Modeling Commodity Futures: Reduced Form vs. Structural Models

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Presentation based on the following papers:

Stochastic Convenience Yield Implied from Interest Rates and Commodity Futures

The Journal of Finance

joint with

Jaime Casassus

Pontificia Universidad Católica de Chile

Equilibrium Commodity Prices with Irreversible Investment and Non-Linear Technologies

joint with

Jaime Casassus

Pontificia Universidad Católica de Chile

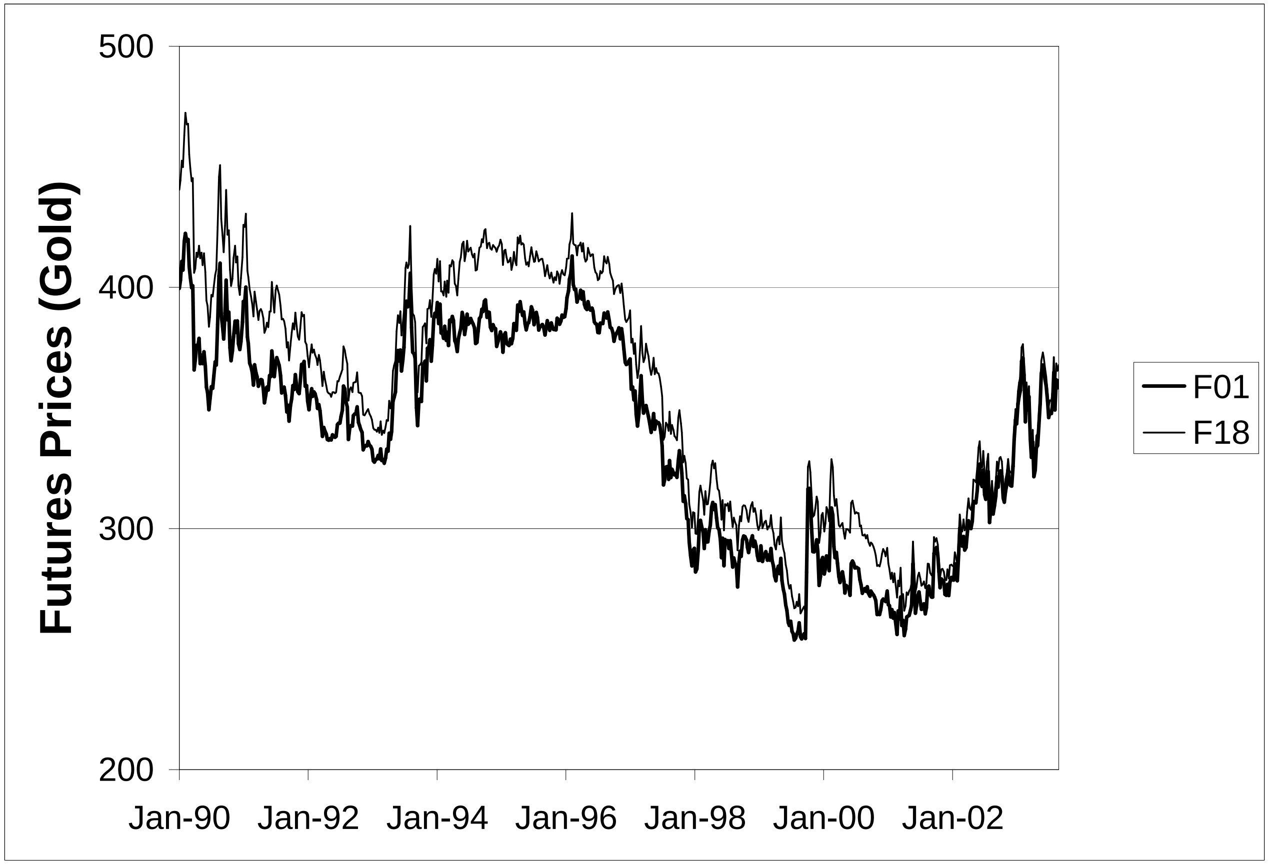
Bryan Routledge

Carnegie Mellon University

Motivation

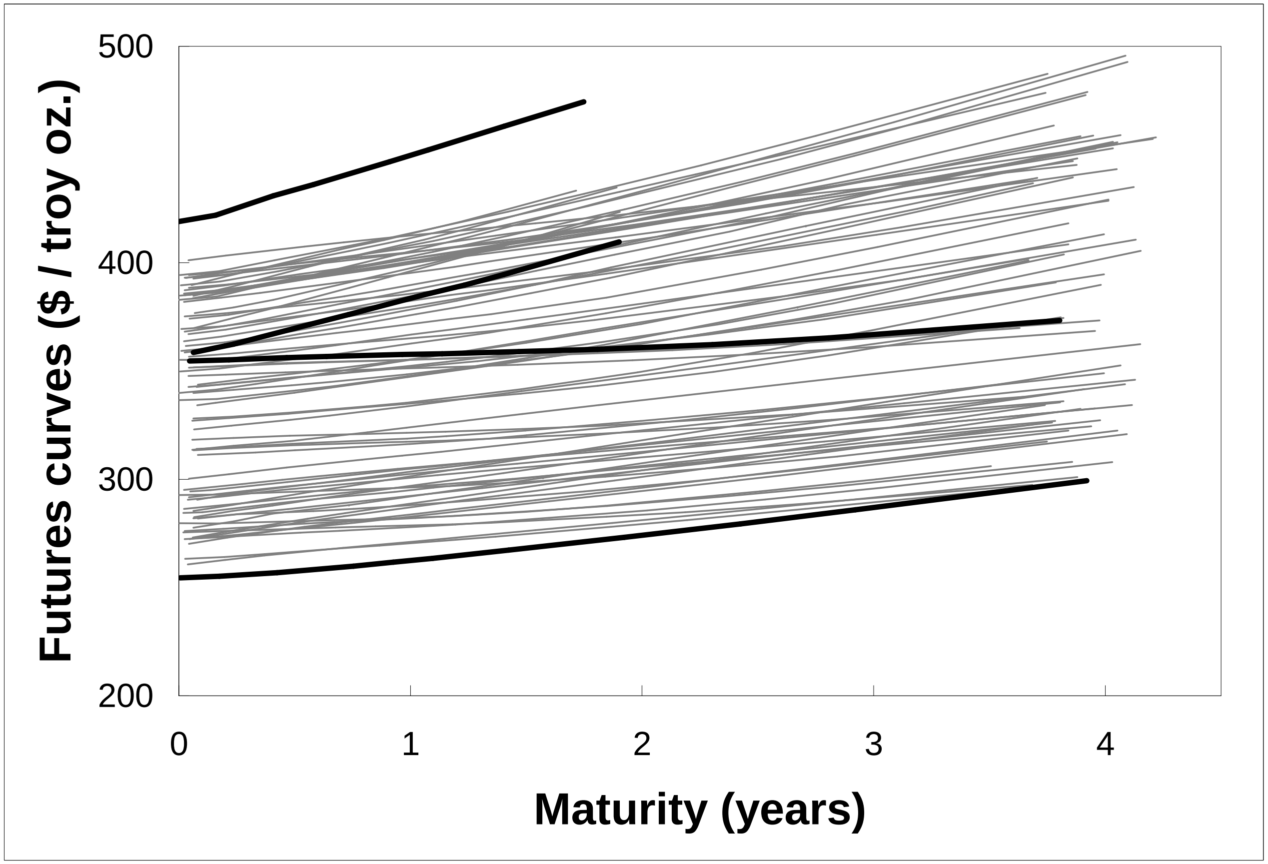
- Dramatic growth in commodity markets
  - trading volume, variety of contracts, number of underlying commodities
- Growth has been accompanied with high levels of volatility
- Commodity spot and futures prices exhibit empirical regularities $\neq$ financial securities (Fama and French (1987), Bessembinder et al. (1995))
  - Mean-reversion,
  - Convenience Yield,
  - 'Samuelson effect.'
- Understanding the behavior of commodity prices important for:
  - Macroeconomic policies
  - Valuation of derivatives (short term)
  - Valuation and exercise of real options (long term)

Gold prices



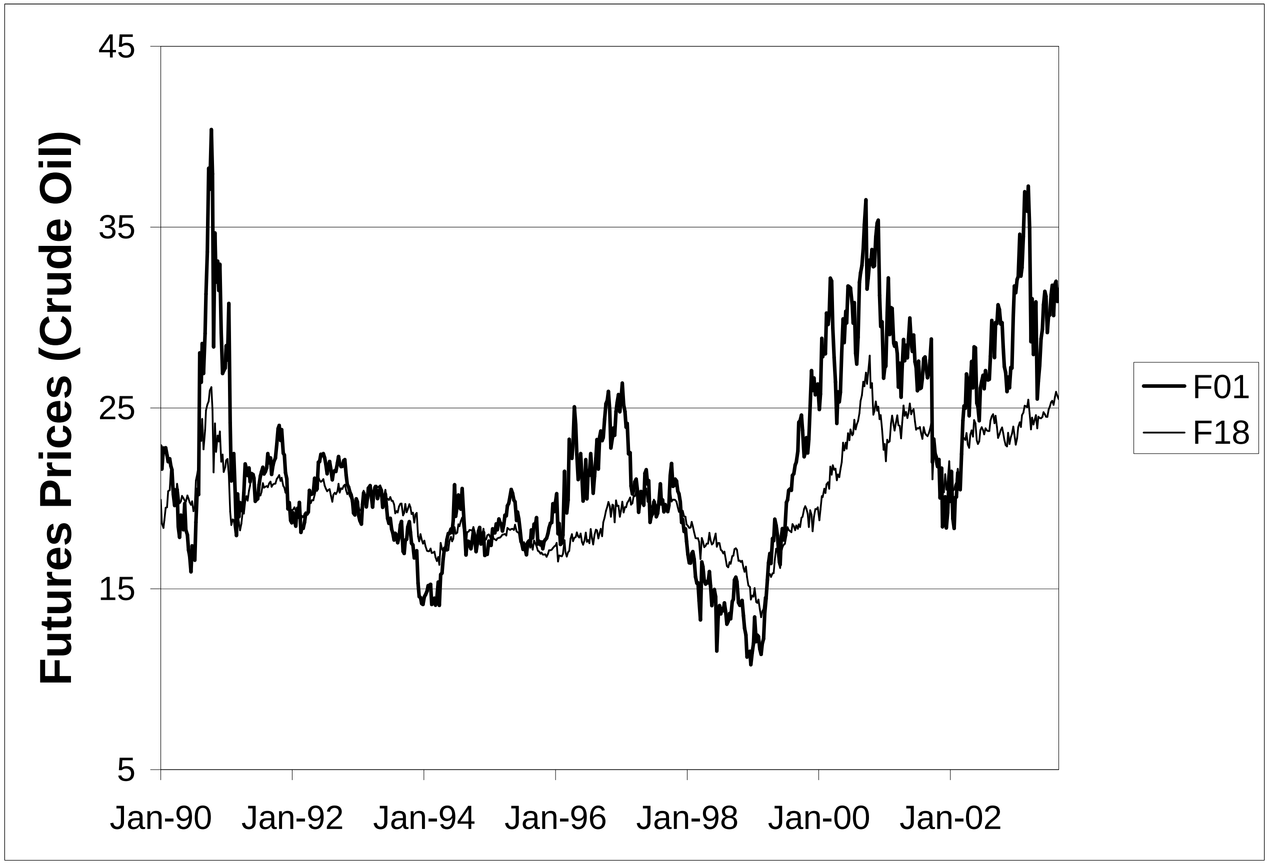
- Stylized facts of spot and futures prices
  → Mean reversion (?), volatility

