

Portfolio Delegation and Market Efficiency ^{*}

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Abstract

We develop a two-period general equilibrium model of portfolio delegation with competitive, differentially skilled managers and convex compensation contracts. We show that convex incentives lead to significant equilibrium mispricing, but reduce price volatility. In particular, price informativeness and volatility may exhibit opposite behaviour. Investors do not internalize the externality that their contract choice has on equilibrium prices. As a result, equilibrium incentives may be too strong or too weak and hurt investors as a whole. For example, investors' utility may be *decreasing* in the average managers' skill. Convex incentives amplify this negative externality. Indirect incentives due to future fund flows may induce investors to choose stronger convex direct incentives, amplifying inefficiencies even further. Inference of skill from performance is asymmetric: past bad performance is indicative of low skill, but past good performance is not indicative of high skill.

JEL CLASSIFICATION:

KEYWORDS:

1 Introduction

The asset management industry experienced a tremendous growth over the last few decades, with assets under management that are currently comparable with the size of the world GDP.¹ This growth in size naturally lead financial institutions to dominate trading activity in essentially all existing financial markets.² The number of different asset management firms has also increased

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¹According to a report titled "Capturing Growth in Adverse Times: Global Asset Management 2012" published by Boston Consulting Group in October 2012, the professionally managed assets of the global asset management industry have remained flat-lined reaching USD 58.3 trillion at year-end 2011, compared to USD 58.8 trillion in 2007. Conventional assets under management of the global fund management industry increased by 10% in 2010, to USD 79.3 trillion. Pension assets accounted for USD 29.9 trillion of the total, with USD 24.7 trillion invested in mutual funds and USD 24.6 trillion in insurance funds. Together with alternative assets (sovereign wealth funds, hedge funds, private equity funds and exchange traded funds) and funds of wealthy individuals, assets of the global fund management industry totalled around USD 117 trillion. See, *Fund Management: The City UK*, 2011-10-05.

²See, e.g., Gompers and Metrick (2001) and Bennett, Sias, and Starks (2003). For example, institutional trading accounted for 96% of New York Stock Exchange equity trading volume in 2002. See, Jones and Lipson (2004). For derivatives markets, the numbers are even stronger: most derivatives are traded over-the-counter and are therefore essentially inaccessible for retail investors.

significantly.³ Their performance is highly heterogeneous even within the same industry sector, which makes it hard even for professional investors to identify skilled managers and overcome potential misalignment of incentives. In addition, very large indirect incentives due to potential future capital inflows distort asset managers' behaviour in a way that cannot be controlled by the incumbent investors. Given the scale at which asset management industry operates, inefficiencies in the incentive provision and capital allocation may have strong consequences for financial markets and the welfare of the society as a whole, as has been demonstrated by the recent financial crisis. Despite all this, we still know very little about the structure of optimal incentive contracts and the induced behaviour of portfolio managers, as well as about the feedback effects of portfolio delegation on asset prices and welfare.

The goal of this paper is to develop a tractable, competitive general equilibrium model of portfolio delegation accounting for convex incentives and heterogeneity in fund manager skill.⁴ We assume that a continuum of investors (principals) and a continuum of risk averse portfolio managers (agents) are matched at time zero. Investors choose convex contracts that optimally incentivise managers to exert an (unobservable) effort and collect fundamental information. To model the heterogeneity in skill across managers, we assume that the cost of information acquisition is randomly distributed in the managers' population. The law of the skill distribution is given by an arbitrary probability density and is common knowledge. The skill is unobservable and managers learn their skill only after the contract is signed and they start trading. Then, given their skill realization, the form of the contractual incentive scheme, as well as their rational expectations about the price informativeness, managers decide whether to acquire information. Conditional on their private information and the price, they make their portfolio choice decision. Each investor selects the optimal contract, rationally anticipating the portfolio manager's behaviour as well as the equilibrium distribution of returns. This leads to the following feedback loop: the optimal contracts chosen by investors determine portfolio manager's behaviour; this behaviour determines equilibrium price distribution; price distribution and anticipated behaviour determine an individual investor's choice of the optimal contract. To capture the empirically observed small dispersion in compensation contracts within the same industry sector,⁵ we only consider symmetric equilibria in which all investors optimally propose the same contracts to the managers. We assume that the contract space consists of a linear component, and a convex, option-like bonus component for returns in excess of a given benchmark.⁶ An investor can thus optimally select the relative strength of both linear (fulcrum) and convex incentive components of the contract, subject to the manager's limited liability and individual rationality constraints. We first solve for equilibrium prices and portfolios in closed form, taking the compensation contracts as given. This allows us to directly

³At the end of 2011, there were over 14,000 mutual funds in the United States, and about 10,000 hedge funds worldwide, according to the Investment Company Institute (ICI).

⁴See, e.g., Li, Zhang and Zhao (2013) for empirical evidence on skill heterogeneity for hedge fund managers.

⁵For example, different variations of the 2-20 contracts in which fund managers receive 2% of assets under management plus 20% of superior performance in excess of the so-called "high-water mark" are used in the majority of hedge funds.

⁶See, Stoughton (1993), Das and Sundaram (2002) for a similar approach. Characterizing the full optimal non-linear contract in the space of all possible state-contingent contracts is a highly non-trivial problem even in partial equilibrium. See, e.g., Dybvig, Farnsworth, and Carpenter (2010).

link the form of incentives with equilibrium price distribution. We show that, in general, there is no direct link between the form of incentives and price informativeness. In fact, convex incentives may lead to significant equilibrium mispricing driven by the “gambling for resurrection” mechanism: even if managers think that, with high probability, fundamentals are low, they will always take a long position whenever the price is sufficiently low, because they will only get the bonus if the realized return is high, and this is only possible if high fundamental value is realized. In equilibrium, this implies that price values that are sufficiently close to the fundamental value are never attained: there is a *strictly positive mispricing gap* because all managers take a contrarian position and the market clears before the price has a chance to approach the fundamental value. We also find that price volatility is decreasing in the strength of convex incentives, but is increasing in the level of the benchmark return. The fact that convex incentives may reduce market efficiency and, at the same time, reduce price volatility, is surprising and cannot be achieved in a standard, linear-Gaussian setup. Indeed, in such a linear equilibrium, price informativeness and volatility are equivalent concepts. These new effects that we identify are purely non-linear in their nature and are driven by non-linear incentives and non-Gaussian price distribution.

As the next step, we endogenize the contract choice by investors and characterize symmetric equilibria of the game. The inability of investors to coordinate implies that, in equilibrium, they do not internalize the externality that their contract choice has on the price distribution. This implies that, in equilibrium, they may choose an inefficient level of incentives. To understand this effect, we consider the case of coordinating investors (or, equivalently, one large representative investor) who internalize the pecuniary externality. We find that non-competitive investor behaviour has very strong *negative* effects on market efficiency: in most cases, coordinating investors decide to provide very weak incentives. As a consequence, equilibrium information acquisition goes down, mispricing significantly increases and price volatility also goes up; furthermore, the magnitude of these effects can be very large. If we assume that noise traders and managers are risk neutral, and investors are risk averse, we can also formulate the full social planner’s problem. As the social planner would like to reduce riskiness of investors’ returns (after fees), he often finds it optimal to offer convex contracts to the managers because convex incentives reduce volatility. This result may have interesting implications for the recent policy debates regarding the introduction of caps on managers’ (convex) bonus contracts.

Our model also allows for incorporating indirect incentives due to convex performance-fund flow relationship.⁷ As Lim, Sensoy and Weisbach (2013) find, indirect incentives for the average hedge fund are about four times as large as direct incentives from incentive fees and returns to managers own investment in the fund.⁸ We do not model the fund flows explicitly, and follow the reduced form approach⁹ by assuming that future fund flows are determined by given convex function. We show that sufficiently large indirect incentives may distort investor’s contract choice and force him to choose convex direct incentives even though investors are risk neutral. This surprising result is based on a simple observation: if the limited liability constraint is not binding,

⁷See, e.g., Chevalier and Ellison (1999), Agarwal, Daniel and Naik (2009).

⁸See also Chung et al. (2012).

⁹See, e.g., Basak and Makarov (2012, 2013) for a similar approach.

it is always optimal for an investor to provide full linear incentives to the manager. However, indirect incentives effectively reduce investors' outside option, making limited liability bind and inducing investors to choose option-like compensation.

Finally, the assumed heterogeneity of skill and the differences in realized performance across managers allow us to address the question of inferring skill from performance. To this end, given the null hypothesis that a given manager is unskilled, we compute the probabilities of type I and type II errors. These are the fraction of skilled managers among all managers with bad performance, and the fraction of non-skilled managers among all managers with good performance. We show that the inference problem is asymmetric: while past bad performance is informative about the absence of skill, past good performance may not be informative of good manager skill. The reason is that, as in the data, fund manager returns are skewed and asymmetrically distributed. Furthermore, we find that convex incentives amplify the inference problem by inducing gambling behaviour.

We now discuss related literature.

Many papers study partial equilibrium optimal contracting in delegated portfolio management. See, Ross (1974), Bhattacharya and Pfleiderer (1985), Starks (1987), Kihlstrom (1988), Huberman and Kandel (1993), Stoughton (1993), Allen (1990), Admati and Pfleiderer (1997), Carpenter (2000), Biais and Germain (2002), Ou-Yang (2003), Cadenillas, Cvitanic, and Zapatero (2007), Li and Tiwari (2009), Axelson, Strömberg, and Weisbach (2009), Dybvig, Farnsworth, and Carpenter (2010), and Makarov and Plantin (2012). Several papers emphasize the importance of benchmarking and relative performance concerns in institutional behaviour.¹⁰ See, Admati and Pfleiderer (1997), Basak, Shapiro and Tepla (2006), Basak, Pavlova and Shapiro (2007, 2008), Basak and Makarov (2012, 2013), Basak and Pavlova (2013), Kaniel and Kondor (2013), and Buffa, Vayanos and Woolley (2013). In particular, Buffa, Vayanos and Woolley (2013) study joint equilibrium determination of optimal contracts and asset prices in a dynamic, continuous time model, and show how an inefficiently high level of benchmarking arises endogenously and may amplify stock market volatility. Since their model is dynamic and is developed in continuous time, they are able to address important dynamic properties of asset prices. By contrast to our paper, prices play only allocational and not informational role; furthermore, contracts are assumed to be linear in past performance, and there is no heterogeneity in managerial skill.

Goldman and Slezak (2003) and Acharya, Pagano and Volpin (2013) investigate the role of fund manager turnover and career concerns within partial equilibrium models of portfolio delegation. Guerrieri and Kondor (2012) incorporate career concerns into a general equilibrium model of portfolio delegation and show that it amplifies price volatility. Glode and Lowery (2013) study how competition for scarce trader talent in OTC markets influences their compensation through a novel externality effect that these traders impose on rival firms.

The huge empirically observed dispersion in the fund managers' performance naturally raises the

¹⁰In fact, relative performance concerns generates strong indirect incentives due to a pronounced convex relationship between past performance and future fund flows (see, e.g., Chevalier and Ellison (1999) and Agarwal, Daniel and Naik (2009)). Lim, Sensoy and Weisbach (2013) find that indirect incentives for the average hedge fund are about four times as large as direct incentives from incentive fees and returns to managers own investment in the fund. See also Chung et al. (2012).

question of identifying managerial skill from trading behaviour. Kacperczyk, Van Nieuwerburgh and Veldkamp (2013a,b) use the theory of rational inattention to show (both theoretically and empirically) that managerial skill can be linked to both stock picking and market timing decisions. Breon-Drish and Sagi (2013) show how skill can be identified from past portfolio choice decisions of mutual fund managers. None of these papers incorporates skill heterogeneity into a general equilibrium model.

Das and Sundaram (2002) analyze a partial equilibrium signalling game where managers compete for investor's money by offering heterogeneous fee structures. They consider exactly the same contract space as we do (a fulcrum (linear) component and an option-like incentive component), and find that convex contracts are beneficial for the investors only when the industry is imperfectly competitive, whereas linear contracts are optimal in a competitive industry. In our model, the industry is competitive. However, without taking into account the externality of contracts on the equilibrium prices, convex contracts are often optimal for the investors. Yet, as we show, this is not true anymore in non-competitive general equilibrium. In fact, if the investors could coordinate on the contract choice, they would be better off by prohibiting the option-like component of the contract. It is important to differentiate our model from the contractual signalling and screening models, such as that of Das and Sundaram (2002). Namely, in our model, we assume that skill is not observable at the time when the contract is signed. Thus, neither signalling nor screening is possible, and all managers receive identical contracts.¹¹

Dow and Gorton (1997) develop a general equilibrium model of portfolio delegation with single strategic manager and a single strategic investor. The manager can be skilled or unskilled; only a skilled manager may receive an informative signal about the asset payoff with some positive probability. The signal is obtained without exerting effort, but there is still an agency problem. Dow and Gorton (1997) show that, even when the manager does not receive an informative signal, he may still trade like a noise trader.

Garcia and Vanden (2009) consider a general equilibrium model in which informed managers set up mutual funds as a means of selling their private information to uninformed managers. However, they assume exogenously specified linear contracts and ignore moral hazard issues.

Kruttili, Patton and Ramadorai (2013) provide empirical evidence that hedge funds have significant impact on asset markets. Hendershott, Livdan, and Schürhoff (2013) find evidence of informed institutional trading. However, Bai, Philippon and Savov (2013) find that, despite the development of the financial industry in the post second world war period, asset markets have not become more informative. Our results may provide a partial explanation for this intriguing phenomenon. Namely, given the significant rise in convex incentives for fund managers over the last few decades (particularly due to increased competition), the increase in informed institutional trading can be (partially) offset by the equilibrium inefficiencies induced by convex incentives.

¹¹One would naturally like to know which model setup better reflects the reality. We believe that both mechanisms may be present in the financial industry. For some hedge funds trading complex derivative strategies, there is indeed some cross-sectional dispersion in the fee structure, and so some signalling and screening may be indeed taking place. But for more conventional mutual and hedge funds, very little dispersion in the fee structure can be observed. Furthermore, it is not clear whether managers actually know whether they are talented or not, which goes along with our assumption that managers only learn their skill after they start trading.

Kyle, Ou-Yang and Wei (2011) (henceforth, KOW (2011)) introduce portfolio delegation and endogenous information acquisition into the Kyle (1985) model of strategic trading. Namely, KOW (2011) consider a model with a single strategic CARA investor and a single strategic CARA manager who trades with competitive market makers and noise traders in the Kyle (1985) setting. KOW (2011) require the contract to be a linear function of trading profits and show that, contrary to the competitive case (see, e.g., Admati and Pfleiderer (1997)), a higher-powered linear contract induces the strategic manager to exert more effort for information acquisition. They also show that, depending on manager’s risk aversion, price informativeness may increase or decrease with the amount of noise trading. Our model is different in many aspects. First, investors and managers are non-strategic and therefore, in contrast to KOW (2011), investors do not internalize the externality that their contract choice generates on equilibrium price distribution. This leads to a new equilibrium feedback loop from the contract choices of other investors on the contract choice of any given investor. Second, KOW (2011) consider linear contracts, whereas we allow for convex contracts with an option-like bonus component resembling the contracts used in the financial industry.¹² Third, we assume a continuum of competitive, non-strategic fund managers, and allow for an arbitrary cross-sectional distribution of skill. To the best of our knowledge, our paper is the first one to introduce convex contracts and distribution of skill into a general equilibrium model with endogenous information acquisition.

The feedback loop that generates interaction between contracts and price informativeness naturally links our paper to the literature on strategic complementarities in information acquisition. Barlevi and Veronesi (2000) were the first to show that the strategic substitutability in information acquisition within the Grossman-Stiglitz (1985) model breaks down if one deviates from the CARA-normal setting. Many other important mechanisms making information acquisition a strategic complement have been proposed afterwards. See, e.g., Veldcamp (2006a,b), Hellwig and Veldcamp (2009), Garcia and Strobl (2011), and Goldstein, Li and Yang (2013). In our model, equilibrium is non-linear.¹³ Moreover, the externality from the contract choice is not only about price informativeness: a change in the form of incentives provision changes the whole probability distribution of the equilibrium price, and not just price informativeness. This additional non-linear feedback mechanism is new to our model.

The paper is organized as follows: Section 2 describes the model with fixed contracts and provides closed form solutions for equilibrium portfolios and price distribution. Section 3 studies endogenous contracts, corresponding externalities, an equilibrium with coordinating investors, and that with a social planner. Section 4 studies an equilibrium in the presence of indirect convex incentives. Section 5 analyzes the problem of inferring skill from performance. Section 6 concludes.

¹²As in KOW (2011), we find that stronger linear incentives increase the amount of information acquisition even though managers are risk neutral. As in KOW (2011), this result is driven by limits to arbitrage: in our model, this is due to leverage constraints, whereas in their model this is due to strategic trading and price impact.

¹³Our model also belongs to the relatively recent literature on non-linear rational expectations models. See, Hellwig, Mukherji and Tsyvinski (2006), and Albagli, Hellwig and Tsyvinski (2013), and Breon-Drish (2013). In particular, the techniques we use to solve for the equilibrium are similar to those developed in Hellwig, Mukherji and Tsyvinski (2006) and Albagli, Hellwig and Tsyvinski (2013).

2 Model Setup with Exogenous Contracts

There are two periods $t = 0, 1$ and two states of the world, H and L realized at time $t = 1$. Managers can trade two assets: a riskless bond with an interest rate normalized to zero, and a risky asset whose payoff θ is realized at time 1 and is given by θ_i , $i = H, L$ in the corresponding state. Without loss of generality, we can normalize $\theta_H = \theta$, $\theta_L = -\theta$.

The economy is populated by three classes of competitive traders: two classes of informed agents (portfolio managers) I and U , and noise traders. The masses of informed and uninformed managers are given by μ_I and μ_U respectively, and we normalize the total mass of all managers to one. A manager of type K , $K = U, I$ receives a noisy signal $\theta + s$, where s is driven from a distribution with a symmetric density η_K and a cumulative distribution function Φ_K , $K = U, I$. We will assume that class I managers are “better” Informed than the Uninformed managers of class U in the sense that the density η_I is less noisy than η_U . Initially, before observing information, all managers have a common prior $\pi = 0.5$ about the probability of state H .

All managers are risk neutral and are compensated according to their performance.¹⁴ We assume that the contract (performance fee) consists of two components: a fulcrum (linear) part and an option-like incentive component.¹⁵ Namely, if return R on manager’s portfolio is realized, the manager receives

$$u(R) = b + aR + d(R - c)^+. \quad (1)$$

Here, b is the constant part of the contract, $a > 0$ is the strength of the fulcrum (symmetric), linear part of the contract, $d > 0$ is the strength of convex, option-like (asymmetric) part of the performance fees, and c is the asymmetric (convex) benchmark level of return.

Each manager is initially endowed with one unit of wealth and is subject to portfolio (leverage) constraints.¹⁶ We model these constraints by assuming that managers cannot take positions that exceed an exogenously given threshold κ of their initial wealth.¹⁷ Given the realization of the price p and his signal s , a manager of class K solves the maximization problem

$$V^K(s, p) = \max_{x \in [-\kappa, \kappa]} E[u(1 + x(\theta - p)) | s, p], \quad K = U, I. \quad (2)$$

In order to compute the expectation on the right-hand side of (2), we need to solve the filtering problem of a manager. Bayes’ rule implies that the posterior probability π^K of state H for a

¹⁴The underlying portfolio delegation problem is studied in the subsequent sections.

¹⁵See, e.g., Das and Sundaram (2002) for the same specification of the contract space. This choice is motivated by the actual contracts observed in the financial industry. It also allows us to isolate the effects of symmetric and asymmetric fee structures in a very clean and precise way.

¹⁶The precise origin of this constraints is not important. This can be any form of limits to arbitrage, margin (collateral) requirements, or other capital constraints.

¹⁷For example, margin requirements are typically based on the expectations of the largest possible move of the asset price. In our model, this is obviously given by $\theta_H - \theta_L$. Therefore, the quotient $\kappa/(\theta_H - \theta_L)$ can be seen as the margin requirement in percentage terms.

manager of type K after observing s and p is given by

$$\pi^K(s, p) = \frac{f_H(p)\eta_K(s - \theta_H)}{f_H(p)\eta_K(s - \theta_H) + f_L(p)\eta_K(s - \theta_L)}, \quad (3)$$

where $f_i(p)$, $i = H, L$ is the density of the equilibrium price conditional on the corresponding state of the world.¹⁸ Given π^K , we can rewrite (2) as

$$V^K(s, p) = \max_{x \in [-\kappa, \kappa]} \left(b + a(1 + x(\pi^K\theta_H + (1 - \pi^K)\theta_L - p)) \right. \\ \left. + d \left(\pi^K(1 - c + x(\theta_H - p))^+ + (1 - \pi^K)(1 - c + x(\theta_L - p))^+ \right) \right), \quad K = U, I. \quad (4)$$

Everywhere in the sequel, we make the following assumption:

Assumption 2.1 *We have $c > 1$ and $\kappa \geq 2\frac{c-1}{\theta_H - \theta_L}$.*

The first assumption guarantees that incentives are not redundant (otherwise the manager could get the incentive part of the compensation by choosing $x = 0$) whereas the second part guarantees that the leverage constraint is sufficiently mild so that, by taking sufficient exposure, the manager can indeed achieve returns sufficient to get incentive payments.

Assumption 2.2 *The densities η_K , $K = H, L$ of the signals satisfy the monotone likelihood property: $\frac{\eta_K(s - \theta_H)}{\eta_K(s - \theta_L)}$ is monotone increasing in s for $K = H, L$.*

Since, by assumption, the agent's compensation contract is convex, he will always take maximal leverage. The following lemma is an immediate consequence of the monotone likelihood property.

Lemma 2.1 *For any $K = U, I$, there exists a threshold $X_K(p)$ such that the solution to the maximization problem (2) for a manager of class K is given by*

$$x = \begin{cases} \kappa, & s \geq X_K(p) \\ -\kappa, & s < X_K(p). \end{cases}$$

Furthermore, the threshold $X_K(p)$ is uniquely determined by the condition

$$\pi^K(X_K(p), p) = \pi^*(p),$$

where

$$\pi^*(p) \equiv \begin{cases} \frac{\alpha(p - \theta_L)}{(\theta_L + \beta - p) + (\alpha + 1)(\theta_H - \theta_L) - 2\beta}, & p \in [\theta_L, \theta_L + \beta] \\ \frac{(\alpha + 1)(p - (\theta_L + \beta)) + \alpha\beta}{(\alpha + 1)(\theta_H - \theta_L) - 2\beta}, & p \in [\theta_L + \beta, \theta_H - \beta] \\ \frac{(\alpha + 1)(p - (\theta_L + \beta)) + \alpha\beta}{(p - (\theta_H - \beta)) + (\alpha + 1)(\theta_H - \theta_L) - 2\beta}, & p \in [\theta_H - \beta, \theta_H], \end{cases} \quad (5)$$

¹⁸We explicitly compute this density below.

and

$$\alpha = \frac{2a}{d}, \quad \beta = \frac{c-1}{\kappa}.$$

The intuition is as follows. A threshold type $X_K(p)$ (i.e., the manager that receives a signal $s = X_K(p)$) is indifferent between taking a long or a short position. Types that are above (below) $X_K(p)$ go long (short) the asset. In the limit of $d = 0$ (linear contract), we get

$$\pi^*(p)|_{d=0} = \frac{p - \theta_L}{\theta_H - \theta_L},$$

which is the posterior probability for which the expected return is identically zero. This case would correspond to a typical mutual fund (purely fulcrum) contract. However, convex compensation distorts manager's behaviour. A direct calculation implies that $\pi^*(p) < \pi^*(p)|_{d=0}$ for $p < \frac{\theta_H + \theta_L}{2}$ and $\pi^*(p) > \pi^*(p)|_{d=0}$ for $p > \frac{\theta_H + \theta_L}{2}$. In particular, in the limit of purely asymmetric contract (i.e., $a = 0$), we get

$$\pi^*(p) \equiv \begin{cases} 0, & p \in [\theta_L, \theta_L + \beta] \\ \frac{p - (\theta_L + \beta)}{(\theta_H - \theta_L) - 2\beta}, & p \in [\theta_L + \beta, \theta_H - \beta] \\ 1, & p \in [\theta_H - \beta, \theta_H]. \end{cases} \quad (6)$$

Using the monotone likelihood property, we arrive at the following result.

