On the Relation Between Credit Spread Puzzles and the Equity Premium Puzzle

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Background

- Investment-grade (IG) firms rarely default.
- Further, recovery rates are substantial:

Moody's 2005 Report:

Exhibit 18 - Average Issuer-Weighted Cumulative Default Rates 1970-2004

Years	1	2	3	4	5	6	7	8	9	10
Aaa	0.00	0.00	0.00	**0.04**	0.12	0.21	0.30	0.41	0.52	**0.63**
Baa	0.19	0.54	0.98	**1.55**	2.08	2.59	3.12	3.65	4.25	**4.89**

Exhibit 27 - Average Recovery Rates by Seniority Class, 1982-2004

Year	Sr. Secured	Sr. Unsecured	Sr. Subordinated	Subordinated Jr.	Subordinated	All bonds
Mean	0.574	**0.449**	0.391	0.320	0.289	0.422

Major Question

Historical spreads are quite large: (Huang and Huang (2003))

years	4	10
Aaa - Treasury	55	63
Baa - Treasury	158	194
Baa - Aaa	103	131

- Treasury rates may be well below ‘risk-free rate’.
- However, (Baa - Aaa) spread should mostly reflect differences in credit risk.

Q? Given historical default rates and recovery rates, can we explain observed (Baa - Aaa) spreads using a pricing kernel consistent with historical equity returns within a structural framework of default?

- How integrated are equity and corporate bond markets?
  – Sharpe ratios across both markets? (Saita (2006))

Major Question

Note: if data includes Great Depression, or if we consider possibility of a Peso problem, then there is much less of a puzzle:

Exhibit 17 - Average Issuer-Weighted Cumulative Default Rates (1920-2004)

years	1	2	3	4	5	6	7	8	9	10
Baa	0.31	0.93	1.69	**2.55**	3.40	4.28	5.12	5.95	6.83	**7.63**

Hence, the magnitude of the ‘credit spread puzzle’ depends upon what we feel agent’s default expectations have been over the past 50 years vs. actual defaults.

Background: Structural Models

Classical framework for modeling credit risk:

- Structural Models
  - Specify firm value and default boundary dynamics.
  - Default occurs if firm value falls below default boundary.
  - equity and debt values/dynamics determined jointly and self-consistently.
    - Merton (1974), Black and Cox (1976), Leland (1994), Longstaff and Schwartz (1996)

Unfortunately, early empirical examinations found structural models don't work well

- Only very small proportion of spread can be attributed to default risk.
  - Jones, Mason and Rosenfeld (1984)
- Changes in spreads not well predicted by structural models.
  - Collin-Dufresne, Goldstein and Martin (2001)
- Structural models are in poor agreement with each other.
  - Eom, Helwege and Huang (2004)

Background: Structural Models

Subsequent literature argues structural models work well at capturing credit component of spreads which should only be a small fraction of observed spread.

- Structural models are in general agreement once they are calibrated to match historical default rates
  - Huang and Huang (2003)

$$\begin{array}{llll} \text{Baa-Treas.} \approx 32\text{bp} & \text{vs.} & \text{actual } 158 \text{ bp} \\ \text{Aaa-Treas.} \approx 1\text{bp} & \text{vs.} & \text{actual } 55 \text{ bp.} \end{array}$$

- Even simple structural models generate hedge ratios similar to empirical values
  - Schaefer and Strebulaev (2005)
- Most of (Aaa - Treasury) may not be due to credit risk, and hence should not be explained by structural models (CDS premia $\neq$ Aaa - Treasury)
  - Grinblatt (1995), Collin Dufresne and Solnik (2001), Hull, Predescu and White (2005), Elton, Gruber, Aggrawal, Mann (2001), Blanco, Brennan, Marsh (2005)

Background: Structural Models

Recent literature actually finds that almost all of the spread can easily be explained within structural model even without liquidity or tax component!

- Spreads are consistent with predictions of a jump diffusion model calibrated to fit equity index options.
  - Cremers, Driessen, Maenhout and Weinbaum (2006)
- Spreads are consistent with a regime switching model where hidden states, which can be inferred from inflation data, affect growth rate of dividends.
  - David (2006)
- Using panel data, structural models perform better than suggested by CGM.
  - Avramov, Jostova, Philipov (2004), Ericsson, Jacobs and Oviedo (2005)

Credit Spread Puzzle

Q? Within structural models, corporate debt is priced using replication arguments of Black/Scholes (i.e., Risk-neutral-measure dynamics). So why is fitting historical default probability/recovery relevant?

A1! HH point out that previous structural models were 'successful' at pricing debt because they never investigated, for a 'reasonable' pricing kernel, what expected default and loss rates looked like.

A2! All structural models need to specify where the 'default boundary' is located. In practice, this boundary is not directly observed.

Q? Why is there a large discrepancy between results of HH and those of subsequent papers (David (2006) and Cremers et al. (2006))

A! fitting historical default/recovery is not sufficient for 'reasonable' calibration. Need to also fit historical Sharpe ratio on corporate bonds.

Intuition for 'reasonable' calibration

- Consider simple Merton (1974) model

$$\frac{dV}{V} + \delta\, dt = (r + \theta\sigma)\, dt + \sigma\, dz$$

where θ is the asset value Sharpe ratio.

- Default occurs at T if $V(T)$ falls below B. in that case recover $1 - L$.
- Spread $(y - r)$ on a date-T zero coupon bond is:

$$(y - r) = -\left(\frac{1}{T}\right) \log\left\{1 - L\ N\left[N^{-1}\left(\pi^P\right) + \theta\sqrt{T}\right]\right\}.$$

$\Rightarrow$ Even though the model is specified by 7 parameters $\{r,\ \mu,\ \sigma,\ \delta, V(0),\ B,\ L\}$, credit spreads only depend on historical default probability, recovery and asset sharpe ratio $\{\pi^P, L, \theta\}$.

- HH only calibrate their models to match historical estimates of $\{\pi^P,\ L\}$!

	T = 4Y			T = 10Y		
Sharpe	Baa	Aaa	Baa-Aaa	Baa	Aaa	Baa-Aaa
0.15	44.0	1.6	42.4	67.7	12.0	55.7
0.20	54.9	2.2	52.7	88.1	17.4	70.7
0.25	68.1	3.0	65.1	112.8	24.6	88.2
0.30	83.7	4.1	79.6	141.7	34.2	107.5
0.35	102.0	5.5	96.5	175.1	46.6	128.5
0.40	123.4	7.4	116.0	212.9	62.2	150.7

Table 1: (Baa - Aaa) spreads as a function of Sharpe ratio. 4Y Baa default rate = 1.55%. 4Y Aaa default rate = 0.04%. 10Y Baa default rate = 4.89%. 10Y Aaa default rate = 0.63%. Recovery rate = 0.449.

- Typical BBB firm asset value Sharpe ratio estimated at $\theta = 0.22$
  $\Rightarrow$ Benchmark (Baa - Aaa) = 57bp

- HH (2003) calibrate model to approximately $\theta \approx 0.17$ (too much idiosyncratic risk!)
  $\Rightarrow$ Benchmark (Baa - Aaa) = 32bp

- David (2006) calibrates model to $\theta = 0.36$ (too much systematic risk).

- Cremers et al. (2006) attribute all the risk-premium on the index to jump-risk.

Can Structural Models Explain Credit Spread Puzzle?

Fundamental pricing formula for discount bond: ($\Lambda \equiv$ pricing kernel)

$$
\begin{aligned}
P &= \mathrm{E}\left[\Lambda\,(1-\mathbf{1}_{\{\tau\le T\}}L_\tau)\right] \\
&= \mathrm{E}\,[\Lambda]\,\mathrm{E}\left[1-\mathbf{1}_{\{\tau\le T\}}L_\tau\right] + \mathrm{Cov}\left[\Lambda\,,(1-\mathbf{1}_{\{\tau\le T\}}L_\tau)\right] \\
&= \frac{1}{R^f}\left(1-E\left[\mathbf{1}_{\{\tau\le T\}}L_\tau\right]\right) - \mathrm{Cov}\left[\Lambda\,,\mathbf{1}_{\{\tau\le T\}}L_\tau\right].
\end{aligned}
$$

Q? Which models can raise credit spreads while matching historical expected recovery and default rates (i.e., holding 1^{st} term on RHS constant)?
and match unconditional sharpe ratios

- To lower prices of risky bonds (and thus raise spreads) need to generate a:

(1) positive covariance between the pricing kernel (Λ) and the default time $\left(\mathbf{1}_{\{\tau\le T\}}\right)$

(2) positive covariance between the pricing kernel (Λ) and loss rates (L_τ).