Gold futures prices



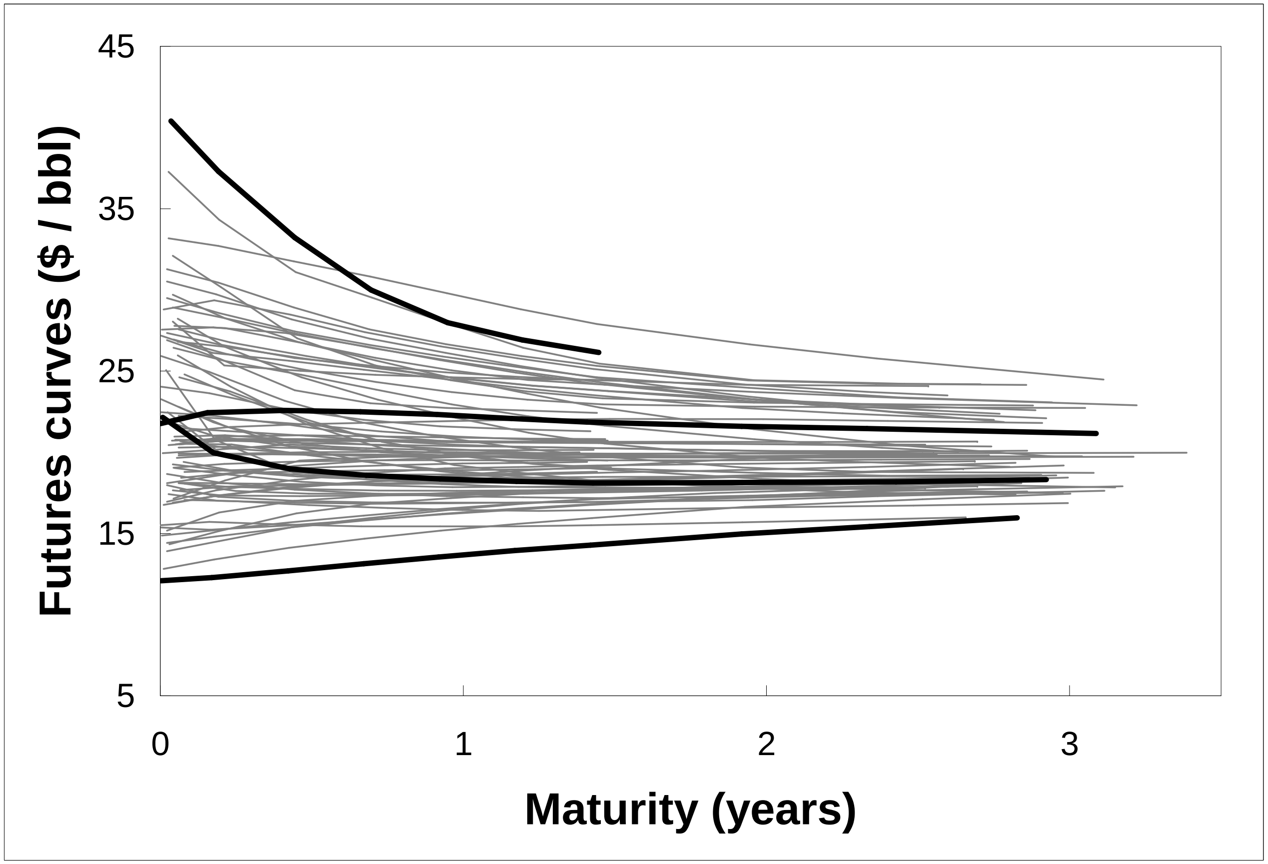
- Stylized facts of futures prices
  - → Weak backwardation (?) & contango
  - → Futures curve not uniquely determined by spot (non Markov)
  - → Samuelson effect (?)

Crude oil prices



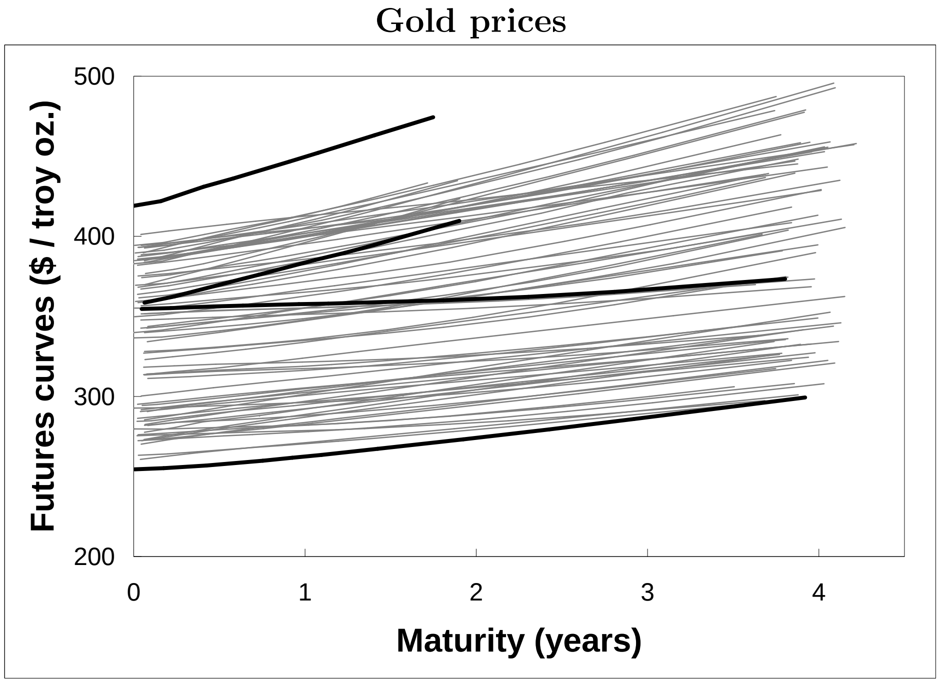
- Stylized facts of spot and futures prices
  → Mean reversion, heteroscedasticity, positive skewness (upward spikes)

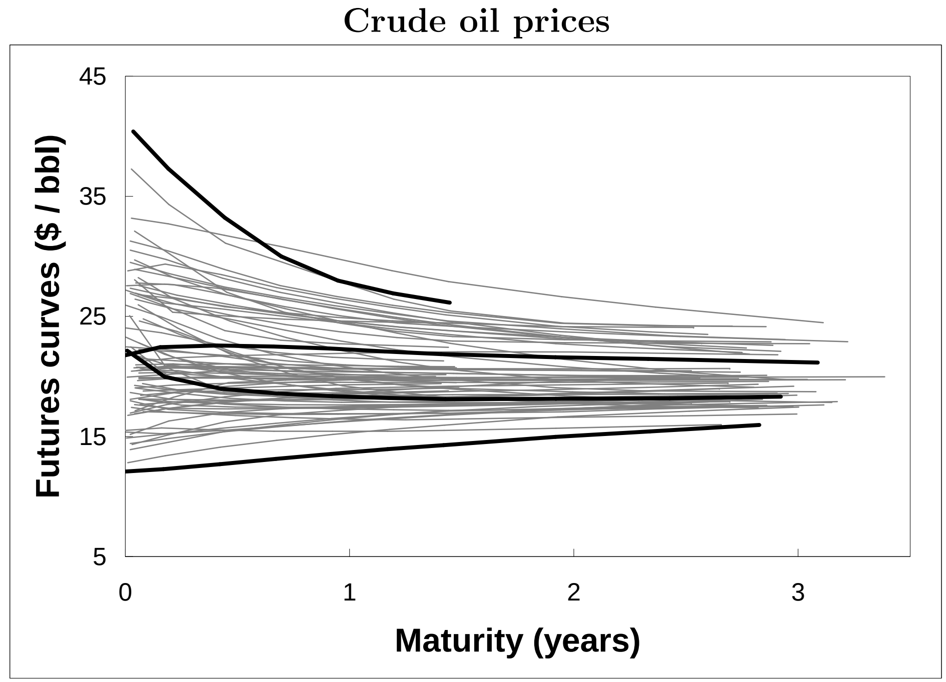
Crude oil futures prices



- Stylized facts of futures prices
  - → Strong (63%) & weak (83%) Backwardation & Contango
  - → Non Markov spot price
  - → Volatility of Futures prices decline with maturity ('Samuelson effect')

Short vs. long maturity futures





- Absence of arbitrage (in frictionless market) implies

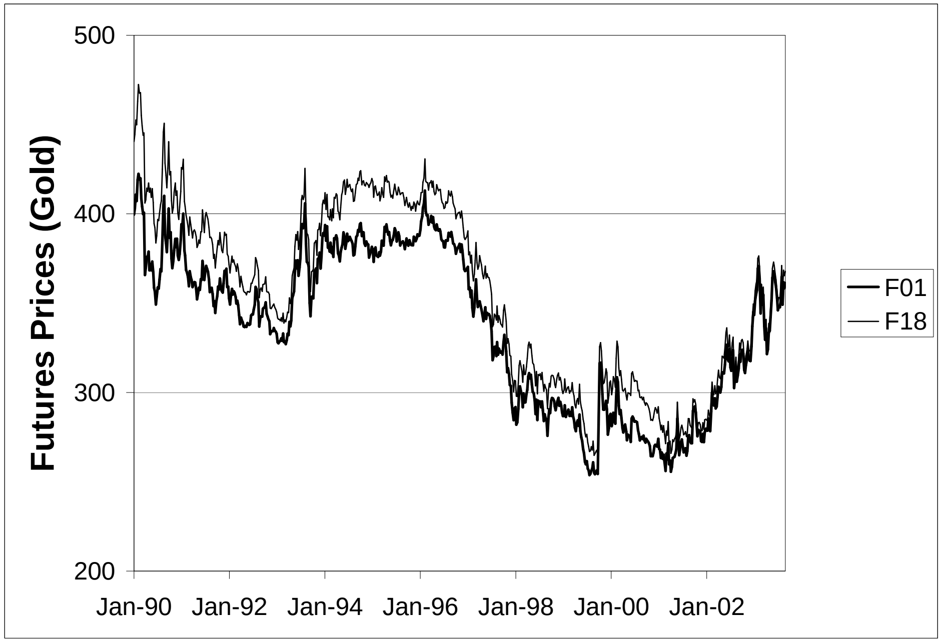
$$F(t,T) = e^{r(t,T)-\delta(t,T)}\,S(t)$$

where $r(t,T)$ is the interest rate and $\delta(t,T)$ is the net convenience yield

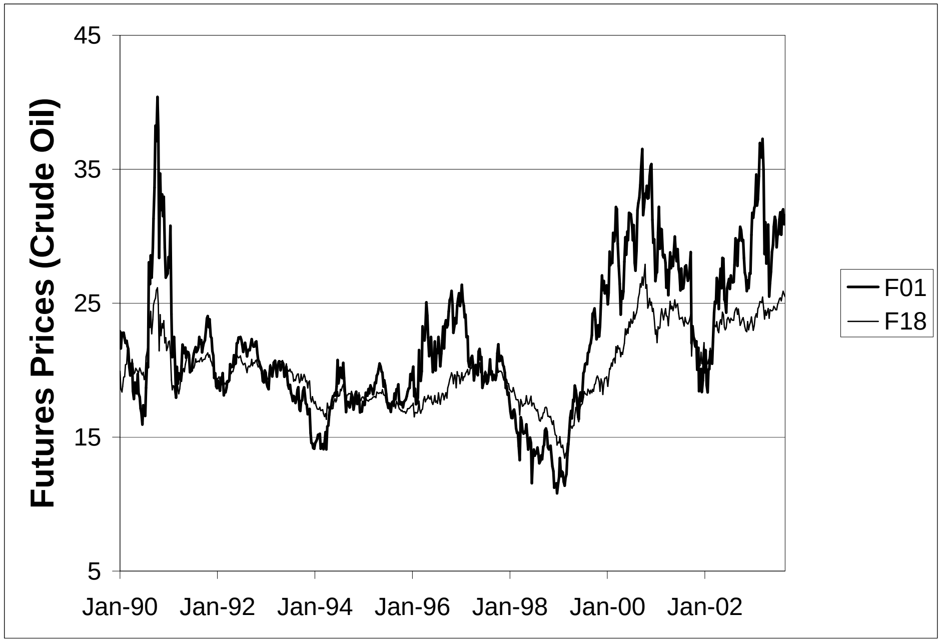
- Convenience yield ≈ ‘dividend’ accruing to holder of commodity (but not of futures)
- Time varying convenience yield ($\delta(t,T) > 0$) necessary to explain backwardation (and possibly mean-reversion? Bessembinder et al. (95))

Expected spot vs. futures prices

Gold prices



Crude oil prices



- Gap between expected spot and futures price is a *risk premium*

$$E_t[S(T)] = F(t,T) + \beta(t,T)$$

- Time-varying expected return (i.e., risk premium), $\beta(t,T)$, can explain mean-reversion in spot if $\beta(t,T) \uparrow$ when $S_t \downarrow$. (Fama & French (88))
- Predictability of futures for expected spots?

