

## On the relative pricing of long maturity S&P 500 index options and CDX tranches

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May 2010

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Motivation  
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Overview  
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CDX Market  
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The model  
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Results  
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Final Thoughts  
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  - Overview
  - CDX Market
  - The model
  - Results
  - Final Thoughts
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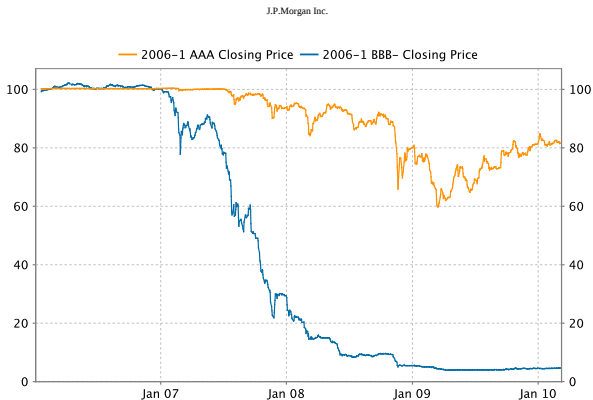
## Securitized Credit Markets Crisis

- ▶ Pre-crisis saw large growth in securitized credit markets (CDO).
- ▶ Pooling and tranching used to create 'virtually risk-free' AAA securities, in response to high demand for highly rated securities.
- ▶ During the crisis **all** AAA markets were hit hard:
  - ▶ Home equity loan CDO prices fell (ABX.HE AAA < 60%).
  - ▶ Super Senior (30-100) tranche spreads > 100bps.
  - ▶ CMBX.AAA (super duper) > 750bps.
- ▶ Raises several questions:
  - Q? Were ratings incorrect (ex-ante default probability higher than expected)?
  - Q? Are ratings sufficient statistics (risk  $\neq$  expected loss)?
  - Q? Were AAA tranches mis-priced (relative to option prices)?
- ▶ Many other surprises:
  - ▶ Corporate Credit spreads widened (CDX-IG > 200bps).
  - ▶ Cash-CDS basis **negative** (-200 bps for IG; -700bps for HY).
  - ▶ LIBOR-Treasury and LIBOR-OIS widened (> 400bps).
  - ▶ Long term Swap spreads became negative (30 year swap over Treasury < -50 bps).
  - ▶ Defaults on the rise (Bear Stearns, Lehman).

## Evidence from ABX markets

- ▶ ABX.HE (subprime) AAA and BBB spreads widened dramatically (prices dropped)

J.P.Morgan DataQuery



## Evidence from CMBX markets

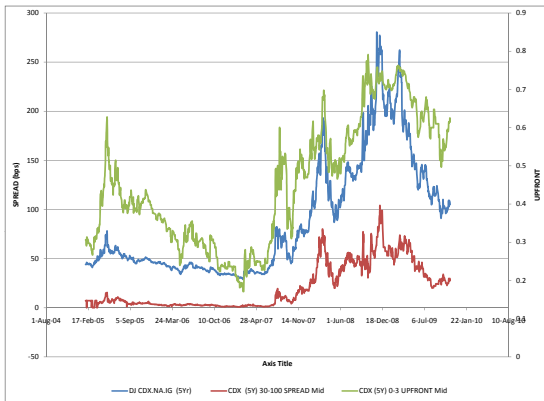
- ▶ CMBX (commercial real estate) AAA spreads widened even more dramatically

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## Corporate IG CDX Tranche spreads

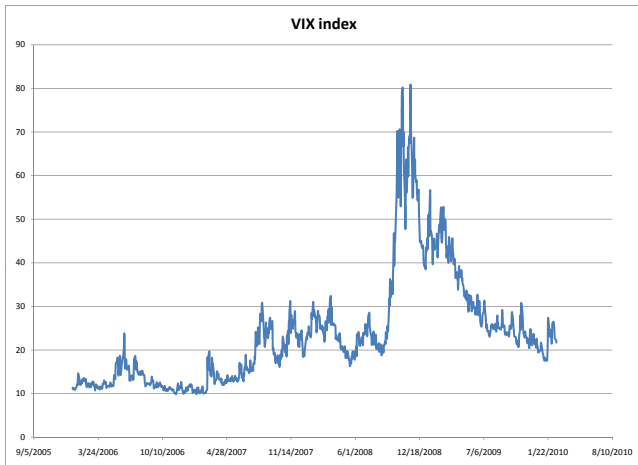
- The impact on tranche prices was dramatic



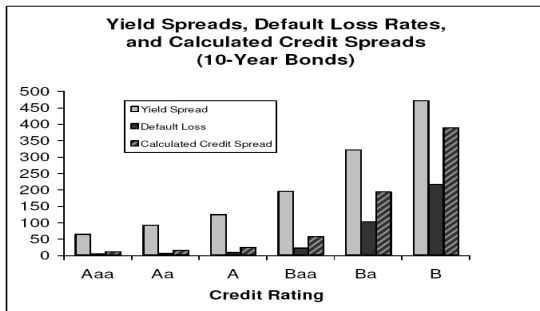
- Implied correlation on equity tranche hit  $> 40\%$
- Correlation on Super-Senior tranches  $> 100\%$ (!) with standard recovery assumption
- Relative importance of expected loss in senior tranche versus in equity tranche indicates increased crash risk.