Can Structural Models Explain Credit Spread Puzzle?

- Structural models define (value triggered) default as first passage of asset value, V_t, at some default boundary, B_t ($\sim$ liabilities):

$$\tau := \inf\{t \; : \; V_t \leq B_t\}$$

$\Rightarrow$ First channel can be broken down into two, implying three separate channels to explain 'credit spread puzzle':

(1a) negative covariance between the pricing kernel (Λ_t) and asset prices (V_t),

(1b) positive covariance between the pricing kernel (Λ_t) and the default boundary (B_t),

(2) positive covariance between the pricing kernel (Λ_t) and loss rates (L_τ).

Relation between Credit puzzles and Equity premium puzzle?

Channel 1a) is identical to that used to explain the 'equity premium puzzle'

As such, we investigate whether pricing kernel models engineered to fit the equity premium can also predict magnitude and time variation of credit spreads

- Campbell and Cochrane (1999) *'habit formation'* model
  - Equity premium explained by *time varying risk-aversion*: Investors become highly risk-averse in bad (i.e., low consumption) states.

- Bansal and Yaron (2004) *'long run risk'* model
  - Equity premium explained by *cash flow risk*: persistent shocks to aggregate consumption & dividend growth rates + stochastic volatility make equity more risky than in an i.i.d. world.

Objectives of Paper

- Explore whether credit spreads can be explained within structural framework using the three channels identified above.

- Validate the common empirical practice of linking credit spread to equity premium

  Chen, Roll, and Ross (86), Keim and Stambaugh (86), Campbell (1987), Fama and French (89, 93), Ammer and Campbell (93), and Jagannathan and Wang (96) ...

- Provide 'out-of-sample' tests for successful equity premium models.

- Use historical consumption data to simulate implied credit spreads and compare with historical credit spread *year by year*.

Results

- The (Aaa - Treasury) spread cannot be explained in terms of credit-risk (HJ bound)
- We can explain the level and time variation in Baa-Aaa credit spreads within a model that (i) has time-varying sharpe ratios (consistent with equity returns as in CC), and (ii) matches the countercylical nature of default rates.
- Countercylical default rates obtain if the default boundary and/or idiosyncratic risk are countercylical.
  - Intuition: $P = E[\Lambda \cdot X]$. CC model implies high risk-aversion in recessions and therefore high state prices (Λ). But recessions are when most defaults occur. Therefore corporate bond cash-flows X are low (high) precisely in the expensive (cheap) states!
- The BY 'long run risk' model with constant risk-premia cannot explain the credit spread puzzle.

Outline of Paper

- Stylized facts.
- Benchmark HH model (constant coefficients).
- CC model
- ‘BY’ model
- Time Series Implications
- Conclusions

Some stylized facts

- Baa-Aaa spreads are high on average (109 bp) and volatile (41 bp std dev).
- Credit spreads are countercyclical.
- Default rates are low on average (4-year default rate Baa = 1.55%)
- Default rates are countercyclical in that the regression of 4-year future default rates on current credit spread yields a significantly positive coefficient of 0.86.
- Typical Baa firm leverage is countercyclical in spite of 'credit refreshment.'
- Typical Baa firm asset value sharpe ratio estimated at 0.22.

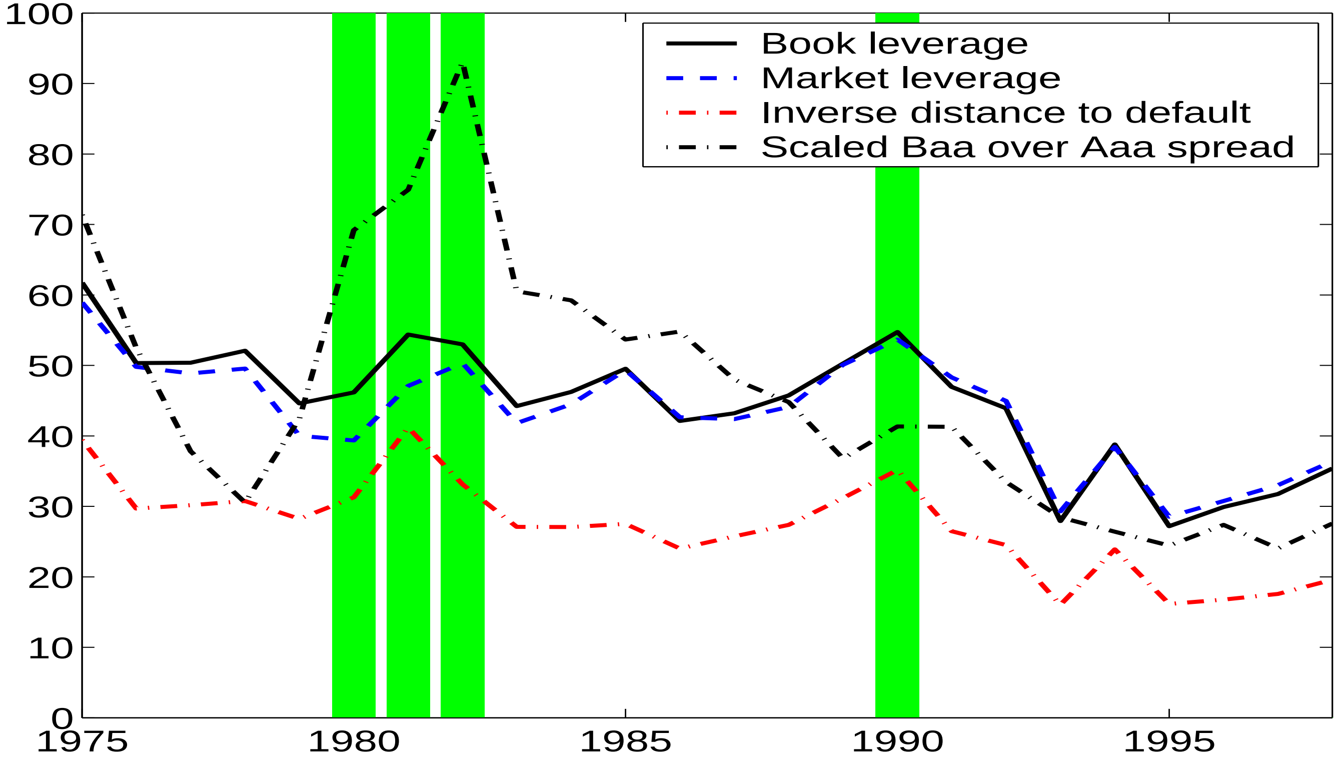


Figure 1: Time series of leverage for Baa rated firms.

Campbell and Cochrane 'countercyclical risk aversion' Model

- CC specifies Pricing Kernel with following dynamics:

$$\Lambda_t \;=\; e^{-\alpha t}\left(C-\hat{C}\right)^{-\gamma} \equiv e^{-\alpha t}\, e^{-\gamma\, s}\, e^{-\gamma\, c},$$

- log consumption (c) and log-surplus/consumption ratio ($s = \log \frac{C-\hat{C}}{C}$) dynamics:

$$\begin{aligned} dc(t) &= g\,dt + \sigma\,dZ(t) \\ ds(t) &= \kappa(\overline{s} - s(t))dt + \sigma\left[\frac{1}{\overline{S}}\sqrt{1-2(s-\overline{s})} - 1\right] dZ(t) \end{aligned}$$

- CC set$\overline{S} = \sigma\sqrt{\frac{\gamma}{\kappa}}$ ($s_{max} \equiv \overline{s} + \frac{1}{2}\left(1-\overline{S}^2\right)$) to obtain a constant real rate of interest

$$r_f = \alpha + \gamma g - \frac{1}{2}\gamma\kappa. \qquad (1)$$

- Calibration: CC choose parameters to match postwar data of $g = 1.88\%$, $\sigma = 1.24\%$, $r_f = 0.94\%$.
- Left with two free parameters (γ, κ) which they choose to match Sharpe ratio (0.43), and the serial correlation of log P/D ratio.

Campbell and Cochrane 'countercyclical risk aversion' Model

⇒ 'out-of-sample predictions' of CC are

- average risk-premium,
- standard deviation of returns,
- average P/D ratio.