Objectives

- Present a three-factor ('maximal') model for commodity prices that nests all Gaussian models (Brennan (91), Gibson Schwartz (90), Ross (97), Schwartz (97), Schwartz and Smith (00))
- Study stylized facts of commodity spot and futures prices prices
- Examine sources of mean-reversion in commodity prices
  - maximal convenience yield vs. time varying risk premia
- Test to what extent the restrictions in existing models are binding
- Illustrate economic significance of maximal model
  - option pricing vs. risk management decisions
- Compare commodities of different nature
  - productive assets: crude oil and copper
  - financial assets: gold and silver

Main results for reduced-form model

→ Three-factors are necessary to explain dynamics of commodity prices

→ In the maximal model the convenience yield is a function of the spot price, interests rates and an idiosyncratic factor

→ Convenience yields are positive and increasing in price level and interest rates (in particular for crude oil and copper)

→ Convenience yields are economically significant for derivative pricing

→ Time-varying risk premium seem more significant for store-of-value assets

→ Risk premia of prices is decreasing in the price level (counter-cyclical)

→ Economically significant implications for risk management (VAR)

Maximal model for commodity prices

- *Maximal*: most general (within certain class) model that is econometrically identified
- Canonical representation of a three-factor Gaussian model for spot prices

$$X(t) := \log S(t) = \phi_0 + \phi_Y^\top Y(t)$$

- $Y(t)$ is a vector of three latent variables

$$dY(t) = -\kappa^Q Y(t)dt + dZ^Q(t)$$

- Maximality implies
  - κ^Q is a lower triangular matrix
  - dZ^Q is a vector of independent Brownian motions
- Futures prices observed for all maturities, obtained in closed-form (Langetieg 80):

$$F^T(t) = \mathrm{E}_t^Q\left[e^{X(T)}\right]$$

Interest rates and convenience yields

- Interest rates follow a one-factor process

$$r(t) = \psi_0 + \psi_1 Y_1(t)$$

- Bond prices observed across maturities obtained in closed-form (Vasicek 77):

$$P^T(t) = \mathrm{E}_t^Q[e^{-\int_t^T r(s)ds}]$$

- Absence of arbitrage implies that (this defines the convenience yield!):

$$E_t^Q[dS(t)] = (r(t) - \delta(t))S(t)dt$$

- The implied convenience yield in the maximal model is affine in $Y(t)$

$$\delta(t) = \psi_0 - \frac{1}{2}\phi_Y^\top \phi_Y + \psi_1 Y_1(t) + \phi_Y^\top \kappa^Q Y(t)$$

Economic representation of the maximal model

- The 'maximal' model is

$$\begin{aligned}
\delta(t) &= \widehat{\delta}(t) + \boxed{\alpha_X} X(t) + \boxed{\alpha_r} r(t) \\
dX(t) &= \left(r(t) - \delta(t) - \frac{1}{2}\sigma_X^2 \right) dt + \sigma_X dZ_X^Q(t) \\
d\widehat{\delta}(t) &= \kappa_{\widehat{\delta}}^Q \left(\theta_{\widehat{\delta}}^Q - \widehat{\delta}(t) \right) dt + \sigma_{\widehat{\delta}} dZ_{\widehat{\delta}}^Q(t) \\
dr(t) &= \kappa_r^Q \left(\theta_r^Q - r(t) \right) dt + \sigma_r dZ_r^Q(t)
\end{aligned}$$

  and the Brownian motions are correlated

- The maximal convenience yield model nests most models in the literature
  e.g. $\alpha_X = \alpha_r = 0 \Rightarrow$ three-factor model of Schwartz (1997)

- $\alpha_X > 0$: mean-reversion in prices under the risk-neutral measure (Samuelson effect)
  – consistent with futures data (empirical) and with 'Theory of Storage' models (theoretical)

- α_r: convenience yield may depend on interest rates
  – if holding inventories becomes costly with high interest rates then $\alpha_r > 0$

Specification of risk premia necessary for estimation

- Risk-premia is a linear function of states variables (Duffee (2002))

$$\beta(t) = \begin{pmatrix} \beta_{0r} \\ \beta_{0\widehat{\delta}} \\ \beta_{0X} \end{pmatrix} + \boxed{\begin{pmatrix} \beta_{rr} & 0 & 0 \\ 0 & \beta_{\widehat{\delta}\widehat{\delta}} & 0 \\ \beta_{Xr} & \beta_{X\widehat{\delta}} & \beta_{XX} \end{pmatrix}} \begin{pmatrix} r(t) \\ \widehat{\delta}(t) \\ X(t) \end{pmatrix}$$

and

$$dZ^Q(t) = \sigma^{-1}\beta(t)dt + dZ^P(t)$$

- Time-varying risk-premia is another source of mean-reversion under historical measure

  $\longrightarrow$ Mean-reversion in commodity prices: $\kappa_X^P = \alpha_X - \beta_{XX}$

  $\longrightarrow$ Mean-reversion in convenience yield: $\kappa_{\widehat{\delta}}^P = \kappa_{\widehat{\delta}}^Q - \beta_{\widehat{\delta}\widehat{\delta}}$

  $\longrightarrow$ Mean-reversion in interest rates: $\kappa_r^P = \kappa_r^Q - \beta_{rr}$

Data and empirical methodology

- Weekly data of futures contracts on crude oil, copper, gold and silver
  - Jan-1990 to Aug-2003
  - with maturities {1,3,6,9,12,15,18} months + some longer contracts
- Build zero-coupon bonds for same period of time
  - with maturities {0.5,1,2,3,5,7,10} years
- Maximum likelihood estimation with time-series and cross-sectional data
  - state variables $\{r, \widehat{\delta}, X\}$ are not directly observed, but futures prices and bonds are observed
  - assume some linear combination of futures and bonds to be observed without error
  - invert for the state variables from observed data
  - first two Principal Components of futures curve are perfectly observed
  - first Principal Component of term structure of interest rate is perfectly observed
  - remaining PCs are observed with errors that follow AR(1) process

Empirical results: sources of mean-reversion

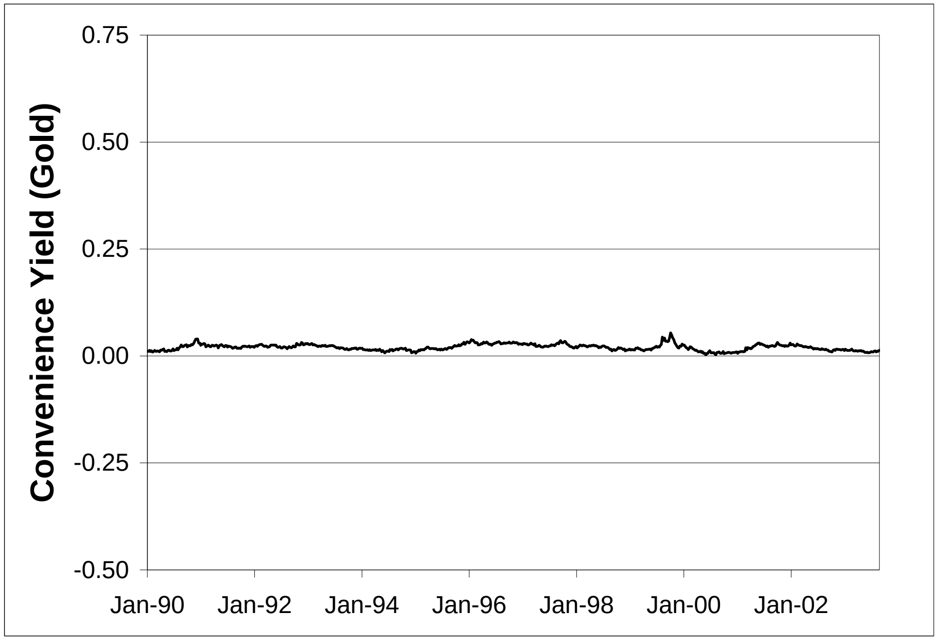
- Convenience yields
  - α_X is significant and positive, and highest for Oil and lowest for Gold
  - α_r is significant and positive for Oil and Gold
- Maximum-likelihood parameter estimates for the model

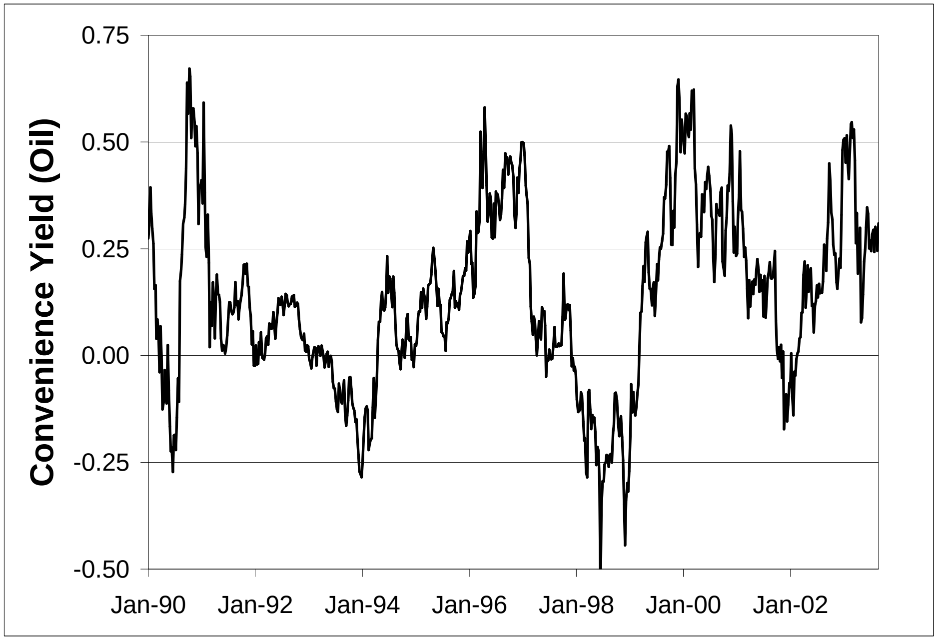
Parameter	Gold Estimate (Std. Error)	Crude Oil Estimate (Std. Error)
α_X	0.000 (0.000)	0.248 (0.010)
α_r	0.332 (0.046)	1.764 (0.083)

- Likelihood ratio test (Prob$\{\chi_2^2 \geq 5.99\} = 0.05$)

Restriction	Gold	Crude Oil
$\alpha_r = \alpha_X = 0$	5.60	1047.20

Empirical results: sources of mean-reversion





- Unconditional moments (convenience yield)

Unconditional Moments	Gold	Crude Oil
$E[\delta]$	0.009	0.109
$\mathrm{Stdev}(\delta)$	0.010	0.210

Empirical results: sources of mean-reversion

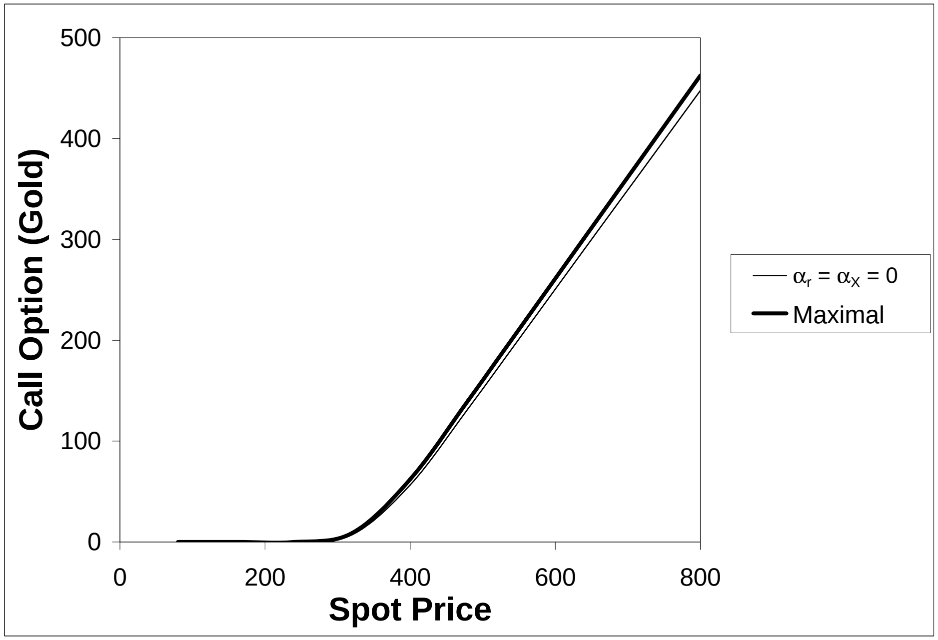
- Time-varying risk premia
  - For metals most risk-premia coefficients associated with prices are significant
  - β_{XX} is always negative, higher mean-reversion under historical measure
- Maximum-likelihood parameter estimates for the model