Proposition 2.1 *Fix the densities $f_H(p)$ and $f_L(p)$ of the equilibrium price. Then, we have $X_K(p) < X_K(p)|_{d=0}$ for $p < \frac{\theta_H + \theta_L}{2}$ and $X_K(p) > X_K(p)|_{d=0}$ for $p > \frac{\theta_H + \theta_L}{2}$. Furthermore, the elasticity of threshold, $\frac{\partial X_K(p)}{\partial p}$ is monotone increasing in the strength of convex incentives d and is decreasing in the benchmark level c . If a/d converges to zero, we have $X_K(p) = -\infty$ for $p \in [\theta_L, \theta_L + \beta]$ and $X_K(p) = +\infty$ for $p \in [\theta_H - \beta, \theta_H]$.*

That is, as expected, convex compensation makes the managers behave more aggressively. This behaviour becomes extreme in the case when the contract is purely asymmetric: for low price levels $p < \theta_L + \beta$, everybody is long the asset, whereas for $p > \theta_H - \beta$ everybody is short the asset.

Now, having solved the maximization problem of a given manager, we can write down the aggregate demand. Since signals are i.i.d. across the managers, Lemma 2.1 and the law of large numbers imply that the mass of class K managers who acquire κ shares is given by $(1 - \Phi_K(X_K(p) - \theta))$, whereas the mass of managers who short sell κ units of the asset is given by $\Phi_K(X_K(p) - \theta)$. Consequently, the net aggregate demand conditional on state θ_i is given by

$$D(p, \theta_i) \equiv \mu_I \kappa (1 - 2\Phi_I(X_I(p) - \theta_i)) + \mu_U \kappa (1 - 2\Phi_U(X_U(p) - \theta_i)). \quad (7)$$

We assume that noise traders' supply ε is distributed on $[-\kappa, \kappa]$ with a smooth, everywhere positive density η_ε .¹⁹ Given a noise trade realization ε , the market clearing condition takes the form

$$D(p, \theta) = \varepsilon. \quad (8)$$

¹⁹The assumption that the support of noise trades is given by $[-\kappa, \kappa]$ is made for simplicity and can be relaxed.

and therefore, assuming that X_I and X_U are monotone increasing in p , the density of the price, $f_i(p)$, conditional on the state θ_i , $i = H, L$ is given by²⁰

$$\begin{aligned} f_i(p) &= \eta_\varepsilon(D(p, \theta_i)) |D_p(p, \theta_i)| \\ &= 2\kappa\eta_\varepsilon(D(p, \theta_i)) (\mu_I\eta_I(X_I(p) - \theta_i) X_I'(p) + \mu_U\eta_U(X_U(p) - \theta_i) X_U'(p)). \end{aligned} \quad (9)$$

Denote by

$$\ell_K(X) \equiv \eta_K(X - \theta_H)/\eta_K(X - \theta_L) \quad (10)$$

the likelihood ratios for the two classes of managers, $K = U, I$. Then, it follows directly from Lemma 2.1 that

$$\ell_U(X_U(p)) = \ell_I(X_I(p)). \quad (11)$$

Indeed, the threshold types are indifferent between taking a long or a short position if and only if they agree on the likelihood of the state H . Let $\mathcal{L}(X) = \ell_U^{-1}(\ell_I(X))$. The following is true.

Theorem 2.1 *Suppose that $a > 0$ and there exists a monotone increasing, absolutely continuous solution $X_I(p)$, $p \in (\theta_L, \theta_H)$ to*

$$\begin{aligned} 2 \log \ell_I(X_I(p)) &= \log \frac{\pi^*(p)}{1 - \pi^*(p)} \\ &- \log \frac{\eta_\varepsilon(\mu_I\kappa(1 - 2\Phi_I(X_I(p) - \theta_H)) + \mu_U\kappa(1 - 2\Phi_U(\mathcal{L}(X_I(p)) - \theta_H)))}{\eta_\varepsilon(\mu_I\kappa(1 - 2\Phi_I(X_I(p) - \theta_L)) + \mu_U\kappa(1 - 2\Phi_U(\mathcal{L}(X_I(p)) - \theta_L))}. \end{aligned} \quad (12)$$

Then, $X_I(p)$ and $X_U(p) = \mathcal{L}(X_I(p))$ form a rational expectations equilibrium.

If $a = 0$ then the same result holds true, but price p can only take values $p \in (\theta_L + \beta, \theta_H - \beta)$.

The result of Theorem 2.1 for $a = 0$ is surprising. It shows an *equilibrium mispricing*: independent of the noise distribution, convex contracts always force deviations of equilibrium price from the fundamental value. In fact, any price realization $p \in [\theta_L, \theta + \beta]$ would cause all managers to go fully long, and hence the market clearing price is not uniquely defined. However, there is a unique clearing price that continuously depends on the noise. We formalize this result in the following corollary.

Corollary 2.1 (Mispricing) *Suppose that the contracts are purely asymmetric (i.e., $a = 0$) and pick a monotone increasing equilibrium $X_I(p)$. Then, there exists a unique market clearing price $p(\varepsilon)$ that continuously depends on the noise ε . This price satisfies $p(\varepsilon) \in [\theta_L + \beta, \theta_H - \beta]$ for all ε .*

The intuition behind this result is as follows: when prices approach $\theta_L + \beta$ from above, the probability of receiving a significant bonus is positive only if the state is high: if the state is low, the realized return will be too small and hence the bonus will be negligible. Thus, even managers who believe that the probability of the high state is very small, decide to “gamble for

²⁰Under the assumption of uniformly distributed noise imposed below, $X_I(p)$ and $X_U(p)$ are indeed always non-decreasing in equilibrium.

resurrection” betting on the realization of the high state. In equilibrium, the demand explodes and the market clears even before the price is able to adjust further to the fundamental value, leading to mispricing. Note that the minimal mispricing β is monotone increasing in the reference return c and is decreasing in the leverage constraint κ . The latter property is intuitive: absent limits to arbitrage, risk neutral traders would immediately correct the mispricing. The dependence on c is also intuitive: larger c reduces the probability of receiving a bonus and hence weakens incentives to acquire information and increases market inefficiency.

One may ask whether this property is robust to introducing a small degree of risk aversion for the managers. As Carpenter (2000) shows, convex incentives are effectively undone to a large extent because the manager optimally gambles to attain a concave utility function. However, this result relies on the manager being unconstrained: when risk aversion is sufficiently small, such a gambling would require huge leverage. In the presence of leverage constraints (as in our model), they will continue to bind for small risk aversion, and the value function will still have a convex part in it, inducing risk loving behaviour and, as a consequence, equilibrium mispricing.

Theorem 2.1 allows us to consider arbitrary forms of signal distributions, as well as arbitrary noise trade distributions. However, to simplify the analysis and isolate the most important economic mechanisms, we will from now on make the following assumption.

Assumption 2.3 *Noise traders’ demand is uniformly distributed on $[-\kappa, \kappa]$ and signals are drawn from normal distributions, so that $\eta_K(X) = \frac{1}{\sqrt{2\pi}\sigma_K} e^{-X^2/(2\sigma_K^2)}$, $K = U, I$, for some $\sigma_U > \sigma_I$.*

Under assumption 2.3, we immediately get the following result.

Proposition 2.2 *Equilibrium exists and is unique. The optimal threshold strategies are given by*

$$X_K(p) = \frac{\sigma_K^2}{2(\theta_H - \theta_L)} \log \left(\frac{\pi^*(p)}{1 - \pi^*(p)} \right), \quad K = U, I. \quad (13)$$

The equilibrium density of the price conditional on a state $i = H, L$ is given by

$$f_i(p) = \mu_I f_i^I(p) + \mu_U f_i^U(p) \quad (14)$$

with

$$f_i^K(p) = \eta_K(X_K(p) - \theta_i) X_K'(p), \quad K = U, I. \quad (15)$$

In the limit as $a \rightarrow 0$, the support of the distribution for $f_i(p)$ converges to $[\theta_L + \beta, \theta_H - \beta]$.²¹

The result of Proposition 2.2 shows that the threshold strategies of two classes of managers are linked via a simple linear relationship. In agreement with Proposition 2.1, the signal threshold elasticity $X_U'(p)$ for the uninformed traders is higher than that for the informed traders. Most importantly, the assumption of uniform noise distribution implies that the density f_i of the price is an affine linear function of the mass μ_I of informed traders: in fact, it is a weighted average of the

²¹Note that, by symmetry, we always have $f_H(p) = f_L(-p)$.

densities f_i^K , $K = U, I$ corresponding to the cases when all traders are uninformed (respectively, informed).

Using the equilibrium characterization, we can now solve the endogenous information acquisition problem. Suppose that, before the trade takes place, all managers are uninformed. However, they may optimally decide to become informed by paying a fixed cost λ . This cost is manager-specific and proxies for his skill: it is more difficult for managers with higher cost (lower skill) to acquire information. Skill is heterogeneous across managers and is distributed according to the cumulative distribution function $H(\lambda)$ supported on \mathbb{R}_+ . After the managers acquire information, all signals are realized, both for informed and uninformed managers, and then trade takes place. Recall formula (4). Then, a manager with skill λ acquires information if and only if $\lambda < \lambda^*$ where we have defined

$$\lambda^* \equiv E[V^I(s, p)] - E[V^U(s, p)] \quad (16)$$

Consequently, the mass of informed traders is given by

$$\mu_I^* = H(E[V^I(s, p)] - E[V^U(s, p)]). \quad (17)$$

Now, conditional on a state θ_i and a price realization p , the expected utility of a manager of class $K = U, I$ is given by

$$\begin{aligned} & \text{Prob}_K[s < X_K(p)|\theta_i, p]u(1 - \kappa(\theta_i - p)) + \text{Prob}_K[s > X_K(p)|\theta_i, p]u(1 + \kappa(\theta_i - p)) \\ &= \Phi_K(X_K(p) - \theta_i)u(1 - \kappa(\theta_i - p)) + (1 - \Phi_K(X_K(p) - \theta_i))u(1 + \kappa(\theta_i - p)). \end{aligned} \quad (18)$$

Therefore, conditional on a given price realization p and a state of the world θ_i , the gain from information acquisition is independent of the mass of informed traders and is given by

$$\begin{aligned} \Delta(\theta_i, p) &= E[V^I(s, p)|p, \theta_i] - E[V^U(s, p)|p, \theta_i] \\ &= \left(-2a\kappa(\theta_i - p) + d(1 - \kappa(\theta_i - p) - c)\mathbf{1}_{p \geq \theta_i + \frac{c-1}{\kappa}} - d(1 + \kappa(\theta_i - p) - c)\mathbf{1}_{p \leq \theta_i - \frac{c-1}{\kappa}} \right) \\ &\quad \times \left(\Phi\left(\frac{X_I(p) - \theta_i}{\sigma_I}\right) - \Phi\left(\frac{\frac{\sigma_I^2}{\sigma_U^2} X_I(p) - \theta_i}{\sigma_U}\right) \right), \end{aligned} \quad (19)$$

where Φ is the cumulative distribution function of a standard Normal variable. Substituting the expression for the density of the equilibrium price, we can rewrite the total gains from information acquisition as

$$E[V^I(s, p)] - E[V^U(s, p)] = \mu_I A + B$$

where we have defined

$$\begin{aligned}
A(a, d) &\equiv 1/2 \sum_{i=H,L} \int_{\theta_H}^{\theta_L} \Delta(\theta_i, p) \left(\eta_I(X_I(p) - \theta_i) - \eta_U \left(\frac{\sigma_U^2}{\sigma_I^2} X_I(p) - \theta_i \right) \frac{\sigma_U^2}{\sigma_I^2} \right) X'_I(p) dp \\
B(a, d) &\equiv 1/2 \sum_{i=H,L} \int_{\theta_H}^{\theta_L} \Delta(\theta_i, p) \eta_U \left(\frac{\sigma_U^2}{\sigma_I^2} X_I(p) - \theta_i \right) \frac{\sigma_U^2}{\sigma_I^2} X'_I(p) dp.
\end{aligned} \tag{20}$$

Coefficient A plays a pivotal role in our analysis: it determines the strategic complementarity of information acquisition. Namely, if $A < 0$, information acquisition is a strategic substitute: an increase in the mass of informed traders reduces the gains from information acquisition. By contrast, if $A > 0$, information acquisition is a strategic complement: an increase in μ_I increases the gains from information acquisition. In this case, strategic complementarities may lead to multiplicity of equilibria. Since gains from information acquisition are always positive, we have $B > 0$ and $A + B > 0$. Our extensive numerical tests indicate that $A < 0$. However, we do not know whether the inequality $A < 0$ always holds.

Summarizing, we arrive at the following characterization of equilibrium information acquisition.

Theorem 2.2 *There exists a one-to-one correspondence between equilibrium masses μ_I of informed managers and solutions $\mu_I \in [0, 1]$ to the equation*

$$\mu_I = H(\mu_I A + B).$$

If information acquisition is a strategic substitute (i.e., $A < 0$) then there exists a unique equilibrium mass of informed traders. Otherwise, there may be multiple equilibria.

As a benchmark example throughout the paper, we will assume that skill is exponentially distributed with the parameter $\lambda > 0$. In addition, we assume that there is a mass $1 - \rho$ of completely unskilled managers. That is, these managers are simply unable to become informed, regardless of their compensation.²² Large (small) λ corresponds to highly skilled (unskilled) managers on average. We will now illustrate equilibrium behaviour with several plots. Since we are investigating rational expectations equilibria and endogenous information acquisition, the most important question is how market efficiency and price informativeness depend on various parameters and, in particular, on the strength a and d of the contractual incentives. As our equilibrium is non-linear and non-Gaussian, conditional variance of the fundamental, $\text{Var}[\theta|p]$ may not be the right measure of efficiency as it depends on the value of p . We suggest to measure efficiency and price informativeness in terms of mispricing: the expected deviation of price from the fundamental, $E[\theta_H - p|\theta = \theta_H] = E[p - \theta_L|\theta = \theta_L]$.²³ This mispricing has a clear economic interpretation: this is the expected loss that an investor has to take if he needs to liquidate the asset in the state H . We therefore could call this mispricing *liquidity*. Proposition 2.2 immediately implies that the following is true.

²²Existing evidence suggests that only a small fraction ρ of fund managers are sufficiently skilled to generate persistent alpha. We use the value of $\rho = 0.3$ in most of our numerical results.

²³This identity holds by symmetry.

Proposition 2.3 *If a/d is sufficiently small then the mispricing satisfies $E[p - \theta_L | \theta = \theta_L] > \beta$. In particular, the larger is the reference return c , the stronger is the mispricing.*

An increase in d may have two effects: first, it incentivizes managers to acquire more information, which decreases mispricing. Second, it induces a more aggressive gambling behaviour, forcing prices deviate further from fundamentals. The competition of these two effects determines the equilibrium response of mispricing to convex incentives. In particular, when managers are highly skilled, they do not need strong incentives to acquire information. Hence, the second effect dominates and mispricing starts increasing in the size of convex incentives. Figure 2 illustrates this phenomenon. As we can see, for small values of d , information acquisition effect dominates, the mass of informed traders increases, and mispricing goes down. But, as almost all managers become informed, the gambling effect dominates and mispricing goes up again.

Figures 1 and 2 illustrate the dependence of the normalized²⁴ mispricing on different parameters of the model. As we can see, it decreases with the precision of both informed and uninformed managers²⁵. Also, the mispricing always increases in the benchmark portfolio parameter c , both because it increases the gap β , and because it reduces incentives to acquire information.

Finally, figures 4 and 5 show the equilibrium behaviour of the normalized price variance, $\text{Var}[p|\theta_H]/(\theta_H - \theta_L)^2 = \text{Var}[p|\theta_L]/(\theta_H - \theta_L)^2$ conditional on any state of the world. As we can see, variance is monotonic in both σ_I , λ , a , d and c , but is non-monotonic in σ_U and θ . This is surprising, given that the mass of informed traders is monotonic in most of these parameters. The reason is that, due to non-linear and non-Gaussian nature of the equilibrium, *there is no direct link between the mass of informed traders and price informativeness*. In fact, in stark contrast to linear-Gaussian models, *price informativeness and conditional variance may exhibit opposite behaviour*: As Figures 3 and 5 show, both the mass of informed traders and the conditional variance decrease in the reference level c . The reason is that, by Proposition 2.3, prices stay closer to their average level as c increases. This result clearly has important implications for empirical measures of price informativeness based on variance.

Insert Figures 1, 2, 3, 4, 5 about here.

3 Endogenous Contracts and Pecuniary Externalities

In this section, we solve for the equilibrium with endogenous contracts. The contracting phase of the game is as follows. At time zero, a continuum of investors, each endowed with unit wealth, are randomly matched one-to-one with managers. Since managerial skill is only realized after they start trading, investors face the risk of being matched with an unskilled manager. They do not internalize the externality that their contract choice has on the price distribution, and take μ_I as

²⁴We always normalize mispricing by $\theta_H - \theta_L$.

²⁵Under the assumption that all the managers can become skilled at some bounded cost ($\rho = 1$), mispricing may be hump shaped with respect to the precision of the uninformed managers. The intuition behind this result is as follows. Lower precision of the uninformed class of managers (higher σ_U) incentivizes them to acquire information and this indirect effect starts to dominate the direct effect of lower precision at sufficiently high σ_U .

given. In this paper, we only consider symmetric equilibria in which all managers have identical contracts; we take the parameter c as given and do not solve the optimization problem with respect to c .²⁶ Given that c is fixed, only parameters a and d matter for the incentives. We will use a and $\alpha = 2a/d$ to parametrize the contract, and we will denote by $\bar{\mathcal{C}} \equiv (\bar{a}, \bar{\alpha})$ the parameters of a contract that investor offers to the manager, and by $\mathcal{C} = (a, \alpha)$ the parameters of the contracts used by all other investors. We first solve the problem of the manager given his own contracts and the contracts of all other managers in the economy.

Lemma 3.1 *For any $K = U, I$, the optimal threshold $\bar{X}_K(p)$ for a manager with a contract $\bar{\mathcal{C}} = (\bar{a}, \bar{b}, \bar{c}, \bar{d})$ given that all the other managers have contracts a, b, c, d , is given by*

$$\bar{X}_K(p; \bar{\alpha}, \mathcal{C}) = \frac{\sigma_K^2}{2(\theta_H - \theta_L)} \left(2 \log \left(\frac{\bar{\pi}^*(p)}{1 - \bar{\pi}^*(p)} \right) - \log \left(\frac{\pi^*(p)}{1 - \pi^*(p)} \right) \right), \quad (21)$$

where π^* and $\bar{\pi}^*$ are the threshold probabilities defined in (5) for the contracts a, b, c, d and $\bar{a}, \bar{b}, \bar{c}, \bar{d}$ respectively.

To decide on the optimal contract, an investor first computes the managers' gains from becoming informed. Let R_K be the return that the manager generates, conditional on his type being K . Clearly, its distribution depends both on the density of the price and the parameter α of the contract. Then, the total gains from information acquisition take the form

$$\begin{aligned} \xi &\equiv \xi(\bar{\mathcal{C}}, \mathcal{C}) \equiv E[V^I(s, p; \bar{\mathcal{C}}, \mathcal{C})] - E[V^U(s, p; \bar{\mathcal{C}}, \mathcal{C})] \\ &= \bar{a} E[R_I - R_U] + \bar{d} E[(R_I - c)^+ - (R_U - c)^+]. \end{aligned} \quad (22)$$

Define

$$\begin{aligned} \Psi_K^M(\bar{\alpha}, \mathcal{C}) &= \int_{\theta_L}^{\theta_H} \left[1 + \kappa(\theta_H - p) \left(1 - 2\Phi \left(\frac{\bar{X}_K(p; \bar{\alpha}, \mathcal{C}) - \theta_H}{\sigma_K} \right) \right) \right] f_H^M(p; \mathcal{C}) dp \\ \Psi_{K, \text{conv}}^M(\bar{\alpha}, \mathcal{C}) &= \int_{\theta_L}^{\theta_H} \kappa(\theta_H - \beta - p)^+ \left(1 - \Phi \left(\frac{\bar{X}_K(p; \bar{\alpha}, \mathcal{C}) - \theta_H}{\sigma_K} \right) \right) f_H^M(p; \mathcal{C}) dp \end{aligned} \quad (23)$$

for $K, M = I, U$. Here, Φ is the cumulative distribution function of a standard Normal variable. Since, by Proposition 2.2, $f_i(p) = \mu_I f_i^I(p) + \mu_U f_i^U(p)$, we can use the symmetry²⁷ to get

$$\begin{aligned} \Psi_K(\bar{\alpha}, \mathcal{C}) &\equiv E[R_K] = \mu_I \Psi_K^I + \mu_U \Psi_K^U \\ \Psi_{K, \text{conv}}(\bar{\alpha}, \mathcal{C}) &\equiv E[(R_K - c)^+] = \mu_I \Psi_{K, \text{conv}}^I + \mu_U \Psi_{K, \text{conv}}^U, \quad K = I, U. \end{aligned} \quad (24)$$

Suppose first that the investors are risk neutral. Denote by V_{pr} the utility of the investor and let V_{pr}^K be the expected utility for an investor whose manager has type $K = U, I$. Given that the manager's skill is only realized ex-post, the ex-ante probability of a manager becoming informed is

²⁶The analysis can be extended to the case of endogenous c , but it gets significantly more complicated.

²⁷By symmetry, $\theta_H = -\theta_L$ and $f_H(p) = f_L(-p)$. Therefore, expected gains from information are the same in the two states of the world. Furthermore, manager receives a bonus in state high only if he is long the asset.

given by $H(\xi(\bar{\mathcal{C}}, \mathcal{C}))$ and hence

$$V_{\text{pr}} = H(\xi(\bar{\mathcal{C}}, \mathcal{C}))V_{\text{pr}}^I + (1 - H(\xi(\bar{\mathcal{C}}, \mathcal{C})))V_{\text{pr}}^U. \quad (25)$$

The expected utility of the investor is given by the expected return net the expected performance fee:

$$V_{\text{pr}}^K = \Psi_K - E[V^K(s, p)], \quad K = U, I.$$

The investor maximizes his utility (25) under the participation constraint of the manager, given the managers outside option U_0 , and subject to the manager's limited liability constraint. Conditional on becoming informed (with probability $H(\xi)$), the manager pays the expected cost $E[\lambda|\lambda < \xi]$ of information acquisition. Therefore, the participation constraint can be rewritten as

$$E[V^I(s, p; \bar{\mathcal{C}}, \mathcal{C})]H(\xi) - E[\lambda|\lambda < \xi] + E[V^U(s, p)](1 - H(\xi)) \geq U_0. \quad (26)$$

The worst possible total return that the manager can generate is given by $R_{\min} \equiv 1 - \kappa(\theta_H - \theta_L)$. Therefore, limited liability constraint is equivalent to

$$b \geq -aR_{\min}. \quad (27)$$

The presence of both constraints (26) and (27) significantly complicates the optimal contracting problem. We will therefore start our analysis by assuming that limited liability does not bind.²⁸

3.1 The case when limited liability does not bind

In this case, individual rationality constraint is always binding.²⁹ Therefore, we get that investor's utility is given by

$$\begin{aligned} V_{\text{pr}}(\xi, \bar{\alpha}) &= H(\xi)V_{\text{pr}}^I + (1 - H(\xi))V_{\text{pr}}^U \\ &= (1 - H(\xi))\Psi_U(\bar{\alpha}, \mathcal{C}) + H(\xi)\Psi_I(\bar{\alpha}, \mathcal{C}) - E[\lambda\mathbf{1}_{\lambda < \xi}] - U_0. \end{aligned} \quad (28)$$

That is, there is no conflict of interest between the investor and the manager, and the investor's goal is to maximize the expected return net the expected cost of information acquisition. The quantities $\xi \geq 0$ and $\bar{\alpha} \geq 0$ represent an equivalent parametrisation of the contract space, and therefore we can rewrite investor's problem as $\max_{\xi, \bar{\alpha} \geq 0} V_{\text{pr}}(\xi, \bar{\alpha})$. Both ξ and $\bar{\alpha}$ have a clear economic meaning: ξ controls the probability of a manager becoming informed, and $\bar{\alpha}$ controls the distortion of the optimal portfolio relative to the risk neutral benchmark. Selecting a non-linear contract would distort the managers' portfolio choice relative to the investor's objective and should therefore be sub-optimal. The following result shows that this is indeed the case and linear contracts are always optimal for risk-neutral investors.

²⁸As we show below, this is always the case when the outside option U_0 is sufficiently large.

²⁹This can always be achieved by adjusting b without changing the incentives.

