Liquidity, Volume, and Volatility

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Motivation

We investigate the relation between the liquidity, volume, and volatility of individual U.S. stocks since 2002 (post-decimalization)

- What drives stock market liquidity?
  - Adverse selection,
  - Inventory risk, and/or
  - Competition

- Dynamics of liquidity, volume, and volatility important for:
  - Dynamic portfolio allocation (Collin-Dufresne, Daniel, and Sağlam (2018))
  - Costs associated with exiting a position

Liquidity and Trading Volume: Theory

Theoretically, high trading volume is generally associated with high liquidity ($\sim$ low spreads)

- Higher volume implies less risk for market makers who can more easily find off-setting trades (Demsetz (1968))
  - Lower cost of trading leads to more trading
- Adverse selection and market breakdown
  - More uninformed trading alleviates the adverse selection problem (Kyle (1985))
- Invariance of Transaction Costs Hypothesis (Kyle and Obizhaeva (2016))
  - $\%\text{spread}_{i,t} \propto \left[\frac{\sigma^2_{i,t}}{P_{i,t}V_{i,t}}\right]^{\frac{1}{3}}$

But, higher volume is also typically associated with higher volatility…

Liquidity and Trading Volume: Empirical Evidence

- Positive volume-liquidity relation supported mostly by cross-sectional evidence (Stoll (2000))
- Only limited (and contradicting) evidence about the time-series relation
  - Spreads widen in response to higher volume (Lee, Mucklow, and Ready (1993))
  - Positive correlation between changes in spread and volume at the market level (Chordia et al. (2001))
  - No relation at market level (Johnson (2008))
- Few studies control for volatility

Main Findings on Liquidity-Volume-Volatility relation

- For large stocks:
  - Spreads positively related to volume *even controlling for volatility*
  - Mostly driven by common component of volume
  - ⇒ Supportive of inventory channel of stock liquidity
- For small stocks:
  - Spreads negatively related to volume
  - Mostly driven by idiosyncratic component of volume
  - Idiosyncratic volatility is strongly positively linked to spreads
  - ⇒ Supportive of adverse selection channel of stock liquidity
- Controlling for volatility of high-frequency order imbalance reconciles 'volume-spread' relation for small and large stocks
  - $\sigma(OI)$ strongly associated with spreads
  - Consistent with a simple inventory model

Related Literature

- Volume and volatility (Clark (1973); Epps and Epps (1976); Tauchen and Pitts (1983); Gallant, Rossi, and Tauchen (1992); Andersen (1996))
- Spreads (Glosten and Harris (1988); Hasbrouck (1991); Foster and Viswanathan (1993); Bollen, Smith, and Whaley (2004))
- Liquidity and volume (Lee, Mucklow, and Ready (1993); Chordia, Roll, and Subrahmaniam (2000); Johnson (2008); Barinov (2010))
- Order imbalance (Chordia, Roll, and Subrahmanyam (2002); Chordia, Hu, Subrahmanyam, and Tong (2018))

Outline

Adverse Selection: Spread-Volume relation

- Continuous-time model of informed trading (Kyle (1985)):
  - Uncertainty about informed trader's signal: σ_{informed}
  - Noise trading volatility: σ_{noise}
    - $\Rightarrow$ Price impact $= \frac{\sigma_{\text{informed}}}{\sigma_{\text{noise}}}$
    - $\Rightarrow$ Price volatility $= \sigma_{\text{informed}}$
    - $\Rightarrow$ Volume ($\sim$ volatility of total order flow) $= \sigma_{\text{noise}}$

- Negative spread-volume relation, even with endogenous informed trading, stochastic volatility and volume: Admati and Pfleiderer (1988); Foster and Viswanathan (1990); Collin-Dufresne and Fos (2016a)

- Must impose direct link between volume and information to obtain positive spread-volume relation: Easley and O'Hara (1992); Collin-Dufresne and Fos (2016b)

Inventory Risk

- High volume makes it easier for market makers to find offsetting trade, thus lowers inventory risk (Demsetz (1968))