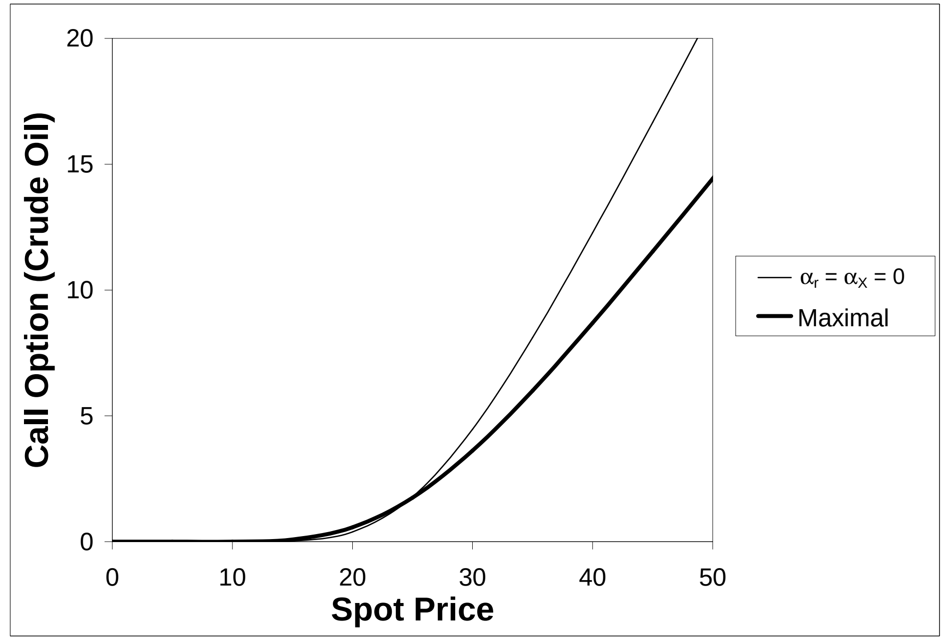
Parameter	Gold Estimate (Std. Error)	Crude Oil Estimate (Std. Error)
β_{0X}	1.858 (1.539)	1.711 (0.964)
β_{Xr}	-2.857 (2.452)	
β_{XX}	-0.301 (0.271)	-0.498 (0.313)

- Likelihood ratio test (Prob$\{\chi^2_5 \geq 11.07\} = 0.05$ and Prob$\{\chi^2_7 \geq 14.07\} = 0.05$)

Restriction	Gold	Crude Oil
$\beta_{1Y} = 0$	20.01	13.16
$\alpha_r = \alpha_X = 0$ and $\beta_{1Y} = 0$	23.96	1057.92

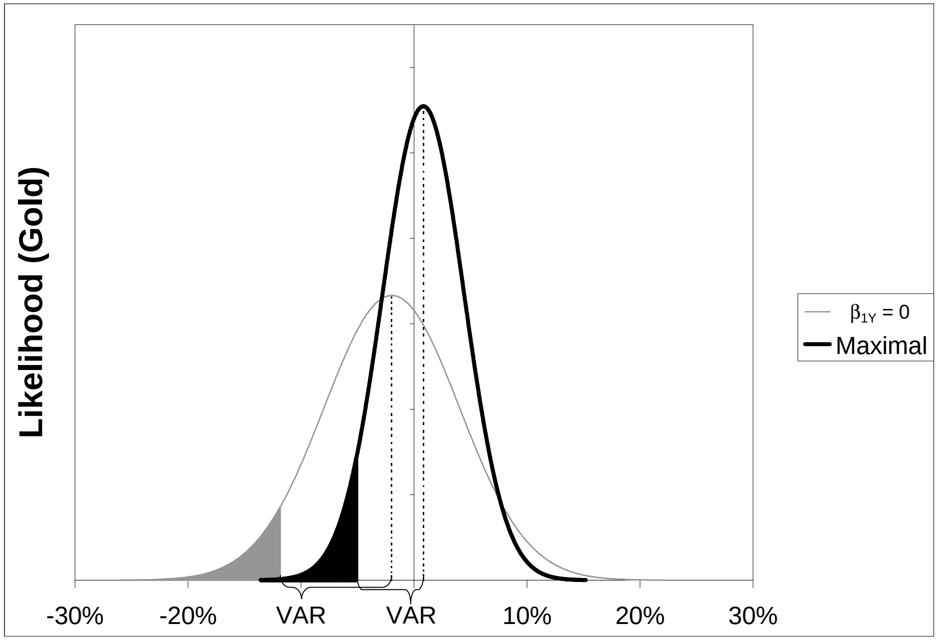
Two-year maturity European call option

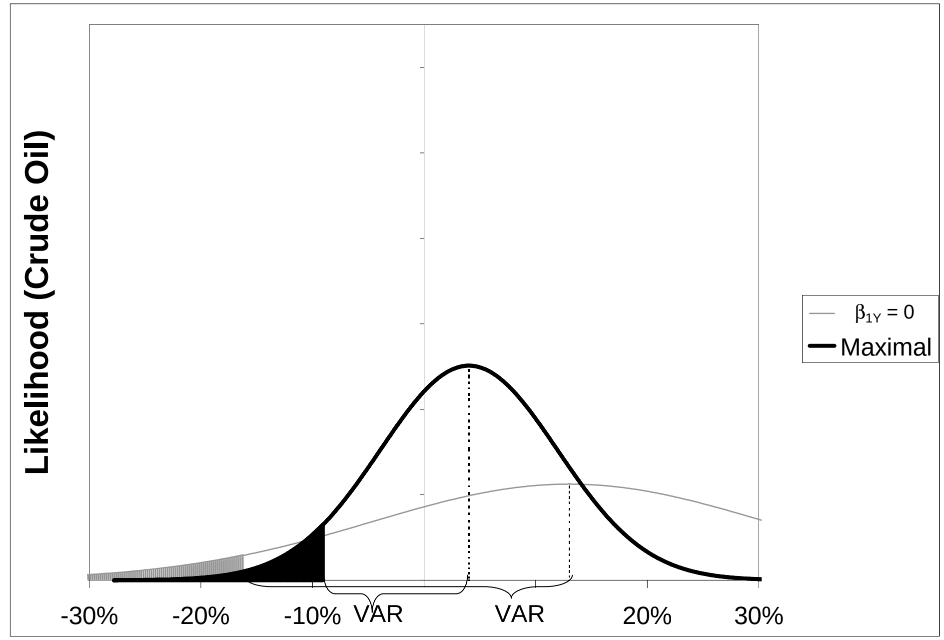




- The strike prices are
  - \$350 per troy ounce for the option on gold
  - \$25 per barrel for the option on crude oil
- Ignoring maximal convenience yield induces overestimation of call option values
  - mean-reversion (under Q) reduces term volatility which decreases option prices
  - convenience yield acts as a stochastic dividend (which increases with S_t)

Value at Risk





- Distribution of returns and VAR for holding the commodity for 5 years
- VAR calculated from the *total* return at 5% significance level
- Ignoring time-varying risk-premia induces overestimation of VAR
  - mean-reversion reduces term volatility which decreases VAR

Conclusions from reduced-form model

- Propose a maximal affine model for commodity prices
  - convenience yield and risk-premia are affine in the state variables
  - disentangles two sources of mean-reversion in prices
  - nests most existing B&S type models
    (Brennan (91), Gibson Schwartz (90), Ross (97) Schwartz (97), Schwartz and Smith (00))
- Three factors are necessary to explain dynamics of commodity prices
- Maximal convenience yield mainly driven by spot price
  - is highly significant for assets used as input to production (i.e. Oil)
  - explains strong backwardation in commodities
  - is economically significant for derivative pricing on productive assets
- Time-varying risk-premia
  - are more significant for store-of-value assets
  - risk premium of commodity prices is decreasing in the price level
  - are economically significant for risk management decisions
- Robust to allowing for jumps in spot dynamics (small impact on futures)

Potential Issues with Reduced-Form Approach

- Reduced-form model:
  - Exogenous specification of spot price process, convenience yield, and interest rate.
  - Arbitrage pricing of Futures contracts.
  - Financial engineering (Black & Scholes) data-driven approach.
- Structural Model useful benchmark to design reduced-form model:
  - endogenous modeling of Convenience Yield.
  - helpful for long horizon decisions (only short term futures data available).
  - provide theoretical foundations for reduced-form dynamics.
  - avoid data-mining, over-parametrization?