Lemma 3.2 *The function $\Psi_K^L(\bar{\alpha}, \mathcal{C})$ is monotone increasing in $\bar{\alpha}$ for $K, L = U, I$. Consequently, investor always chooses $\bar{\alpha} = \infty \Leftrightarrow \bar{d} = 0$.*

By Lemma 3.2, the maximization problem of the investor reduces to finding the optimal threshold type ξ . The first order conditions with respect to ξ take the form

$$\Psi_I - \Psi_U = \xi,$$

where the left-hand side is evaluated at $\bar{\alpha} = \infty$. That is, investors optimally choose the threshold type ξ that equalizes marginal gain from information acquisition ($\Psi_I - \Psi_U$) with the marginal loss ξ . However, by (22) we have $\xi = \bar{a}(\Psi_I - \Psi_U)$. Thus, $\bar{a} = 1$. This is intuitive: since there is no conflict of interest between investors and managers, it is optimal for the investor to provide 100% incentives to the manager, and adjust the constant b to satisfy the manager's IR constraint. Even though we have derived this result assuming that investors are risk neutral, it is straightforward to show that it does not depend on investor's risk aversion. Indeed, the optimal contract with $a = 1$ implies that the agent takes on the whole risk and hence the investor's profits are risk-free. Hence, even if investors are risk averse, they will choose the same optimal contract.³⁰

By (24), we have

$$\Psi_I - \Psi_U = \mathcal{A}\mu_I + \mathcal{B} \tag{29}$$

where \mathcal{A} and \mathcal{B} are independent of the contract parameters and are given by

$$\mathcal{A} = (\Psi_I^I - \Psi_I^U) - (\Psi_U^I - \Psi_U^U), \quad \mathcal{B} = \Psi_U^I - \Psi_U^U,$$

and the equilibrium mass μ_I of informed managers satisfies $\mu_I = H(\xi)$. It remains to verify that the limited liability constraint indeed does not bind for the optimal contract. Since $b = U_0 - (H(\xi)\Psi_I + (1 - H(\xi))\Psi_U - E[\lambda \mathbf{1}_{\lambda < \xi}])$, limited liability does not bind if and only if $U_0 > U_0^*$, where

$$U_0^*(\mu_I) = (H(\Psi_I - \Psi_U)\Psi_I + (1 - H(\Psi_I - \Psi_U))\Psi_U - E[\lambda \mathbf{1}_{\lambda < \Psi_I - \Psi_U}])|_{\bar{\alpha}=\alpha=\infty} - R_{\min}. \tag{30}$$

Summarizing, we arrive at the following result.

Theorem 3.1 *Suppose that there exists a solution μ_I to*

$$\mu_I = H(\mathcal{A}\mu_I + \mathcal{B}).$$

such that $U_0 > U_0^(\mu_I)$. Then, independent of investors' preferences, the linear contract with $a = 1$ is optimal and is the equilibrium contract.*

Uniqueness of equilibrium in Theorem 3.1 depends on whether information acquisition is a strategic substitute or a strategic complement, which is determined by the sign of \mathcal{A} . In the case

³⁰This follows directly from the Jensen's inequality.

when $\mathcal{A} < 0$ (which is what we always find numerically), uniqueness of equilibrium is achieved, just as in the basic information acquisition game (see, Theorem 2.2).

Proposition 3.1 *If $\mathcal{A} < 0$ then there exists a unique symmetric equilibrium in which limited liability does not bind.*

Recall that, for two cumulative distribution functions H_1, H_2 , the first one is dominated by the second one in the sense of first order stochastic dominance if $H_1(x) \geq H_2(x)$ for all x , in which case we write $H_1 \prec_{fbsd} H_2$ and say that managers in market 1 are more skilled than those in the market 2. The following result shows that, when information acquisition is a strategic substitute, more skilled managers always acquire more information.

Proposition 3.2 *Suppose that $\mathcal{A} < 0$. Consider two markets with different distributions of skill, H_1 and H_2 , and suppose that managers in the market one are more skilled: $H_1 \prec_{fbsd} H_2$. Then, $\mu_I^1 > \mu_I^2$. That is, more skilled managers always acquire more information in equilibrium.*

Finally, we note that in the case when informed signal precision is sufficiently high, we can show analytically that $\mathcal{A} < 0$. The following is true.

Proposition 3.3 *If either σ_I or θ_H is sufficiently small then information acquisition is a strategic substitute. That is, $\mathcal{A} < 0$.*

The competitive behaviour of investors may lead to a potential inefficiency: investors take the equilibrium price distribution as given without internalizing the pecuniary externality that their contracts choice has on the prices. If investors could coordinate on their contracts (e.g., by creating some general industry-wide rules³¹), they might potentially be strictly better off.³² Let us consider a “representative investor” whose objective is to maximize ex-ante utility of all investors in the economy.³³ The representative investor internalizes the externality which his choice of the threshold type, ξ , has on the equilibrium mass $\mu_I = H(\xi)$. Suppose for simplicity that investors are risk neutral. By (28), the value function of the representative investor, $V_{pr}^c(\xi, \bar{\alpha})$, can be rewritten as

$$\begin{aligned} V_{pr}^c(\xi, \alpha) &= \mathcal{A}_U(\alpha) H(\xi) + \mathcal{B}_U(\alpha) - U_0 \\ &+ H(\xi) ((\mathcal{A}_I(\alpha) - \mathcal{A}_U(\alpha))H(\xi) + (\mathcal{B}_I(\alpha) - \mathcal{B}_U(\alpha)) - E[\lambda|\lambda < \xi]). \end{aligned} \tag{31}$$

Note first that the optimality of linear contracts (corresponding to $\alpha = \infty$) is not all clear in this case. Extensive numerical experiments suggest that this is indeed the case. From now on, we will

³¹The existence of regulations in the form of permissible fee contracts in some segments of the industry suggests that investors might be using these regulations as a coordination device to internalize some of the contractual externalities.

³²It is known (see, e.g., Geanakoplos and Polemarchakis (1985)) that pecuniary externalities matter when there is some form of market incompleteness. In our model, the mechanism is similar: manager skill is a form of uninsurable idiosyncratic risk that cannot be contracted upon because it is unobservable.

³³This does not necessarily have to be a regulator. We could also think of this representative investor as a single, strategic investor who hires all (a continuum of) managers and is therefore fully strategic, as in KOW (2011).

therefore assume that $\alpha = \infty$ and omit α in the notation. Then, the representative investor only has the option to choose a threshold type ξ^c . The first order conditions take the form

$$\mathcal{A}_U + 2(\mathcal{A}_I - \mathcal{A}_U)H(\xi^c) + (\mathcal{B}_I - \mathcal{B}_U) = \xi^c,$$

and the optimal level of incentives, a^c , is given by

$$a^c = 1 + \frac{\mathcal{A}_U + (\mathcal{A}_I - \mathcal{A}_U)H(\xi)}{(\mathcal{A}_I - \mathcal{A}_U)H(\xi) + (\mathcal{B}_I - \mathcal{B}_U)}.$$

Assuming that information acquisition is a strategic substitute, there exists a unique solution, ξ^c , to this equation. Otherwise, the true global maximum has to be selected from potentially multiple local extrema. Ex-ante, it is not clear whether a^c is above or below 1, and, similarly, whether the mass of informed managers optimal for the representative investor, $H(\xi^c)$, is larger or smaller than the equilibrium mass of informed traders in the case of non-coordinating investors. The analysis for the case of risk averse investors is similar.

Figure 11 shows that for most parameter values, the optimal value of a that coordinating investors would agree on is much lower than the competitive full-delegation result with $a = 1$. This means that competitive fund industry makes investors choose a much higher proportional fee a than the one that would be optimal for them if they could coordinate. As a direct consequence, with competitive investors the mass of informed managers is much higher than that with coordinating investors (see Figure 8). However, when precisions of two classes of managers are sufficiently close to each other, coordinating investors may choose a linear compensation a greater than 1. Furthermore, our extensive numerical results suggest that linear contracts are always optimal, even when investors coordinate.

Figures 9 and 10 show the ex-ante utilities of the coordinating and competitive investors for the two cases $\rho = 0.3$ (few good managers) and $\rho = 1$ (many good managers). As we can see, the gains from coordination are high when the difference in the precision of the signals of the two classes is not too high. Quite surprisingly, in the case of $\rho = 1$, the utility of competitive investors may be *decreasing in the average skill level* λ . That is, the pecuniary externality implies that better managers are bad for investors as a whole. The reason is that, as the mass of informed managers increases, prices become more revealing, reducing inefficiencies and, as a consequence, reducing the excess returns that the skilled managers can generate. By contrast, if investors coordinate, they internalize this externality and their utility always increases in managers' quality.

Figures 6 and 7 show that the competitive market reveals information better than the market with coordinating investors. Mispricing is lower with competitive investors because, absent coordination, investors choose higher proportional fees a and (as a result) higher information acquisition and a higher mass of informed managers μ_I in the market. Surprisingly, mispricing may be decreasing in σ_U , the quality of the signals of uninformed managers. The reason is that an increase in σ_U increases the gains from information acquisition, which in turn increases the mass of informed managers and improves price informativeness.

Insert Figures 6, 7, 8, 9, 10, 11 about here.

The presence of noise traders makes it difficult to clearly interpret our results from the social welfare perspective. Indeed, the managers are making profits, capitalising on the inelastic demand of noise traders. Suppose however that noise traders and managers are risk neutral, whereas investors have mean-variance preferences. Then, total social welfare can be computed: trade between managers and noise traders does not influence welfare, and therefore the social planner wants to minimize the variance of investors returns after fees plus the total cost of information acquisition. Our numerical results indicate that *the social planner always finds it optimal to provide convex incentives*. The reason is that, as we discuss above, convex incentives are a very efficient way of reducing aggregate volatility. Figures 12, 13 illustrate the results. We can see that the optimal contract from the point of view of the social planner consists of nonzero fulcrum and asymmetric parts. The dependence of the socially optimal contract on model parameters is highly non-trivial and non-monotonic. Furthermore, interestingly enough, the utility of the investors under the social planner's contracts' choice is typically higher than the utility of investors in the competitive market equilibrium.

Insert Figures 12, 13 about here.

3.2 The case when limited liability binds

As above, we only consider symmetric equilibria. Fix the contract parameters (ξ, α) of all managers in the economy. By the above, the unconstrained optimal contract is given by $a = 1$, $d = 0$. This contract satisfies the limited liability constraint if and only if $U_0 \geq U_0^*(H(\xi), \alpha)$ where

$$U_0^*(H(\xi), \alpha) = (H(\Psi_I - \Psi_U)\Psi_I + (1 - H(\Psi_I - \Psi_U))\Psi_U - E[\lambda \mathbf{1}_{\lambda < \Psi_I - \Psi_U}]) .$$

Otherwise, the optimal contract will have $b = -aR_{\min}$. Denote $\hat{\Psi}_K = E[R_K - R_{\min}]$. As above, it will be convenient to parametrize the optimal contract by the gains from information acquisition, $\bar{\xi}$, and the portfolio distortion parameter, $\bar{\alpha} = 2\bar{a}/\bar{d}$. Then, a direct calculation implies that

$$\bar{\xi} = \bar{d}G(\bar{\alpha})$$

with

$$G(\bar{\alpha}) = (0.5\bar{\alpha}(\Psi_I(\bar{\alpha}) - \Psi_U(\bar{\alpha})) + (\Psi_{I,\text{conv}}(\bar{\alpha}) - \Psi_{U,\text{conv}}(\bar{\alpha}))) \geq 0. \quad (32)$$

and the manager's IR (participation) constraint can be rewritten as

$$H(\bar{\xi})\bar{\xi} + \frac{\bar{\xi}}{G(\bar{\alpha})} \left(0.5\bar{\alpha}\hat{\Psi}_U(\bar{\alpha}) + \Psi_{U,\text{conv}}(\bar{\alpha}) \right) - E[\lambda \mathbf{1}_{\lambda < \bar{\xi}}] \geq U_0. \quad (33)$$

The following is true.

Lemma 3.3 Let $\zeta(x)$ be the unique solution to

$$H(\zeta)\zeta + \zeta x - E[\lambda \mathbf{1}_{\lambda < \zeta}] = U_0.$$

and let

$$\xi^*(\bar{\alpha}) = \zeta \left(\left(0.5\bar{\alpha}\hat{\Psi}_U(\bar{\alpha}) + \Psi_{U,\text{conv}}(\bar{\alpha}) \right) / G(\bar{\alpha}) \right).$$

Then, manager's participation constraint is fulfilled if and only if $\bar{\xi} \geq \xi^*(\bar{\alpha})$.³⁴

Everywhere in this section we will assume for simplicity that investors are risk neutral. Then, we can rewrite investor's maximization problem as

$$\max_{\bar{\xi} \geq \xi^*(\bar{\alpha}), \bar{\alpha} \geq 0} \left\{ H(\bar{\xi})\Psi_I(\bar{\alpha}) + (1 - H(\bar{\xi}))\Psi_U(\bar{\alpha}) + (1 - H(\bar{\xi}))\bar{\xi} - \frac{\bar{\xi}}{G(\bar{\alpha})} \left(0.5\bar{\alpha}\hat{\Psi}_I(\bar{\alpha}) + \Psi_{I,\text{conv}}(\bar{\alpha}) \right) \right\} \quad (34)$$

We will need the following technical condition that simplifies our analysis.

Assumption 3.1 The density $h(\xi)$ and the inverse hazard rate $(1 - H(\xi))/h(\xi)$ are nonincreasing.

In particular, Assumption 3.1 is fulfilled for the benchmark case $h(\xi) = \rho\lambda e^{-\lambda\xi}$. We can now characterize the parameter $\bar{\xi}$ of the optimal contract.

Lemma 3.4 If

$$(\Psi_I(\bar{\alpha}) - \Psi_U(\bar{\alpha})) - \xi^*(\bar{\alpha}) + \frac{1 - H(\xi^*(\bar{\alpha}))}{h(\xi^*(\bar{\alpha}))} - \frac{1}{h(\xi^*(\bar{\alpha}))G(\bar{\alpha})} \left(0.5\bar{\alpha}\hat{\Psi}_I(\bar{\alpha}) + \Psi_{I,\text{conv}}(\bar{\alpha}) \right) \leq 0 \quad (35)$$

then IR binds and the optimal $\bar{\xi}$ is given by $\bar{\xi} = \xi^*(\bar{\alpha})$. Otherwise, the optimal $\bar{\xi} = \xi(\bar{\alpha})$ is the unique solution

$$\max_{\bar{\alpha} \geq 0} \left\{ \hat{\Psi}_U(\bar{\alpha}) + (H(\xi(\bar{\alpha})) - \xi(\bar{\alpha})h(\xi(\bar{\alpha}))) (\Psi_I(\bar{\alpha}) - \Psi_U(\bar{\alpha})) + (\xi(\bar{\alpha}))^2 h(\xi(\bar{\alpha})) \right\}. \quad (36)$$

It is interesting to note that condition (35) is always violated if managers' skill is very low. Namely, the following is true.

Lemma 3.5 Suppose that $h(\xi)$ is sufficiently small for all $\xi > 0$. Then, the constraint $\xi \geq \xi^*(\bar{\alpha})$ always binds.

For example, if $h(\xi) = \lambda e^{-\lambda\xi}$ and λ is sufficiently small, most managers have very low skill (i.e., very high cost of information acquisition), and hence it is too expensive to provide corresponding incentives.

Substituting optimal $\bar{\xi}$ into investors utility (34), we can now optimize over the portfolio distortion parameter $\bar{\alpha}$, and then solve for the equilibrium fixed point. We first discuss the properties

³⁴Note that $\zeta(x) = 0$ if $U_0 = 0$, in which case IR never binds.

of the equilibrium optimal contracts. Recall that, in the case when limited liability does not bind, the optimal contract is always characterized by $a = 1$, $d = 0$. As we can see from Figures 14, 15, limited liability changes the shape of the optimal contract dramatically: both linear and convex parts are present. Interestingly enough, as Figure 15 shows, convex incentives are hump-shaped in the parameter λ controlling the average skill of the managers. That is, the model predicts that we should observe weaker asymmetric incentives if managers are either unskilled or highly skilled.

Quite surprisingly, despite the limited liability being binding, *coordinating investors always find it optimal to choose linear contracts*. The reason is that convex incentives significantly distort equilibrium price distribution, reducing investors profits, and investors prefer to avoid this when they internalize the corresponding externality.

Insert Figures 14, 15 about here.

Figures 16, 17, 18, 19 illustrate other equilibrium properties. Comparing the results with the case when limited liability does not bind, we clearly see several patterns:

- *Mispricing* (Figures 6 versus 16): Consistent with the above, we have a more pronounced mispricing. In fact, as we can see from the way mispricing depends on λ , endogenous convex incentives imply that mispricing is always uniformly higher when limited liability binds, independently of the average skill level λ . This is driven by two effects: First, binding limited liability means a low outside option for the investors. With a lower outside option investors get paid less, get weaker incentives and, as a result, equilibrium μ_I is lower. Second, convex incentives lead to a more pronounced mispricing due to the “gambling effect” explained above.
- *Conditional variance* (Figures 7 versus 17): as in the linear contract case, the conditional variance is *increasing* in the average skill level λ . However, in agreement with our general results, convex incentives reduces the overall volatility level.
- *Mass of informed managers* (Figures 8 versus 18): Since the outside option of the managers needs to be small to make limited liability bind, investors provide weaker incentives, which leads to a decrease in the mass of informed managers. Interestingly enough, monotonicity in θ does not hold for the case when limited liability binds: the mass increases in θ for the case when LL does not bind, and has a hump shape when it binds. The exact mechanism behind this is hard to disentangle: an increase in θ both increases potential profits from trade (by alleviating the leverage constraint) and increases signal informativeness for both manager classes (it is easier to distinguish high value from low value if the difference is larger).
- *Manager’s outside option* U_0 is a new parameter that starts playing a role when limited liability binds. It also has a clear economic interpretation: the outside option depends on how experienced a manager is (measured by the length of his rack record). In particular, we expect that in relatively young (non-mature) fund industry sectors, U_0 is relatively small, whereas in mature sectors managers should have a higher U_0 . Furthermore, the size of the

outside option U_0 is also directly linked to manager’s own capital. Figures 20 and 21 show the dependence of equilibrium characteristics on manager’s outside option. Figure 21 is the most intriguing: it shows that the size of convex compensation has a U-shape with respect to U_0 . The intuition behind this finding is as follows: when U_0 is very small, it is optimal to provide the manager with just a little bit of purely convex incentives. As U_0 increases but remains small, the contract parameters do not change because the participation constraint does not bind yet. As U_0 increases further, limited liability still binds, but the total amount of compensation has to go up to satisfy the manager’s participation constraint, which starts binding. Hence the increase in d . However, as U_0 becomes sufficiently large, limited liability stops binding and hence the optimal contract becomes the linear one with $a = 1$ and $d = 0$. As Figures 20 shows, an increase in U_0 always leads to an increase in equilibrium incentives for information acquisition and hence to an increase in the mass of informed traders and a decrease in the mispricing. At the same time, as the ratio d/a essentially decreases with U_0 , conditional variance goes up.

Insert Figures 16, 17, 18, 19 about here

Insert Figures 20 and 21 about here

4 Indirect Convex Incentives

It is well known that indirect incentives induced by the well documented convex relationship between past performance and fund-flows play a very important role in the fund managers’ decisions. See, for example, Chevalier and Ellison (1999) and Agarwal, Daniel and Naik (2009). In fact, as Lim, Sensoy and Weisbach (2013) find, indirect incentives for the average hedge fund are about four times as large as direct incentives from incentive fees and returns to managers own investment in the fund.³⁵

Indirect convex incentives can be easily incorporated into our model. Namely, suppose that a manager receives $a^F R + d^F (R - c)^+$ at time 1 due to the convex performance-fund flow relationship, with some $a^F > 0$, $d^F > 0$. Then, an investor can choose a contract $(\bar{a}, \bar{b}, \bar{d})$, and the full (direct+indirect) incentives are given by $(\bar{a} + a^F, \bar{d} + d^F)$. We will use (a, b, c, d) to denote the contracts used by other investors in the economy. As above, we assume that incumbent investors are risk neutral. In this case, investor’s value function is given by

$$V_{\text{pr}}^K = \Psi_K - E[u(R_K)], \quad K = U, I$$

where u is the performance contract that the investor is using. By contrast, manager’s value function consists of two parts:

$$V^K = E[u(R_K)] + E[u^F(R_K)]$$

³⁵See also Chung et al. (2012).

where u^F is the part coming from the indirect incentives, and manager's participation constraint takes the form

$$\begin{aligned} & (E[V^I(s, p; \bar{\mathcal{C}}, \mathcal{C})] - E[\lambda | \lambda < \xi])H(\xi) + E[V^U(s, p)](1 - H(\xi)) \\ & \geq U_0 - (H(\xi)E[u^F(R^I)] + (1 - H(\xi))E[u^F(R^U)]). \end{aligned} \quad (37)$$

As above, we only consider symmetric equilibria. If the reservation utility U_0 is low enough, limited liability binds and we have $b = -aR_{\min}$. Denote $\hat{\Psi}_K = E[R_K - R_{\min}]$.³⁶ As above, it will be convenient to parametrize the optimal contract by the total gains from information acquisition (both direct and indirect),

$$\begin{aligned} \bar{\xi} & \equiv \xi(\bar{\mathcal{C}}, \mathcal{C}) \equiv E[V^I(s, p; \bar{\mathcal{C}}, \mathcal{C})] - E[V^U(s, p; \bar{\mathcal{C}}, \mathcal{C})] \\ & = (\bar{a} + a^F)E[R_I - R_U] + (\bar{d} + d^F)E[(R_I - c)^+ - (R_U - c)^+]. \end{aligned} \quad (38)$$

and the portfolio distortion parameter will be $\bar{\alpha} = 2(\bar{a} + a^F)/(\bar{d} + d^F)$. Then, a direct calculation implies that

$$\bar{\xi} = (\bar{d} + d^F)G(\bar{\alpha})$$

with $G(\bar{\alpha})$ defined in (32) and manager's IR (participation) constraint can still be written as in (33). Therefore, Lemma 3.3 translates to the setup with indirect incentives. The nonnegativity constraint on the original contract parameters, $(\bar{a}, \bar{d}) \geq 0$ transform into a non-linear constraint on $(\bar{\xi}, \bar{\alpha})$. Namely,

$$\bar{\xi} \geq \xi^{**}(\bar{\alpha}) \equiv \max\{d^F G(\bar{\alpha}), 2a^F G(\bar{\alpha})/\bar{\alpha}\}.$$

Then, we can rewrite investor's maximization problem as

$$\begin{aligned} \max_{\bar{\xi} \geq \max\{\xi^*(\bar{\alpha}), \xi^{**}(\bar{\alpha})\}} & \left\{ H(\bar{\xi})\Psi_I(\bar{\alpha}) + (1 - H(\bar{\xi}))\Psi_U(\bar{\alpha}) \right. \\ & + (1 - H(\bar{\xi}))(\bar{\xi} - a^F(\Psi_I(\bar{\alpha}) - \Psi_U(\bar{\alpha})) - d^F(\Psi_{I,\text{conv}}(\bar{\alpha}) - \Psi_{U,\text{conv}}(\bar{\alpha}))) \\ & \left. - \frac{\bar{\xi}}{G(\bar{\alpha})} \left(0.5\bar{\alpha}\hat{\Psi}_I(\bar{\alpha}) + \Psi_{I,\text{conv}}(\bar{\alpha}) \right) + a^F\Psi_I(\bar{\alpha}) + d^F\Psi_{I,\text{conv}}(\bar{\alpha}) \right\} \end{aligned} \quad (39)$$

As above, we assume that Assumption 3.1 holds. The following lemma holds.

Lemma 4.1 *If*

$$\begin{aligned} & (\Psi_I(\bar{\alpha}) - \Psi_U(\bar{\alpha})) - \xi^M(\bar{\alpha}) + a^F(\Psi_I(\bar{\alpha}) - \Psi_U(\bar{\alpha})) + d^F(\Psi_{I,\text{conv}}(\bar{\alpha}) - \Psi_{U,\text{conv}}(\bar{\alpha})) \\ & + \frac{1 - H(\xi^M(\bar{\alpha}))}{h(\xi^M(\bar{\alpha}))} - \frac{1}{h(\xi^M(\bar{\alpha}))G(\bar{\alpha})} \left(0.5\bar{\alpha}\hat{\Psi}_I(\bar{\alpha}) + \Psi_{I,\text{conv}}(\bar{\alpha}) \right) \leq 0 \end{aligned} \quad (40)$$

then IR binds and the optimal $\bar{\xi}$ is given by $\bar{\xi} = \xi^M(\bar{\alpha}) \equiv \max\{\xi^*(\bar{\alpha}), \xi^{**}(\bar{\alpha})\}$. Otherwise, the

³⁶Under the benchmark parameters, we always assume $R_{\min} = 0$.

optimal $\bar{\xi} = \xi(\bar{\alpha})$ is the unique solution to

$$\max_{\bar{\alpha} \geq 0} \left\{ \Psi_U(\bar{\alpha}) + (H(\xi(\bar{\alpha})) - \xi(\bar{\alpha})h(\xi(\bar{\alpha}))) \left((1 + a^F) (\Psi_I(\bar{\alpha}) - \Psi_U(\bar{\alpha})) + d^F (\Psi_{I,\text{conv}}(\bar{\alpha}) - \Psi_{U,\text{conv}}(\bar{\alpha})) \right) \right. \\ \left. + (\xi(\bar{\alpha}))^2 h(\xi(\bar{\alpha})) + a^F \Psi_U(\bar{\alpha}) + d^F \Psi_{U,\text{conv}}(\bar{\alpha}) \right\}. \quad (41)$$

It is interesting to note that condition (40) is always violated if managers' skill and indirect incentives are very low. Namely, the following is true.