## Evidence from S&P500 Option markets

- Implied volatility index widened dramatically: increased market and crash risk.



## The Credit spread puzzle (pre-crisis)



source: Huang and Huang (2003)

- ▶ Huang and Huang (2003) find that Structural models, when calibrated to match average loss rate, tend to underpredict yield spreads
- ▶ Chen, Collin-Dufresne, Goldstein (2008) find that standard models cannot explain the level of observed spreads because:
  - ▶ (i) historical expected loss rates have been low, **and**
  - ▶ (ii) Idiosyncratic risk on typical IG bonds is very high ( $\sim 3/4$  of total risk).

## CDO collateral typically have high beta due to diversification

- ▶ Coval, Jurek, Stafford propose theory for large growth in structured product markets:
  - ▶ Posit that ratings are sufficient statistic for expected loss.
  - ▶ Tranching process pools risky securities (e.g., BBB) to create lower risk (e.g., AAA) and higher risk (e.g., Z) securities by creating different levels of subordination (tranches).
  - ▶ By nature of that process senior tranches have more systematic risk and therefore should have higher expected return for given expected loss ( $\sim$  rating).
  - ▶ However investors focus only on expected loss ( $\sim$  rating).
- ⇒ Effectively, according to CJS, the banking sector exploits 'naive' investors by manufacturing portfolios with same expected loss as generic AAA, but different systematic risk and selling them at identical prices.
- ▶ CJS find evidence for their story using CDX.IG synthetic tranche prices:
  - ▶ Use pricing model for tranches based on the one-factor Gaussian copula market standard.
  - ▶ Instead of assuming that the common factor has a Gaussian density (as in the standard model), the authors extract its density from long-term S&P500 option prices.
  - ▶ Their results suggest that observed market spreads on all mezzanine and senior tranches are substantially lower than model-implied 'fair' spreads.

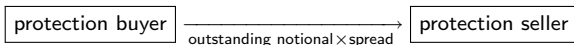
## Overview and main results of our paper

- ▶ Revisit the relative pricing of tranches and SP500 options
- ▶ Same market: CDX-IG tranches
- ▶ Propose a structural model to price both SP500 options and CDO tranches written on portfolio of single names.
- ▶ Allows us to model the **dynamics** of default and investigate the term structure of credit spreads.
- ▶ Main findings:
  - ▶ The model consistently prices tranches and options when calibrated:
    - ▶ to SP500 options to match market dynamics (systematic risk).
    - ▶ to the **term structure of credit spreads** to capture idiosyncratic dynamics.
  - ▶ Timing of default has first order impact on tranche spreads (especially on difference between equity and senior tranches). This cannot be captured in a one-period model.
  - ▶ The ratio of idiosyncratic to market wide jump risk is crucial to capture the tail properties of the loss distribution.
  - ▶ Quoted index options are not informative about pricing of senior tranches (too 'narrow' strike range). Difficult to extrapolate much about fair-pricing of AAA tranches based on quoted SP500 options.

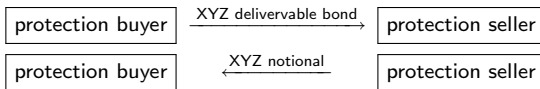
## The CDX index

- ▶ The CDX index is an insurance contract against credit events of a portfolio of counterparties (e.g., 125 names in CDX.IG):

- ▶ Prior to credit event:



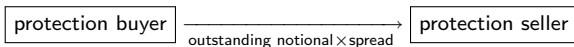
- ▶ Upon arrival of credit event of XYZ:



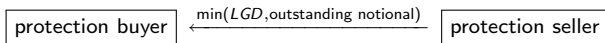
- ▶ Following credit event outstanding notional is reduced by notional of XYZ in portfolio (i.e.,  $\frac{1}{125}$  in CDX.IG).
  - ▶ Contract expires at maturity or when notional exhausted.
- ▶ N.B.: CDX contract  $\approx$  equally weighted portfolio of single name CDS contracts  
CDX spread  $\approx$  average of single name CDS spreads

## Synthetic CDO Tranches

- ▶ Selling protection on CDO tranche with attachment points  $[L, U]$  (i.e., notional =  $U - L$ ) written on underlying basket of 125 single names (CDX):
  - ▶ Prior to a credit event:



- ▶ Upon arrival of credit event ( $LGD = \text{notional} - \text{deliverable bond price}$ ), if cumulative loss exceeds lower attachment point (i.e.,  $\mathcal{L}_t = \sum_{i=1}^{125} LGD_i \mathbf{1}_{\{\tau_i \leq t\}} > L$ ) then



- ▶ Following credit event outstanding tranche notional is reduced by LGD (up to exhaustion of outstanding notional).
  - ▶ Also, super senior tranche notional is reduced by recovery (to satisfy 'adding up constraint').
  - ▶ Contract expires at maturity or when tranche notional is exhausted.
  - ▶ Tranche payoff is call spread on cumulative loss:  $\max(\mathcal{L}_t - L, 0) - \max(\mathcal{L}_t - U, 0)$ .
- ⇒ Tranche valuation depends on entire distribution of cumulative portfolio losses and crucially on default event correlation model.

## Market Model: Implied Gaussian Copula Correlation

- ▶ Market standard for quoting CDO tranche prices is the *implied correlation* of the Gaussian Copula framework.
  - ▶ Intuition builds on structural model of default (CDO model due to Vasicek 1987 who combines Merton (1974) with CAPM idea):
    - ▶ Each name in basket characterized by an 'asset value' driven by two factors: a common market factor and an idiosyncratic factor  

$$(V_i = \beta_i M + \sqrt{1 - \beta_i^2} \epsilon_i \text{ with } M, \epsilon_i \text{ independent centered Gaussian}).$$
    - ▶ Pairwise 'asset correlation' is the product of the individual asset betas ( $\rho_{ij} = \beta_i \beta_j$ ).
    - ▶ Default occurs when asset value falls below a constant barrier ( $\text{DefProb} = P(V_i \leq B_i)$ ).
  - ▶ Market convention for quoting tranche values in terms of *implied correlation* assumes:
    - ▶ The individual beta is identical across all names in the basket.
    - ▶ The default boundary is identical and calibrated to CDX level.
    - ▶ All firms have identical LGD of 60%.
- ⇒ With these heroic assumptions, a single number, the *implied correlation* ( $= \rho$ ), allows to match a given tranche's model price with the market price (for a given CDX level).

## The implied correlation smile

- ▶ Market Quotes on Aug. 4, 2004 (CDX index spread 63.25 bp)

| Tranche      | 0-3% | 3-7% | 7-10% | 10-15% | 15-30% |
|--------------|------|------|-------|--------|--------|
| CDX.IG (bps) | 4138 | 349  | 135   | 46     | 14     |

- ▶ The market displays an *implied correlation smile*:

|          |       |      |       |       |       |
|----------|-------|------|-------|-------|-------|
| Imp Corr | 21.7% | 4.1% | 17.8% | 18.5% | 29.8% |
|----------|-------|------|-------|-------|-------|

⇒ The smile shows that the Gaussian copula model is mis-specified (~ option skew).

- ▶ Market quotes on June 1st 2005 IG4-5Y (CDX index spread of 42 bp):

| Tranche  | 0-3%  | 3-7% | 7-10%  | 10-15% | 15-30% |
|----------|-------|------|--------|--------|--------|
| CDX.IG   | 3050  | 66   | 9.5    | 7.5    | 4      |
| Imp Corr | 9.08% | 5.8% | 10.02% | 16.77% | 27.62% |

- ▶ Market quotes on June 4, 2008 IG9-5Y (CDX index ref 118 bp):

| Tranche  | 0-3% | 3-7%   | 7-10% | 10-15% | 15-30% | 30-100% |
|----------|------|--------|-------|--------|--------|---------|
| CDX.IG   | 5150 | 435    | 232   | 130    | 70     | 41      |
| Imp Corr | 40%  | 88.23% | 4.31% | 13.47% | 32.06% | 88.35%  |

## A structural model for pricing long-dated S&P500 options

- ▶ The market model is the **Stochastic Volatility Common Jump (SVCJ)** model of Broadie, Chernov, Johannes (2009):

$$\frac{dM_t}{M_t} = (r - \delta) dt + \sqrt{V_t} dw_1^Q + (e^y - 1) dq - \bar{\mu}_y \lambda^Q dt - (e^{y_C} - 1) (dq_C - \lambda_C^Q dt)$$

$$dV_t = \kappa_V (\bar{V} - V_t) dt + \sigma_V \sqrt{V_t} (\rho dw_1^Q + \sqrt{1 - \rho^2} dw_2^Q) + y_V dq$$

$$d\delta_t = \kappa_\delta (\bar{\delta} - \delta_t) dt + \sigma_\delta \sqrt{V_t} (\rho_1 dw_1^Q + \rho_2 dw_2^Q + \sqrt{1 - \rho_1^2 - \rho_2^2} dw_3^Q) + y_\delta dq.$$