- Our results show that CC model generates expected returns and standard deviations of only about half historical levels while average P/D ratio about twice the historical levels.
- Fortunately, with slightly different calibration ($\gamma = 2.54$, $g_d = 0.04$) results for dividend vs. consumption) claim are pretty good.

Note: Both CC and BY assume $g_c = g_d \approx 0.0189$

However, this ignores leveraged nature of equity. (Abel 1999, 2005)

Chen et al (2006b) shows that one can account for leveraged nature of equity simply by raising the growth rate on dividends.

Panel A:	**Parameter Inputs**			
CF type	g_d	σ_d	γ	ρ_{cd}
Dividends	.040	.080	2.45	.60
Output	.0189	.063	2.45	.48

Panel B:	**Model Outputs:**				
CF type	$\exp\left(\mathrm{E}\left[p-d\right]\right)$	$\sigma(p-d)$	Sharpe	$\mathrm{E}\left[r-r_f\right]$	$\sigma\left(r-r_f\right)$
Claim to Dividends	24	.21	.44	.073	.17
Historical equity	25	.26	.43	.067	.16
Claim to output	23	.15	.44	.053	.12
Historical (debt + equity)	19	.20	.43	.050	.10

Panel A: Parameter Calibrations for dividend and output processes. Panel B: Sample Moments of claims to dividends vs. historical values; and claims to (dividends plus interest) vs. historical values.

Implication for Credit Spreads

- CC model predicts that market portfolio has dynamics:

$$\frac{dV(t)}{V(t)} = \Big(\theta(s_t) + r - \delta(s_t)\Big)\, dt + \sigma(s_t)\, dz_V(t). \tag{2}$$

  with endogenous *stochastic* $\theta(s)$, $\delta(s)$, $\sigma(s)$

- As in benchmark case assume that typical Baa firm value is:

$$\frac{dP(t)}{P(t)} = \frac{dV(t)}{V(t)} + \sigma_{idio}\, dz_{idio}(t). \tag{3}$$

- Pick default boundary to match historical default rates.

- Pick σ_{idio} to match the sharpe ratio on typical Baa firm asset value (0.22).

Constant Default Boundary

- For **constant** default boundary (set equal to 60% of average leverage) find:
  - Average (Baa-Aaa) spread ($\mathbf{77 \pm 11}$)**bp,** vs. historical (109 ± 41)bp.
  - **Procyclical** default rates (because risk-premia are countercyclical).

		Baa			Aaa			
$s(0)$	Population Distr. (%)	Spread over Treas. (bp)	Q-Default Rate (%)	P-Default Rate (%)	Spread over Treas. (bp)	Q-Default Rate (%)	P-Default Rate (%)	(Baa - Aaa) Spread (bp)
-3.66	0.01	100.5	7.19	0.84	6.6	0.44	0.02	93.9
-3.56	0.01	97.8	7.00	0.90	6.5	0.44	0.02	91.3
-3.46	0.02	97.1	6.95	0.97	6.5	0.43	0.02	90.6
-3.36	0.02	95.3	6.75	1.00	6.4	0.43	0.02	88.9
-3.26	0.03	94.3	6.74	1.03	6.3	0.42	0.02	88.0
-3.16	0.04	93.9	6.72	1.09	6.2	0.41	0.03	87.7
-3.06	0.05	93.6	6.69	1.13	6.1	0.41	0.03	87.5
-2.96	0.06	91.3	6.61	1.22	6.0	0.40	0.03	85.3
-2.86	0.07	90.1	6.50	1.36	5.9	0.39	0.04	84.2
-2.76	0.09	88.0	6.30	1.43	5.8	0.37	0.04	82.2
-2.66	0.11	85.5	6.12	1.54	5.3	0.34	0.04	80.2
-2.56	0.13	80.5	5.77	1.75	5.0	0.32	0.05	75.5
-2.46	0.15	76.0	5.46	1.89	4.8	0.31	0.05	71.2
-2.36	0.14	66.2	4.76	2.08	4.0	0.25	0.06	62.2
-2.27	0.04	52.9	3.82	2.20	3.0	0.17	0.06	49.9
Average		82.3	5.90	1.55	5.2	0.34	0.04	77.1
Std. Dev.		12.7	0.90	0.41	1.0	0.07	0.01	11.7

Table 2: Model generated 4-year Baa and Aaa credit spreads when the nominal default boundary is a constant (B = 0.356 (0.208) for Baa (Aaa) firms).

$s(0)$	Population Distr. (%)	**Baa** Spread over Treas. (bp)	**Baa** Q-Default Rate (%)	**Baa** P-Default Rate (%)	**Aaa** Spread over Treas. (bp)	**Aaa** Q-Default Rate (%)	**Aaa** P-Default Rate (%)	(Baa - Aaa) Spread (bp)
-3.66	0.01	195.8	33.84	2.47	72.4	14.01	0.24	123.4
-3.56	0.01	188.5	32.64	2.69	67.5	13.07	0.27	121.0
-3.46	0.02	183.6	31.82	2.82	63.9	12.38	0.29	119.7
-3.36	0.02	176.5	30.65	2.91	59.7	11.56	0.31	116.8
-3.26	0.03	171.2	29.77	3.05	56.7	10.98	0.34	114.5
-3.16	0.04	166.2	28.93	3.14	53.2	10.28	0.36	113.0
-3.06	0.05	160.9	28.02	3.39	50.5	9.77	0.39	110.4
-2.96	0.06	155.0	27.05	3.74	47.3	9.15	0.42	107.7
-2.86	0.07	149.9	26.22	3.86	44.5	8.58	0.47	105.4
-2.76	0.09	144.7	25.33	4.40	42.4	8.17	0.54	102.3
-2.66	0.11	138.5	24.33	4.72	39.4	7.60	0.59	99.1
-2.56	0.13	131.8	23.21	5.20	36.6	7.05	0.66	95.2
-2.46	0.15	124.9	22.08	5.86	34.2	6.59	0.76	90.7
-2.36	0.14	116.4	20.71	6.64	30.8	5.93	0.91	85.6
-2.27	0.04	104.7	18.86	7.36	26.0	5.03	1.05	78.7
Average		141.8	24.88	4.89	42.5	8.19	0.63	99.4
Std. Dev.		26.8	4.44	1.39	13.9	2.69	0.22	13.5

Table 3: Model generated 10-year Baa and Aaa credit spreads when the nominal default boundary is a constant. B = 0.295 (0.171) for Baa (Aaa) firms.

Countercyclical Initial Distance to Default

- Even with constant default boundary, the initial distance to default for the median Baa firm may change across the business cycle: $(\frac{V(0)}{B} = \frac{V(0)}{F} \cdot \frac{F}{B})$.
- Indeed, regressing leverage on surplus consumption ratio we find:

$$MLV_{Baa}[S(0)] = 0.52 - .61S(0)$$

- Thus we set **countercyclical** *nominal* initial default boundary equal to

$$B_{Baa}[S(0)] \;=\; \psi_{Baa}\,[0.52 - .61S(0)] \tag{4}$$
$$B_{Aaa}[S(0)] \;=\; \psi_{Aaa}\,[0.52 - .61S(0)]\,. \tag{5}$$

where $\psi_{Baa} = 0.750$ & $\psi_{Aaa} = 0.435$ set to match historical default rates.