Lemma 4.2 *Suppose that a^F and d^F are small and $h(\xi)$ is sufficiently small for all $\xi > 0$. Then, the constraint $\xi \geq \xi^M(\bar{\alpha})$ always binds.*

For example, if $h(\xi) = \rho\lambda e^{-\lambda\xi}$ and λ is sufficiently small, most managers have very low skill (i.e., very high cost of information acquisition), and hence it is too expensive to provide corresponding incentives.

Substituting optimal ξ into investors utility, we can now optimize over the portfolio distortion parameter $\bar{\alpha}$, and then solve for the equilibrium fixed point. Interesting enough, Figures 22, 23 show that linear indirect incentives may lead to a (slight) *increase in the direct convex incentives*. The reason is that linear indirect incentives reduce the conflict of interest between managers and investors, and at the same time relax the IR constraint, making limited liability bind even stronger. However, as the linear indirect incentives become sufficiently large, it gets optimal for the incumbent investors not to provide any incentives at all, making the manager “work for the future”. In contrast, indirect convex incentives decrease both the linear and the convex parts of the managerial contracts: convex direct incentives are reduced to overcome the portfolio distortion, and overall incentives reduce because the IR constraint is relaxed.

Insert Figures 22, 23 about here.

Figures 24, 25, 26, 27 show the behaviour of equilibrium prices. Figure 24 shows that, in complete agreement with Proposition 2.3, mispricing is monotone increasing in convex indirect incentives. At the same time, Figure 25 shows that convex indirect incentives reduce price variance. Figure 27 shows that indirect incentives increase investors' utility. However, Figure 26 shows that, while convex indirect incentives induce more information acquisition, *the effect of linear indirect incentives on information acquisition is ambiguous*. Namely, while information acquisition increases for small values of a_F , it experiences a large drop as a_F hits the critical value at which equilibrium a drops to zero. Interestingly enough, this drop is absent in the case when investors internalize the pecuniary externality: the mass of informed managers simply stays constant (and low) until a_F hits the critical level, and then steadily increases, purely due to stronger indirect incentives.

Insert Figures 24, 25, 26, 27 about here.

5 Inferring Skill from Performance

In our model, both good and bad managers may be able to capture the right direction of the trade. Since they always take the same maximal leverage, they will also have identical performance. Under the null hypothesis that the manager is unskilled (i.e., his type is above ξ), we can compute the corresponding Type I and Type II errors: the fraction of bad managers among all managers with good performance (i.e., those with positive returns), and the fraction of good managers among all managers with bad performance (i.e., those with negative returns). These fractions correspond to Type I and Type II errors for the null hypothesis.

For a given price realization, denote by $M_{\pm}^K(p; \theta_i)$ the fraction of type-K managers who have positive (respectively, negative) excess returns in the state θ_i , conditional on the price p . By symmetry, we always have $M_{\pm}^K(p; \theta_H) = M_{\pm}^K(-p; \theta_L)$. Therefore, it suffices to consider $M_{\pm}^K(p; \theta_L)$. Conditional on the state θ_L , only managers that go short have positive excess returns. Hence, in equilibrium, we have

$$M_{+}^K(p; \theta_L) = \Phi\left(\frac{X_K(p) - \theta_L}{\sigma_K}\right), \quad M_{-}^K(p; \theta_L) = 1 - M_{+}^K(p; \theta_L).$$

We can now compute the probabilities of Type I and Type II errors if we use performance for inference about the skill. The probability for a randomly picked manager to be uninformed, conditional on him having positive returns, is given by

$$\frac{\mu_U M_{+}^U(p; \theta_L)}{\mu_I M_{+}^I(p; \theta_L) + \mu_U M_{+}^U(p; \theta_L)}.$$

Similarly, the probability for a randomly picked manager to be informed conditional on him having negative returns is given by

$$\frac{\mu_I M_{-}^I(p; \theta_L)}{\mu_I M_{-}^I(p; \theta_L) + \mu_U M_{-}^U(p; \theta_L)}.$$

We can also compute the unconditional probabilities ex-ante, before the price and the state are realized.

Lemma 5.1 *The probability \Pr_{+}^U for an uninformed manager to have a positive excess return is given by the following expressions*

$$\Pr_{+}^U = E[M_{+}^U(p; \theta_L)] = \frac{\mu_U}{2} + \mu_I \int_{-\infty}^{+\infty} \Phi\left(\frac{\frac{\sigma_U^2}{\sigma_I^2} X - \theta_L}{\sigma_U}\right) \frac{1}{\sqrt{2\pi}\sigma_I} e^{-\frac{(X-\theta_L)^2}{2\sigma_I^2}} dX$$

whereas the corresponding probability \Pr_{+}^I for an informed manager is

$$\Pr_{+}^I = E[M_{+}^I(p; \theta_L)] = \frac{\mu_I}{2} + \mu_U \int_{-\infty}^{+\infty} \Phi\left(\frac{X - \theta_L}{\sigma_I}\right) \frac{1}{\sqrt{2\pi}\sigma_U} e^{-\frac{(\frac{\sigma_U^2}{\sigma_I^2} X - \theta_L)^2}{2\sigma_U^2}} \frac{\sigma_U^2}{\sigma_I^2} dX.$$

In the case when σ_I is sufficiently small (i.e., skilled managers get signals with a very high precision), these probabilities can be computed in closed form.

Proposition 5.1 *Suppose that σ_I is sufficiently small. Then,*

$$\frac{\mu_U E[M_+^U(p; \theta_L)]}{\mu_I E[M_+^I(p; \theta_L)] + \mu_U E[M_+^U(p; \theta_L)]} \approx \mu_U^2 \quad (42)$$

and

$$\frac{\mu_I E[M_-^I(p; \theta_L)]}{\mu_I E[M_-^I(p; \theta_L)] + \mu_U E[M_-^U(p; \theta_L)]} \approx \mu_I^2 \quad (43)$$

We can interpret (42) and (43) as average measures of Type I and Type II errors under the null hypothesis of managers being unskilled. Past positive performance is informative about skill only if the probability of Type I error is below 50%; similarly, past negative performance is informative about absence of skill only if the probability of Type II error is below 50%. Proposition 5.1 implies that, when σ_I is sufficiently small, past positive (negative) performance is informative about skill only if $\mu_U < \sqrt{0.5}$ ($\mu_I < \sqrt{0.5}$). Making the common assumption that only a small fraction (less than 20%) of managers are skilled, we get that the probability of Type I error is above 60% on average. By contrast, the probability of Type I error is extremely low (below 5% in this case). This asymmetry obviously has important implications for potential investors.

Figures 28-32 illustrate these phenomena. Interestingly enough, we can see that the two errors typically exhibit completely opposite behaviour: when one is increasing, the other is decreasing. Also, perhaps surprisingly, Type I errors are always increasing in the average skill λ , whereas Type II errors are decreasing in λ . However, when λ is sufficiently small, Type I error is much smaller than Type II error. Furthermore, as Figure 30 shows, more mature fund sectors (i.e, those with large U_0) have significantly lower Type I errors and at the same time much higher Type II errors. Finally, as indirect linear incentives have an ambiguous effect on μ_I , the same effect is present for the inference errors.

Insert Figures 28, 29, 30, 31, 32 about here.

It is important to note that our model does not make any predictions about the time series of an individual manager's returns. By contrast, our predictions are cross-sectional and should be tested accordingly.³⁷

6 Conclusions

We have developed a non-linear rational expectations model of portfolio delegation with convex compensation contracts and differentially skilled portfolio managers. Managers face a two-dimensional moral hazard problem due to endogenous and unobservable portfolio choice and the

³⁷See, e.g., Buraschi, Kosowski, and Trojani (2013) for a recent empirical study of the cross-section of hedge fund returns.

effort to acquire information. In equilibrium, both dimensions of the moral hazard problem influence equilibrium price distribution and price informativeness, creating a link between managerial incentives and market efficiency. Investors do not internalize the externality that their contract choice has on the market price and, as a result, often choose contractual incentives that are inefficiently strong. In particular, when the managers' limited liability binds, competitive investors select very strong convex incentives. By contrast, if the investors were allowed to coordinate, they would always prefer linear contracts.

We show that convex incentives have negative effects on market efficiency, forcing prices to systematically deviate from fundamentals. Indirect incentives due to future fund flows driven by a convex performance-fund flow relationship may further amplify these problems: By relaxing the managers' participation constraint, they make limited liability bind even stronger. This makes it optimal for investors to choose convex compensation contracts, leading to further inefficiencies.

Our results indicate the important role of non-linearities in general equilibrium and the mutual feedback effects between portfolio managers' incentives and equilibrium prices: by contrast to linear equilibria in the standard CARA-normal setting, there is no direct link between volatility and market efficiency. In particular, a social planner who cares about volatility will prefer convex incentives. However, nonlinearities imply that the effects on higher order moments (e.g., skewness and kurtosis) also need to be evaluated. Furthermore, non-linearities imply an asymmetric link between managerial skill and past performance.

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A Proofs

Proof of Lemma 2.1.

Convexity of the utility function $u(1 + x(\theta - p))$ in x guarantees that the maximum is reached on the boundary of domain of definition. That is,

$$\begin{aligned} V^K(s, p) &= \max_{x \in [-\kappa, \kappa]} E[u(1 + x(\theta - p)) | s, p] = \max_{x = \pm \kappa} \pi^K u(1 + x(\theta_H - p)) + (1 - \pi^K) u(1 + x(\theta_L - p)) \\ &= \max_{x = \pm \kappa} \Phi(x, \pi^K(s), p), \end{aligned}$$

where

$$\Phi(x, s, p) = u(1 + x(\theta_L - p)) + \pi^K [u(1 + x(\theta_H - p)) - u(1 + x(\theta_L - p))]$$

and

$$\pi^K = \frac{1}{1 + \frac{f_L(p)}{f_H(p)} \frac{\eta_K(s - \theta_L)}{\eta_K(s - \theta_H)}}.$$

We can compute the differences in expected utilities for $x = \pm \kappa$,

$$\begin{aligned} &\frac{\partial [\Phi(\kappa, s, p) - \Phi(-\kappa, s, p)]}{\partial s} \\ &= \frac{\partial \pi^K}{\partial s} \left[(u(1 + \kappa(\theta_H - p)) - u(1 - \kappa(\theta_H - p))) - (u(1 + \kappa(\theta_L - p)) - u(1 - \kappa(\theta_L - p))) \right] > 0. \end{aligned}$$

The inequality holds because (1) maximum likelihood property implies that π^K increases in s , (2) utility u is monotone increasing, and (3) price p belongs to the interval $[\theta_L, \theta_H]$.

For $s \rightarrow +\infty$, the probability of high realization $\pi^K \rightarrow 1$ and thus $\Phi(\kappa, +\infty, p) - \Phi(-\kappa, +\infty, p) > 0$. Similarly, for $s \rightarrow -\infty$, the probability of high realization $\pi^K \rightarrow 0$ and thus $\Phi(\kappa, -\infty, p) - \Phi(-\kappa, -\infty, p) < 0$. This completes the first part of the proof.

To find the form of $\pi^*(p)$ which determines the signal threshold $X_K(p)$, we first have to compute the expected utility $V(\pi, p, x)$ of a manager given price p , posterior probability of high state π and asset holdings x .³⁸ The function $V(\pi, p, x)$ is defined piecewise:

1. $x < -\frac{c-1}{p-\theta_L}$. Only for θ_L is the incentive part non-zero.

$$V_{(1)}(\pi, p, x) = a(1 + x(\theta_L + (\theta_H - \theta_L)\pi - p) - b) + d(1 - c + x(\theta_L - p))(1 - \pi)$$

2. $x \in \left[-\frac{c-1}{p-\theta_L}, \frac{c-1}{\theta_H-p}\right]$. Independently of θ , the incentive part is 0.

$$V_{(2)}(\pi, p, x) = a(1 + x(\theta_L + (\theta_H - \theta_L)\pi - p) - b)$$

3. $x > \frac{c-1}{\theta_H-p}$. Only for θ_H is the incentive part non-zero.

$$V_{(3)}(\pi, p, x) = a(1 + x(\theta_L + (\theta_H - \theta_L)\pi - p) - b) + d(1 - c + x(\theta_H - p))\pi$$

³⁸Note that given π , the maximization problems of the managers of classes $K = I, U$ are the same.

At the signal threshold $X_K(p)$, which corresponds to the threshold probability of high state π^* , a risk-neutral manager of type K is indifferent between submitting demands κ and $-\kappa$. Depending on the price p , the following cases can be realized.

1. $\kappa \leq \min\left(\frac{c-1}{p-\theta_L}, \frac{c-1}{\theta_H-p}\right)$. The equation which determines π^* is then

$$V_{(2)}(\pi^*, p, \kappa) = V_{(2)}(\pi^*, p, -\kappa)$$

We can rewrite

$$\begin{aligned} p - (\theta_H - \theta_L)\pi^* - \theta_L &= 0, \\ \pi^* &= \frac{p - \theta_L}{\theta_H - \theta_L}. \end{aligned} \quad (44)$$

2. $\frac{c-1}{p-\theta_L} < \kappa \leq \frac{c-1}{\theta_H-p}$. The equation which determines π^* is then

$$V_{(2)}(\pi^*, p, \kappa) = V_{(1)}(\pi^*, p, -\kappa),$$

$$\begin{aligned} a(1 + \kappa(\theta_L + (\theta_H - \theta_L)\pi^* - p) - b) &= \\ a(1 - \kappa(\theta_L + (\theta_H - \theta_L)\pi^* - p) - b) + d(1 - c - \kappa(\theta_L - p))(1 - \pi^*), & \\ \pi^* &= \frac{\left(\frac{2a}{d} + 1\right)(p - \theta_L) - \frac{c-1}{\kappa}}{p + \frac{2a}{d}(\theta_H - \theta_L) - \frac{c-1}{\kappa} - \theta_L}. \end{aligned} \quad (45)$$

3. $\frac{c-1}{\theta_H-p} < \kappa \leq \frac{c-1}{p-\theta_L}$. The equation which determines π^* is then

$$V_{(3)}(\pi^*, p, \kappa) = V_{(2)}(\pi^*, p, -\kappa),$$

$$\begin{aligned} a(1 + \kappa(\theta_L + (\theta_H - \theta_L)\pi^* - p) - b) + d(1 - c + \kappa(\theta_H - p))\pi^* &= \\ a(1 - \kappa(\theta_L + (\theta_H - \theta_L)\pi^* - p) - b), & \\ \pi^* &= \frac{\frac{2a}{d}(p - \theta_L)}{-p + \frac{2a}{d}(\theta_H - \theta_L) - \frac{c-1}{\kappa} + \theta_H}. \end{aligned} \quad (46)$$

4. $\kappa > \max\left(\frac{c-1}{p-\theta_L}, \frac{c-1}{\theta_H-p}\right)$. The equation which determines π^* is then

$$V_{(3)}(\pi^*, p, \kappa) = V_{(1)}(\pi^*, p, -\kappa),$$

$$\begin{aligned} a(1 + \kappa(\theta_L + (\theta_H - \theta_L)\pi^* - p) - b) + d(1 - c + \kappa(\theta_H - p))\pi^* &= \\ a(1 - \kappa(\theta_L + (\theta_H - \theta_L)\pi^* - p) - b) + d(1 - c - \kappa(\theta_L - p))(1 - \pi^*), & \\ \pi^* &= \frac{\left(\frac{2a}{d} + 1\right)(p - \theta_L) - \frac{c-1}{\kappa}}{\left(\frac{2a}{d} + 1\right)(\theta_H - \theta_L) - 2\frac{c-1}{\kappa}}. \end{aligned} \quad (47)$$

The cases can be summarized as follows,

1. If $\kappa \geq 2\frac{c-1}{\theta_H-\theta_L}$, then

$$\pi^*(p) \equiv \begin{cases} \frac{\alpha(p-\theta_L)}{-p+\alpha(\theta_H-\theta_L)+\theta_H-\beta}, & p \in [\theta_L, \theta_L + \beta] \\ \frac{(\alpha+1)(p-\theta_L)-\beta}{(\alpha+1)(\theta_H-\theta_L)-2\beta}, & p \in [\theta_L + \beta, \theta_H - \beta] \\ \frac{(\alpha+1)(p-\theta_L)-\beta}{p+\alpha(\theta_H-\theta_L)-\theta_L-\beta}, & p \in [\theta_H - \beta, \theta_H] , \end{cases} \quad (48)$$

2. If $\kappa < 2\frac{c-1}{\theta_H-\theta_L}$, then

$$\pi^*(p) \equiv \begin{cases} \frac{\alpha(p-\theta_L)}{-p+\alpha(\theta_H-\theta_L)+\theta_H-\beta}, & p \in [\theta_L, \theta_H - \beta] \\ \frac{p-\theta_L}{\theta_H-\theta_L}, & p \in [\theta_H - \beta, \theta_L + \beta] \\ \frac{(\alpha+1)(p-\theta_L)-\beta}{p+\alpha(\theta_H-\theta_L)-\theta_L-\beta}, & p \in [\theta_L + \beta, \theta_H] , \end{cases} \quad (49)$$

where

$$\alpha = \frac{2a}{d}, \quad \beta = \frac{c-1}{\kappa}.$$

Under Assumption 2.1, the first case is realized. This completes the proof. ■

Proof of Theorem 2.1. Suppose that all the managers in the economy have identical optimal probability threshold $\pi^*(p)$ and optimal signal thresholds $X_I(p), X_U(p)$. Denote the optimal signal thresholds of an agent with probability threshold $\bar{\pi}^*(p)$ by $\bar{X}_I(p), \bar{X}_U(p)$.

From the Bayes' rule in (3) and Lemma 2.1, it follows that

$$\pi^I(\bar{X}_I, p) = \pi^U(\bar{X}_U, p) = \bar{\pi}^*(p) = \frac{1}{1 + \frac{f_L(p)\eta_I(\bar{X}_I-\theta_L)}{f_H(p)\eta_I(X_I-\theta_H)}}. \quad (50)$$

Equivalently, using the definition of likelihood and rearranging terms we have

$$\log\left(\frac{\bar{\pi}^*(p)}{1 - \bar{\pi}^*(p)}\right) = \log\left(\frac{f_H(p)}{f_L(p)}\right) + \log \ell_K(\bar{X}_K(p)), \quad K = I, U. \quad (51)$$

Identity $\ell_U(X_U(p)) = \ell_I(X_I(p))$ implies that $X_U(p) = \mathcal{L}(X_I(p))$. Denote $A(p) = \mathcal{L}'(X_I(p))$. Then,

$$\begin{aligned} \log\left(\frac{f_H(p)}{f_L(p)}\right) &= \log\left(\frac{\mu_I\eta_I(X_I(p) - \theta_H)X_I'(p) + \mu_U\eta_U(X_U(p) - \theta_H)X_U'(p)}{\mu_I\eta_I(X_I(p) - \theta_L)X_I'(p) + \mu_U\eta_U(X_U(p) - \theta_L)X_U'(p)}\right) \\ &= \log\left(\frac{\mu_I\eta_I(X_I(p) - \theta_H)X_I'(p) + \mu_U\eta_U(X_U(p) - \theta_H)A(p)X_I'(p)}{\mu_I\eta_I(X_I(p) - \theta_L)X_I'(p) + \mu_U\eta_U(X_U(p) - \theta_L)A(p)X_I'(p)}\right) \\ &= \log\left(\frac{\eta_I(X_I(p) - \theta_H)(\mu_I X_I'(p) + \mu_U\eta_U(X_U(p) - \theta_H)/\eta_I(X_I(p) - \theta_H)A(p)X_I'(p))}{\eta_I(X_I(p) - \theta_L)(\mu_I X_I'(p) + \mu_U\eta_U(X_U(p) - \theta_L)/\eta_I(X_I(p) - \theta_L)A(p)X_I'(p))}\right) \\ &= \log\left(\frac{\eta_I(X_I(p) - \theta_H)}{\eta_I(X_I(p) - \theta_L)}\right) = \log \ell_I(X_I(p)), \end{aligned} \quad (52)$$

where we have used that $\ell_U(X_U(p)) = \ell_I(X_I(p))$ is equivalent to

$$\frac{\eta_U(X_U(p) - \theta_H)}{\eta_I(X_I(p) - \theta_H)} = \frac{\eta_U(X_U(p) - \theta_L)}{\eta_I(X_I(p) - \theta_L)}.$$

■

Proof of Lemma 3.1. The proof follows directly from formula (51). ■

Proof of Lemma 3.2.

The claim is based on the following observation

Lemma A.1

$$\frac{p - \theta_L}{\theta_H - p} \leq \frac{\bar{\pi}(p)}{1 - \bar{\pi}(p)}$$

if and only if $p > 0$.