$\Rightarrow$ Negative spread-volume relation

- Supply shocks are risky to absorb for risk-averse market makers (Grossman and Miller (1988)):
  - Liquidity providers with risk aversion γ
  - Price impact $\propto \gamma\sigma^2_{\text{ret}}$
  - Volume $\propto \sigma^2_{\text{noise}}$
  - Since noise trading moves prices, $\frac{\partial}{\partial\sigma^2_{\text{noise}}}\sigma^2_{\text{ret}} > 0$

$\Rightarrow$ Positive spread-volume relation, but

  - without controlling for the variance of order imbalance, or the return volatility

Data and Variables

Sample:

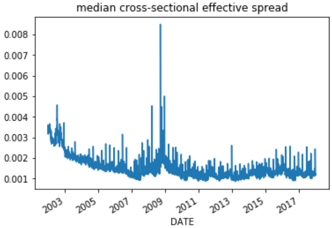
- U.S. common stocks; 2002-2017
  - Price $>$ \$5 and market capitalization $>$ \$100 million

Main variables:

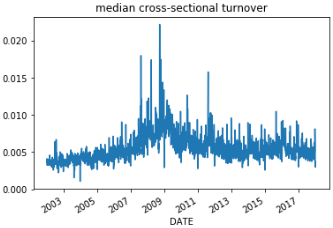
- Effective spread: $2|\ln P_{i,t} - \ln M_{i,t}|$ dollar/share-weighted over the trading day
  - Similar results with dollar effective spread
- Volume: share turnover (during trading hours)
  - Similar results with CRSP turnover
- Volatility: average absolute return over the past five trading days or realized volatility
  - Similar results with $|r_t|$, $|r_{t,\text{intraday}}|$

Spreads, Volume, and Volatility ▸ descr. ▸ corr.

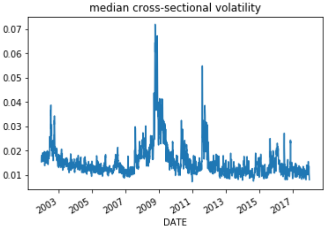
Daily cross-sectional average



Spread



Volume



Volatility

Methodology

Volume and volatility elasticities of spread:

$$\log s_{i,t} = \alpha_i + \beta_\tau \log \tau_{i,t} + \epsilon_{i,t}$$
$$\log s_{i,t} = \alpha_i + \beta_\sigma \log \sigma_{i,t} + \epsilon_{i,t}$$
$$\log s_{i,t} = \alpha_i + \beta_\tau \log \tau_{i,t} + \beta_\sigma \log \sigma_{i,t} + \text{controls} + \epsilon_{i,t}$$

- Levels, changes, and vector autoregressions
- Invariance (Kyle and Obizhaeva (2016)): $s_{i,t} \propto \left[\frac{\sigma^2_{i,t}}{P_{i,t} V_{i,t}}\right]^{\frac{1}{3}}$, where V is the share volume and P is the share price
- *Controls:* daily price and market capitalization; day-of-the-week and month-of-the-year indicators
- Estimated each month/year on stocks sorted into market capitalization quintiles

Methodology (2)

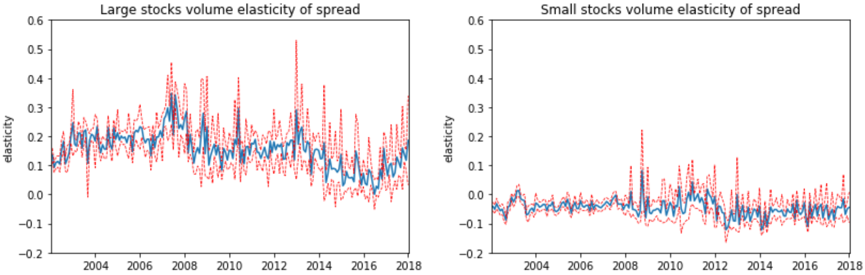
- First-difference specification:

$$\Delta s_{i,t} = \alpha_i + \beta_\tau \Delta \tau_{i,t} + \beta_\sigma \Delta \sigma_{i,t} + \text{controls} + u_{i,t},$$

  where $\Delta x_t \equiv \log(\frac{x_t}{x_{t-1}})$
- Trend-adjusted variables details
- Vector autoregressions
  - Both volatility and volume tend to Granger-cause spreads for the median large stock
  - Spreads tend not to Granger-cause volatility and volume (cannot reject the null for around 4/5 of stocks)
  - Volume Granger-causes volatility for around 3/4 of the stocks, but volatility Granger-causes volume for only around 1/5 of the stocks