		Baa			**Aaa**			
$s(0)$	Population Distr. (%)	Spread over Treas. (bp)	Q-Default Rate (%)	P-Default Rate (%)	Spread over Treas. (bp)	Q-Default Rate (%)	P-Default Rate (%)	(Baa - Aaa) Spread (bp)
-3.46	0.02	123.0	8.74	1.34	8.5	0.59	0.03	114.5
-3.36	0.02	119.0	8.46	1.34	7.5	0.51	0.03	111.5
-3.26	0.03	117.6	8.36	1.41	8.0	0.55	0.03	109.6
-3.16	0.04	111.7	7.95	1.48	7.3	0.50	0.03	104.4
-3.06	0.05	108.9	7.76	1.43	7.6	0.52	0.03	101.3
-2.96	0.06	103.9	7.41	1.47	6.9	0.46	0.04	97.0
-2.86	0.07	98.5	7.03	1.48	6.5	0.43	0.04	92.0
-2.76	0.09	92.4	6.61	1.57	6.0	0.40	0.04	86.4
-2.66	0.11	87.6	6.27	1.60	5.7	0.38	0.04	81.9
Average		86.8	6.20	1.55	5.6	0.37	0.04	81.2
Std. Dev.		23.2	1.63	0.18	1.7	0.13	0.01	21.5

Countercyclical Default Boundary

- In addition to the initial distance to default of the typical Baa firm varying over the business cycle, the default boundary of each firm may be stochastic.
- We assume:

$$B_{Baa}(S(t), S(0)) = \psi^*_{Baa}(0.52 - 0.61S(0)))(1 - slope * (S(t) - \overline{S})) \quad (6)$$
$$B_{Aaa}(S(t), S(0)) = \psi^*_{Aaa}(0.52 - 0.61S(0)))(1 - slope * (S(t) - \overline{S})). \quad (7)$$

- We choose $\psi_{Baa} = 0.696$ and $\psi_{Aaa} = 0.394$ to perfectly match historical default rates.
- We set $slope = 4$ to match the estimated historical regression coefficient between 4-year future default probabilities and (Baa - Aaa) spreads ($\beta \sim .86 \pm .42$)
- With this calibration, we find the model matches the data well:
  - average (Baa - Aaa) credit spread of 107bp (4-year) and 124bp (10-year),
  - a regression coefficient of +0.585, and
  - a standard deviation of spreads of 46bp.

$s(0)$	Population Distr. (%)	**Baa** Spread over Treas. (bp)	**Baa** Q-Default Rate (%)	**Baa** P-Default Rate (%)	**Aaa** Spread over Treas. (bp)	**Aaa** Q-Default Rate (%)	**Aaa** P-Default Rate (%)	(Baa - Aaa) Spread (bp)
-3.66	0.01094	216.8	15.02	1.95	16.3	1.17	0.04	200.5
-3.56	0.0138	206.6	14.35	1.98	15.8	1.13	0.06	190.8
-3.46	0.01743	197.8	13.76	1.96	15.2	1.08	0.06	182.6
-3.36	0.02273	190.6	13.29	2.00	14.4	1.02	0.05	176.2
-3.26	0.02864	181.3	12.67	1.88	13.9	0.98	0.06	167.4
-3.16	0.0364	170.5	11.94	1.93	13.3	0.94	0.05	157.2
-3.06	0.04615	163.4	11.47	1.84	12.4	0.88	0.05	151.0
-2.96	0.05657	151.1	10.65	1.74	11.4	0.80	0.06	139.7
-2.86	0.07159	140.4	9.92	1.72	10.4	0.73	0.05	130.0
-2.76	0.08983	127.6	9.05	1.65	9.4	0.65	0.05	118.2
-2.66	0.10887	112.4	8.00	1.57	7.9	0.54	0.05	104.5
-2.56	0.12837	94.6	6.77	1.48	6.8	0.46	0.04	87.8
-2.46	0.14735	78.2	5.62	1.34	5.5	0.36	0.02	72.7
-2.36	0.14429	56.1	4.05	1.21	3.9	0.24	0.04	52.2
-2.27	0.03753	33.6	2.43	0.98	2.1	0.11	0.02	31.5
Average		115.6	8.18	1.55	8.5	0.59	0.04	107.1
Std. Dev.		50.3	3.45	0.29	4.0	0.30	0.01	46.3

Model generated 4-year Baa and Aaa credit spreads

$s(0)$	Population Distr. (%)	**Baa** Spread over Treas. (bp)	**Baa** Q-Default Rate (%)	**Baa** P-Default Rate (%)	**Aaa** Spread over Treas. (bp)	**Aaa** Q-Default Rate (%)	**Aaa** P-Default Rate (%)	(Baa - Aaa) Spread (bp)
-3.66	0.01	277.5	44.73	4.08	106.1	19.90	0.45	171.4
-3.56	0.01	268.3	43.43	4.18	99.7	18.74	0.46	168.6
-3.46	0.02	261.4	42.43	4.15	94.3	17.75	0.47	167.1
-3.36	0.02	251.3	40.99	4.16	88.7	16.72	0.47	162.6
-3.26	0.03	241.9	39.62	4.27	83.4	15.75	0.49	158.5
-3.16	0.04	231.0	38.06	4.33	78.5	14.83	0.51	152.5
-3.06	0.05	222.0	36.78	4.41	73.5	13.91	0.53	148.5
-2.96	0.06	211.4	35.26	4.55	68.6	13.01	0.55	142.8
-2.86	0.07	201.4	33.76	4.59	64.0	12.14	0.57	137.4
-2.76	0.09	189.0	31.98	4.73	59.2	11.25	0.59	129.8
-2.66	0.11	177.7	30.31	4.95	53.9	10.28	0.61	123.8
-2.56	0.13	163.0	28.09	5.12	48.7	9.31	0.65	114.3
-2.46	0.15	147.0	25.69	5.33	42.9	8.24	0.69	104.1
-2.36	0.14	130.3	23.13	5.38	36.4	7.02	0.73	93.9
-2.27	0.04	109.2	19.83	5.67	28.7	5.58	0.77	80.5
Average		181.8	30.79	4.87	58.0	11.03	0.61	123.8
Std. Dev.		48.1	7.13	0.56	22.4	4.13	0.10	26.3

Model generated 10-year Baa and Aaa credit spreads

Time-varying Idiosyncratic Risk

- How realistic is the specification of time-varying default boundary?
- Alternative mechanism to generate correlation between spreads and default rates: We specify time varying idiosyncratic risk

$$\sigma_{idio}(S_t) = 0.212 - 1.1(S_t - \bar{S}), \tag{8}$$

to match regression coefficient of default rates on spreads. (coefficients within one-standard deviation of empirical estimates obtained from regressing idiosyncratic volatility measures on consumption surplus ratio.)

- Find a Baa-Aaa spread of 91 bps and std dev of 39 bps.

⇒ As long as the model can capture countercyclical nature of default rates (regardless of mechanism) then CC pricing kernel can generate Baa-Aaa spread broadly consistent with historical data.

Time-varying Recovery Rates

- Recent empirical evidence that recovery rate rates are procyclical (Altman, Resti, Sironi (2005), Moody's (2005)).
- We specify recovery rate as

$$recovery = 0.418 + 0.59S(\tau), \tag{9}$$

  to match average historical recovery rates and cyclicality in recovery rates.
- Find it increases spreads by only a few basis points.

Implied premia and historical spread

- Use historical consumption surplus ratio to back out model-implied credit spreads and equity premia and compare with historical credit spreads
- Historical consumption surplus ratio strikingly (inversely) related to historical credit spreads