We have that

$$\begin{aligned} \Psi_K = \frac{1}{2} \sum_{\theta_i = \pm\theta} \int_{-\infty}^{+\infty} & \left[1 + \kappa(\theta_i - p) \left(1 - 2\Phi \left(\frac{\bar{X}_K(p; \bar{\alpha}) - \theta_i}{\sigma_K} \right) \right) \right] \\ & \left(\frac{\mu_I}{\sqrt{2\pi}\sigma_I} e^{-\frac{(X-\theta_i)^2}{2\sigma_I^2}} + \frac{\mu_U}{\sqrt{2\pi}\sigma_U} e^{-\frac{(X\sigma_U^2/\sigma_I^2 - \theta_i)^2}{2\sigma_U^2}} \frac{\sigma_U^2}{\sigma_I^2} \right) dX. \end{aligned} \quad (53)$$

and

$$\begin{aligned} \frac{\partial \Psi_K}{\partial \bar{\alpha}} &= \frac{1}{2} \sum_{\theta_i = \pm\theta} \int_{-\infty}^{+\infty} -2\kappa(\theta_i - p) \frac{1}{\sigma_K} \phi \left(\frac{\bar{X}_K(p; \bar{\alpha}) - \theta_i}{\sigma_K} \right) \frac{\partial \bar{X}_K(p; \bar{\alpha})}{\partial \bar{\alpha}} f(\theta_i, X) dX \\ &= -\kappa \int_{-\infty}^{+\infty} \frac{1}{\sigma_K} \left[\sum_{\theta_i = \pm\theta} (\theta_i - p) \phi \left(\frac{\bar{X}_K(p; \bar{\alpha}) - \theta_i}{\sigma_K} \right) f(\theta_i, X) \right] \frac{\partial \bar{X}_K(p; \bar{\alpha})}{\partial \bar{\alpha}} dX, \end{aligned} \quad (54)$$

where ϕ is a pdf of standard normal distribution and

$$\begin{aligned}
f(\theta_i, X) &= \left(\frac{\mu_I}{\sqrt{2\pi}\sigma_I} e^{-\frac{(X-\theta_i)^2}{2\sigma_I^2}} + \frac{\mu_U}{\sqrt{2\pi}\sigma_U} e^{-\frac{(X\sigma_U^2/\sigma_I^2-\theta_i)^2}{2\sigma_U^2}} \frac{\sigma_U^2}{\sigma_I^2} \right), \\
\bar{X}_K(p; \bar{\alpha}) &= \frac{\sigma_K^2}{4\theta} \left(2 \ln \left(\frac{\bar{\pi}(p)}{1-\bar{\pi}(p)} \right) - \ln \left(\frac{\pi(X)}{1-\pi(X)} \right) \right), \\
\bar{\pi}(p) &\equiv \begin{cases} \frac{\bar{\alpha}(p+\theta)}{-p+2\theta\bar{\alpha}+\theta-\beta}, & p \in [-\theta, -\theta+\beta] \\ \frac{(\bar{\alpha}+1)(p+\theta)-\beta}{2\theta(\bar{\alpha}+1)-2\beta}, & p \in [-\theta+\beta, \theta-\beta] \\ \frac{(\bar{\alpha}+1)(p+\theta)-\beta}{p+2\theta\bar{\alpha}+\theta-\beta}, & p \in [\theta-\beta, \theta], \end{cases} \\
\bar{\alpha} &= \frac{2\bar{a}}{\bar{d}}, \quad \beta = \frac{c-1}{\kappa}, \\
p(X) &= \begin{cases} \frac{(2\theta\alpha+\theta-\beta)\pi(X)-\alpha\theta}{\pi(X)+\alpha}, & \pi(X) \in \left[0, \frac{\alpha\beta}{-2\beta+2\theta(\alpha+1)} \right] \\ \left(2\theta - \frac{2\beta}{\alpha+1} \right) \pi(X) + \frac{\beta}{\alpha+1} - \theta, & \pi(X) \in \left[\frac{\alpha\beta}{-2\beta+2\theta(\alpha+1)}, \frac{-\beta+(\alpha+1)(2\theta-\beta)}{-2\beta+2\theta(\alpha+1)} \right] \\ \frac{(2\theta\alpha+\theta-\beta)\pi(X)-(\alpha+1)\theta+\beta}{-\pi(X)+\alpha+1}, & \pi(X) \in \left[\frac{-\beta+(\alpha+1)(2\theta-\beta)}{-2\beta+2\theta(\alpha+1)}, 1 \right], \end{cases} \\
\pi(X) &= \frac{1}{1 + e^{-\frac{4\theta}{\sigma_I^2} X}}.
\end{aligned} \tag{55}$$

At $\alpha = +\infty$, the price function is

$$p(X) = \theta(2\pi(X) - 1), \tag{56}$$

and the term in brackets is simplified as follows,

$$\begin{aligned}
&\sum_{\theta_i=\pm\theta} (\theta_i - p) \phi \left(\frac{\bar{X}_K(p) - \theta_i}{\sigma_K} \right) f(\theta_i, X) \\
&= (\theta_L - p) \theta \phi \left(\frac{\bar{X}_K(p) + \theta}{\sigma_K} \right) f(-\theta, X) + (\theta_H - p) \theta \phi \left(\frac{\bar{X}_K(p) - \theta}{\sigma_K} \right) f(\theta, X) \\
&= \left((\theta_L - p) \phi \left(\frac{\bar{X}_K(p) + \theta}{\sigma_K} \right) \left(\frac{\mu_I}{\sqrt{2\pi}\sigma_I} \phi \left(\frac{X + \theta}{\sigma_I} \right) + \frac{\mu_U}{\sqrt{2\pi}\sigma_U} \phi \left(\frac{X\sigma_U^2/\sigma_I^2 + \theta}{\sigma_U} \right) \frac{\sigma_U^2}{\sigma_I^2} \right) \right. \\
&+ \left. (\theta_H - p) \phi \left(\frac{\bar{X}_K(p) - \theta}{\sigma_K} \right) \left(\frac{\mu_I}{\sqrt{2\pi}\sigma_I} \phi \left(\frac{X - \theta}{\sigma_I} \right) + \frac{\mu_U}{\sqrt{2\pi}\sigma_U} \phi \left(\frac{X\sigma_U^2/\sigma_I^2 - \theta}{\sigma_U} \right) \frac{\sigma_U^2}{\sigma_I^2} \right) \right) \\
&= \frac{\mu_I}{\sqrt{2\pi}\sigma_I} \left((\theta_L - p) \phi \left(\frac{\bar{X}_K(p) + \theta}{\sigma_K} \right) \phi \left(\frac{X + \theta}{\sigma_I} \right) \right. \\
&\quad \left. + (\theta_H - p) \phi \left(\frac{\bar{X}_K(p) - \theta}{\sigma_K} \right) \phi \left(\frac{X - \theta}{\sigma_I} \right) \right) \\
&+ \frac{\mu_U\sigma_U}{\sqrt{2\pi}\sigma_I^2} \left((\theta_L - p) \phi \left(\frac{\bar{X}_K(p) + \theta}{\sigma_K} \right) \phi \left(\frac{X\sigma_U^2/\sigma_I^2 + \theta}{\sigma_U} \right) \right. \\
&\quad \left. + (\theta_H - p) \phi \left(\frac{\bar{X}_K(p) - \theta}{\sigma_K} \right) \phi \left(\frac{X\sigma_U^2/\sigma_I^2 - \theta}{\sigma_U} \right) \right).
\end{aligned} \tag{57}$$

Let us prove now that this is a global maximum.

Lemma A.2 *We have $\partial\bar{\pi}/\partial\bar{\alpha} < 0$ for $p > 0$ and $\partial\bar{\pi}/\partial\bar{\alpha} > 0$ for $p < 0$.*

Proof. The proof is direct. ■

We also have that

$$\frac{\partial\bar{X}_K(p; \bar{\alpha})}{\partial\bar{\alpha}} = \frac{\sigma_K^2}{2\theta\bar{\pi}(p)(1-\bar{\pi}(p))} \frac{\partial\bar{\pi}(p)}{\partial\bar{\alpha}}, \quad (58)$$

so due to Lemma A.2 this derivative is nonnegative for $p < 0$ and nonpositive for $p > 0$.

Furthermore,

$$\begin{aligned} & (\theta_L - p)\phi\left(\frac{\bar{X}_K(p) + \theta}{\sigma_K}\right)\phi\left(\frac{X_i(p) + \theta}{\sigma_i}\right) + (\theta_H - p)\phi\left(\frac{\bar{X}_K(p) - \theta}{\sigma_K}\right)\phi\left(\frac{X_i(p) - \theta}{\sigma_i}\right) \\ &= (\theta_H - p)\phi\left(\frac{\bar{X}_K(p) - \theta}{\sigma_K}\right)\phi\left(\frac{X_i(p) - \theta}{\sigma_i}\right)\left(-\frac{(p - \theta_L)\phi\left(\frac{\bar{X}_K(p) + \theta}{\sigma_K}\right)\phi\left(\frac{X_i(p) + \theta}{\sigma_i}\right)}{(\theta_H - p)\phi\left(\frac{\bar{X}_K(p) - \theta}{\sigma_K}\right)\phi\left(\frac{X_i(p) - \theta}{\sigma_i}\right)} + 1\right) \\ &= (\theta_H - p)\phi\left(\frac{\bar{X}_K(p) - \theta}{\sigma_K}\right)\phi\left(\frac{X_i(p) - \theta}{\sigma_i}\right)\left(-\frac{(p - \theta_L)}{(\theta_H - p)}e^{-2\bar{X}_K(p)\theta/\sigma_K^2 - 2X_i(p)\theta/\sigma^2} + 1\right) \\ &= (\theta_H - p)\phi\left(\frac{\bar{X}_K(p) - \theta}{\sigma_K}\right)\phi\left(\frac{X_i(p) - \theta}{\sigma_i}\right)\left(-\frac{(p - \theta_L)}{(\theta_H - p)}\frac{1 - \bar{\pi}(p)}{\bar{\pi}(p)} + 1\right) \end{aligned} \quad (59)$$

Combining this with the sign of the derivative $\frac{\partial\bar{X}_K(p; \bar{\alpha})}{\partial\bar{\alpha}}$, we have that the sign of the FOC (54) is positive for $\bar{\alpha} < \alpha$ and is negative for $\bar{\alpha} > \alpha$. Thus, $\bar{\alpha} = +\infty$ is a global maximum. ■

Proof of Proposition 3.3. Now, suppose we only send $\sigma_I \rightarrow 0$. Then,

$$\begin{aligned}
A_I &= \frac{1}{2} \sum_{i=H,L} \int_{-\infty}^{+\infty} \kappa \left(\theta_i - \theta_L - (\theta_H - \theta_L) \frac{1}{1 + e^{-\frac{2(\theta_H - \theta_L)X}{\sigma_I^2}}} \right) \left(1 - 2\Phi \left(\frac{X - \theta_i}{\sigma_I} \right) \right) \\
&\quad \left(\frac{1}{\sqrt{2\pi}\sigma_I} e^{-\frac{(X - \theta_i)^2}{2\sigma_I^2}} - \frac{1}{\sqrt{2\pi}q\sigma_I} e^{-\frac{(X - (\sigma_I/\sigma_U)^2 \theta_i)^2}{2(\sigma_I^2/\sigma_U)^2}} \right) dX \\
&= \frac{1}{2} \sum_{i=H,L} \int_{-\infty}^{+\infty} \kappa \left(\theta_i - \theta_L - (\theta_H - \theta_L) \frac{1}{1 + e^{-\frac{2(\theta_H - \theta_L)}{\sigma_I^2}(\theta_i + \sigma_I y)}} \right) (1 - 2\Phi(y)) \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy \\
&\quad - \frac{1}{2} \sum_{i=H,L} \int_{-\infty}^{+\infty} \kappa \left(\theta_i - \theta_L - (\theta_H - \theta_L) \frac{1}{1 + e^{-2\frac{(\theta_H - \theta_L)}{\sigma_I^2}(\theta_i \sigma_U^2 / \sigma_U^2 + y \sigma_U^2 / \sigma_U)}} \right) \\
&\quad \times \left(1 - 2\Phi \left(\frac{\theta_i (\sigma_U^2 / \sigma_U^2 - 1) + y \sigma_U^2 / \sigma_U}{\sigma_I} \right) \right) \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy \\
&\approx \frac{1}{2} \sum_{i=H,L} \int_{-\infty}^{+\infty} \kappa (\theta_i - \theta_L - (\theta_H - \theta_L) \mathbf{1}_{i=H}) (1 - 2\Phi(y)) \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy \\
&\quad - \frac{1}{2} \sum_{i=H,L} \int_{-\infty}^{+\infty} \kappa \left(\theta_i - \theta_L - (\theta_H - \theta_L) \frac{1}{1 + e^{-2(\theta_H - \theta_L)(\theta_i / \sigma_U^2 + y / \sigma_U)}} \right) (1 - 2\mathbf{1}_{i=L}) \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy \\
&= -(\theta_H - \theta_L) \frac{1}{2} \int_{\mathbb{R}} \left(\frac{1}{1 + e^{-2(\theta_H - \theta_L)(\theta_L / \sigma_U^2 + y / \sigma_U)}} + \frac{e^{-2(\theta_H - \theta_L)(\theta_H / \sigma_U^2 + y / \sigma_U)}}{1 + e^{-2(\theta_H - \theta_L)(\theta_H / \sigma_U^2 + y / \sigma_U)}} \right) \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy
\end{aligned} \tag{60}$$

Similarly,

$$\begin{aligned}
A_U &= \frac{1}{2} \sum_{i=H,L} \int_{-\infty}^{+\infty} \kappa \left(\theta_i - \theta_L - (\theta_H - \theta_L) \frac{1}{1 + e^{-\frac{2(\theta_H - \theta_L)X}{\sigma_I^2}}} \right) \left(1 - 2\Phi \left(\frac{\sigma_U^2/\sigma_I^2 X - \theta_i}{\sigma_U} \right) \right) \\
&\quad \left(\frac{1}{\sqrt{2\pi}\sigma_I} e^{-\frac{(X-\theta_i)^2}{2\sigma_I^2}} - \frac{1}{\sqrt{2\pi}q\sigma_I} e^{-\frac{(X-(\sigma_I/\sigma_U)^2\theta_i)^2}{2(\sigma_I^2/\sigma_U)^2}} \right) dX \\
&= \frac{1}{2} \sum_{i=H,L} \int_{-\infty}^{+\infty} \kappa \left(\theta_i - \theta_L - (\theta_H - \theta_L) \frac{1}{1 + e^{-\frac{2(\theta_H - \theta_L)}{\sigma_I^2}(\theta_i + \sigma_I y)}} \right) \\
&\quad \times \left(1 - 2\Phi \left(\frac{((\sigma_U^2/\sigma_I^2) - 1)\theta_i + (\sigma_U^2/\sigma_I)y}{\sigma_U} \right) \right) \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy \\
&\quad - \frac{1}{2} \sum_{i=H,L} \int_{-\infty}^{+\infty} \kappa \left(\theta_i - \theta_L - (\theta_H - \theta_L) \frac{1}{1 + e^{-2\frac{(\theta_H - \theta_L)}{\sigma_I^2}(\theta_i\sigma_I^2/\sigma_U^2 + y\sigma_I^2/\sigma_U)}} \right) \\
&\quad \times (1 - 2\Phi(y)) \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy \\
&\approx \frac{1}{2} \sum_{i=H,L} \int_{-\infty}^{+\infty} \kappa (\theta_i - \theta_L - (\theta_H - \theta_L) \mathbf{1}_{i=H}) (1 - 2\mathbf{1}_{i=H}) \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy \\
&\quad - \frac{1}{2} \sum_{i=H,L} \int_{-\infty}^{+\infty} \kappa \left(\theta_i - \theta_L - (\theta_H - \theta_L) \frac{1}{1 + e^{-2(\theta_H - \theta_L)(\theta_i/\sigma_U^2 + y/\sigma_U)}} \right) (1 - 2\Phi(y)) \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy \\
&= -(\theta_H - \theta_L) \frac{1}{2} \int_{\mathbb{R}} \left(-\frac{1}{1 + e^{-2(\theta_H - \theta_L)(\theta_L/\sigma_U^2 + y/\sigma_U)}} + \frac{e^{-2(\theta_H - \theta_L)(\theta_H/\sigma_U^2 + y/\sigma_U)}}{1 + e^{-2(\theta_H - \theta_L)(\theta_H/\sigma_U^2 + y/\sigma_U)}} \right) \frac{1}{\sqrt{2\pi}} (1 - 2\Phi(y)) e^{-y^2/2} dy
\end{aligned} \tag{61}$$

Thus, the required inequality $A_U > A_I$ can be rewritten

$$\int_{\mathbb{R}} \frac{2}{1 + e^{4\psi^2 - 4\psi y}} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy + 1 > \int_{\mathbb{R}} \left(\frac{1}{1 + e^{4\psi^2 - 4\psi y}} + \frac{1}{1 + e^{-4\psi^2 - 4\psi y}} \right) 2\Phi(y) \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy$$

where we have used that

$$\theta_L = -\theta_H = -\theta$$

and defined $\psi = \theta/\sigma_U$.

Now, in order to prove this inequality, we will make use of the fact $\Phi(-y) = 1 - \Phi(y)$ and the identity $\int f(y)dy = \int (f(y) + f(-y))dy$. Then, we can rewrite the desired inequality as

$$\int_{\mathbb{R}} \left(\frac{1 - \Phi(y)}{1 + e^{4\psi^2 - 4\psi y}} + \frac{\Phi(y)}{1 + e^{4\psi^2 + 4\psi y}} \right) \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy + 1 > \int_{\mathbb{R}} \left(\frac{\Phi(y)}{1 + e^{-4\psi^2 - 4\psi y}} + \frac{1 - \Phi(y)}{1 + e^{-4\psi^2 + 4\psi y}} \right) \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy
\end{aligned} \tag{62}$$

We will now use the identities

$$\frac{1}{1 + e^{4\psi^2 - 4\psi y}} + \frac{1}{1 + e^{-4\psi^2 + 4\psi y}} = 1, \quad \frac{1}{1 + e^{-4\psi^2 - 4\psi y}} + \frac{1}{1 + e^{4\psi^2 + 4\psi y}} = 1$$

that can be verified by direct calculation. It follows that

$$(1 - \Phi(y)) \left(\frac{1}{1 + e^{4\psi^2 - 4\psi y}} + \frac{1}{1 + e^{-4\psi^2 + 4\psi y}} \right) + \Phi(y) \left(\frac{1}{1 + e^{-4\psi^2 - 4\psi y}} + \frac{1}{1 + e^{4\psi^2 + 4\psi y}} \right) = 1,$$

and hence

$$\begin{aligned} & \int_{\mathbb{R}} \left(\Phi(y) \left(\frac{1}{1 + e^{-4\psi^2 - 4\psi y}} - \frac{1}{1 + e^{4\psi^2 + 4\psi y}} \right) \right. \\ & \left. + (1 - \Phi(y)) \left(\frac{1}{1 + e^{-4\psi^2 - 4\psi y}} - \frac{1}{1 + e^{4\psi^2 + 4\psi y}} \right) \frac{1}{\sqrt{2\pi}} \right) e^{-y^2/2} dy \\ & < \int_{\mathbb{R}} \left(\Phi(y) \left(\frac{1}{1 + e^{-4\psi^2 - 4\psi y}} + \frac{1}{1 + e^{4\psi^2 + 4\psi y}} \right) \right. \\ & \left. + (1 - \Phi(y)) \left(\frac{1}{1 + e^{-4\psi^2 - 4\psi y}} + \frac{1}{1 + e^{4\psi^2 + 4\psi y}} \right) \frac{1}{\sqrt{2\pi}} \right) e^{-y^2/2} dy \\ & \leq 1 \end{aligned} \tag{63}$$

and the claim follows. ■

Proof of Lemma 5.1. Managers choose the right direction of the trade if they take a short position when the fundamental is low and a long one when the fundamental is high. In equilibrium, the probability of avoiding a mistake for the fund with manager of type $K = I, U$ is

$$\begin{aligned} Pr_+^K &= \frac{1}{2} \int_{-\infty}^{+\infty} \Phi \left(\frac{X_K + \theta}{\sigma_K} \right) \\ & \quad \left(\frac{\mu_I}{\sqrt{2\pi}\sigma_I} e^{-\frac{(X+\theta)^2}{2\sigma_I^2}} + \frac{\mu_U}{\sqrt{2\pi}\sigma_U} e^{-\frac{(X\sigma_U^2/\sigma_I^2 + \theta)^2}{2\sigma_U^2}} \frac{\sigma_U^2}{\sigma_I^2} \right) dX \\ & + \frac{1}{2} \int_{-\infty}^{+\infty} \left(1 - \Phi \left(\frac{X_K - \theta}{\sigma_K} \right) \right) \\ & \quad \left(\frac{\mu_I}{\sqrt{2\pi}\sigma_I} e^{-\frac{(X-\theta)^2}{2\sigma_I^2}} + \frac{\mu_U}{\sqrt{2\pi}\sigma_U} e^{-\frac{(X\sigma_U^2/\sigma_I^2 - \theta)^2}{2\sigma_U^2}} \frac{\sigma_U^2}{\sigma_I^2} \right) dX \\ & = \int_{-\infty}^{+\infty} \Phi \left(\frac{X_K + \theta}{\sigma_K} \right) \left(\frac{\mu_I}{\sqrt{2\pi}\sigma_I} e^{-\frac{(X+\theta)^2}{2\sigma_I^2}} + \frac{\mu_U}{\sqrt{2\pi}\sigma_U} e^{-\frac{(X\sigma_U^2/\sigma_I^2 + \theta)^2}{2\sigma_U^2}} \frac{\sigma_U^2}{\sigma_I^2} \right) dX, \end{aligned} \tag{64}$$

where the last equality holds due to symmetry of the model. The probabilities could be further

simplified,

$$\begin{aligned}
Pr_+^I &= \int_{-\infty}^{+\infty} \Phi\left(\frac{X+\theta}{\sigma_I}\right) \left(\frac{\mu_I}{\sqrt{2\pi}\sigma_I} e^{-\frac{(X+\theta)^2}{2\sigma_I^2}} + \frac{\mu_U}{\sqrt{2\pi}\sigma_U} e^{-\frac{(X\sigma_U^2/\sigma_I^2+\theta)^2}{2\sigma_U^2}} \frac{\sigma_U^2}{\sigma_I^2} \right) dX \\
&= \int_{-\infty}^{+\infty} \Phi\left(\frac{X+\theta}{\sigma_I}\right) \mu_I d\Phi\left(\frac{X+\theta}{\sigma_I}\right) \\
&+ \int_{-\infty}^{+\infty} \Phi\left(\frac{X+\theta}{\sigma_I}\right) \frac{\mu_U}{\sqrt{2\pi}\sigma_U} e^{-\frac{(X\sigma_U^2/\sigma_I^2+\theta)^2}{2\sigma_U^2}} \frac{\sigma_U^2}{\sigma_I^2} dX \\
&= \frac{\mu_I}{2} \Phi^2\left(\frac{X+\theta}{\sigma_I}\right) \Big|_{X=-\infty}^{X=+\infty} \\
&+ \int_{-\infty}^{+\infty} \Phi\left(\frac{X+\theta}{\sigma_I}\right) \frac{\mu_U}{\sqrt{2\pi}\sigma_U} e^{-\frac{(X\sigma_U^2/\sigma_I^2+\theta)^2}{2\sigma_U^2}} \frac{\sigma_U^2}{\sigma_I^2} dX \\
&= \frac{\mu_I}{2} + \int_{-\infty}^{+\infty} \Phi\left(\frac{X+\theta}{\sigma_I}\right) \frac{\mu_U}{\sqrt{2\pi}\sigma_U} e^{-\frac{(X\sigma_U^2/\sigma_I^2+\theta)^2}{2\sigma_U^2}} \frac{\sigma_U^2}{\sigma_I^2} dX.
\end{aligned} \tag{65}$$

Similarly,

$$\begin{aligned}
Pr_+^U &= \int_{-\infty}^{+\infty} \Phi\left(\frac{X\sigma_U^2/\sigma_I^2+\theta}{\sigma_U}\right) \left(\frac{\mu_I}{\sqrt{2\pi}\sigma_I} e^{-\frac{(X+\theta)^2}{2\sigma_I^2}} + \frac{\mu_U}{\sqrt{2\pi}\sigma_U} e^{-\frac{(X\sigma_U^2/\sigma_I^2+\theta)^2}{2\sigma_U^2}} \frac{\sigma_U^2}{\sigma_I^2} \right) dX \\
&= \frac{\mu_U}{2} + \int_{-\infty}^{+\infty} \Phi\left(\frac{X\sigma_U^2/\sigma_I^2+\theta}{\sigma_U}\right) \frac{\mu_I}{\sqrt{2\pi}\sigma_I} e^{-\frac{(X+\theta)^2}{2\sigma_I^2}} dX.
\end{aligned} \tag{66}$$

■

Proof of Proposition 5.1. Let us now look at the probabilities when $\sigma_I \rightarrow 0$. Change of variable gives

$$\begin{aligned}
Pr_+^I &= \frac{\mu_I}{2} + \int_{-\infty}^{+\infty} \Phi\left(\frac{X+\theta}{\sigma_I}\right) \frac{\mu_U}{\sqrt{2\pi}\sigma_U} e^{-\frac{(X\sigma_U^2/\sigma_I^2+\theta)^2}{2\sigma_U^2}} \frac{\sigma_U^2}{\sigma_I^2} dX \\
&= \frac{\mu_I}{2} + \int_{-\infty}^{+\infty} \Phi\left(\frac{\frac{\sigma_I^2}{\sigma_U^2}X_U+\theta}{\sigma_I}\right) \frac{\mu_U}{\sqrt{2\pi}\sigma_U} e^{-\frac{(X_U+\theta)^2}{2\sigma_U^2}} dX_U.
\end{aligned} \tag{67}$$

We have that

$$\Phi\left(\frac{\frac{\sigma_I^2}{\sigma_U^2}X_U+\theta}{\sigma_I}\right) \xrightarrow{\sigma_I \rightarrow 0} \Phi(+\infty) = 1 \tag{68}$$

and thus

$$Pr_+^I \xrightarrow{\sigma_I \rightarrow 0} \frac{\mu_I}{2} + \mu_U = 1 - \frac{\mu_I}{2}. \tag{69}$$

Similarly,

$$\begin{aligned}
Pr_+^U &= \frac{\mu_U}{2} + \int_{-\infty}^{+\infty} \Phi\left(\frac{X\sigma_U^2/\sigma_I^2 + \theta}{\sigma_U}\right) \frac{\mu_I}{\sqrt{2\pi}\sigma_I} e^{-\frac{(X+\theta)^2}{2\sigma_I^2}} dX \\
&= \frac{\mu_U}{2} + \int_{-\infty}^{+\infty} \Phi\left(\frac{\sigma_U^2/\sigma_I^2(\sigma_I y - \theta) + \theta}{\sigma_U}\right) \frac{\mu_I}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy \\
&\xrightarrow{\sigma_I \rightarrow 0} \frac{\mu_U}{2} + \int_{-\infty}^{+\infty} \Phi(-\infty) \frac{\mu_I}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy
\end{aligned} \tag{70}$$

and thus

$$Pr_+^U \xrightarrow{\sigma_I \rightarrow 0} \frac{\mu_U}{2} = \frac{1}{2} - \frac{\mu_I}{2}. \tag{71}$$

Substituting $Pr_+^I = E[M_+^I(p; \theta_L)]$ and $Pr_+^U = E[M_+^U(p; \theta_L)]$ into expressions (42) and (43) concludes the proof. ■

B Plots

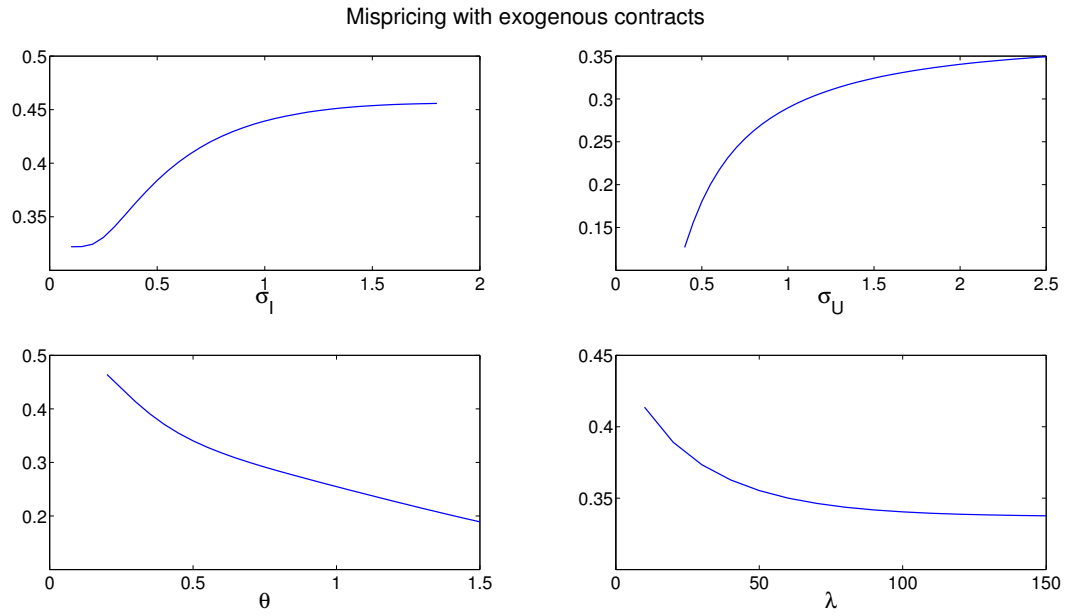


Figure 1: Mispricing as a function of σ_I , σ_U , θ and λ . Exogenous contracts. Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $a = 0.05$, $d = 0.3$, $c = 1.1$, $\lambda = 100$, $\rho = 0.3$.