Results for **Large vs. Small** Stocks **Volume**

$$\log s_{i,t} = \alpha_i + \beta_\tau \log \tau_{i,t} + \beta_\sigma \log \sigma_{i,t} + \text{controls} + \epsilon_{i,t}$$



Volume elasticity of spread

Results for **Large vs. Small** Stocks **Volatility**

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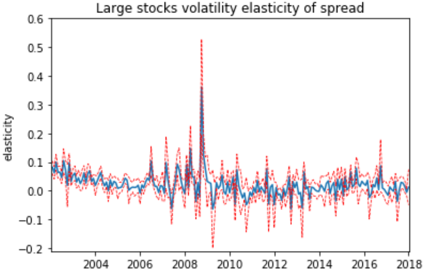
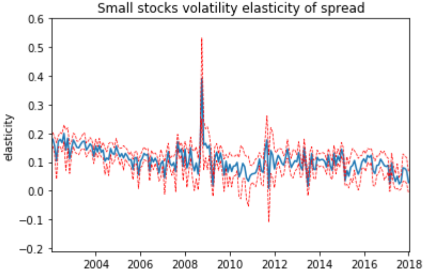




Volatility elasticity of spread

Decomposing Volume and Volatility

Systematic vs. idiosyncratic volume and volatility

- *Adverse selection channel:*
  - Idiosyncratic volatility is naturally linked to ‘insider information’ and adverse selection
  - Idiosyncratic volume is more linked to ‘information events’ that trigger more informed trading
  - Systematic component can be relevant if adverse-selection due to differential interpretation of public news
- *Inventory risk channel:*
  - If liquidity providers under-diversified then idiosyncratic volatility could matter.
  - Systematic volume shock consumes liquidity everywhere.

Decomposing Volume and Volatility

- Decompose volume into common and idiosyncratic components
  - For stock i, regress daily (log) turnover on a common turnover measure:

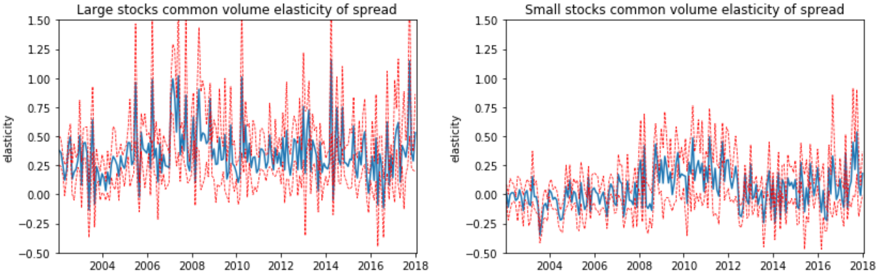
$$\log \tau_{i,t} = a_i + b_i \tau_{m,t} + \tau^I_{i,t},$$

    where $\tau_{m,t}$ is the equal-weighted average daily (log) turnover of stocks in the same size quintile as stock i (excluding stock i)
  - Commonality in liquidity (Chordia et al. (2000))

- Decompose volatility into common and idiosyncratic components
  - Regress stock i's return on the equal-weighted average return of stocks in the same size quintile (excluding stock i)

Large vs Small Stocks: **Common Volume** Component

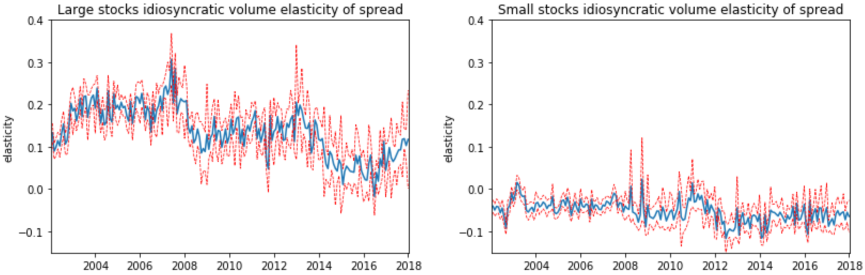
$$\log s_{i,t} = \alpha_i + \beta_{\tau,C}\tau^C_{i,t} + \beta_{\tau,I}\tau^I_{i,t} + \beta_{\sigma,C}\sigma^C_{i,t} + \beta_{\sigma,I}\sigma^I_{i,t} + \text{controls} + \epsilon_{i,t}$$



Common volume elasticity of spread

Large vs Small Stocks: **Idiosyncratic Volume** Comp.

