamical Properties of Stochastic Differential Equation with Shear - A Case Study

Abstract:

In this poster/talk I consider a seemingly simple class of planar SDEs driven by additive white noise, which can exhibit rich dynamical behavior. In particular I will present conditions for the (non-)existence of a global stochastic flow (so-called strong completeness), for the (non-)existence of pull-back attractors and for (the lack of) strong synchronization. The mechanism responsible non-existence of a global stochastic flow is (to the best of my knowledge) new and could be relevant for larger classes of S(P)DEs.

This is based on joint work with Maximilian Engel and Michael Scheutzow.

**El Mehdi Haress**

Numerical approximation of the stochastic heat equation with distributional reaction term

**Gergely Bodó**

Title: Stochastic integration with respect to cylindrical Levy processes in Hilbert space (Joint work with Markus Riedle)

Abstract:

In this talk, we introduce a theory of stochastic integration with respect to arbitrary cylindrical Levy processes in Hilbert space. Since cylindrical Levy processes do not enjoy a semi-martingale

decomposition, our approach is based on a decoupling inequality for the tangent sequence of the Radonified increments. This enables us to characterise the largest space of predictable Hilbert-Schmidt operator-valued processes which are integrable with respect to a cylindrical Levy process. As a by-product of our construction, we obtain an explicit analytic condition for the integrability of a predictable process, which is expressible in terms of the cylindrical characteristics of the integrator. We consider separately the special case of standard symmetric  $\alpha$ -stable cylindrical Levy processes. Here, our theory simplifies significantly and it is possible to identify the largest space of predictable Hilbert-Schmidt operator-valued integrands with the collection of all predictable processes that have paths in the Bochner space  $L_d$ .

## Harprit Singh

Rough Geometric Integration

## Matteo Ferrari

Title : New a Priori Estimates for 2D Navier-Stokes Equations with Applications to Invariant Measure.

Abstract :

We consider the stochastic two-dimensional Navier-Stokes equation for incompressible fluids, set in a bounded domain with Dirichlet boundary conditions. We consider additive noise in the form  $G \, dW$ , where  $W$  is a cylindrical Wiener process and  $G$  a bounded linear operator with range dense in the domain of  $A^\gamma$ ,  $A$  being the Stokes operator.

While it is known that existence of invariant measure holds for  $\gamma > 1/4$ , previous results show its uniqueness only for  $\gamma > 3/8$ . We fill this gap and prove uniqueness and strong mixing property in the range  $\gamma \in (1/4, 3/8]$  by adapting the so-called Sobolevskii-Kato-Fujita approach to the stochastic N-S equations. This method provides new a priori estimates, which entail both better regularity in space for the solution and strong Feller and irreducibility properties for the associated Markov semigroup.

## Samuel Gallay

Weak existence for SDEs with singular drifts and fractional Brownian or Lévy noise beyond the subcritical regime.

Work with Oleg Butkovsky.

We study a multidimensional stochastic differential equation with additive noise:

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$$dX_t = b(t, X_t) dt + d\xi_t,$$

\$\$

where the drift  $b$  is integrable in space and time, and  $\xi$  is either a fractional Brownian motion or an  $\alpha$ -stable process. We show weak existence of solutions to this equation under the optimal condition on integrability indices of  $b$ , going beyond the subcritical Krylov-Röckner (Prodi-Serrin-



Ladyzhenskaya) regime. This extends the recent results of Krylov (2020) to the fractional Brownian and Lévy cases. We also construct a counterexample to demonstrate the optimality of this condition. Our methods are built upon a version of the stochastic sewing lemma of Lê and the John--Nirenberg inequality.

### **Tobias Hurth**

Title: Entropy for Conditioned Random Dynamical Systems

Abstract:

We introduce a notion of metric entropy for random dynamical systems conditioned on staying within a certain region. The entropy is defined in terms of a probability measure, recently introduced by Castro, Chemnitz, Chu, Engel, Lamb, and Rasmussen, which is ergodic - even mixing - with respect to the skew product and which is singular in the sense that it assigns mass 1 to the set of tuples  $(\omega, x)$  for which the prescribed region is never left. We discuss - in part through conjectures - the relationship between entropy and conditioned Lyapunov exponents and study the example of the doubling map on the unit circle with additive noise. This talk is based on work with Maximilian Engel, Alex Blumenthal, and Dennis Chemnitz.

### **Toyomu Matsuda**

Shifted stochastic sewing and its applications

### **Wenhao Zhao**

Title: Sharp interface limit for 1D stochastic Allen-Cahn equation in full small noise regime

Abstract:

We study the sharp interface limit for the  $d$ -dimensional stochastic Allen-Cahn equation, and extend earlier work by Funaki[1] to the full small noise regime. The main new idea is the construction of a series of functional correctors, which are designed to recursively cancel potential divergences. In addition, in order to show these correctors are well behaved, we develop a systematic decomposition of functional derivatives of the deterministic Allen-Cahn flow of all orders. This decomposition is of its own interest, and may be useful in other situations as well.

[1]: Funaki, T. The scaling limit for a stochastic PDE and the separation of phases. *Probab. Th. Rel. Fields* 102, 221–288 (1995). <https://doi.org/10.1007/BF01213390>

### **Yassine Nachit**

Malliavin Calculus for SDEs driven by non-Gaussian processes

### **Younes/Zine**

Hyperbolic sine-Gordon model beyond the first threshold.