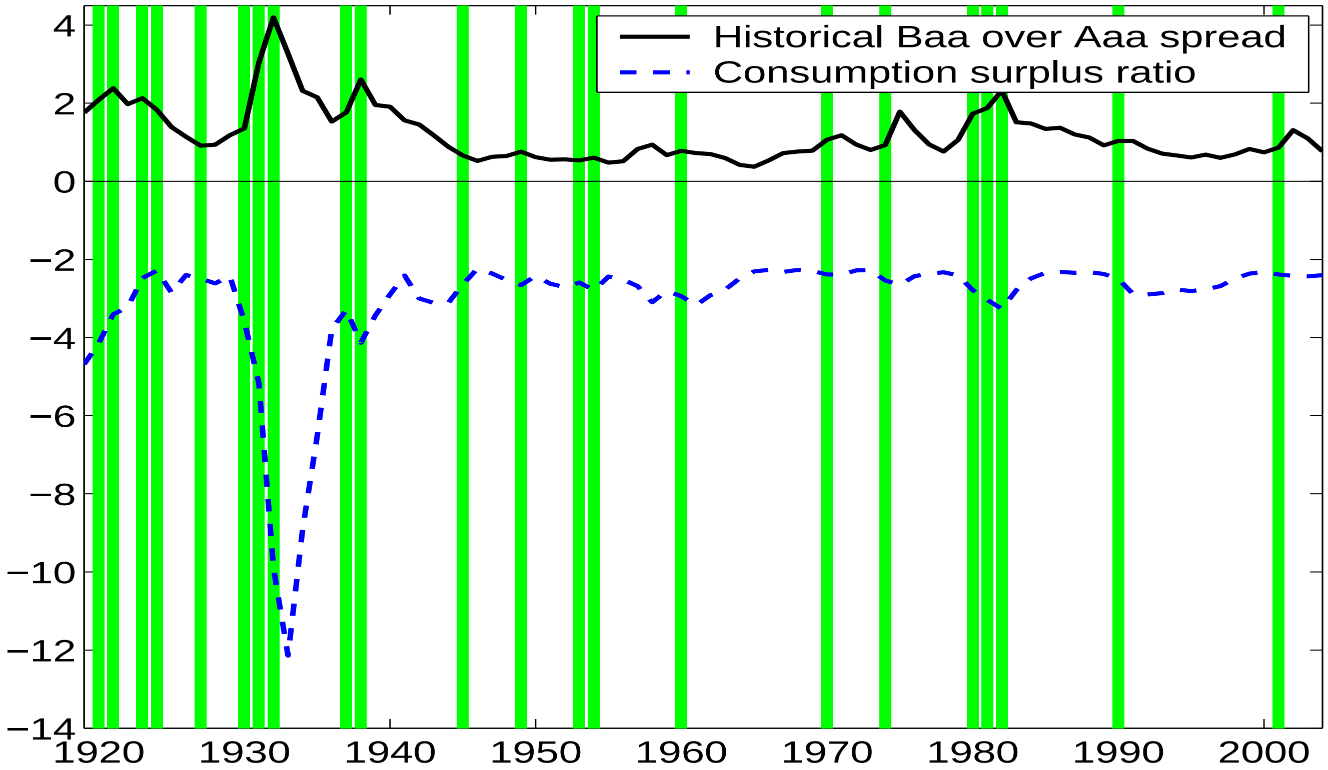


Figure 2: The relation between historical credit spread and consumption surplus ratio.

Implied premia and historical spread

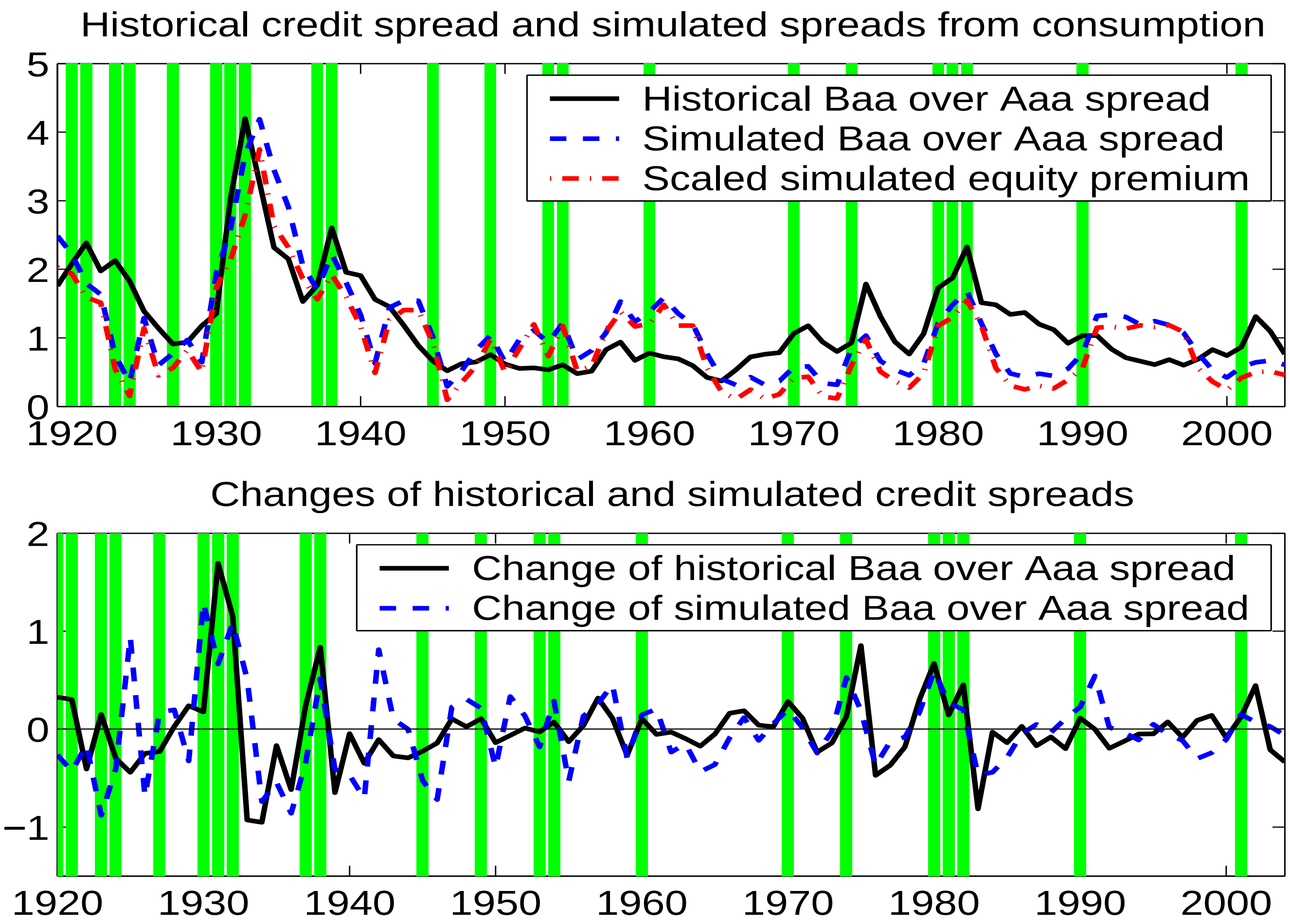


Figure 3: The levels and changes of historical and simulated credit spreads and simulated equity premium.

Implied premia and historical spread

- For the 1919-2004 period, the CC model can match the max, min, mean, and std of historical credit spreads
- The implied and historical credit spreads are 71% correlated for the long sample
- The changes of the implied and historical credit spreads are 46% for 1919-2004 and increases to 54% for 1946-2004.

Panel A: Summary Statistics

	1919-2004				**1946-2004**			
Variable	Mean	Std.	Min	Max	Mean	Std.	Min	Max
SS	1.13	0.76	0.29	4.19	0.84	0.38	0.29	1.68
SEP	7.55	5.48	0.77	29.99	5.55	3.39	0.77	12.37
Spread	1.20	0.70	0.37	4.20	0.90	0.39	0.37	2.33