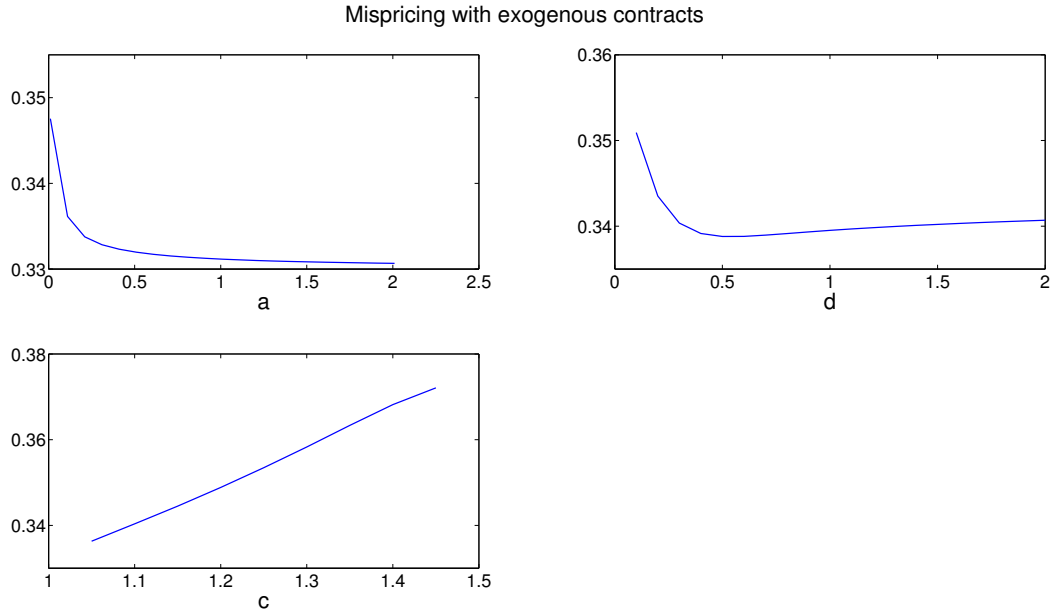


Figure 2: Mispricing as a function of parameters of contracts a , d and c . Exogenous contracts. Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $a = 0.05$, $d = 0.3$, $c = 1.1$, $\lambda = 100$, $\rho = 0.3$.

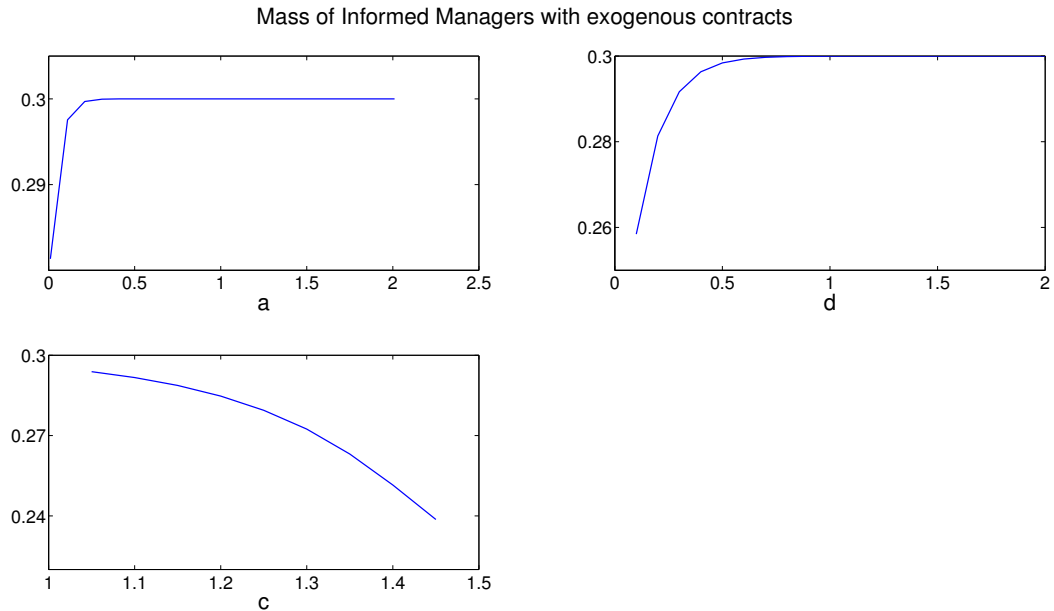


Figure 3: Mass of Informed Managers μ_I as a function of parameters of contracts a , d and c . Exogenous contracts. Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $a = 0.05$, $d = 0.3$, $c = 1.1$, $\lambda = 100$, $\rho = 0.3$.

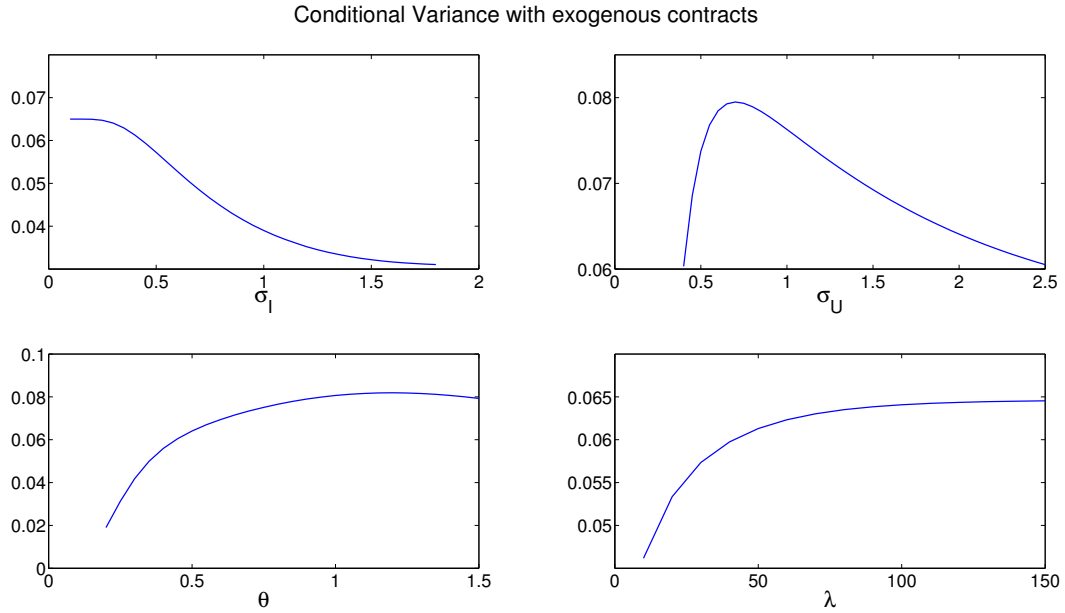


Figure 4: Normalized Conditional Variance as a function of σ_I , σ_U , θ and λ . Exogenous contracts. Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $a = 0.05$, $d = 0.3$, $c = 1.1$, $\lambda = 100$, $\rho = 0.3$.

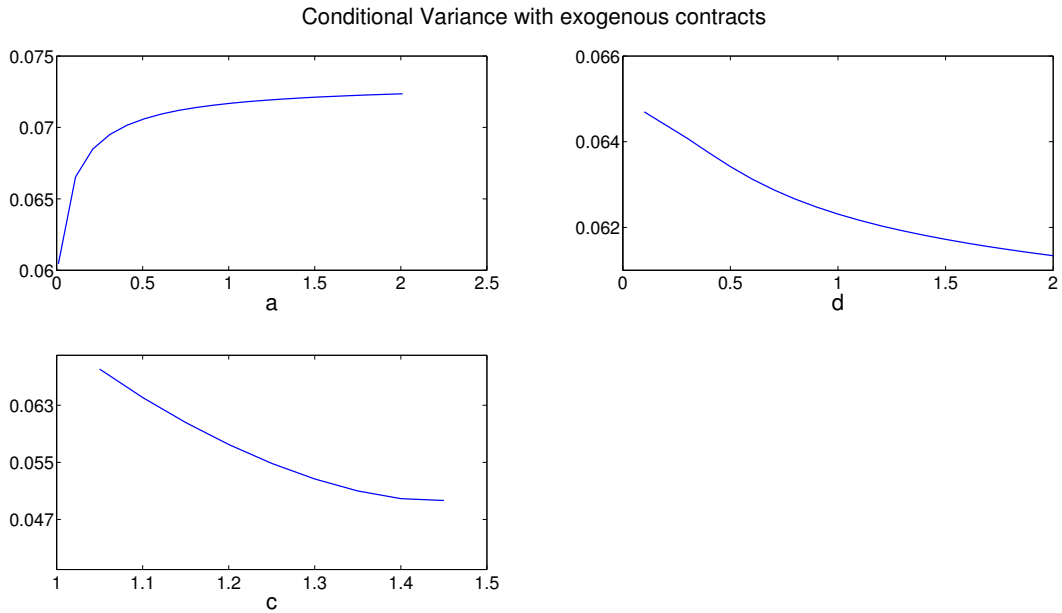


Figure 5: Normalized Conditional Variance as a function of parameters of contracts a , d and c . Exogenous contracts. Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $a = 0.05$, $d = 0.3$, $c = 1.1$, $\lambda = 100$, $\rho = 0.3$.

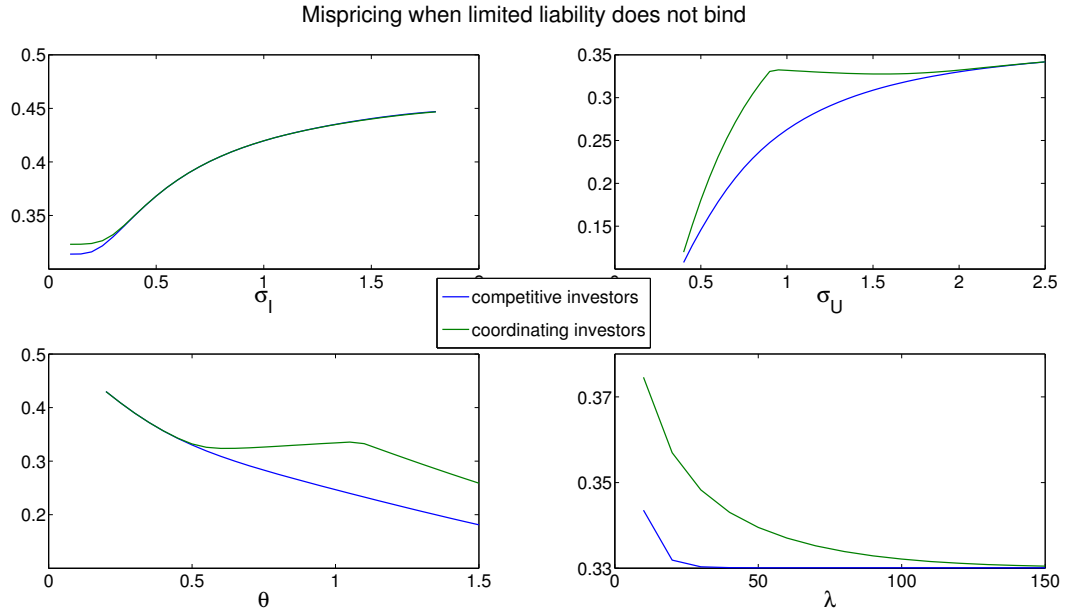


Figure 6: Mispricing as a function of σ_I , σ_U , θ and λ . Endogenous contracts with competitive (blue line) and coordinating risk-neutral investors (green line). Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $\lambda = 100$, $\rho = 0.3$.

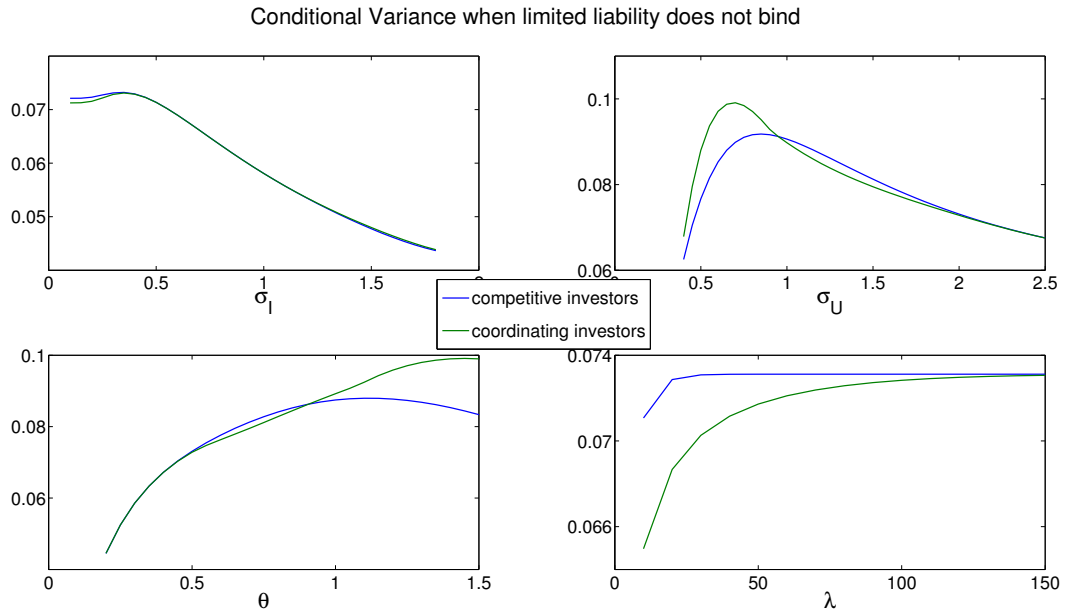


Figure 7: Normalized Conditional Variance as a function of σ_I , σ_U , θ and λ . Endogenous contracts with competitive (blue line) and coordinating risk-neutral investors (green line). Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $\lambda = 100$, $\rho = 0.3$.

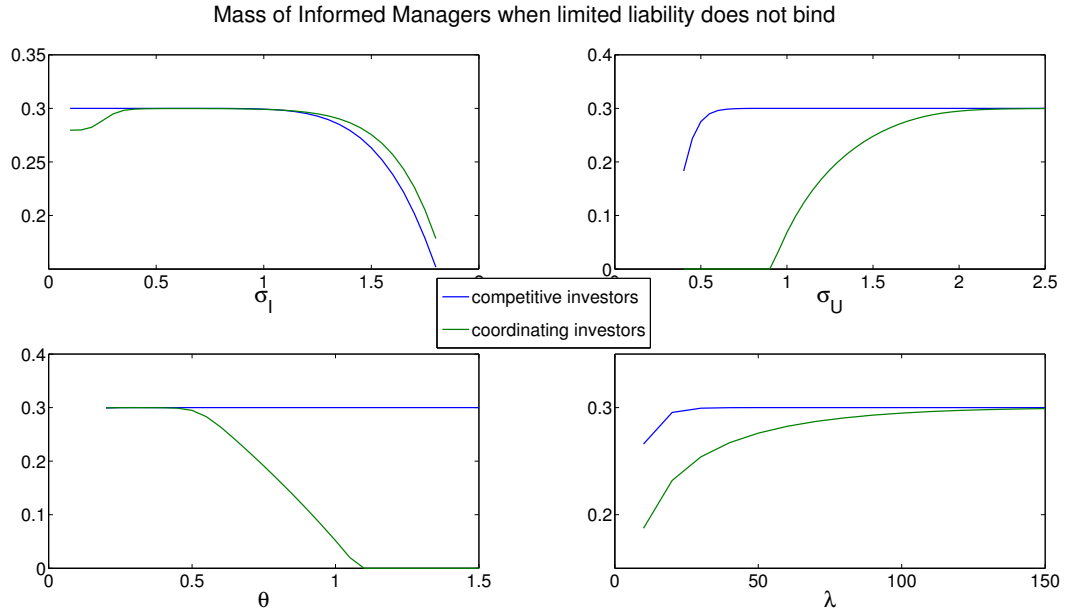


Figure 8: Mass of Informed Managers μ_I as a function of σ_I , σ_U , θ and λ . Endogenous contracts with competitive (blue line) and coordinating risk-neutral investors (green line). Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $\lambda = 100$, $\rho = 0.3$.

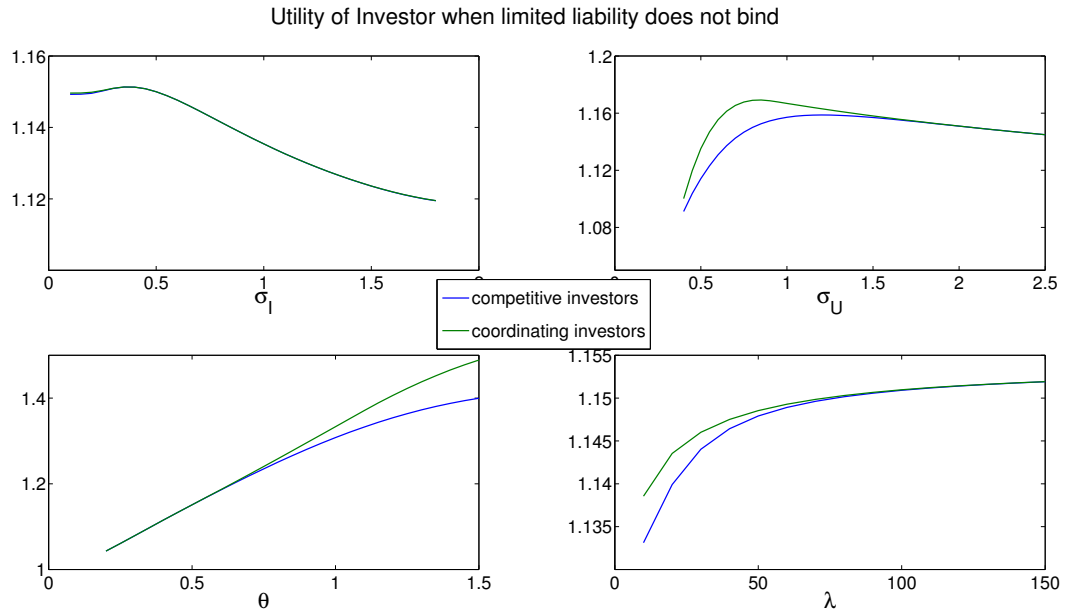


Figure 9: Investor's Utility as a function of σ_I , σ_U , θ and λ . Endogenous contracts with competitive (blue line) and coordinating risk-neutral investors (green line). Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $\lambda = 100$, $\rho = 0.3$.

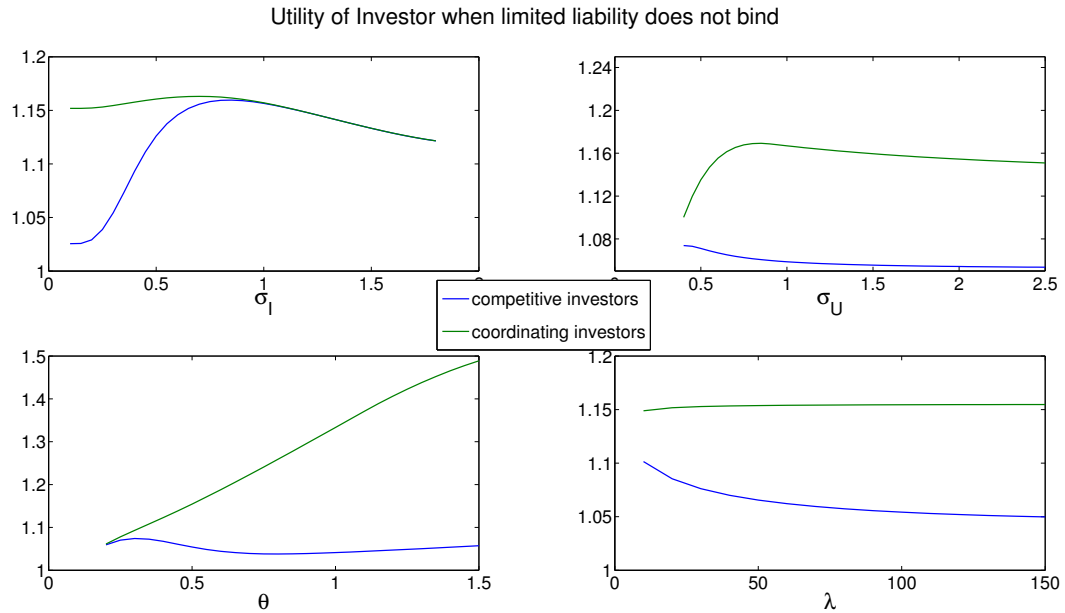


Figure 10: Investor's Utility as a function of σ_I , σ_U , θ and λ . Endogenous contracts with competitive (blue line) and coordinating risk-neutral investors (green line). Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $\lambda = 100$, $\rho = 1$.

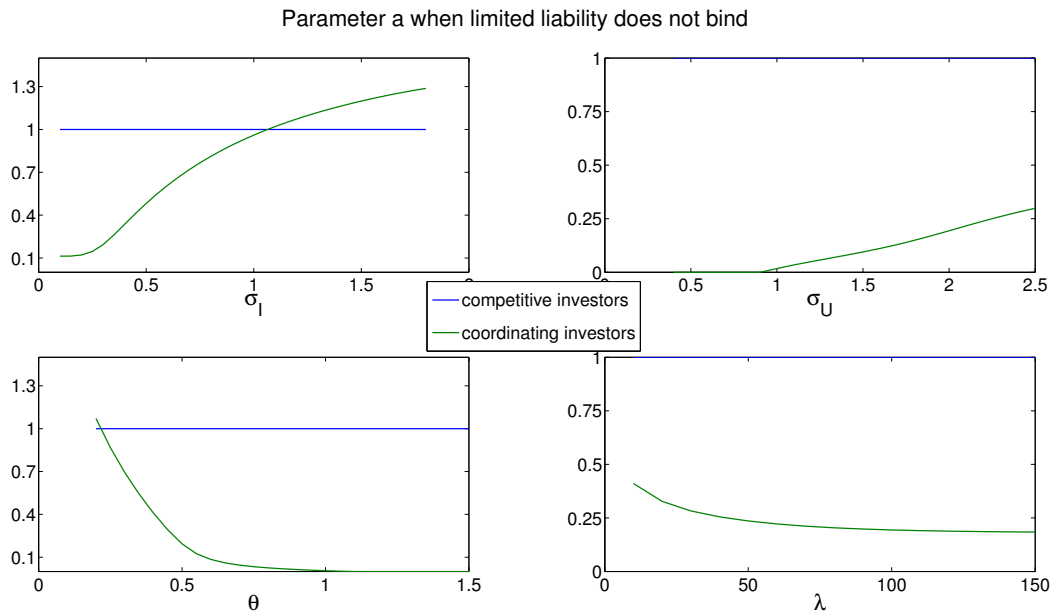


Figure 11: Contract parameter a as a function of σ_I , σ_U , θ and λ . Endogenous contracts with competitive (blue line) and coordinating risk-neutral investors (green line). Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $\lambda = 100$, $\rho = 0.3$.