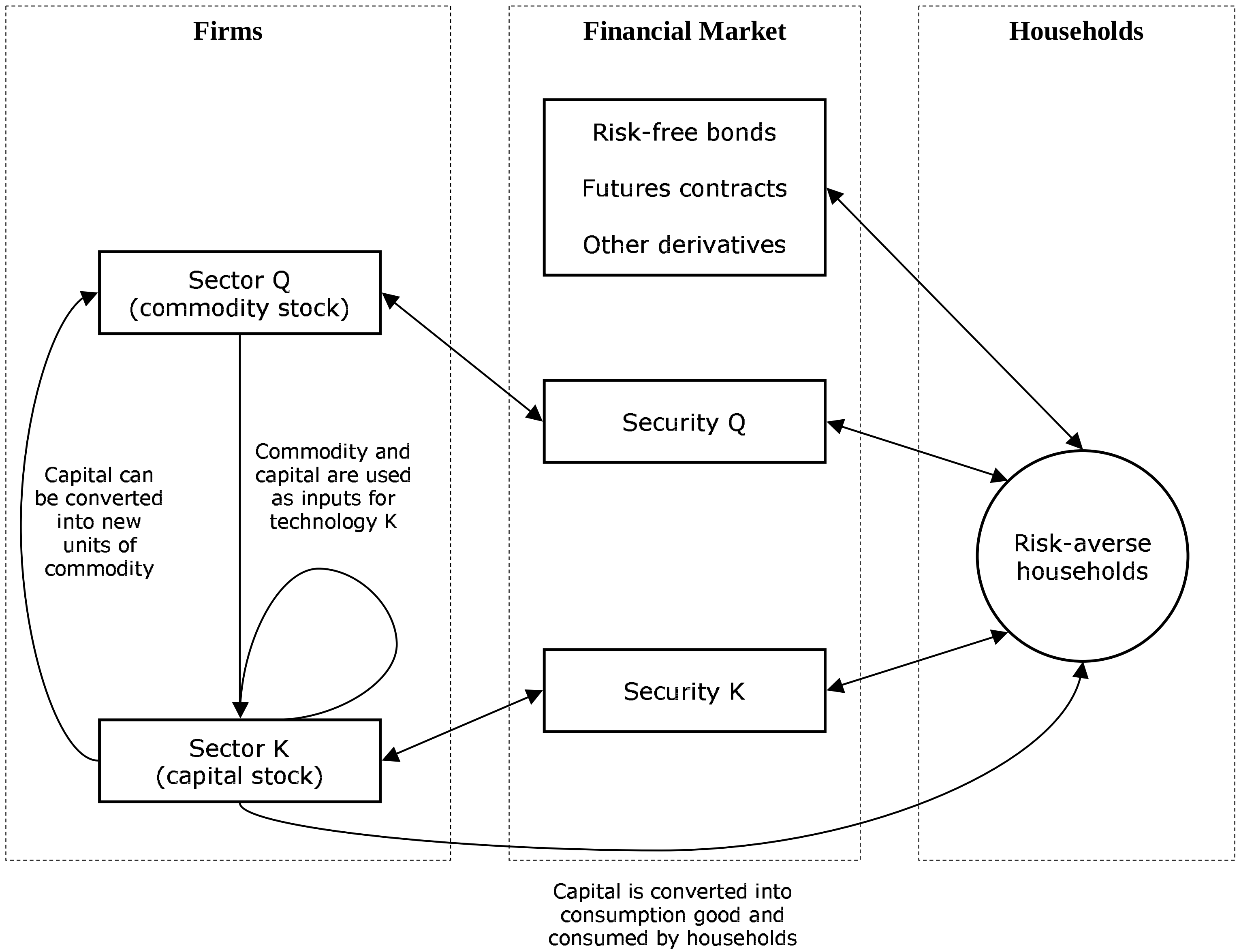
Existing Theories of Convenience Yield

- 'Theory of Storage': (Kaldor (1939), Working (1948), Brennan (1958))
  - Why are inventories high when futures prices are below the spot?
  - Inventories are valuable because help smooth demand/supply shocks.
  - Used by the Reduced-form literature to justify informally 'dividend.'
- – 'Stockout' literature (Deaton and Laroque (1992), Routledge, Seppi and Spatt (2000) (RSS))
  - Competitive rational expectation models with risk neutral agents.
  - Stockouts (i.e., non-negativity constraint on inventories) explain 'Backwardation.'
  - Inconsistent with frequency of backwardation in data?
- 'Option' approach (Litzenberger and Rabinowitz (1995))
  - Oil in the ground as a call option on oil price with strike equal to extraction cost.
  - Convenience yield must exist in equilibrium for producers to extract (i.e., exercise their call).
  - Predicts Backwardation 100% of the time (flexibility of production technology?).
- 'Technology' approach (Sundaresan and Richard (1978))
  - Convenience yield is similar to a real interest on foreign currency (where commodity is numeraire).

A Structural Model

- Equilibrium Model of a Commodity - Input to Production:
  - Oil is produced by oil wells with variable flow rate (adjustment costs).
  - Investment in new oil wells is costly (fixed and variable costs).
  - Single consumption good produced with two inputs: Oil and Consumption good.
- Main results:
  - Mean-reverting, heteroscedastic, positively skewed prices.
  - Price is non-Markov (regime switching): depends on 'distance to investment.'
  - Price can exceed its marginal production costs (fixed costs).
  - Generates Backwardation at observed frequencies.
  - Convenience yields arises endogenously (adjustment costs).
- Empirical 'implementation:'
  - Quasi-Maximum Likelihood estimation of regime-switching model.
  - Estimation is consistent with the predictions from structural model.

A general equilibrium model for commodity prices



Representative Agent in a Two-sector economy

The RA owns the technologies of sectors Q and K and maximices

$$J(K,Q) = \sup_{\{C_t, X_t, dI_t\}} E_0 \left[\int_0^\infty e^{-\rho t} U(C_t) dt \right]$$

- X_t: How much to invest in commodity sector
- I_t: When to invest in commodities
- Capital stock:

$$dK_t = \left(f\left(K_t, \bar{i} Q_t\right) - C_t\right) dt + \sigma K_t db_t - \beta(K_t, Q_t, X_t) dI_t$$

- Commodity stock:

$$dQ_t = -(\bar{i} + \delta) Q_t dt + X_t dI_t$$

- Flow rate $\bar{i}$ is fixed (∞ adjustment cost - relaxed below).
- Irreversible investment with increasing returns to scale (fixed costs)

$$\beta(K_t, Q_t, X_t) = \beta_K K_t + \beta_Q Q_t + \beta_X X_t$$

$\Rightarrow$ Investment in commodity sector is intermittent and lumpy

Solution using standard Dynamic programming

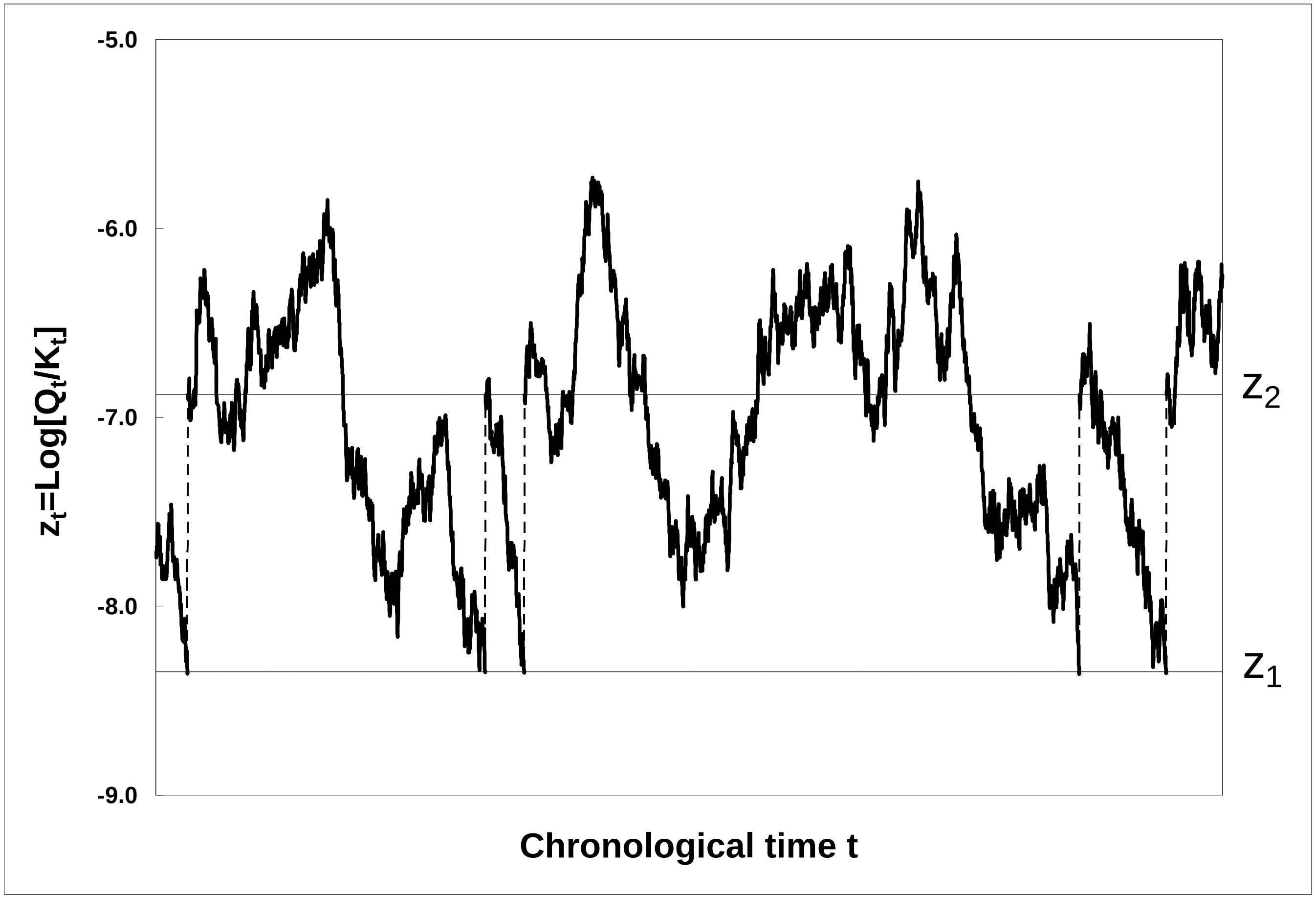
- If investment is perfectly reversible ($X_t \leq 0$ possible, no fixed costs: $\beta_K = \beta_Q = 0$ and $\beta_X > 0$) then
  - The optimal policy is simply to keep a constant Q/K ratio:

$$\frac{Q_t^*}{K_t^*} = Z^*$$

  - And Consume a constant fraction of total wealth $C_t = a(K_t + \beta_X Q_t)$.
  - The oil price is simply $S_t = \beta_X$.
- If investment is irreversible then the optimal policy is discrete and 'lumpy'
  - *no-investment* region: $J(K_t - \beta_t, Q_t + X_t) < J(K_t, Q_t)$
  - *investment* region: $J(K_t - \beta_t, Q_t + X_t) \geq J(K_t, Q_t)$
- Under some technical condition can solve for the HJB equation for optimal policy.

Simulation for state variable $z_t = \log(Q_t/K_t)$

- Regulated dynamics at the investment boundary
- $dz_t = \mu_{zt}dt - \sigma db_t + \Lambda_z dI_t$ where $\Lambda_z = z_2 - z_1$ if $dI_t = 1$



Equilibrium Asset Prices

- In equilibrium, financial assets are characterized by:

$$
\begin{aligned}
\xi_t &= e^{-\rho t} \frac{J_K(K_t, Q_t)}{J_K(K_0, Q_0)} \\
r_t &= f_K(K_t, \bar{i} Q_t) - \sigma \lambda_t \\
\lambda_t &= -\sigma \frac{K_t J_{KK}}{J_K} \\
\Lambda_B &= -\frac{\beta_K}{1 - \beta_K}
\end{aligned}
$$

- Any financial claim satisfies:

$$
\frac{dH_t}{H_t} = \mu_{Ht} dt + \sigma_{Ht} db_t + \Lambda_B dI_t
$$

- Subject to the equilibrium conditions

$$
\mu_{Ht} = r_t + \lambda_t \sigma_{Ht} \tag{1}
$$

$\Rightarrow$ All financial securities jump by fixed amount at investment date (wealth effect).

Commodity price

- The equilibrium commodity price is the transfer price from sector Q to sector K, i.e. the representative agent's shadow price for that unit

$$J(K_t, Q_t) = J(K_t + S_t\epsilon, Q_t - \epsilon) \;\text{ or }\; S_t = \frac{J_Q}{J_K}$$

- Dynamics of the commodity price process

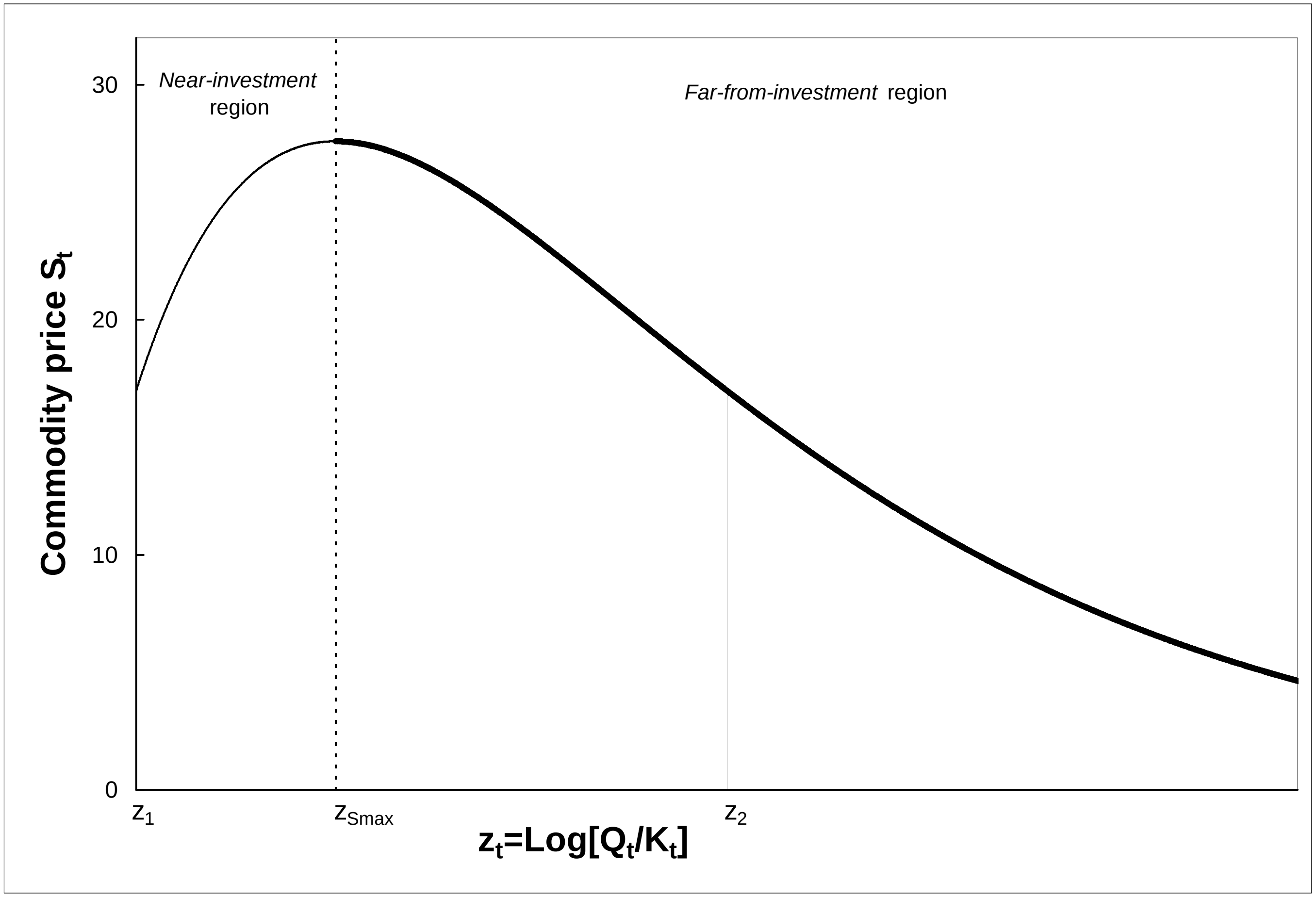
$$\frac{dS_t}{S_t} = \mu_{St}dt + \sigma_{St}db_t + \Lambda_S dI_t$$

- From first order conditions and stochastic discount factor

$$\Lambda_S \;=\; \frac{\beta_Q - \beta_K\beta_X}{1 - \beta_K}$$

- Note $\Lambda_S \neq \Lambda_B$, but does not imply arbitrage (!)