Panel B: Cross Correlations of Levels

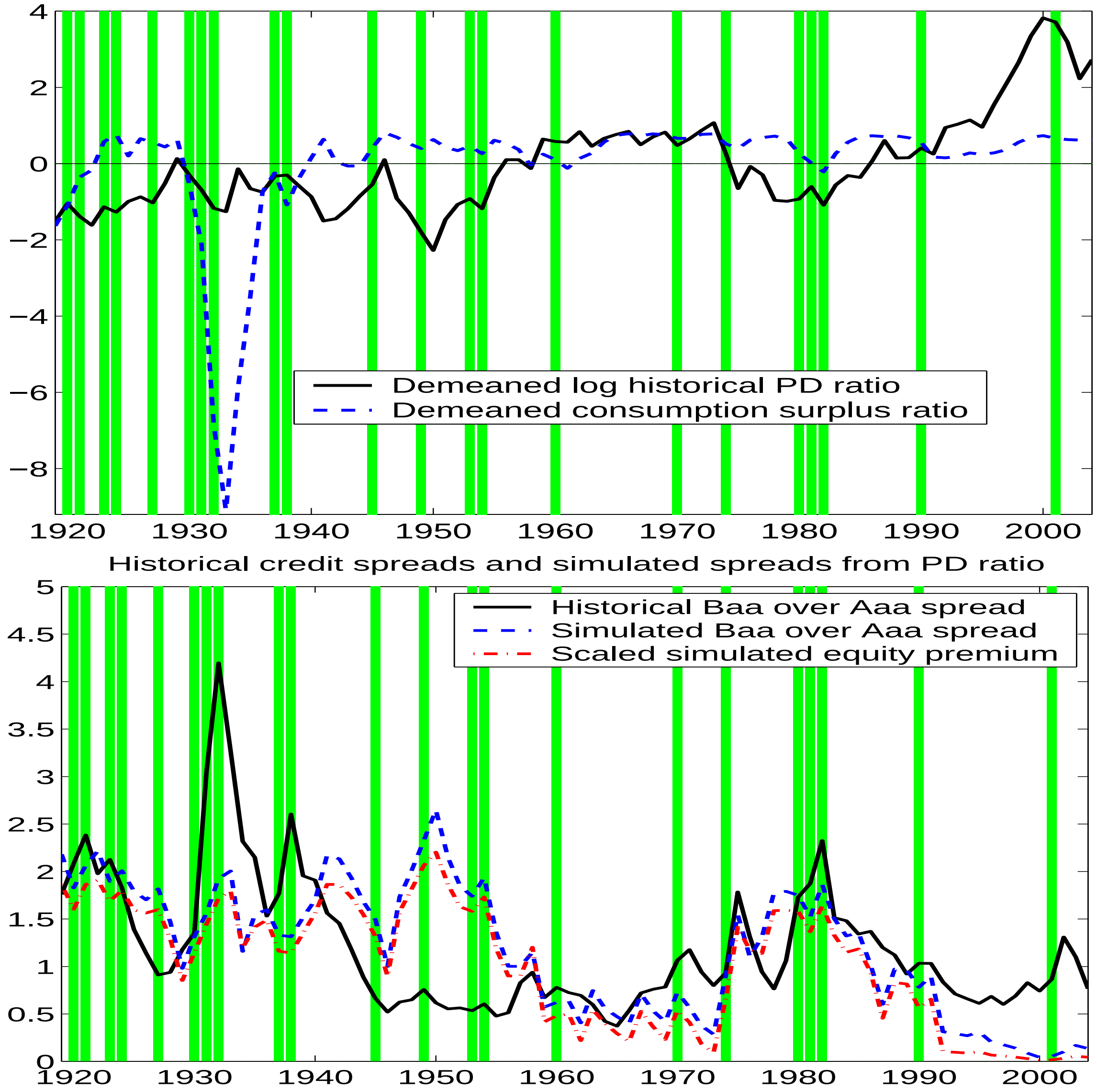
	1919-2004				**1946-2004**			
	CS	SS	SEP	PD	CS	SS	SEP	PD
PD	0.21	-0.30	-0.32		0.22	-0.22	-0.22	
Spread	-0.72	0.71	0.64	-0.31	-0.23	0.19	0.18	-0.13

Panel C: Cross Correlations of the changes

	1919-2004				**1946-2004**			
	CS	SS	SEP	PD	CS	SS	SEP	PD
PD	0.16	-0.25	-0.24		0.35	-0.38	-0.36	
Spread	-0.49	0.46	0.35	-0.20	-0.58	0.54	0.54	-0.28

Table 4: CS is consumption surplus ratio. SS is simulated Baa over Aaa spread; SEP is simulated equity premium; Spread is the actual Baa over Aaa spread; PD is the actual Price dividend ratio.

Implying consumption surplus ratio from Price-dividend ratio



Conclusions

- We argue that to be economically meaningful we should compare spread predictions of models that generate identical historical default rates, recovery rates **and** Sharpe ratios.
- Then to explain credit spread puzzles, we have to focus on different models of the pricing kernel (risk-premia).
- We compare two models of pricing kernel that are engineered to explain the Equity premium puzzle using very different mechanisms:
  1. Time varying risk-aversion generating countercyclical risk-premia (CC),
  2. Time varying stochastic drift generating long-run cash-flow risk (BY).
- The CC model can explain level and time variation in credit spreads, provided the structural model generates sufficient countercyclicality in default rates. We provide two mechanisms for the latter: countercyclical default boundary (unobservable) and idiosyncratic risk (some evidence).
  - Intuition: CC note that representative agent with habit formation utility is extremely risk averse to recessions. But this is exactly when defaults occur!

Conclusions

- The BY model generates too low an average level of spreads.
  - Intuition: Corporate bonds are not nearly as exposed to long-run risk as are equities
- Results suggests that to reconcile equity returns with credit spreads need:
  - high time-variation in risk-premia (i.e., not just large and constant!).
  - countercyclical default rates.
- Credit spread is highly systematic in that it covaries closely with aggregate consumption

  Campbell and Taksler (02) and Jagannathan and Wang (96)
- The (Aaa - Treasury) spread does not seem to be explained in terms of credit-risk

Bansal-Yaron 'Long Run Risks' Model

- The BY model generates the equity premium via cash flow risk as opposed to time varying risk-aversion. It adds stochastic volatility and consumption growth:

$$\begin{aligned} dc_t &= (\mu + x_t)dt + (v_t + \bar{v})\, dZ_c(t) \\ dd_t &= (\mu_d + \phi x_t)dt + \sigma_d (v_t + \bar{v})\, dZ_c(t) \\ dx_t &= -\kappa x_t\, dt + \sigma_x (v_t + \bar{v})\, dZ_x(t) \\ dv_t &= -\nu v_t\, dt + \sigma_v\, dZ_v(t) \end{aligned}$$

where c, d are the log consumption and dividend process respectively.

- Representative agent has recursive utility:

$$J(t) = \mathrm{E}_t[\textstyle\int_t^\infty f(c_s, J_s)ds]$$

where the 'normalized' aggregator function (with $\theta = \frac{1-\gamma}{1-\rho}$):

$$f(c, J) = \frac{\beta u_\rho(c)}{((1-\gamma)J)^{1/\theta - 1}} - \beta\theta J$$

- The pricing kernel in this economy is simply:

$$\Pi(t) = e^{\int_0^t f_J(c_s, J_s)ds} f_c(c_t, J_t) \tag{10}$$

- The pricing kernel can be approximated by:

$$\begin{aligned}\frac{d\Lambda_t}{\Lambda_t} &= -r_t dt - (\lambda_{c0} + \lambda_{c1} v_t) dZ_c(t) - (\lambda_{v0} + \lambda_{v1} v_t) dZ_v(t) - (\lambda_{x0} + \lambda_{x1} v_t) dZ_x(t) \\ r_t &= \alpha_0 + \alpha_x x_t + \alpha_v v_t + \alpha_{vv} v_t^2\end{aligned}$$

- All parameters are calibrated following BY to match the dividend claim return characteristics.

- BY model predicts that market portfolio has dynamics:

$$\frac{dV(t)}{V(t)} = \Big(\theta(x_t, v_t) + r_t - \delta(x_t, v_t)\Big)\, dt + \sigma_d(x_t, v_t)\, dz_d(t) + \sigma_x(x_t, v_t)\, dz_x(t) + \sigma_v(x_t, v_t)\, dz_v(t). \tag{11}$$

with endogenous *stochastic* $\theta(x, v)$, $\delta(x, v)$, $\sigma(x, v)$

- As in benchmark case assume that typical Baa firm value is:

$$\frac{dP(t)}{P(t)} = \frac{dV(t)}{V(t)} + \sigma_{idio}\, dz_{idio}(t). \tag{12}$$

Implications for Credit spreads?

- Consider three cases to distinguish cash flow risk from time varying risk-premia:
  - Case I: growth rate risk (i.e., we set the volatility and risk-premia to be constant: $\sigma_v = 0, v_t = 0$ and $\lambda_{j1} = 0$, $j = c, v, x$).
  - Case II: growth rate and volatility risk (i.e., we set the risk-premia to be constant: $\lambda_{j1} = 0$, $j = c, v, x$)).
  - Case III: growth rate and volatility risk as well as time varying risk-premia.

CASE I							
Stochastic growth - Constant volatility and risk-premia							
P def prob	P-DP Std Dev	Q def prob	Q-DP Std Dev	Average Spread	Std Dev. Spread	Reg Coef	σ_{idio}^{Baa}
0.0155	0.0031	0.036	0.0066	0.0045	0.00082	4.04	0.245
(0.0005)	(0.0009)						
CASE II							
Stochastic growth and volatility - Constant risk-premia							
P def prob	P-DP Std Dev	Q def prob	Q-DP Std Dev	Average Spread	Std Dev. Spread	Reg Coef	σ_{idio}^{Baa}
0.0153	0.0057	0.0394	0.0145	0.0046	0.0017	3.2589	0.242
(0.0004)	(0.0006)						
CASE III							
Stochastic growth and volatility - Time varying risk-premia							
P def prob	P-DP Std Dev	Q def prob	Q-DP Std Dev	Average Spread	Std Dev. Spread	Reg Coef	σ_{idio}^{Baa}
0.0155	0.0049	0.0492	0.0204	0.0058	0.0025	1.853	0.241
(0.0004)	(0.0007)						

BY Model

- Pure cash flow risk (Case I) generates spread similar to benchmark.
- Introducing countercyclical volatility (Case II) only slightly increases the spread.
- Introducing time varying risk premium (Case III) substantially increases spread, though still far short of historical average.
- Introducing time varying default boundary not likely to improve performance: trade-off between fitting average default probability and co-variance between default probability and spreads.