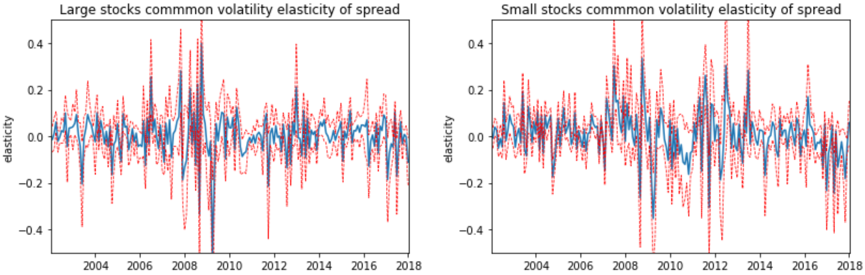
$$\log s_{i,t} = \alpha_i + \beta_{\tau,C}\tau^C_{i,t} + \beta_{\tau,I}\tau^I_{i,t} + \beta_{\sigma,C}\sigma^C_{i,t} + \beta_{\sigma,I}\sigma^I_{i,t} + \text{controls} + \epsilon_{i,t}$$



Idiosyncratic volume elasticity of spread

Large vs Small Stocks **Common Volatility** Comp.

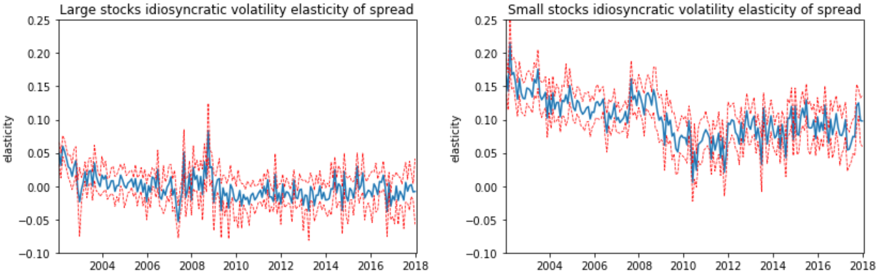
$$\log s_{i,t} = \alpha_i + \beta_{\tau,C}\tau^C_{i,t} + \beta_{\tau,I}\tau^I_{i,t} + \beta_{\sigma,C}\sigma^C_{i,t} + \beta_{\sigma,I}\sigma^I_{i,t} + \text{controls} + \epsilon_{i,t}$$



Common volatility elasticity of spread

Large vs Small Stocks: **Idiosyncratic Volatility** Comp.

$$\log s_{i,t} = \alpha_i + \beta_{\tau,C}\tau^C_{i,t} + \beta_{\tau,I}\tau^I_{i,t} + \beta_{\sigma,C}\sigma^C_{i,t} + \beta_{\sigma,I}\sigma^I_{i,t} + \text{controls} + \epsilon_{i,t}$$



Idiosyncratic volatility elasticity of spread

Decomposition: Key Takeaways

- Support for *inventory effects* for **large stocks**
  - Common volume elasticity is large and positive
  - Idiosyncratic volume elasticity is positive but tends to decrease over time ($\sim$ robustness $P \geq \$100$)
  - Common and Idiosyncratic volatility elasticity are mostly insignificant
- Support for *adverse selection* for **small stocks**
  - Common volume elasticity is mostly insignificant
  - Idiosyncratic volume elasticity is negative
  - Common volatility elasticity is in general insignificant while idiosyncratic volatility elasticity is large and positive

$\Rightarrow$ Standard adverse selection intuition for dynamics of liquidity works well for small stocks, but not for large ones!