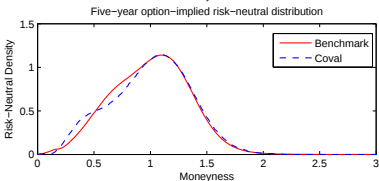
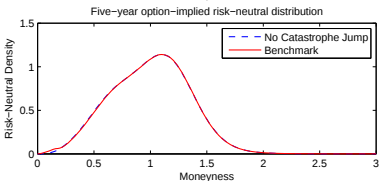
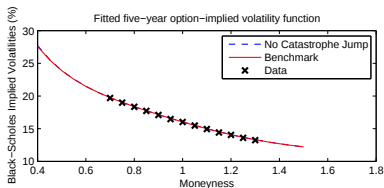
- ▶ We add stochastic dividend yield (**SVDCJ**) to be help fit long-dated options as well.
- ▶ The parameters of the model are calibrated to 5-year index option prices obtained from CJS.
- ▶ State variables are extracted given parameters from time-series of short maturity options (obtained from OptionMetrics).
- ▶ Advantage of using structural model: Arbitrage-free extrapolation into lower strikes (needed for senior tranches).

## Calibration of option pricing model to long-dated S&P500 options

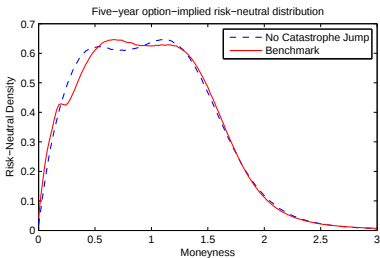
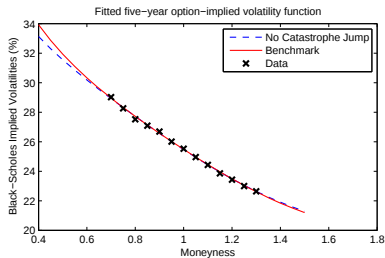
| Parameter       | Pre-crisis (< Sept. 2007) |              | Post-Crisis  |              |
|-----------------|---------------------------|--------------|--------------|--------------|
|                 | Estimation 1              | Estimation 2 | Estimation 3 | Estimation 4 |
| $\rho$          | -0.48                     | -0.48        | -0.48        | -0.48        |
| $\sigma_V$      | 0.2016                    | 0.2016       | 0.2016       | 0.2016       |
| $\lambda$       | 0.1534                    | 0.1608       | 0.1743       | 0.2465       |
| $\rho_q$        | 0.0203                    | 0.0199       | -0.0509      | -0.0576      |
| $\mu_y$         | -0.2991                   | -0.2843      | -0.4726      | -0.3479      |
| $\sigma_y$      | 0.2445                    | 0.2441       | 0.4609       | 0.3915       |
| $\bar{V}$       | 0.0037                    | 0.0038       | 0.0132       | 0.0094       |
| $\mu_V$         | 0.0035                    | 0.0033       | 0.0099       | 0.0056       |
| $\kappa_V$      | 5.4368                    | 5.3644       | 1.5442       | 2.1596       |
| $V_0$           | 0.0037                    | 0.0038       | 0.0132       | 0.0094       |
| $\kappa_\delta$ | -0.5914                   | -0.5903      | -0.4816      | -0.4953      |
| $\bar{\delta}$  | 0.04                      | 0.04         | 0.04         | 0.04         |
| $\sigma_\delta$ | 0.0454                    | 0.0423       | 0.0405       | 0.0304       |
| $\rho_1$        | -0.9054                   | -0.8968      | -0.5056      | -0.4135      |
| $\rho_2$        | -0.0032                   | -0.0036      | -0.0078      | -0.0066      |
| $\mu_d$         | 0.0002                    | 0.0002       | 0.003        | 0.0007       |
| $\sigma_d$      | 0.0007                    | 0.0008       | 0.0006       | 0.0006       |
| $\delta_0$      | 0.04                      | 0.04         | 0.04         | 0.04         |
| $r$             | 0.05                      | 0.05         | 0.05         | 0.05         |
| $y_C$           | 0                         | -2           | 0            | -2           |
| $\lambda_C^Q$   | 0                         | 0.00076      | 0            | 0.0066       |

- ▶ Excellent fit
- ▶ Note: (risk-neutral) mean-reversion coefficient on dividend yield negative.