Conclusions:

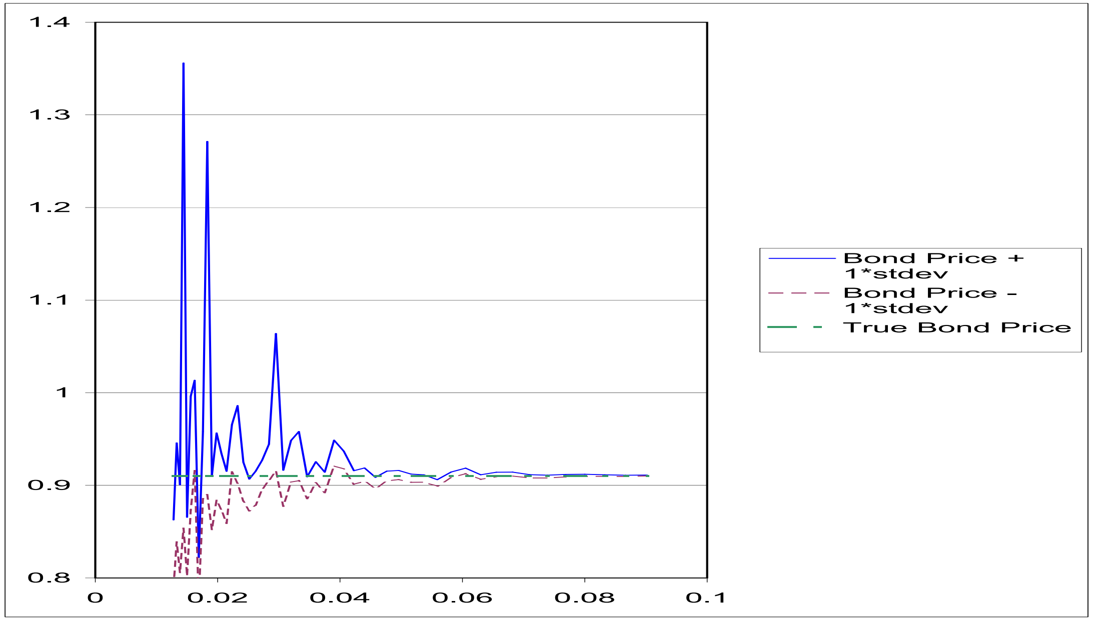
- Time varying risk premium is essential.
- Note that Corporate bonds do not share in the upside risk to cash flows as does equity. This might explain why CC model seems to outperform BY model for corporate bonds.

- Intuition for why our results differ from CC, similar to ‘importance sampling’ issue for simulating default events.
- In CC model, tail events happen with very low probability: difficult to simulate prices accurately.
- Example: Try to estimate the risk-free bond price,

$$\mathrm{E}[\frac{\Lambda_T}{\Lambda_0}] = \frac{1}{n}\sum_{i=1}^{n}\frac{\Lambda_T(\omega_i)}{\Lambda_0}$$

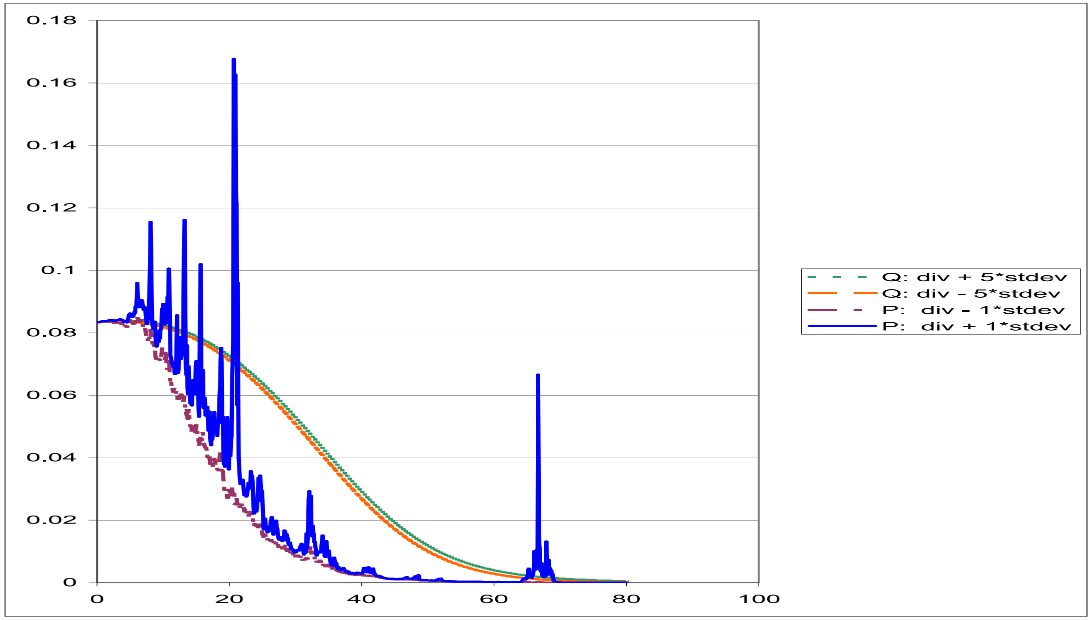
  We know the correct answer in closed form since the interest rate is constant in the model:

$$\mathrm{E}[\frac{\Lambda_T}{\Lambda_0}] = e^{-r_f T}$$



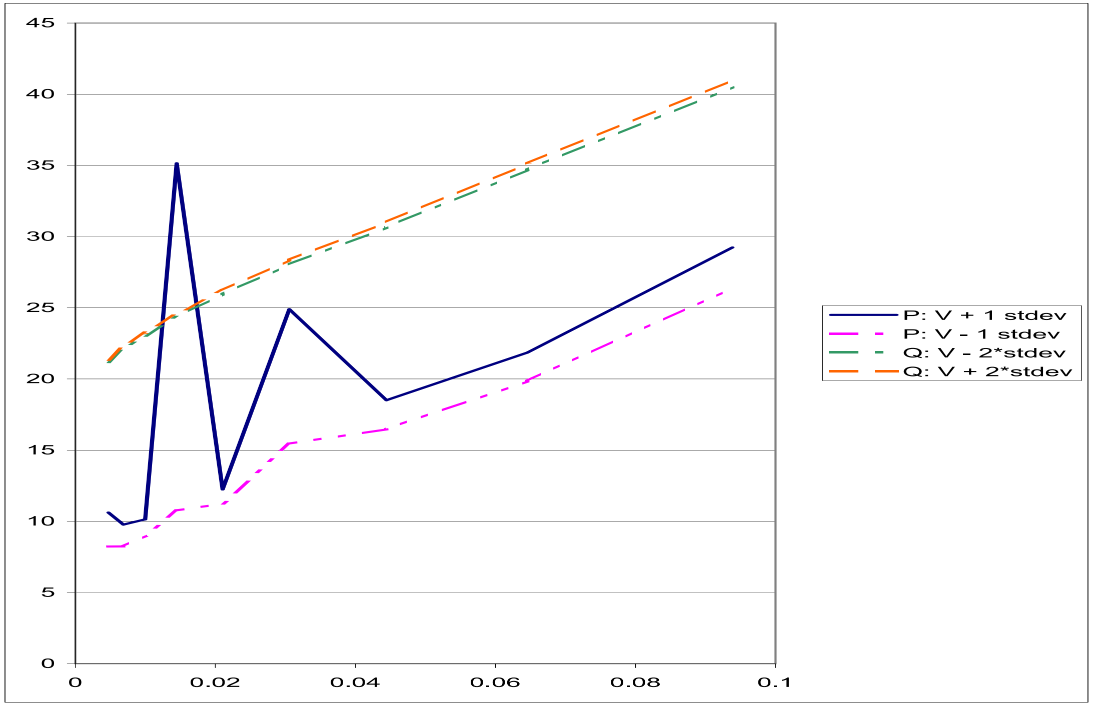
P-estimation for the value of the one-period riskless bond. The three curves are the true value, and the estimated value plus or minus one standard error. Parameter values are $g = 0.0189$, $\gamma = 2$, $\sigma = 0.015$, $\kappa = 0.138457$, $r = 0.0094$ and $\Delta t = 10$. 100,000 sample paths are used for the P-measure estimates.