Inventory Model

Natural to distinguish between volume and order imbalance (one-sided volume) (e.g., Chordia et al. (2002))

- Long-lived liquidity provider with CARA

$$\max_{c_t, n_t} \mathrm{E}[\int_0^\infty -e^{-\beta t - \alpha c_t}]$$

- One dividend-paying asset and one risk-free asset
- The liquidity provider absorbs supply shocks so that her inventory follows a Markov Chain with transition intensities $\lambda_{i,j}$

Inventory Model

$$dS_t + \delta_t dt = \mu_t dt + \sigma_t dZ_t + \sum_{i=1}^{M} 1_{\{N_{t-}=i\}} \sum_{j \neq i} \eta_{ij}(dN_{ij}(t) - \lambda_{ij} dt)$$

$$d\delta_t = \kappa_\delta(\overline{\delta}(N_t) - \delta_t)dt + \sigma_\delta dZ(t)$$

$$dN_t = \sum_{i=1}^{M} 1_{\{N_{t-}=i\}} \sum_{j \neq i}(j - i)(dN_{ij}(t) - \lambda_{ij} dt)$$

The states characterize

- the fundamental value $\overline{\delta}(N_t) := \sum_{i=1}^{M} \overline{\delta}_i 1_{\{N_t=i\}}$ and
- the *total supply* or inventory that the market maker must hold in equilibrium $\theta(N_t) := \sum_{i=1}^{M} \theta_i 1_{\{N_t=i\}}$
- adverse selection if $\overline{\delta}(N)$ inversely related to $\theta(N)$

Inventory Model (two states without adverse selection)

- Symmetric model: $\lambda_{i,j} = \lambda$

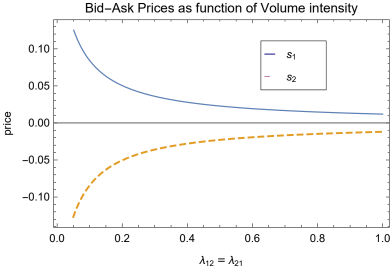
$$\frac{\alpha r \sigma^2}{2\lambda + r} < \text{Price Impact} < \alpha\sigma^2$$

- Asymmetric model: $\lambda_{1,2} \neq \lambda_{1,2}$
  - Holding volume constant, the variance of order imbalance varies with $\lambda_{1,2}$

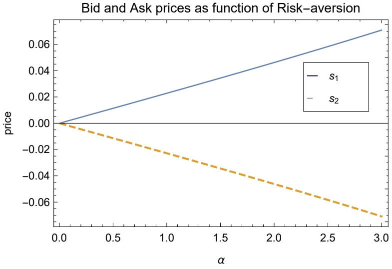
- A high trading intensity lowers the holding period but may increase the variance of shocks to inventory

Inventory Model

Bid-Ask Spread Comparative Statics



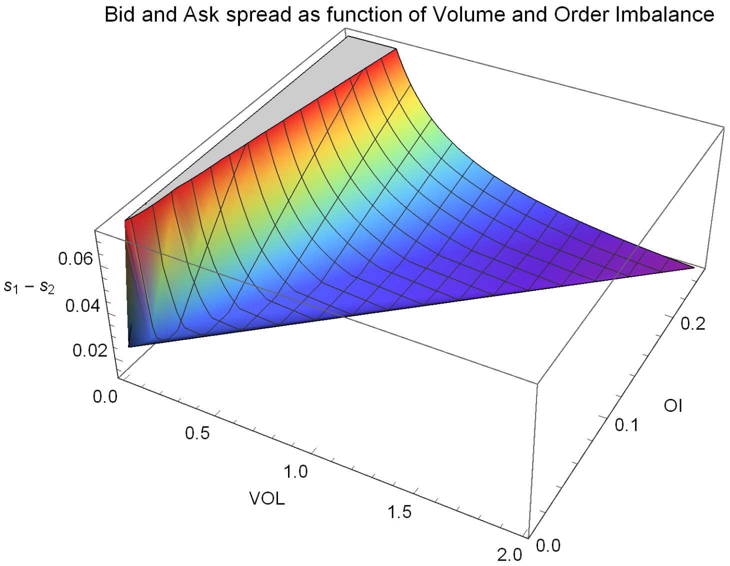
Bid-Ask and Volume



Bid-Ask and Risk Aversion

Inventory Model

Bid-Ask spread as a function of Volume and Variance of Order Imbalance



Volatility of Order Imbalance

Simple inventory model suggests to distinguish volume from order imbalance (to capture 'one-sided' volume)