Commodity prices as a function of the state variable



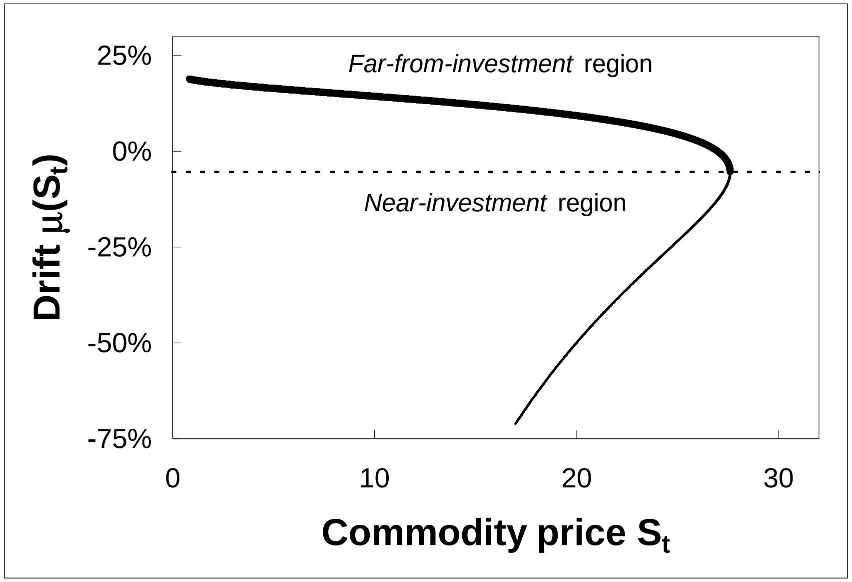
- Two opposite forces: demand / depreciation vs. investment probability
- The spot price process itself is not a Markov process
- The spot price follows a two-regime process: $\varepsilon_t = \begin{cases} 1 \text{ if } & z > z_{Smax} \\ 2 \text{ if } & z_1 < z \leq z_{Smax} \end{cases}$

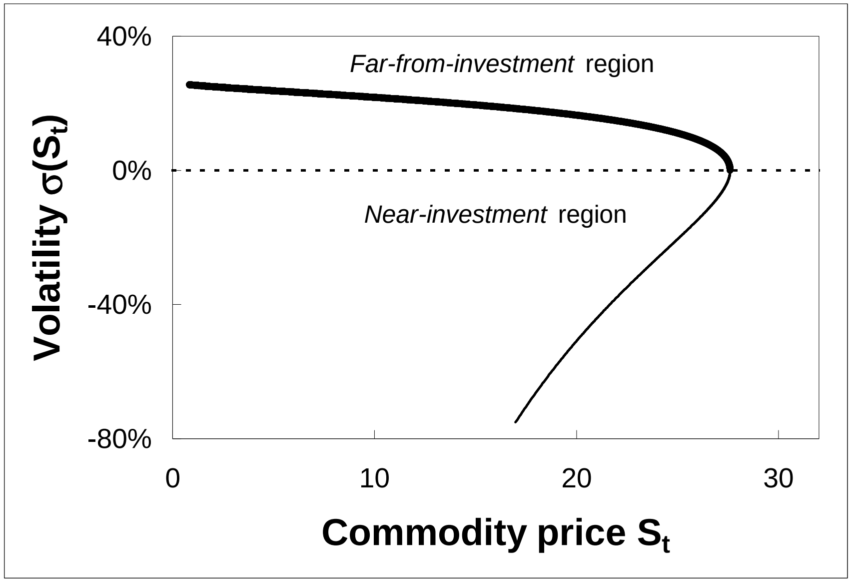
Commodity price process from the equilibrium model

- Regime switching model ($\varepsilon_t = 1, 2$)

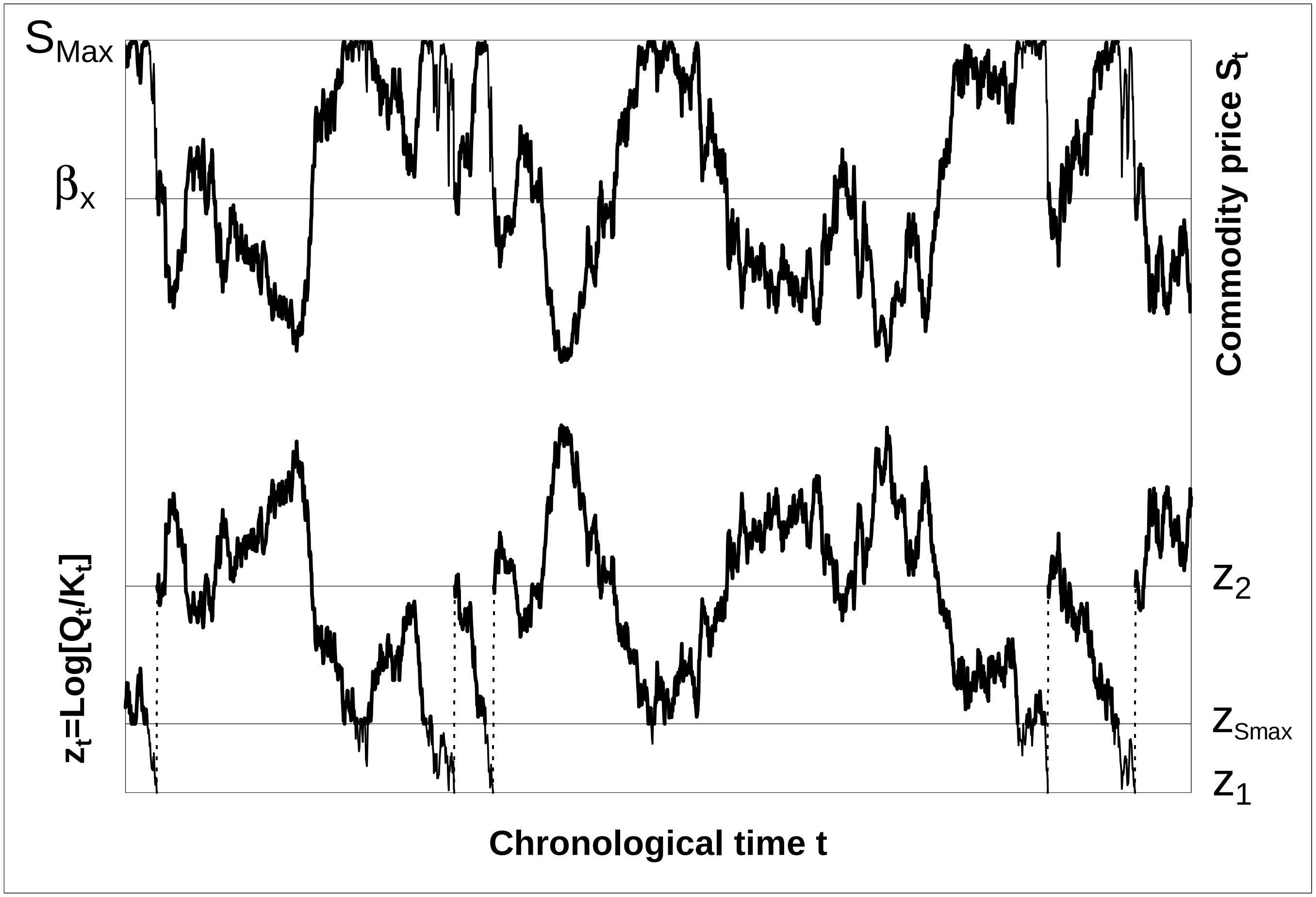
$$\frac{dS_t}{S_t} = \mu_S(S_t, \varepsilon_t)dt + \sigma_S(S_t, \varepsilon_t)db_t + \Lambda_{St}dI_t$$

- Predictions about the dynamics of the commodity price process





Simulation of commodity prices



- Price can be well above its marginal production cost (β_X)
- Mean-reversion.

Futures prices

- The stochastic process for the futures prices $H(z_t, T)$ is

$$\frac{dH_t}{H_t} = \mu_{Ht}dt + \sigma_{Ht}db_t + \Lambda_{Ht}dI_t$$

subject to the equilibrium conditions

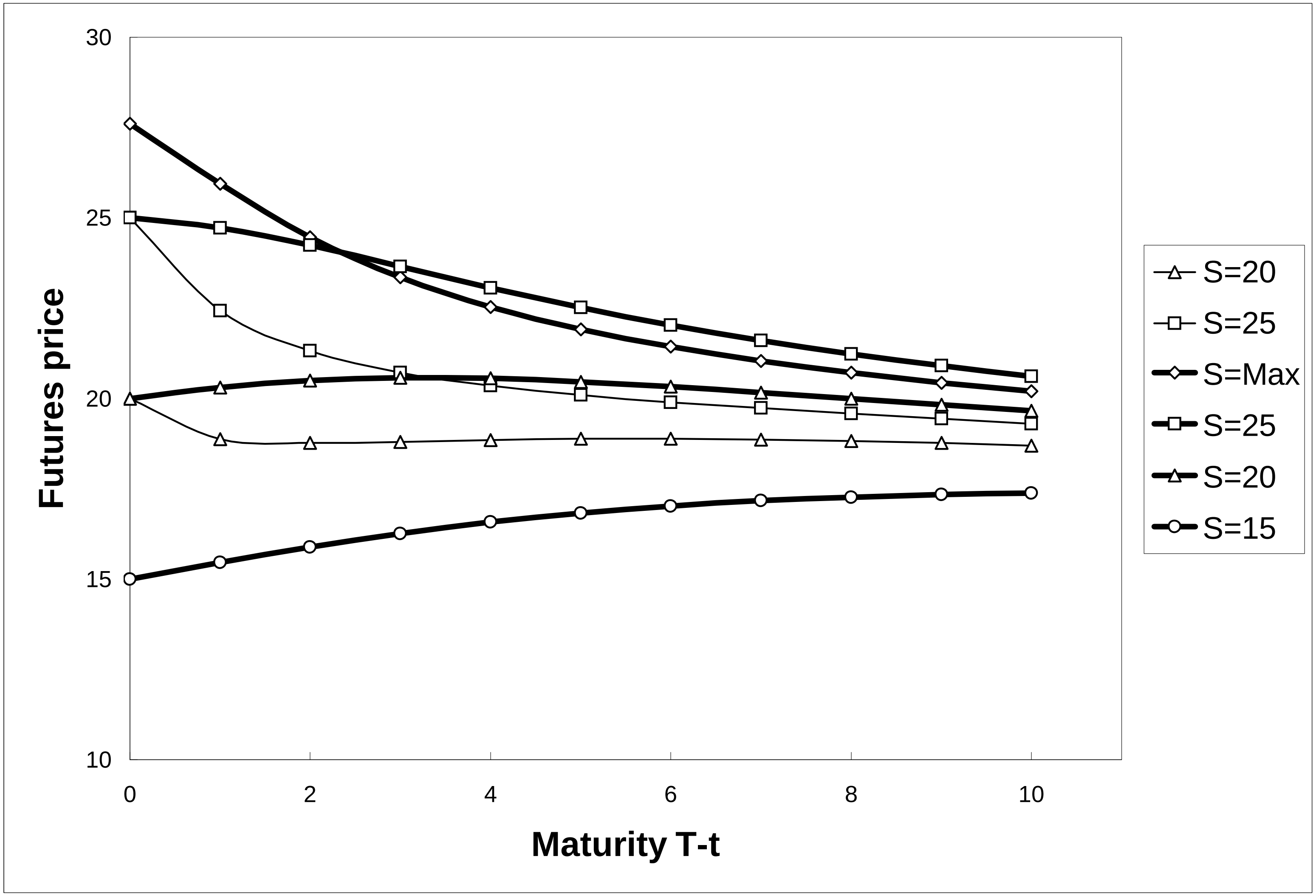
$$\frac{\mu_{Ht}}{\sigma_{Ht}} = \lambda_t = \text{market price of risk} \;\; \text{and} \;\; H(z_1, t) = H(z_2, t)$$