- Results are worse for the claim to individual dividends (also known in closed-form).



Estimation of the value of the individual consumption claims using Monte Carlo methods under both the P and Q-measures. Parameter values are $g = 0.0189$, $\gamma = 2$, $\sigma = 0.015$, $\kappa = 0.138457$, and $r = 0.0094$. 100,000 sample paths are used for the P-measure estimates, whereas only 10,000 sample paths are used for the Q-measure estimates.

- However, simulating under the risk-neutral measure (analogous to 'importance sampling' idea) seems to improve results.



Estimation of the price-consumption ratio using Monte Carlo methods under both the P and Q-measures. Parameter values are $g = 0.0189$, $\gamma = 2$, $\sigma = 0.015$, $\kappa = 0.138457$, and $r = 0.0094$. 100,000 sample paths are used for the P-measure estimates, whereas only 10,000 sample paths are used for the Q-measure estimates.

A note on exchange economy prices with recursive utility

- A few recent papers (BY, Campbell et alii.) solve for asset prices (or optimal portfolio) in models where representative agent has recursive utility using Campbell-Shiller Log-linearization.
- Instead we (CDG 2005) propose improvement to their approximation.
- Consider simple Channel I dynamics:

$$\begin{aligned} dc_t &= (\mu_c + x(t))\, dt + \sigma_c\, dz_c(t) \\ dx_t &= -\kappa_x x\, dt + \sigma_x\, dz_x(t), \end{aligned}$$

- Representative agent has recursive utility:

$$J(t) = \mathrm{E}_t[\int_t^\infty f(c_s, J_s) ds]$$

where the 'normalized' aggregator function:

$$f(c, J) = \frac{\beta u_\rho(c)}{((1-\gamma)J)^{1/\theta - 1}} - \beta\theta J$$

where $\theta = \frac{1-\gamma}{1-\rho}$.

- The pricing kernel in this economy is simply:

$$\Pi(t) = e^{\int_0^t f_J(c_s, J_s) ds} f_c(c_t, J_t) \qquad (13)$$

- We obtain an explicit solution for the value function (and the pricing kernel):

$$J(t) = u_\gamma(c_t)(\beta I(x_t))^\theta$$

where $I(x)$ is equilibrium price-consumption ratio. It solves a non-linear ODE:

$$I\left((1-\gamma)(\mu_c + x) + (1-\gamma)^2\frac{\sigma_c^2}{2} - \beta\theta\right) + \theta(\frac{\sigma_x^2}{2}I_{xx} - \kappa_x x I_x) + \theta(\theta-1)\frac{(I_x)^2}{I}\frac{\sigma_x^2}{2} = -\theta$$

- Continuous time equivalent of Campbell-Shiller log-linearization is to guess that $I(x) = e^{A+Bx}$ and 'replace' the RHS by an exponential function:

$$f(x) = (n_0 + n_1 x)e^{A+Bx} \approx -\theta$$

where n_0, n_1 are chosen 'locally' so that $f(\mathrm{E}[x]) = -\theta$, and $f'(\mathrm{E}[x]) = 0$.

- Instead, we propose an approximation that is global in the sense that it uses information from the unconditional distribution of the state variables ($\sim$ 'Feynman-Kac'). Choose a parametric function $f(x;\Theta)$ such that:

$$\min_{\Theta} \quad \mathrm{E}[(f(x_t;\Theta) + \theta)^2] \text{ subject to}$$

$$f(x) = I\left((1-\gamma)(\mu_c + x) + (1-\gamma)^2\frac{\sigma_c^2}{2} - \beta\theta\right) + \theta(\frac{\sigma_x^2}{2}I_{xx} - \kappa_x x I_x) + \theta(\theta-1)\frac{(I_x)^2}{I}\frac{\sigma_x^2}{2}$$

- Essentially treat ODE as constraint and choose Θ in 'global' sense.
- For case, where $\gamma = \rho$ (Constant relative risk-aversion) we can compare both approximation (CS vs. CDG) vs. the exact closed form:

$$I(x) = \int_0^\infty e^{A(\tau)-B(\tau)x} d\tau$$

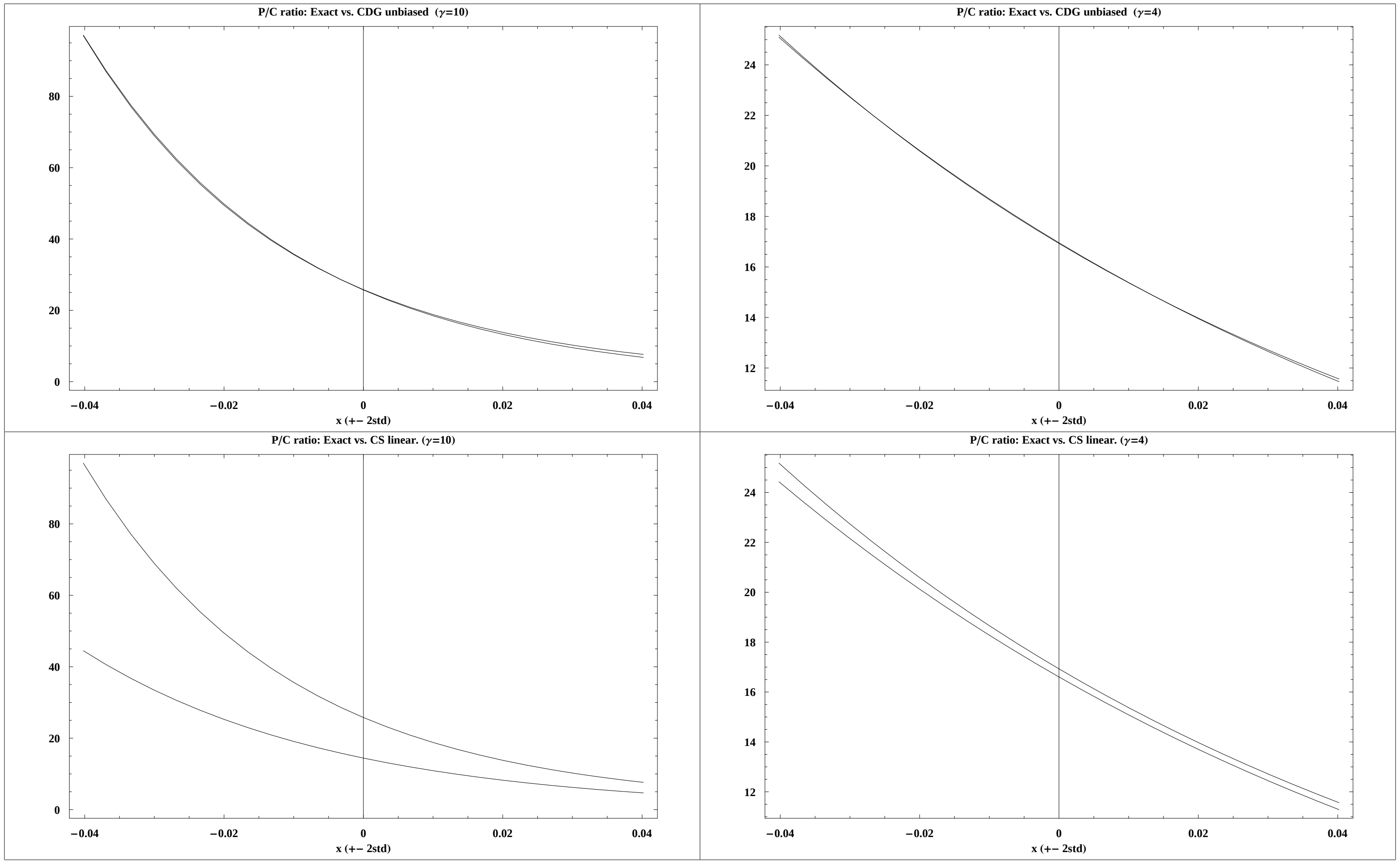


Figure 4: **Comparison of Approximations vs. Exact solution Equilibrium Price Dividend ratio**

- Our approach can be extended in several directions:
  - Refine approximation by going to higher order.
  - Works for incomplete market portfolio choice problem.
  - Can be extended to solve finite maturity portfolio choice problems.