- Compute order imbalance as a proportion of shares outstanding over every 5mn interval of the trading day
  - High frequency market making
- σ(OI) is the standard deviation of the 5mn imbalance, computed each day
  - Control: realized volatility ▸ details

Volatility of Order Imbalance (large stocks)

$$\log s_{i,t} = \alpha_i + \beta_x \log x_{i,t} + \epsilon_{i,t}$$

Year	β_τ	β_{RVol}	$\beta_{\sigma(\text{OI})}$
2002	0.14*** (18.00)	0.44*** (16.55)	0.18*** (16.01)
2003	0.17*** (14.98)	0.47*** (41.98)	0.17*** (21.30)
2004	0.17*** (23.54)	0.42*** (39.94)	0.17*** (20.32)
2005	0.17*** (24.66)	0.40*** (36.48)	0.18*** (20.78)
2006	0.16*** (21.20)	0.35*** (36.96)	0.18*** (24.37)
2007	0.24*** (19.32)	0.42*** (26.62)	0.24*** (19.27)
2008	0.17*** (10.09)	0.46*** (18.40)	0.24*** (12.91)
2009	0.11*** (10.80)	0.27*** (17.05)	0.21*** (15.01)
2010	0.12*** (11.66)	0.30*** (15.89)	0.19*** (16.94)
2011	0.13*** (13.97)	0.32*** (27.03)	0.17*** (20.59)
2012	0.14*** (10.56)	0.32*** (19.97)	0.19*** (12.06)
2013	0.13*** (12.27)	0.34*** (23.22)	0.18*** (20.25)
2014	0.09*** (6.46)	0.33*** (26.40)	0.18*** (9.93)
2015	0.05*** (3.76)	0.35*** (16.49)	0.14*** (12.30)
2016	0.04*** (4.07)	0.31*** (18.75)	0.15*** (12.51)
2017	0.09*** (7.36)	0.33*** (32.89)	0.17*** (10.97)
$\bar{R}^2$	14.32	19.97	20.63

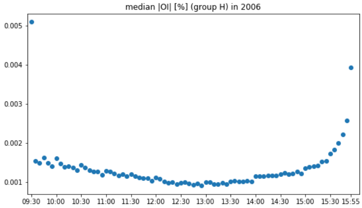
Volatility of Order Imbalance (large stocks)

$$\log s_{i,t} = \alpha_i + \beta_\tau \log \tau_{i,t} + \beta_\sigma \log \sigma_{i,t} + \beta_{\sigma(\text{OI})} \log \sigma(\text{OI})_{i,t} + \text{controls} + \epsilon_{i,t}$$

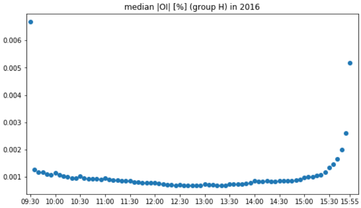
Year	β_τ	β_{RVol}	$\beta_{\sigma(\text{OI})}$
2002	-0.26*** (-12.39)	0.51*** (13.85)	0.30*** (14.60)
2003	-0.25*** (-13.74)	0.52*** (54.16)	0.29*** (18.90)
2004	-0.25*** (-15.80)	0.47*** (42.07)	0.28*** (19.07)
2005	-0.27*** (-18.06)	0.45*** (40.77)	0.30*** (20.26)
2006	-0.27*** (-23.77)	0.41*** (54.66)	0.29*** (27.35)
2007	-0.28*** (-16.59)	0.49*** (31.72)	0.33*** (19.30)
2008	-0.40*** (-19.20)	0.55*** (27.57)	0.37*** (18.29)
2009	-0.32*** (-18.99)	0.37*** (29.47)	0.33*** (19.57)
2010	-0.29*** (-22.26)	0.38*** (22.91)	0.30*** (21.73)
2011	-0.27*** (-26.55)	0.40*** (34.40)	0.26*** (26.88)
2012	-0.28*** (-15.38)	0.38*** (25.66)	0.27*** (13.80)
2013	-0.30*** (-28.21)	0.41*** (25.93)	0.28*** (26.71)
2014	-0.42*** (-20.59)	0.48*** (36.59)	0.32*** (13.92)
2015	-0.43*** (-33.39)	0.52*** (28.04)	0.29*** (24.11)
2016	-0.44*** (-27.83)	0.48*** (24.90)	0.30*** (21.35)
2017	-0.41*** (-25.57)	0.51*** (46.57)	0.29*** (14.64)
$\bar{R}^2$(%)		30.91 (vs 20.38 without σ(OI))	