# Pre-crisis Option pricing fit



# During-Crisis Option pricing fit



## A structural model of individual firm's default

- ▶ Given market dynamics, we assume individual firm  $i$  dynamics:

$$\begin{aligned} \frac{dA_i(t)}{A_i(t)} + \delta_A dt - rdt &= \beta_i \left( \sqrt{V_t} dw_1^Q + (e^y - 1) dq - \bar{\mu}_y \lambda^Q dt \right) + \sigma_i dw_i \\ &\quad + (e^{y_C} - 1) (dq_C - \lambda_C^Q dt) + (e^{y_i} - 1) (dq_i - \lambda_i^Q dt). \end{aligned}$$

- ▶ Note

- ▶  $\beta$ : exposure to market excess return (i.e., systematic diffusion **and** jumps).
- ▶  $dq_C$ : 'catastrophic' market wide jumps.
- ▶  $dq_i$ : idiosyncratic firm specific jumps.
- ▶  $dw_i$ : idiosyncratic diffusion risks.
- ▶ Default occurs the first time firm value falls below a default barrier  $B_i$  (Black (1976)):

$$\tau_i = \inf\{t : A_i(t) \leq B_i\}. \quad (1)$$

- ▶ Recovery upon default is a fraction of the remaining asset value:  $(1 - \ell)B_i$ .

## Pricing of the CDX index via Monte-Carlo

- ▶ The running spread on the CDX index is closely related to a weighted average of CDS spreads.
- ▶ Determined such that the present value of the **protection leg** ( $V_{idx, prot}$ ) equals the PV of the **premium leg** ( $V_{idx, prem}$ ):

$$V_{idx, prem}(S) = SE \left[ \sum_{m=1}^M e^{-rt_m} (1 - n(t_m)) \Delta + \int_{t_{m-1}}^{t_m} du e^{-ru} (u - t_{m-1}) dn_u \right]$$

$$V_{idx, prot} = E \left[ \int_0^T e^{-rt} dL_t \right].$$

- ▶ We have defined:
  - ▶ The (percentage) defaulted notional in the portfolio:  $n(t) = \frac{1}{N} \sum_i \mathbf{1}_{\{\tau_i \leq t\}}$ ,
  - ▶ The cumulative (percentage) loss in the portfolio:  $L(t) = \frac{1}{N} \sum_i \mathbf{1}_{\{\tau_i \leq t\}} (1 - R_i(\tau_i))$

## Pricing of the CDX Tranches via Monte-Carlo

- ▶ The tranche loss as a function of portfolio loss is

$$T_j(L(t)) = \max [L(t) - K_{j-1}, 0] - \max [L(t) - K_j, 0] .$$

- ▶ The initial value of the protection leg on tranche- $j$  is

$$Prot_j(0, T) = E^Q \left[ \int_0^T e^{-rt} dT_j(L(t)) \right]$$

- ▶ For a tranche spread  $S_j$ , the initial value of the premium leg on tranche- $j$  is

$$Prem_j(0, T) = S_j E^Q \left[ \sum_{m=1}^M e^{-rt_m} \int_{t_{m-1}}^{t_m} du (K_j - K_{j-1} - T_j(L(u))) \right] .$$

- ▶ Appropriate modifications to the cash-flows
  - ▶ Equity tranche (upfront payment),
  - ▶ Super-senior tranche (recovery accounting).

## Calibration of firms' asset value processes

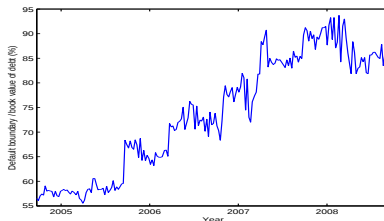
- ▶ Calibrate 7 (unlevered) asset value parameters ( $\beta, \sigma, B, \lambda_1, \lambda_2, \lambda_3, \lambda_4$ ) to match median CDX-series firm's:
  - ▶ Market beta
  - ▶ Idiosyncratic risk (estimated from rolling regressions for CDX series constituents using CRSP-Compustat)
  - ▶ Term structure of CDX spreads (1 to 5 year)
- ▶ Set jump size to -2 ( $\sim$  jump to default).
- ▶ Calibrate catastrophic jump intensity  $\lambda_C = 0.00076$  (less than 1 event per 1000 years) to match super-senior tranche spread (or set to zero for comparison).
- ▶ Set loss given default  $\ell$  to 40% ( $\sim$  match historical average) in normal times.
- ▶ Set  $\ell = 20\%$  if catastrophe jump occurs ( $\sim$  Altman et al.).
- ▶ Market volatility, jump-risk, dividend-yield all estimated from S&P500 option data in previous step.