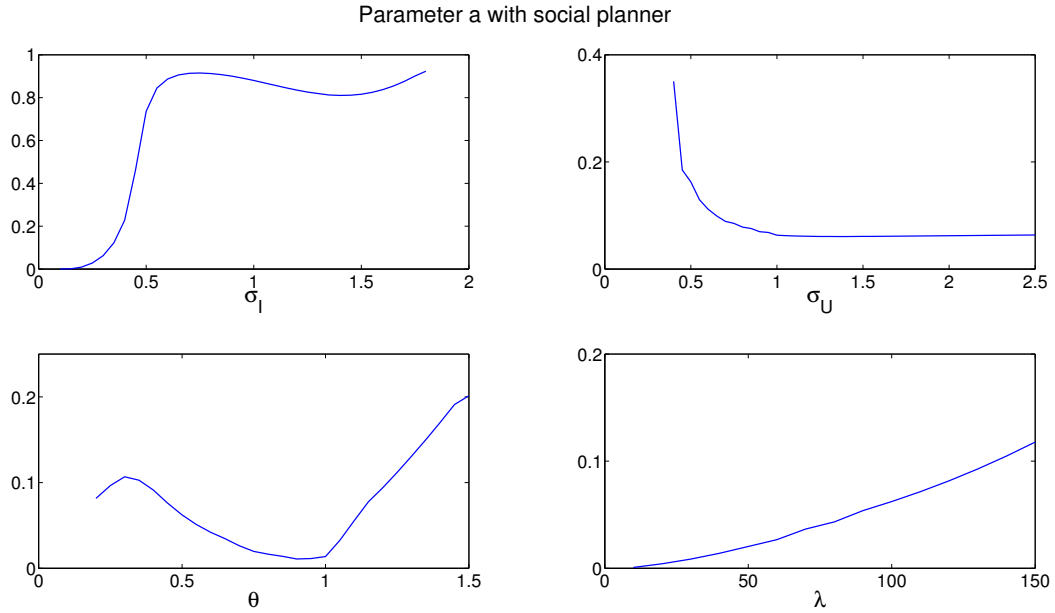


Figure 12: Contract parameter a as a function of σ_I , σ_U , θ and λ . Endogenous contracts with social planner. Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $\lambda = 100$, $\rho = 1$, $\gamma = 0.2$.

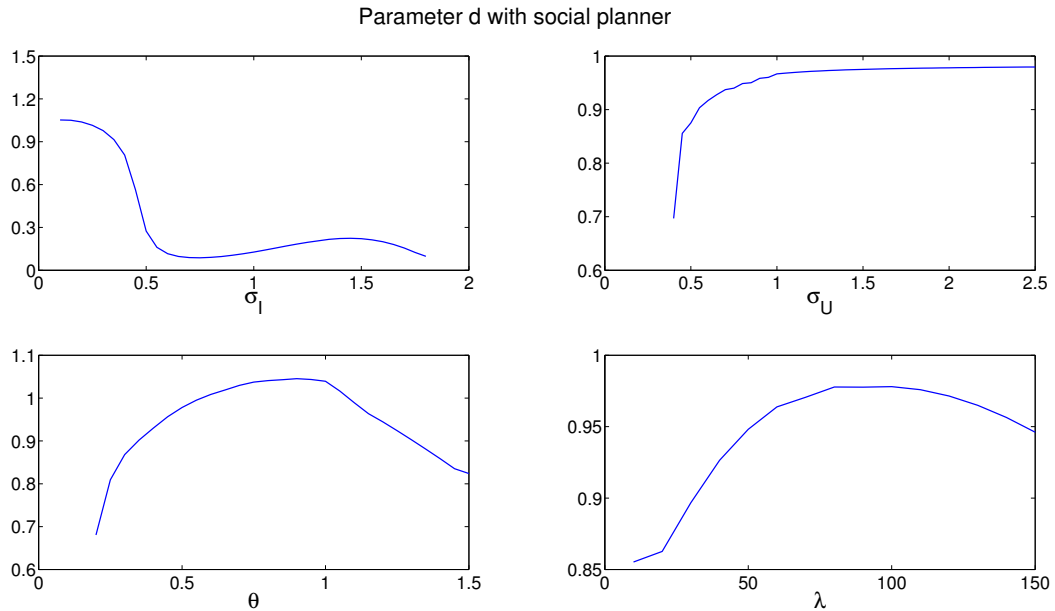


Figure 13: Contract parameter d as a function of σ_I , σ_U , θ and λ . Endogenous contracts with social planner. Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $\lambda = 100$, $\rho = 1$, $\gamma = 0.2$.

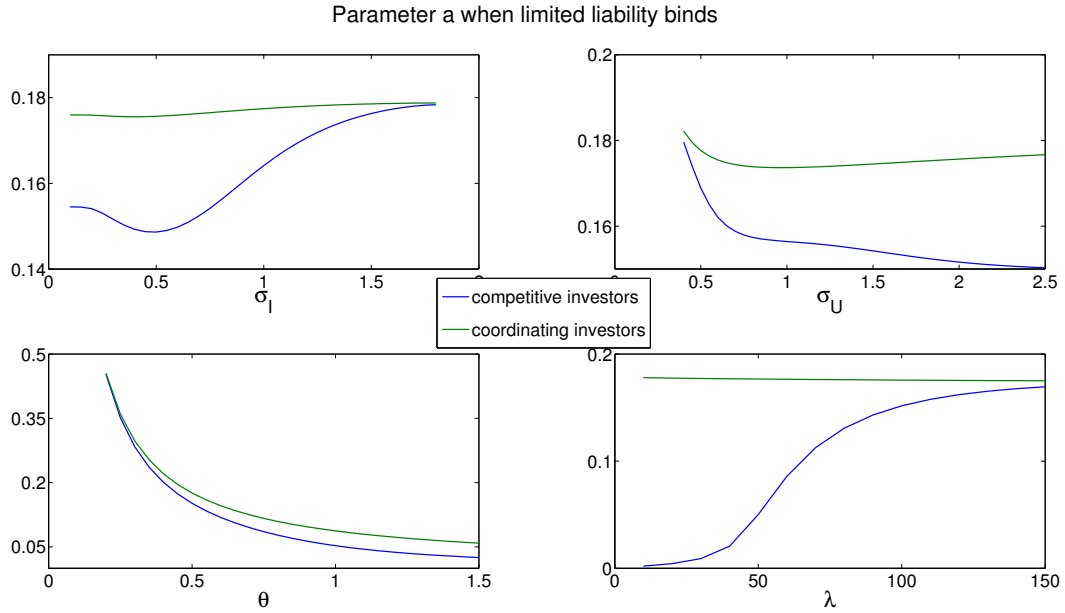


Figure 14: Contract parameter a as a function of σ_I , σ_U , θ and λ . Endogenous contracts when limited liability binds. Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $\lambda = 100$, $\rho = 0.3$.

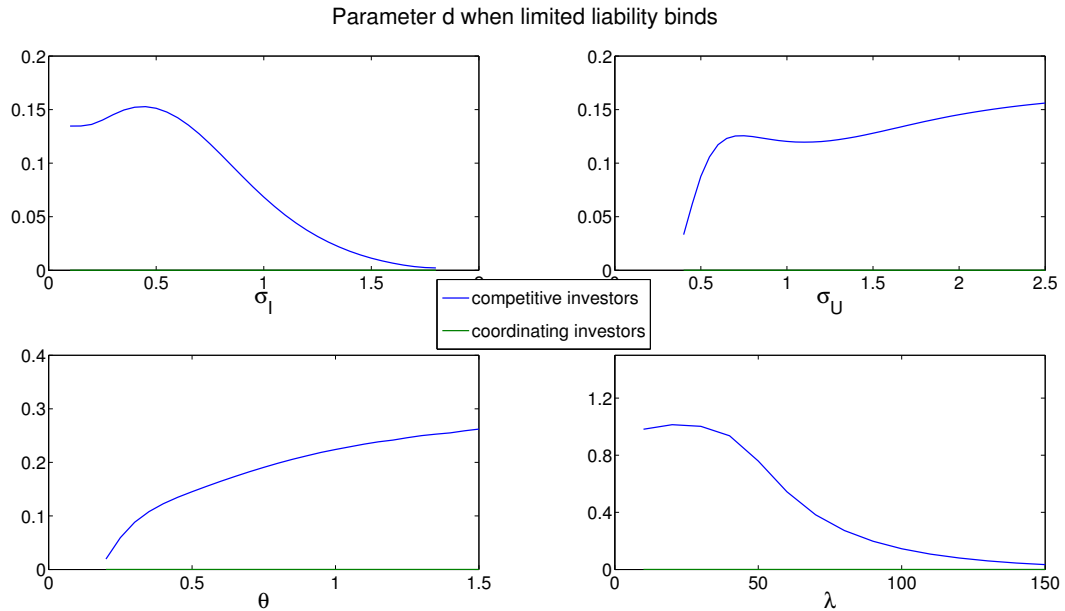


Figure 15: Contract parameter d as a function of σ_I , σ_U , θ and λ . Endogenous contracts when limited liability binds. Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $\lambda = 100$, $\rho = 0.3$.

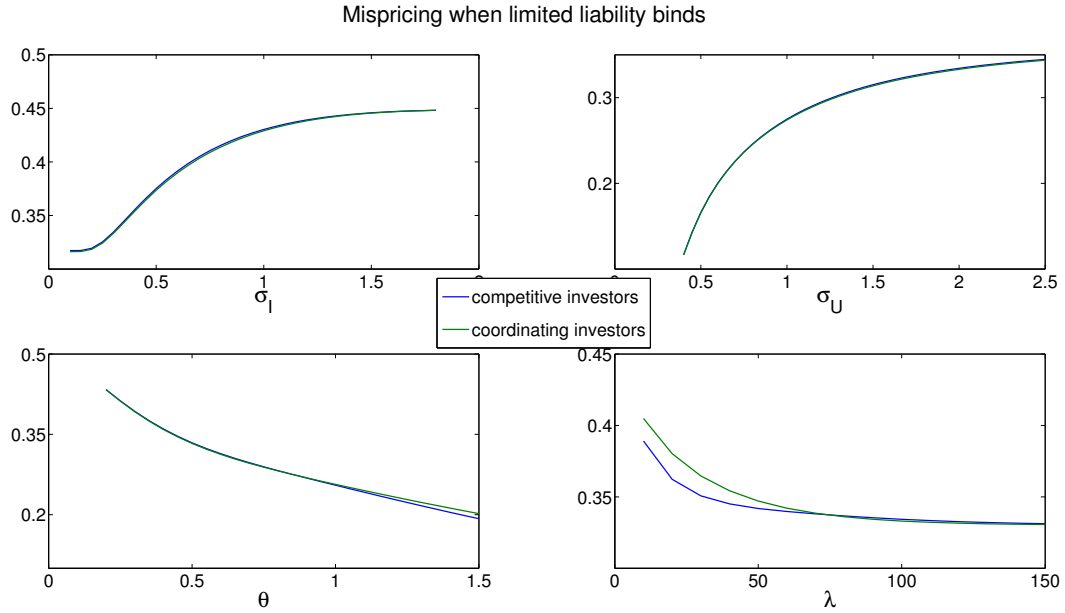


Figure 16: Mispricing as a function of σ_I , σ_U , θ and λ . Endogenous contracts when limited liability binds. Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $\lambda = 100$, $\rho = 0.3$.

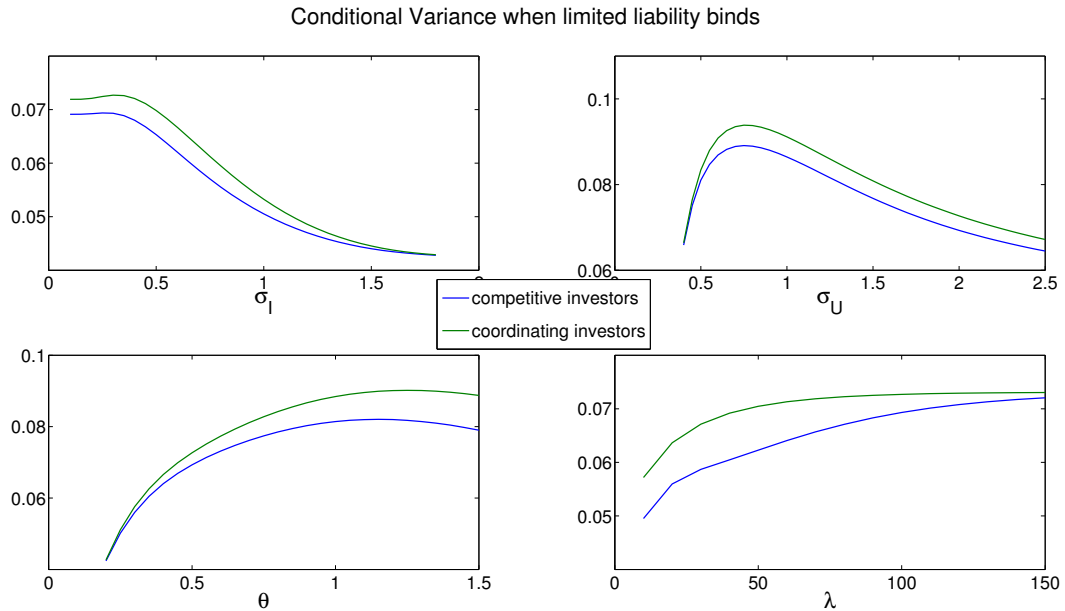


Figure 17: Normalized Conditional Variance as a function of σ_I , σ_U , θ and λ . Endogenous contracts when limited liability binds. Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $\lambda = 100$, $\rho = 0.3$.

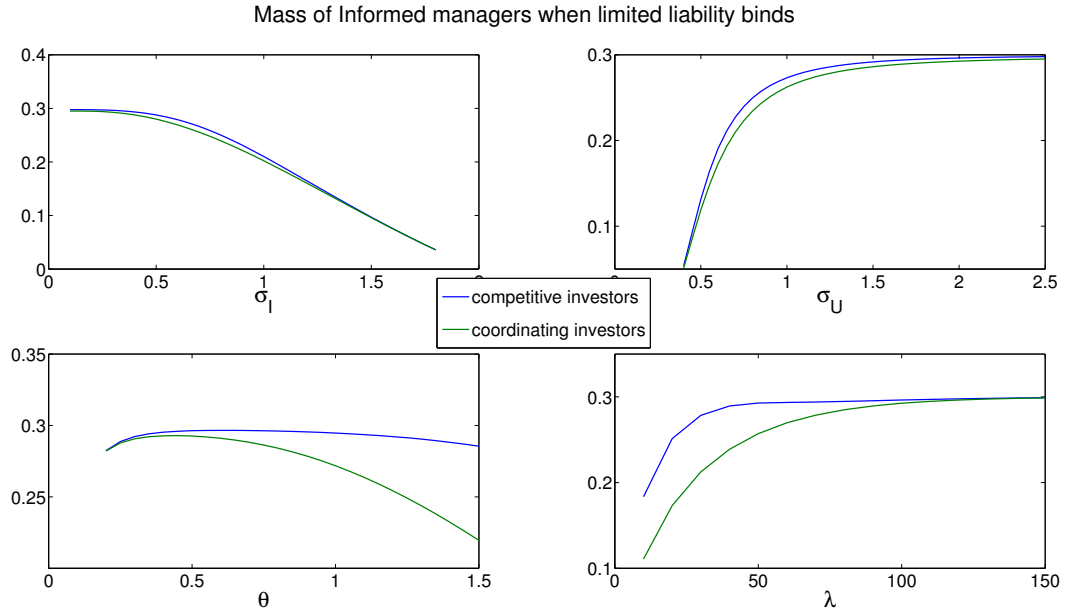


Figure 18: Mass of Informed Managers μ_I as a function of σ_I , σ_U , θ and λ . Endogenous contracts when limited liability binds. Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $\lambda = 100$, $\rho = 0.3$.

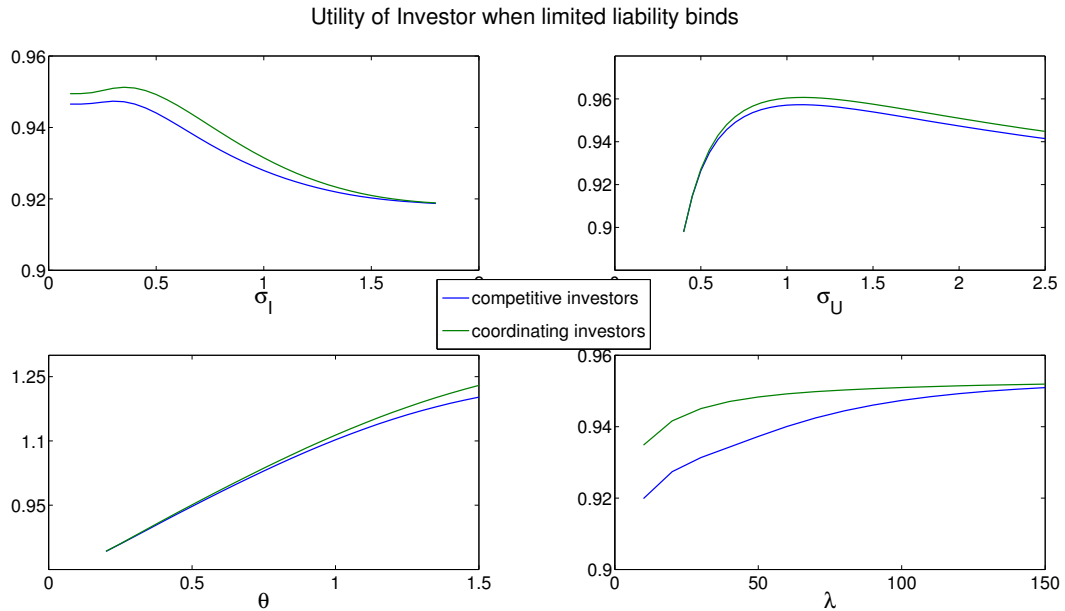


Figure 19: Investor's Utility as a function of σ_I , σ_U , θ and λ . Endogenous contracts when limited liability binds. Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $\lambda = 100$, $\rho = 0.3$.

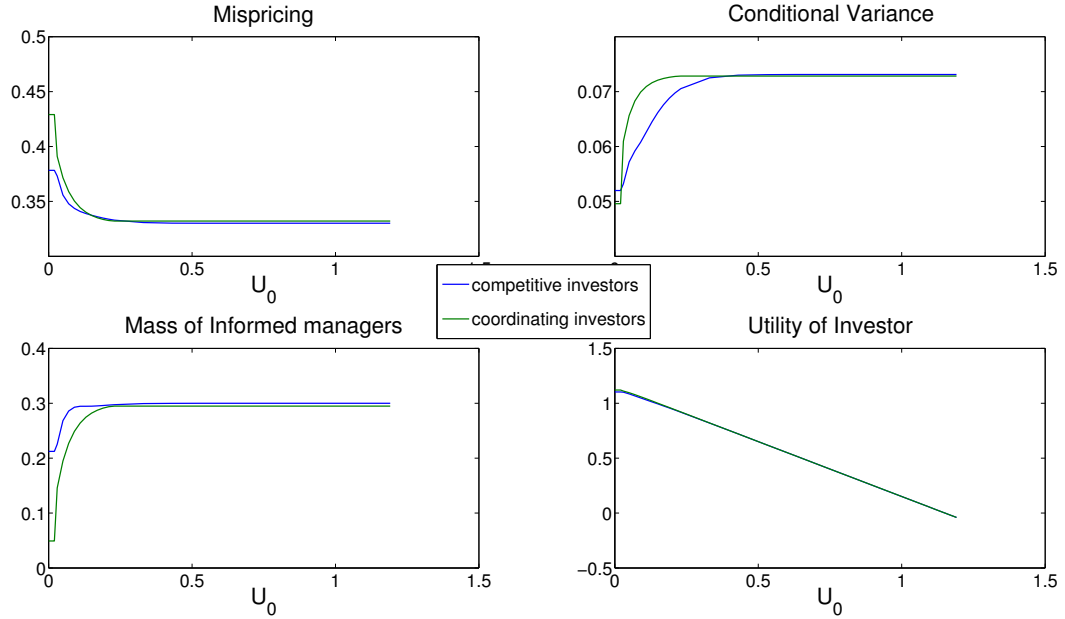


Figure 20: Mispricing, Normalized Conditional Variance, Mass of Informed Managers and Investor's Utility as functions of U_0 . Endogenous contracts with competitive (blue line) and coordinating risk-neutral investors (green line). Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $\lambda = 100$, $\rho = 0.3$.

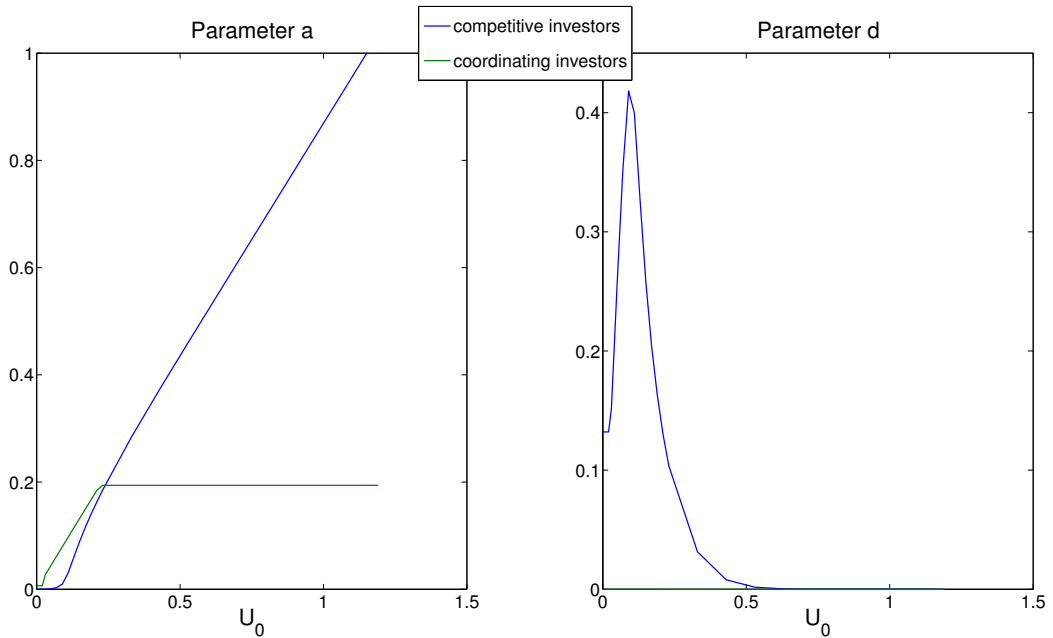


Figure 21: Contract parameters a and d as functions of U_0 . Endogenous contracts with competitive (blue line) and coordinating risk-neutral investors (green line). Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $\lambda = 100$, $\rho = 0.3$.