- The futures price satisfies the following PDE

$$\frac{1}{2}\sigma^2 H_{zz} + (\mu_z - \sigma\Lambda_b)H_z - H_t = 0$$

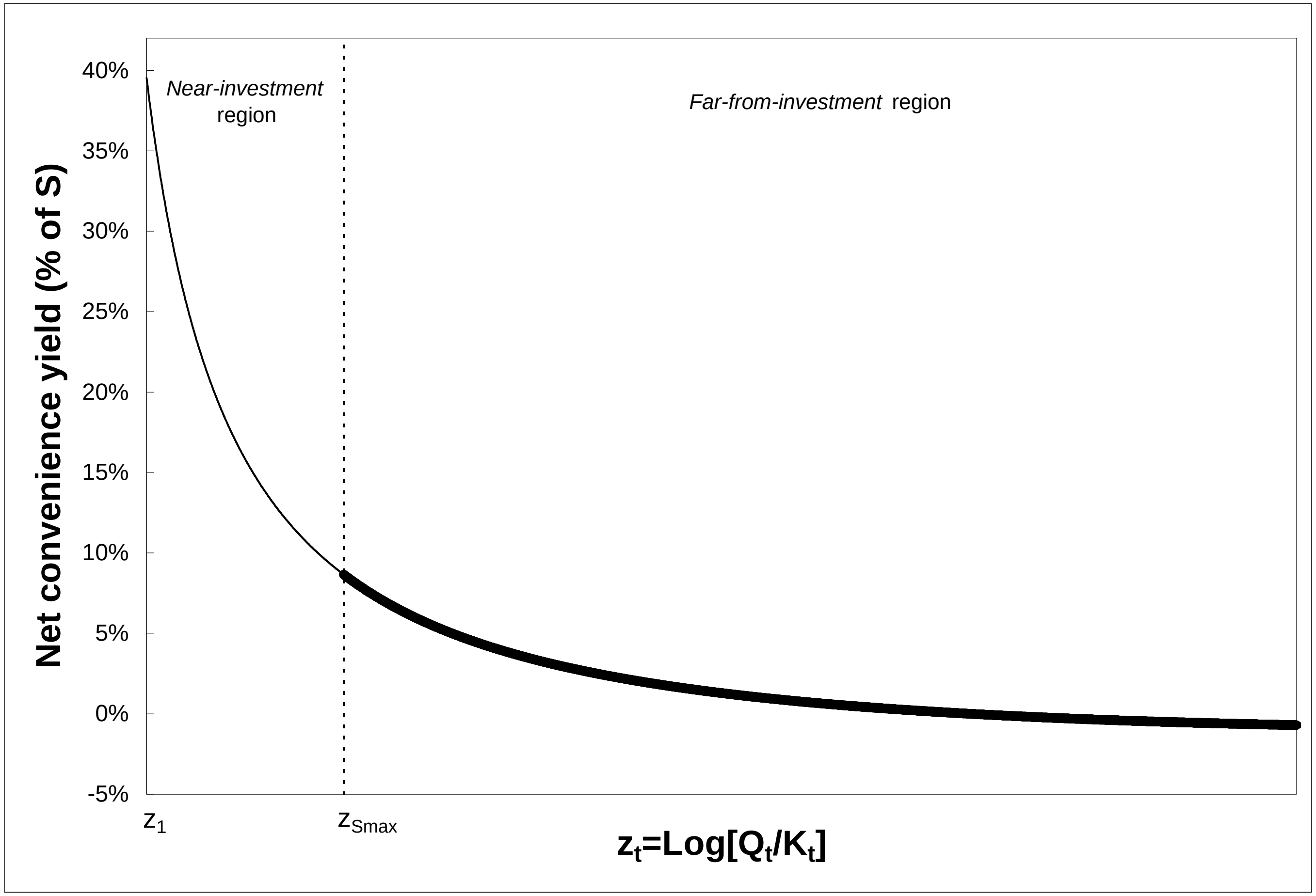
and boundary condition $H(z_t, 0) = S(z_t)$

Futures prices on the commodity for different maturities



Net convenience yield

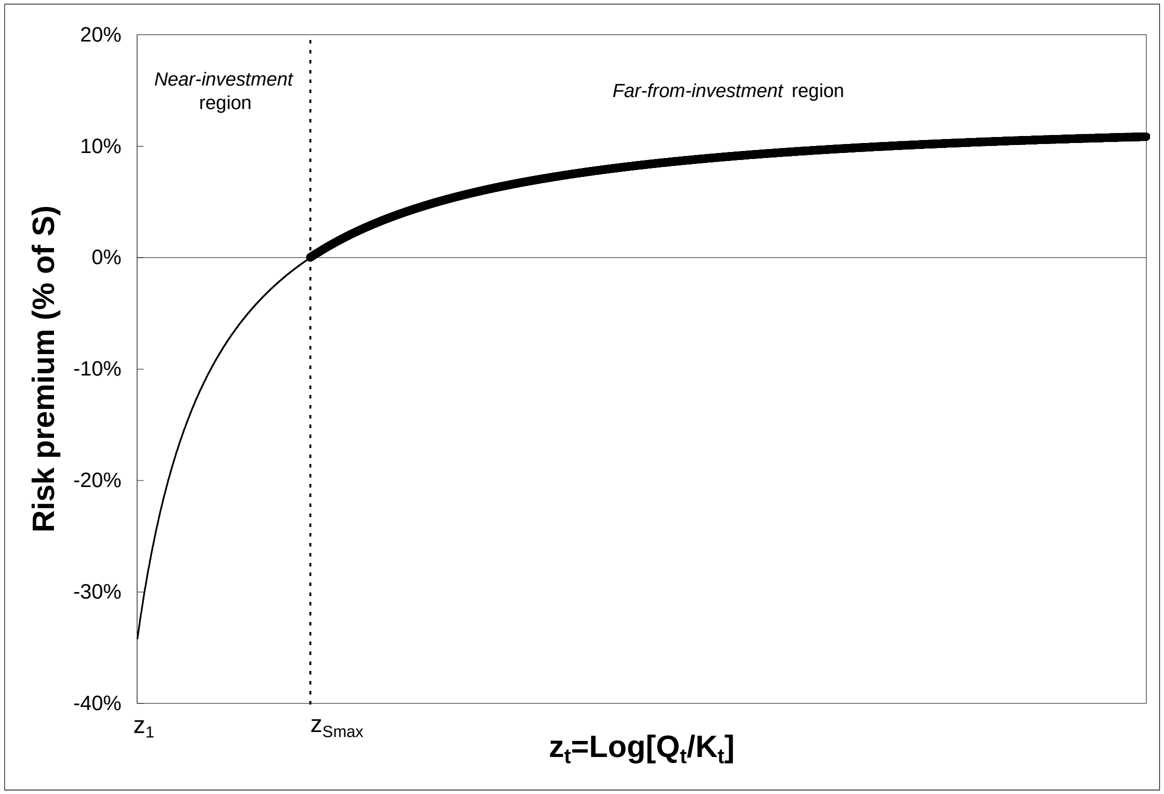
- $y_t = \frac{\bar{i}}{S_t}(f_q(K_t, \bar{i}Q_t) - S_t) - \delta$



Risk premium for commodity prices

- Risk premium is: $\sigma_{St}\lambda_t = \gamma \operatorname{cov}\left(\frac{dS_t}{S_t}, dC_t\right)$

$$\frac{dS_t}{S_t} = (r_t - y_t + \sigma_{St}\lambda_t)dt + \sigma_{St}dw_t + \Lambda_{St}dI_t$$



- consistent with empirical findings

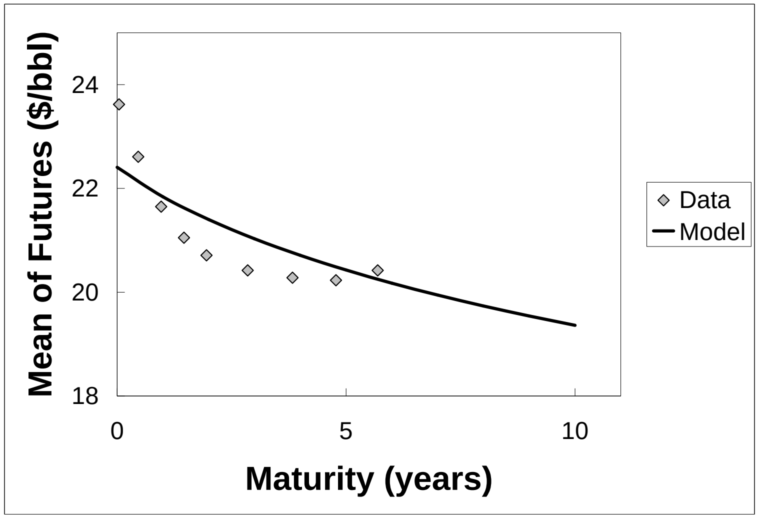
Calibration of Model Parameters

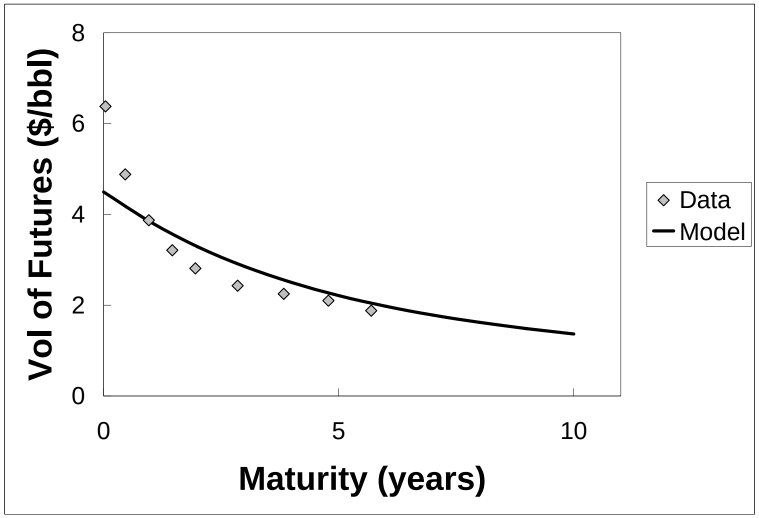
- Fix $\eta = 0.04$ consistent with recent RBC studies (Finn (00), Wei (03))
- Fix δ, ρ to reasonable numbers (for identification).
- Estimate $\alpha, \bar{i}, \sigma, \gamma$ and costs $\beta_X, \beta_K, \beta_Q$ to fit a few moments:

1. US annual Oil Consumption /US GDP $\approx \bar{i}QS/f(K, \bar{i}Q)$.
2. US Consumption (non-durables + services) / US GDP $\approx C/f(K, \bar{i}Q)$
3. Nine futures prices historical Means and Variances.
4. Data averages for Cons/GDP from 49-02 and from 97 to 03 for Futures.
5. Model averages estimated by simulating stationary distribution of z.