## Results of Calibration

- Systematic risk increased a lot:

| Series | Period        | Equity Beta | Leverage Ratio | Market Volatility | Idiosyncratic Asset Volatility |
|--------|---------------|-------------|----------------|-------------------|--------------------------------|
| 3      | 9/2004-3/2005 | 0.82        | 0.36           | 10.34             | 27.08                          |
| 4      | 3/2005-9/2005 | 0.83        | 0.36           | 10.38             | 25.29                          |
| 5      | 9/2005-3/2006 | 0.87        | 0.33           | 10.02             | 23.86                          |
| 6      | 3/2006-9/2006 | 0.92        | 0.33           | 11.35             | 21.84                          |
| 7      | 9/2006-3/2007 | 0.94        | 0.32           | 9.80              | 20.93                          |
| 8      | 3/2007-9/2007 | 0.94        | 0.32           | 15.67             | 19.90                          |
| 9      | 9/2007-3/2008 | 0.98        | 0.31           | 21.86             | 18.64                          |
| 10     | 3/2008-9/2008 | 0.99        | 0.29           | 23.42             | 18.61                          |

- Estimates of default boundary rise from 57% to almost 95% (Davydenko (2008), Leland (2004) estimate range (56%, 70%) pre-crisis).



## Average tranche spreads predicted for pre-crisis period

- ▶ We report six tranche spreads averaged over the pre-crisis period Sep 04 - Sep 07:
  - ▶ The historical values;
  - ▶ Benchmark model: Catastrophic jumps calibrated to match the super-senior tranche; Idiosyncratic jumps and default boundary calibrated to match the 1 to 5 year CDX index.
  - ▶  $\lambda_C^Q = 0$ : No catastrophic jumps; Idiosyncratic jumps and default boundary calibrated to match 1 to 5 year CDX index;
  - ▶  $\lambda_i^Q = 0$ : Catastrophic jumps calibrated to match the super-senior tranche; No idiosyncratic jumps; Default boundary calibrated to match only the 5Y CDX index.
  - ▶  $\lambda_C^Q = 0, \lambda_i^Q = 0$ : No catastrophic jumps; No idiosyncratic jumps; Default boundary calibrated to match only the 5Y CDX index;
  - ▶ The results reported by CJS

|                                       | 0-3% | 3-7% | 7-10% | 10-15% | 15-30%   | 30-100%  | 0-3% Upftr |
|---------------------------------------|------|------|-------|--------|----------|----------|------------|
| data                                  | 1472 | 135  | 37    | 17     | 8        | 4        | 0.34       |
| benchmark                             | 1449 | 113  | 25    | 13     | 8        | 4        | 0.33       |
| $\lambda_C^Q = 0$                     | 1669 | 133  | 21    | 6      | 1        | 0        | 0.40       |
| $\lambda_i^Q = 0$                     | 1077 | 206  | 70    | 32     | 12       | 4        | 0.22       |
| $\lambda_C^Q = 0, \lambda_i^Q = 0$    | 1184 | 238  | 79    | 31     | 6        | 0        | 0.26       |
| CJS                                   | 914  | 267  | 150   | 87     | 28       | 1        | na         |
| $\frac{CJS - Data}{Benchmark - Data}$ | 24.3 | 6    | 9.4   | 17.5   | $\infty$ | $\infty$ |            |

## Interpretation

- ▶ Errors are an order of magnitude smaller than those reported by CJS.
- ▶ However, model without jumps ( $\lambda_C^Q = 0$ ,  $\lambda_i^Q = 0$ ) generates similar predictions to CJS.
- ▶ Why? Problem is two-fold:
  - ▶ **Backloading** of defaults in standard diffusion model:

| Average CDX index spreads for different models |        |        |        |        |        |
|------------------------------------------------|--------|--------|--------|--------|--------|
|                                                | 1 year | 2 year | 3 year | 4 year | 5 year |
| Data                                           | 13     | 20     | 28     | 36     | 45     |
| Benchmark                                      | 13     | 20     | 28     | 36     | 45     |
| $\lambda_C^Q = 0$                              | 13     | 20     | 28     | 36     | 45     |
| $\lambda_i^Q = 0$                              | 6      | 7      | 16     | 29     | 45     |
| $(\lambda_C^Q = 0, \lambda_i^Q = 0)$           | 0      | 3      | 13     | 28     | 45     |

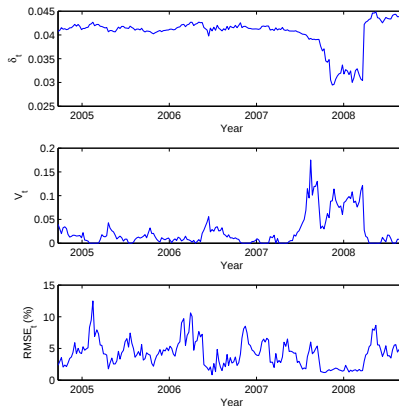
- ▶ Idiosyncratic jumps generates a five-year loss distribution that is **more peaked** around the risk-neutral expected losses of 2.4%.  
 (loss distribution with  $\lambda_C^Q = 0$ ,  $\lambda_i^Q = 0$  has std dev of 2.9%, whereas loss distribution with  $(\lambda_i^Q > 0, \lambda_C^Q = 0)$  has std dev of 1.7%).