Parameter a when there are indirect incentives

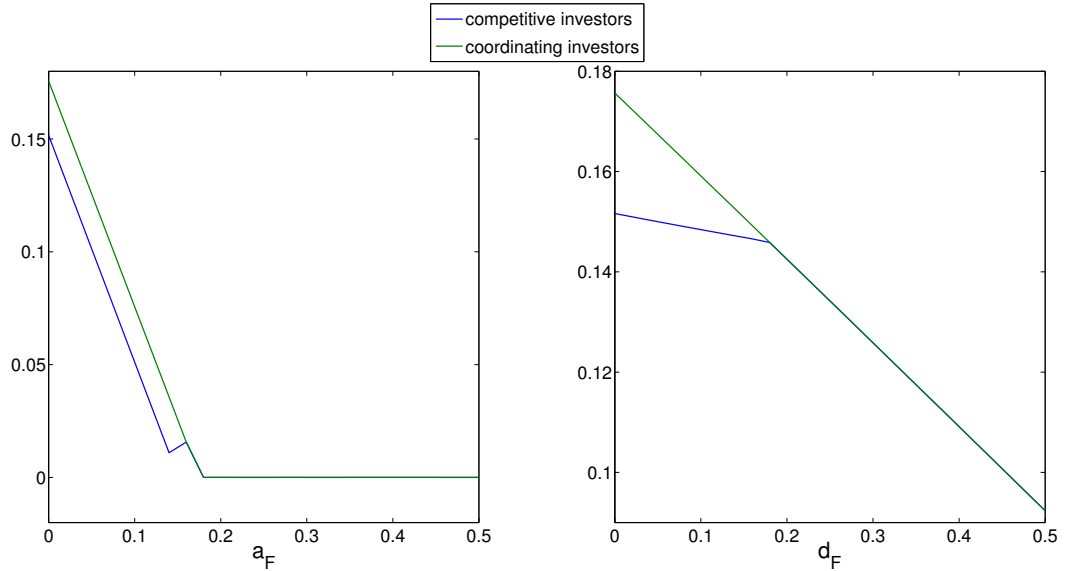


Figure 22: Contract parameter a as a function of a_F and d_F . Endogenous contracts when limited liability binds and there are indirect incentives. Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $\lambda = 100$, $\rho = 0.3$.

Parameter d when there are indirect incentives

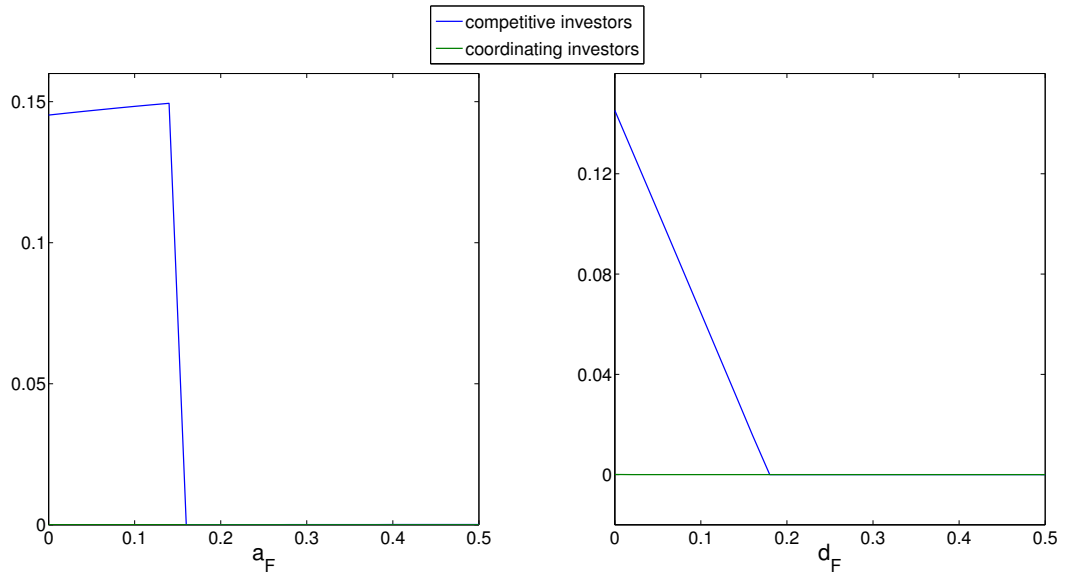


Figure 23: Contract parameter d as a function of a_F and d_F . Endogenous contracts when limited liability binds and there are indirect incentives. Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $\lambda = 100$, $\rho = 0.3$.

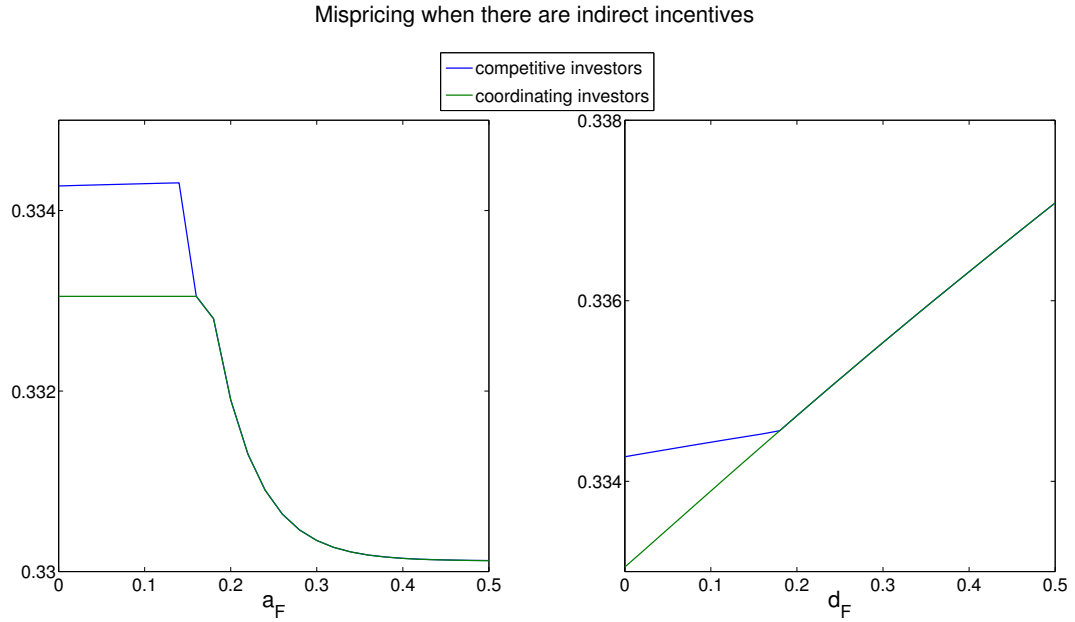


Figure 24: Mispricing as a function of a_F and d_F . Endogenous contracts when limited liability binds and there are indirect incentives. Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $\lambda = 100$, $\rho = 0.3$.

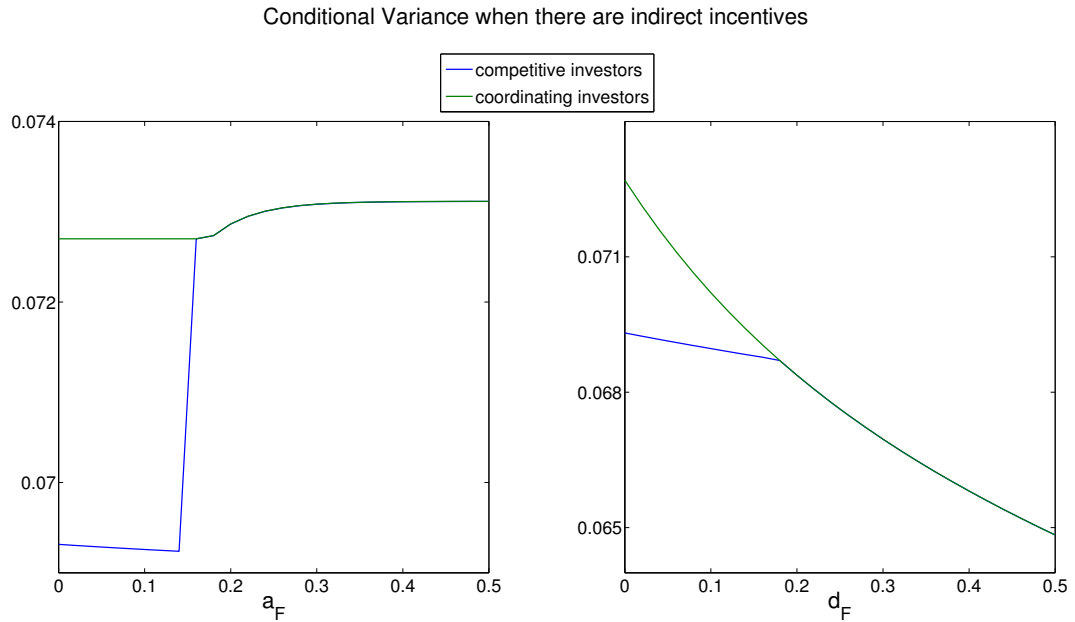


Figure 25: Normalized Conditional Variance as a function of a_F and d_F . Endogenous contracts when limited liability binds and there are indirect incentives. Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $\lambda = 100$, $\rho = 0.3$.

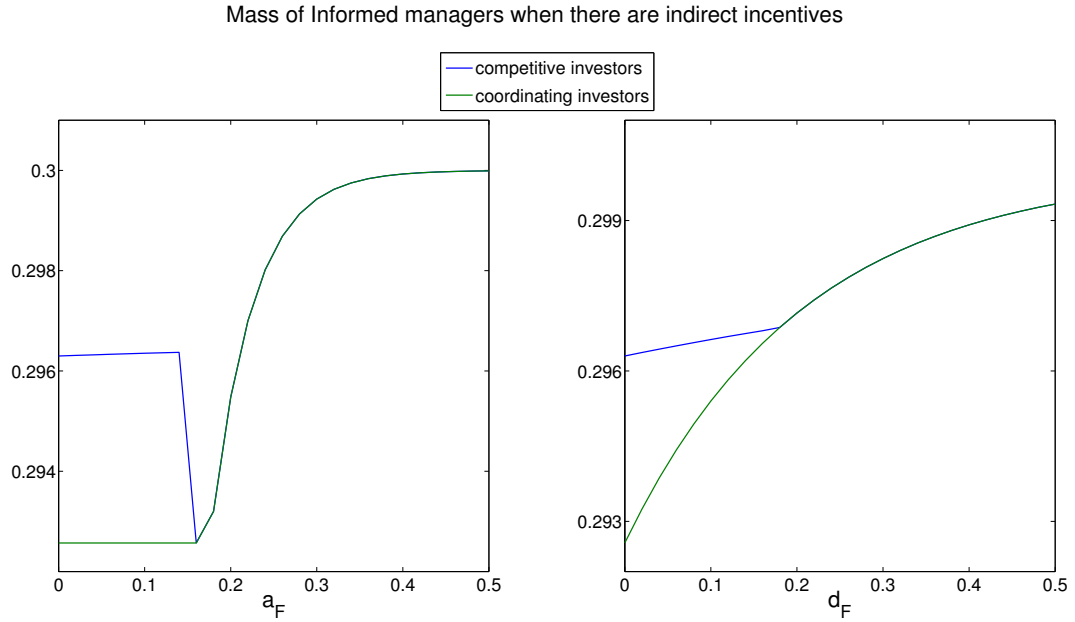


Figure 26: Mass of Informed Managers μ_I as a function of a_F and d_F . Endogenous contracts when limited liability binds and there are indirect incentives. Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $\lambda = 100$, $\rho = 0.3$.

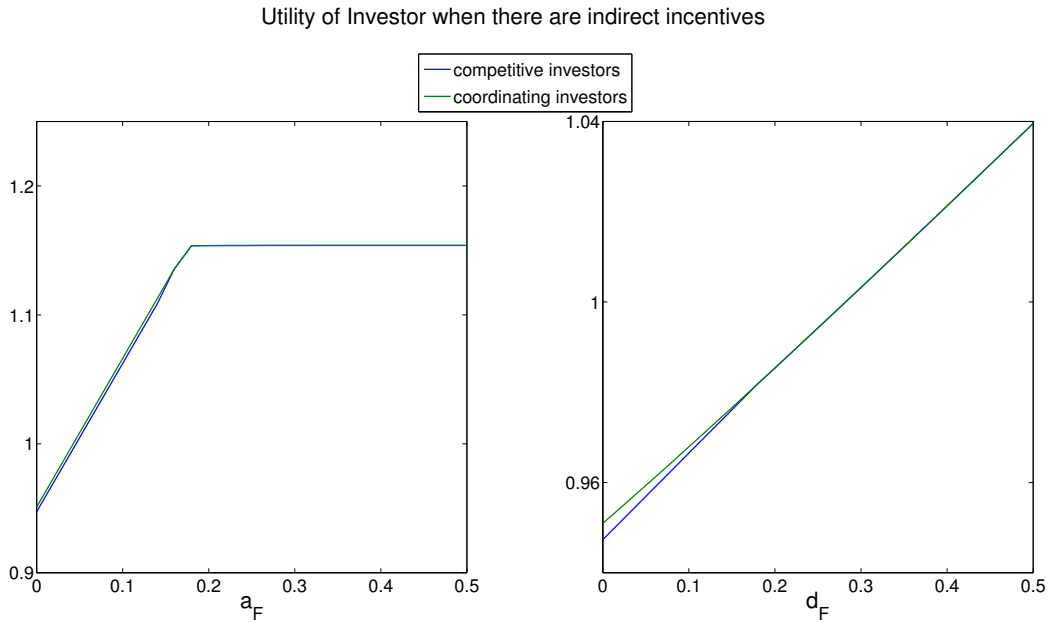


Figure 27: Investor's Utility as a function of a_F and d_F . Endogenous contracts when limited liability binds and there are indirect incentives. Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $\lambda = 100$, $\rho = 0.3$.

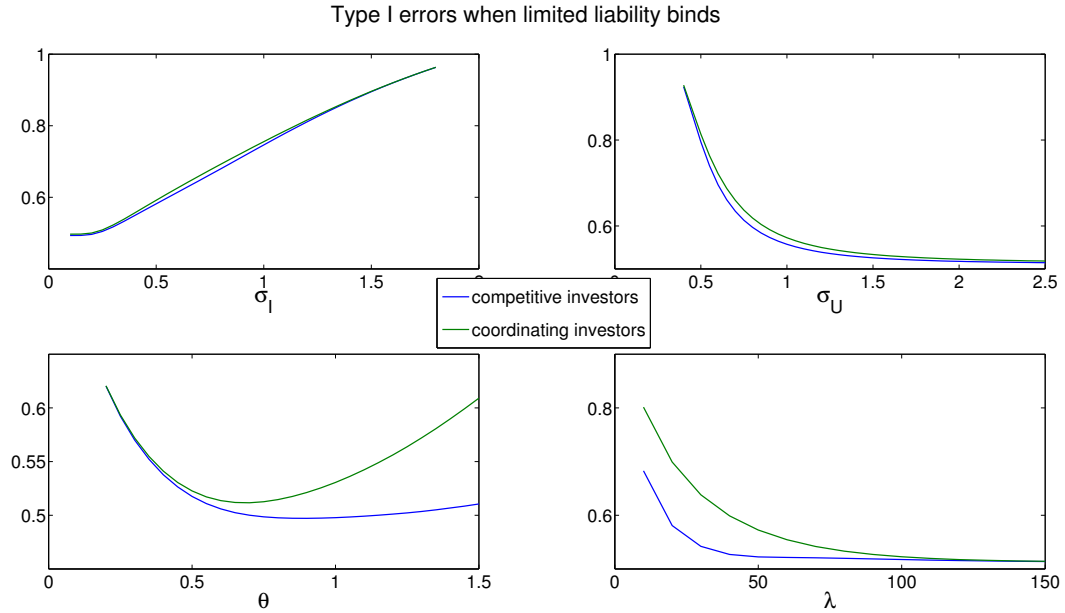


Figure 28: Type I error as a function of σ_I , σ_U , θ and λ . Endogenous contracts when limited liability binds. Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $\lambda = 100$, $\rho = 0.3$.

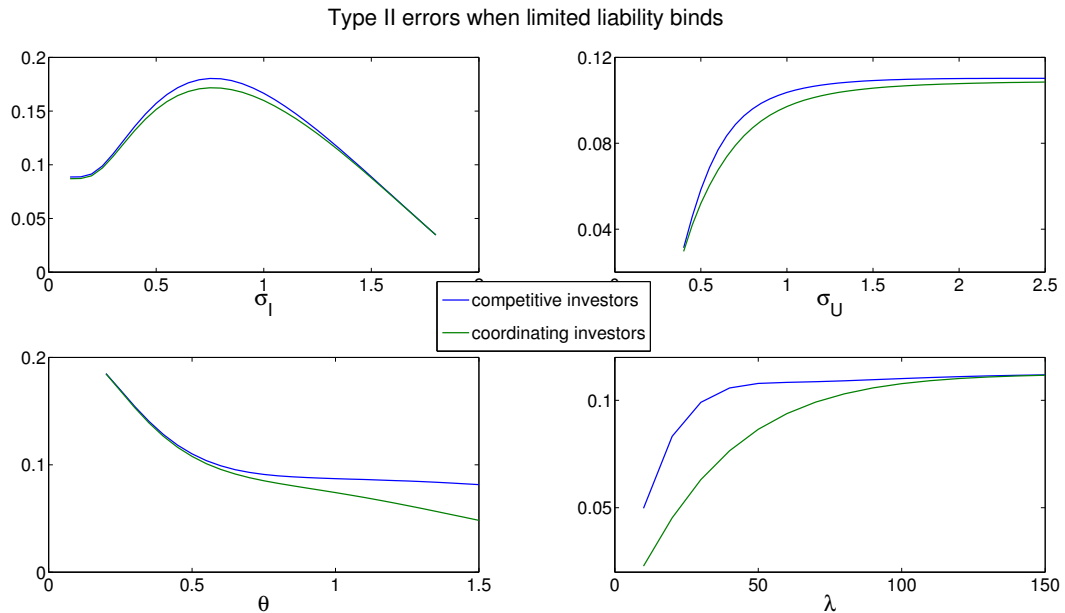


Figure 29: Type II error as a function of σ_I , σ_U , θ and λ . Endogenous contracts when limited liability binds. Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $\lambda = 100$, $\rho = 0.3$.

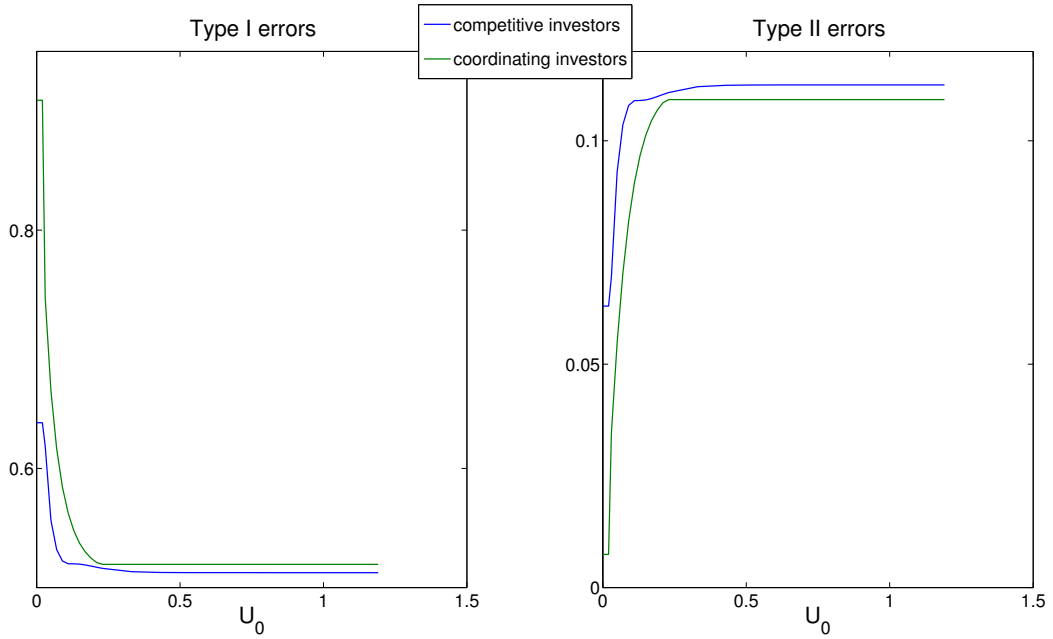


Figure 30: Type I and Type II errors as functions of U_0 . Endogenous contracts with competitive (blue line) and coordinating risk-neutral investors (green line). Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $\lambda = 100$, $\rho = 0.3$.

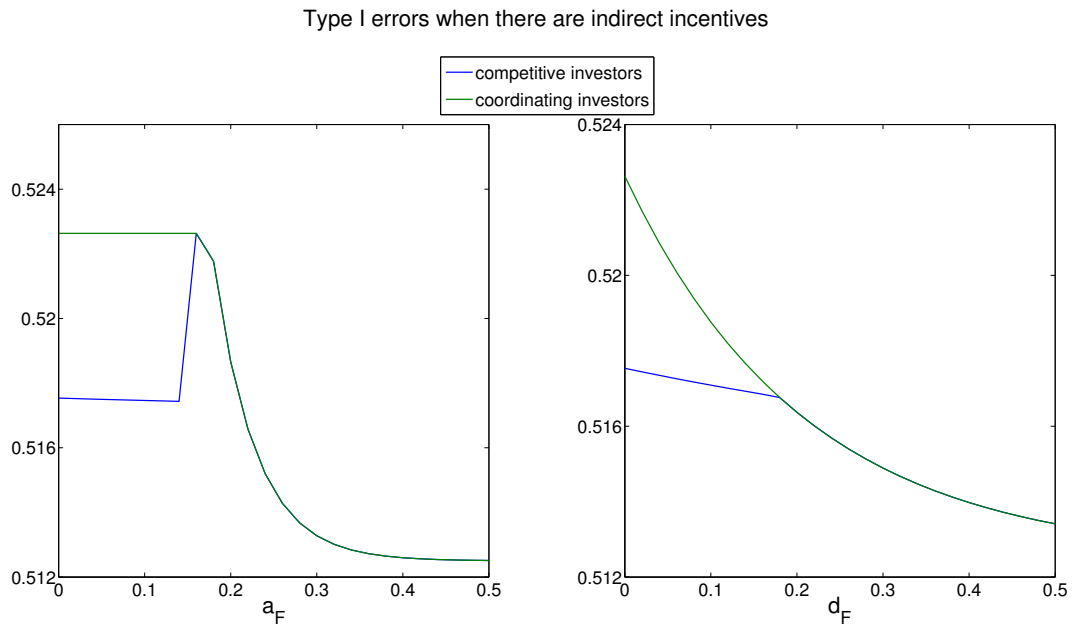


Figure 31: Type I error as a function of a_F and d_F . Endogenous contracts when limited liability binds and there are indirect incentives. Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $\lambda = 100$, $\rho = 0.3$.

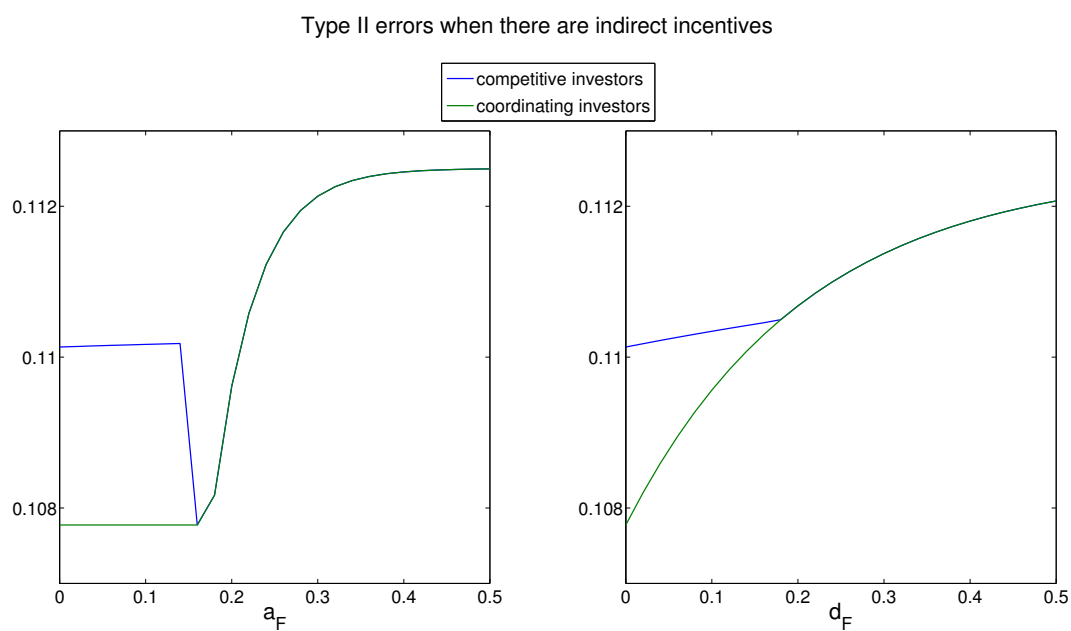


Figure 32: Type II error as a function of a_F and d_F . Endogenous contracts when limited liability binds and there are indirect incentives. Benchmark parameter values: $\sigma_I = 0.3$, $\sigma_U = 2$, $\kappa = 1$, $\theta_H = -\theta_L = 0.5$, $\lambda = 100$, $\rho = 0.3$.