Production technologies		
Productivity of capital K,	α	0.23
Oil share of output,	η	0.04
Demand rate for oil,	$\bar{i}$	0.07
Volatility of return on capital,	σ	0.263
Depreciation of oil,	δ	0.02
Irreversible investment		
Fixed cost (K component),	β_K	0.016
Fixed cost (Q component),	β_Q	0.272
Marginal cost of oil,	β_X	17
Agents preferences		
Patience,	ρ	0.05
Risk aversion,	γ	1.8

	Historical data		Model	
	Mean	Vol	Mean	Vol
Crude oil futures prices (US$/bbl)				
1-months contract	23.62	6.38	22.38	4.47
6-months contract	22.61	4.88	22.15	4.19
12-months contract	21.65	3.87	21.86	3.86
18-months contract	21.05	3.21	21.64	3.58
24-months contract	20.71	2.81	21.42	3.31
36-months contract	20.42	2.43	21.09	2.92
48-months contract	20.28	2.25	20.77	2.56
60-months contract	20.23	2.10	20.49	2.27
72-months contract	20.42	1.88	20.25	2.05
Macroeconomic ratios				
consumption of oil-output ratio	2.16%	0.01	2.1%	0.01
output-consumption of capital ratio	1.8	0.08	2.2	0.01





Reduced-Form model with two regimes

- Estimate a Reduced Form model that is consistent with the equilibrium model

$$dS_t = \mu_S(S_t, \varepsilon_t)S_t dt + \sigma_S(S_t, \varepsilon_t)S_t db_t$$

where

$$\begin{aligned} \mu_S(S_t, \varepsilon_t) &= \alpha + \kappa_\varepsilon(\log[S_{Max}] - \log[S_t]) \\ \sigma_S(S_t, \varepsilon_t) &= \sigma_\varepsilon \sqrt{\log[S_{Max}] - \log[S_t]} \end{aligned}$$

and ε_t is a two-state Markov chain with transition (Poisson) probabilities

$$P_t = \begin{bmatrix} 1 - \lambda_1 dt & \lambda_1 dt \\ \lambda_2 dt & 1 - \lambda_2 dt \end{bmatrix}$$

- Define $\varepsilon_t = \begin{cases} 1 & \text{in the } \textit{far-from-investment} \text{ region} \\ 2 & \text{in the } \textit{near-investment} \text{ region} \end{cases}$

Estimation and predictions

- Maximum Likelihood (weekly crude oil prices from 1/1982 to 8/2003)
- Estimate $\Theta = \{\alpha, \kappa_1, \kappa_2, \sigma_1, \sigma_2, S_{Max}, \lambda_1, \lambda_2\}$

far-from-investment **state**			***near-investment*** **state**			**Common parameters**		
Parameter	Estimate	t-ratio	Parameter	Estimate	t-ratio	Parameter	Estimate	t-ratio
λ_1	0.984	3.2	λ_2	5.059	3.2	α	-0.248	-2.4
$1/\lambda_1$	1.017		$1/\lambda_2$	0.198		S_{Max}	39.797	99.7
$\lambda_2/(\lambda_1+\lambda_2)$	83.7%		$\lambda_1/(\lambda_1+\lambda_2)$	16.3%				
κ_1	0.319	2.7	κ_2	0.055	0.4			
σ_1	0.251	25.8	σ_2	0.808	12.9			

- Predictions from the structural model

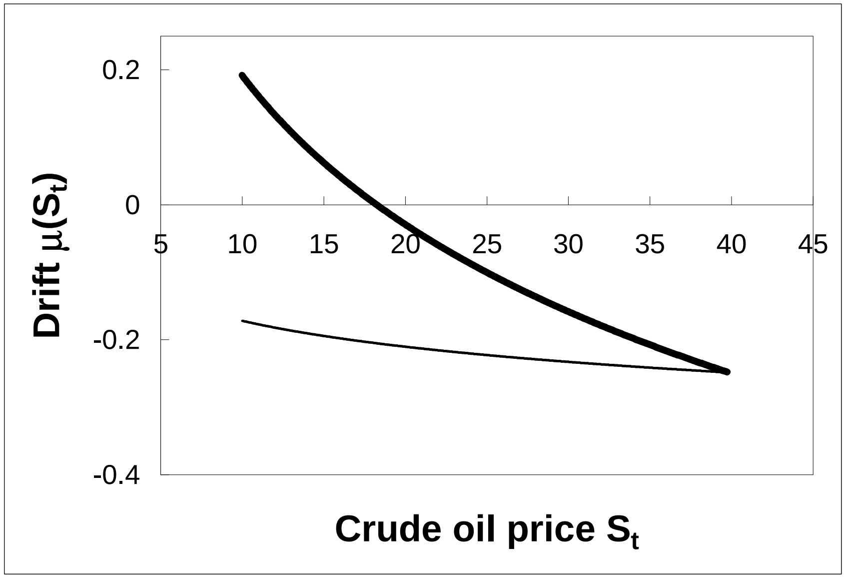
$$\mu_S(S_{Max}, \varepsilon_t) < 0 \quad \Leftrightarrow \quad \alpha < 0$$
$$\mu_S(0,1) > 0 \ \text{ and } \ \kappa_1 > 0$$
$$\mu_S(S_t,2) < 0 \ \text{ and } \ \kappa_2 < 0$$
$$\lambda_1 \ll \lambda_2$$

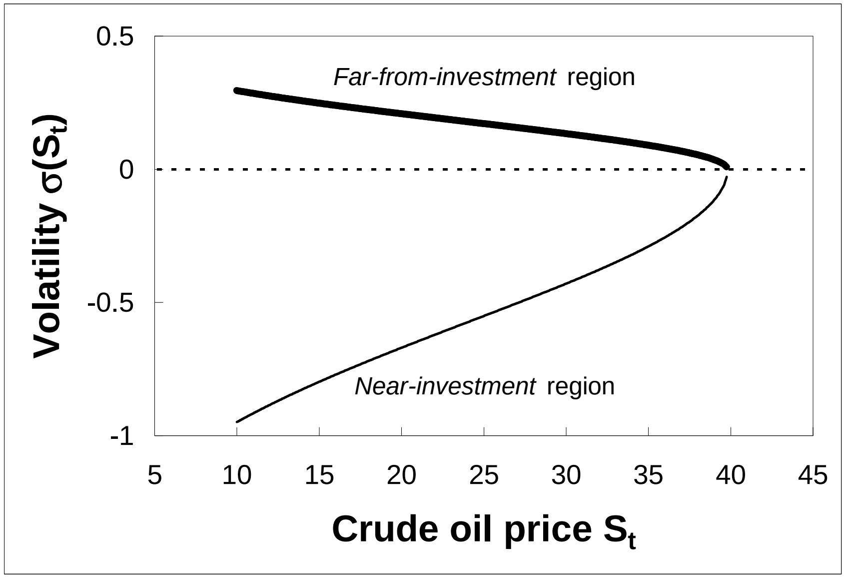
Regime-switching estimation of commodity price process

- Regime-switching model

$$dS_t = (\alpha + \kappa_\varepsilon(\log[S_{Max}] - \log[S_t]))S_t dt + \sigma_\varepsilon \sqrt{\log[S_{Max}] - \log[S_t]} S_t db_t$$

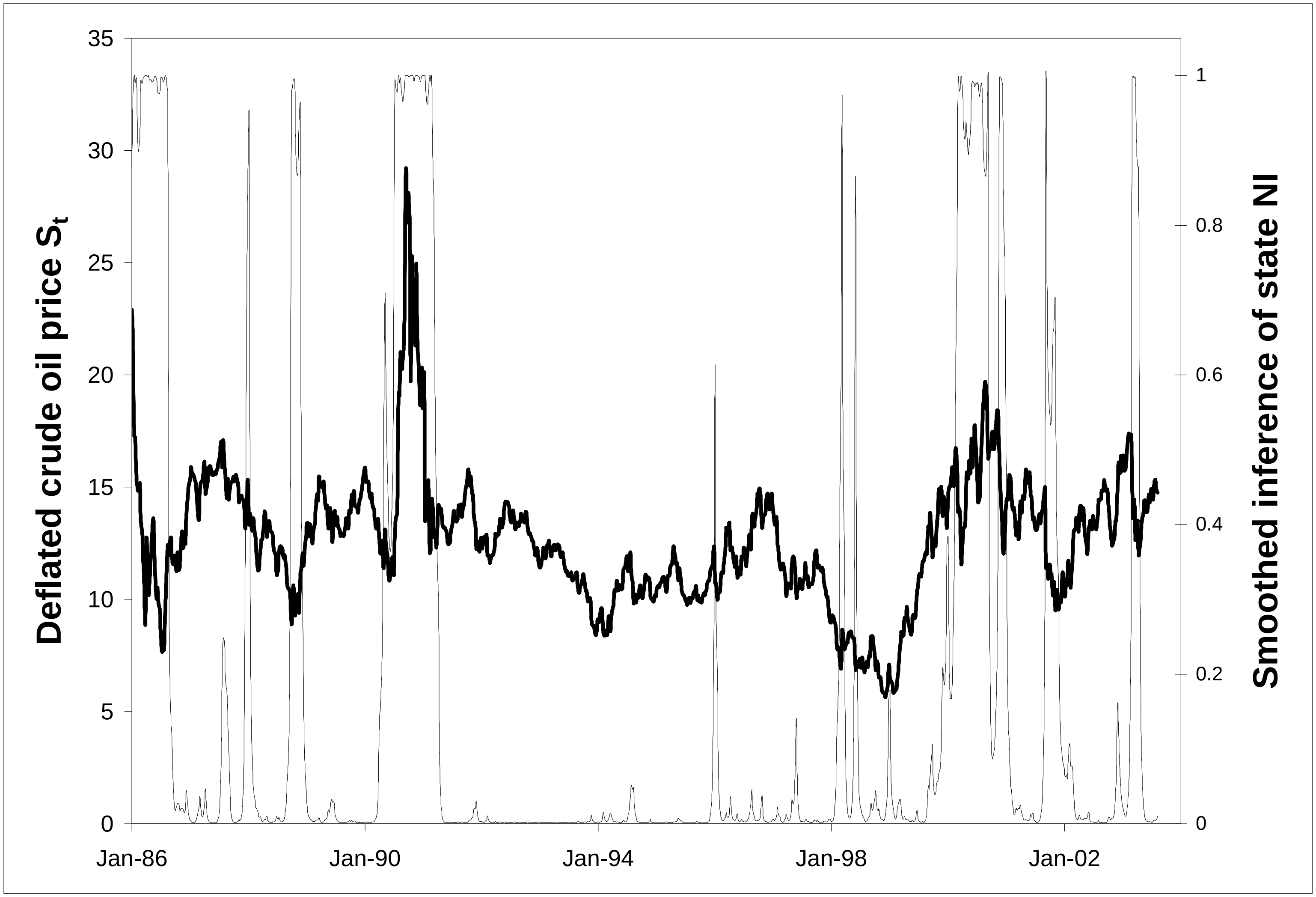
- Estimated drift and volatility of crude oil returns





Smoothed inferences for the regime switching model

- Historical crude oil price
- Inferred probability of *near-investment* state



Conclusion

- Structural model of commodity whose primary use is as an input to production.
- Infrequent lumpy investment in commodity determines two regimes for the commodity price, depending on the distance to the investment trigger.
- The spot price exhibits mean reversion, heteroscedasticity, and regime switching.
- Convenience yield has two endogenous components which arise because the commodity helps smooth production in response to demand/supply shocks.
- The model can generate the frequency of backwardation observed in the data.
- Estimates of a reduced-form regime switching model seem consistent with the predictions of the model
- Future work:
  - Investigate predictability of commodity return:
    Results show that beta of oil w.r.t to S&P 500 is related to regime as predicted by the model (negative in *near-investment* regime, positive in *far-from-investment* regime). Regime estimate different from slope of futures curve.
  - Implication of model prediction for reduced-form modeling, pricing and hedging of options.