## In Summary:

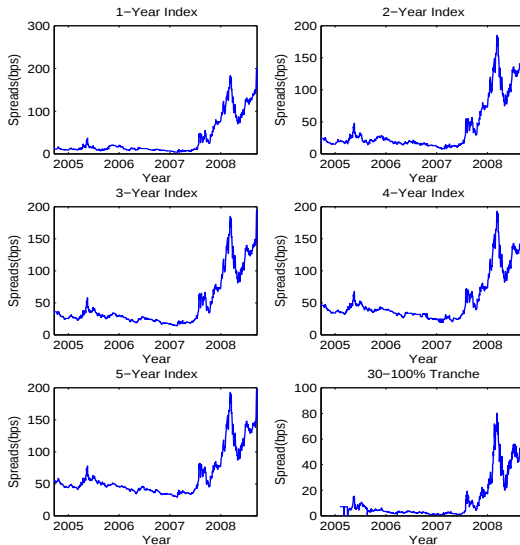
- ▶ In order to estimate tranche spreads, it is necessary that the model be calibrated to match the term structure of credit spreads.
- ▶ Specifying a model with idiosyncratic dynamics driven only by diffusive risks generates a model where:
  - ▶ the timing of defaults is **backloaded**.
  - ⇒ Counter-factually low spreads/losses at short maturities, which biases down the equity tranche spread.
  - ▶ the ratio of systematic to idiosyncratic default risk is too high.
  - ⇒ Excessively fat-tailed loss distribution, which biases senior tranche spreads up.
- ▶ In addition, the super-senior tranche spread (and therefore, spreads on other senior tranches) cannot be *extrapolated* from option prices alone.
- ▶ However, spreads on other tranches can be *interpolated* reasonably well given option prices and super-senior tranche spreads.
- ▶ S&P 500 options and CDX tranche prices market can be fairly well reconciled within our arbitrage-free model.

## Time Series Results

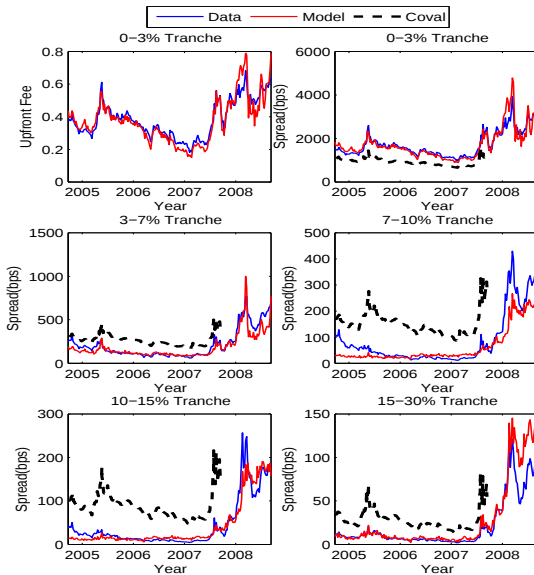
- Keeping parameters of the option pricing model fixed, each week, we fit
  - the state variables  $V_t$  and  $\delta_t$  to match quoted option prices.
  - The intensity of the catastrophic jump to match the super-senior tranche,
  - The default barrier and idiosyncratic jump intensity parameters to match the term structure of CDX index spreads with maturities of one-year to five-years.



## Series that we match 'in-sample' in benchmark model



# 'Out of Sample' Time Series Predictions of benchmark model



## Robustness Analysis

- ▶ We study the effects of relaxing some of our simplifying assumptions:
  - ▶ firm homogeneity,
  - ▶ no changes in capital structure,
  - ▶ uncorrelated idiosyncratic shocks (i.e., “no industry effects”),
  - ▶ constant firm-level asset dividend yield, (with stochastic market equity dividend yield)
  - ▶ constant interest rates.
- ▶ We still calibrate the model to 5-year option implied volatilities, 1-5 year CDX indices, and the super-senior tranche spreads.

|                                      | 0-3% | 3-7% | 7-10% | 10-15% | 15-30% | 30-100% | 0-3% Upf |
|--------------------------------------|------|------|-------|--------|--------|---------|----------|
| data                                 | 1472 | 135  | 37    | 17     | 8      | 4       | 0.34     |
| benchmark                            | 1449 | 113  | 25    | 13     | 8      | 4       | 0.33     |
| Dynamic capital structure            | 1452 | 116  | 27    | 14     | 8      | 4       | 0.34     |
| Stochastic firm payout               | 1441 | 122  | 29    | 14     | 9      | 4       | 0.33     |
| SVCJ                                 | 1330 | 138  | 47    | 26     | 12     | 4       | 0.30     |
| Heterogeneous initial credit spreads | 1406 | 133  | 28    | 13     | 8      | 4       | 0.32     |
| Stochastic short-term rate           | 1484 | 114  | 22    | 11     | 8      | 4       | 0.36     |
| Industry Correlations                | 1370 | 153  | 31    | 16     | 10     | 5       | 0.31     |

Table: Robustness check

## Details of robustness checks

- ▶ Dynamic capital structure: We assume that if a firm performs well, it will issue additional debt, in turn raising the default boundary  $A_B(t + dt) = \max[A_B(t), c A(t)]$
- ▶ Stochastic asset dividend yield at the firm level: We specify the firm payout ratio as  $\delta_A(t) = \bar{\delta}_A + \xi(\delta_t - \bar{\delta})$ , where  $\bar{\delta}_A = 0.05$  is the average payout ratio, and  $\xi = 0.7$  measures the correlation of dynamics of the firm payout ratio and market dividend-price ratio.
- ▶ Constant market equity dividend yield: we specify market dynamics using the SVCJ option model so that both the market dividend price ratio and the firm payout ratio are constants in this scenario.
- ▶ Heterogeneity in initial credit spreads: We use our model to back out the default boundaries for each firm based on their average 5-year CDS spreads in the on-the-run period of Series 4. The 5-year CDS spreads are from Datastream. The cross-sectional mean and the standard deviation of the log default boundaries are -1.59 and 0.344. we specify a distribution for the log default boundaries of the 125 firms using a normal distribution with the above parameters.
- ▶ Stochastic interest rates: We specify the spot rate to follow Vasciek (1977).
- ▶ Industry Correlations: we assume that there are approximately two firms per industry with dynamics that are perfectly correlated. As such, instead of modeling 125 firms, we consider only 60 “industries”.

## Conclusion of our analysis

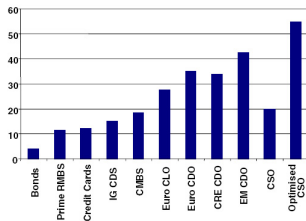
- ▶ It is possible to reconcile pricing of SP500 options and CDX-IG tranches within an arbitrage-free structural model of default.
- ▶ It is crucial to calibrate the model to the **term structure** of credit spreads to correctly account for the **timing** of defaults and the **ratio** of idiosyncratic to systematic default risk.
- ▶ Difference between the equity and senior tranche 'fair spreads' are sensitive to the timing of default. This is not easily captured in a static model.
- ▶ The ratio of idiosyncratic to systematic default risk varied much during the pre to post crisis period. More systematic risk implied from S&P 500 options actually lead to senior tranche spreads predicted by the model being larger during the crisis than observed (given that the model fits super-senior).
- ▶ If anything the model suggests that relative prices of tranches and options were 'more consistent' pre-crisis than during the crisis (in contrast to CJS (2009a,b)).
- ▶ Quoted index options are not very informative about pricing of super senior tranches: Quoted strike range is too 'narrow.'
- ▶ **Caveat:** The recalibration of model parameters (default intensities) over time is not internally consistent.

## Are senior tranches priced inefficiently by naive investors?

- ▶ Investors care only about expected losses ( $\sim$  ratings) and not about covariance (ironic since they trade in correlation markets!).

⇒ Spreads across AAA assets should be equalized. Are they?

AAA spreads by asset  
5y generic, bp



Source: Citi

- ⇒ All spreads should converge to **Physical** measure expected loss.
  - ▶ We observe large risk-premium across the board ( $\lambda^Q/\lambda^P > 6$ .)
  - ▶ Large time-variation in that risk-premium.
- ⇒ Time-variation in spreads should be similar to that of rating changes (smoother?).
  - ▶ Evidence seems inconsistent with marginal price setters caring only about expected loss ( $\sim$  ratings).

## What drives differences between structured AAA spreads?

- ▶ 'Reaching for yield' by rating constrained investors who **want** to take more risk because their incentives (limited liability) and **can** because ratings simply do not reflect risk and/or expected loss.
- ▶ Taking more risk by loading on systematic risk was the name of the game (agency conflicts).
- ▶ Possible that excess 'liquidity'/leverage lead to spreads being 'too' narrow **in all** markets, but little evidence that markets were ex-ante mis-priced on a relative basis.
- ▶ Ex-post (during the crisis) other issues, such as availability of collateral and funding costs, seem more relevant to explain cross-section of spreads across markets.
- ▶ Indeed, how to explain negative and persistent:
  - ▶ swap spreads?
  - ▶ cds basis?