Additional Evidence

- Other liquidity measures:
  - Order imbalance volatility positively associated with price impact (Amihud) and negatively associated with depth
    details
- Intraday patterns:

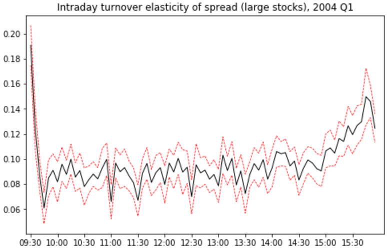


2006

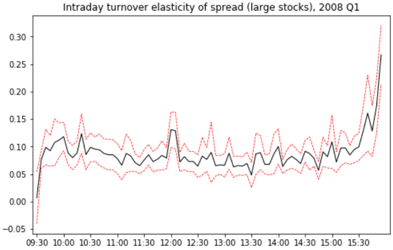


2016

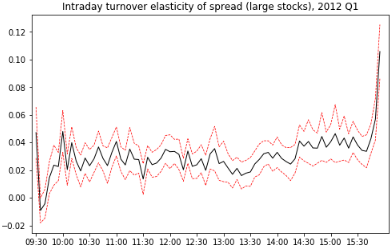
Intraday Elasticities of Spread



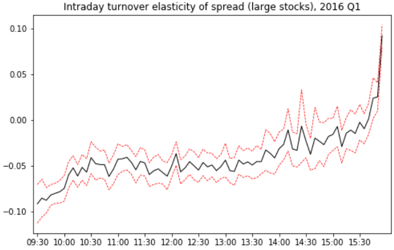
2004 Q1



2008 Q1



2012 Q1



2016 Q1

Conclusion

- New evidence about the time-series (and cross-sectional) relation between liquidity, volume, and volatility
- Adverse selection theories fit well the day-to-day variation in spread, volume, and volatility of small stocks
- Inventory risk seems more important for the day-to-day variation in spread, volume, and volatility of large stocks
- Controlling for Volatility of (high-frequency) Order Imbalance reconciles evidence between large and small stocks,
  - $\Rightarrow$ is consistent with simple inventory risk model, and
  - $\Rightarrow$ adds substantial explanatory power

Appendix

Descriptive Statistics (Small Stocks)

		2004	2008	2012	2016
Small caps					
spread [bp]	mean	70.18	96.68	62.69	70.32
	median	51.33	50.35	40.85	44.66
	σ (within)	48.62	103.16	49.98	63.32
turnover [%]	mean	0.50	0.52	0.42	0.48
	median	0.19	0.27	0.23	0.25
	σ (within)	1.38	0.83	0.89	1.28
volatility [%]	mean	1.83	3.06	1.72	1.87
	median	1.53	2.44	1.50	1.51
	σ (within)	1.06	2.13	1.02	1.58
obs.		146,897	132,182	119,480	126,515

Descriptive Statistics (Large Stocks) ▸ back

		2004	2008	2012	2016
Large caps					
spread [bp]	mean	8.27	8.29	4.65	4.77
	median	6.59	6.20	3.65	3.63
	σ (within)	5.95	10.23	3.04	4.31
turnover [%]	mean	0.67	1.42	0.90	0.82
	median	0.46	1.03	0.67	0.61
	σ (within)	0.58	1.22	0.74	0.63
volatility [%]	mean	1.17	2.70	1.16	1.23
	median	1.01	2.03	1.01	1.01
	σ (within)	0.57	1.99	0.58	0.72
obs.		151,157	137,730	121,479	129,411

Correlations

cross-sectional averages of the stocks' time-series correlations

back

Small caps

	τ	σ	$\lvert r\rvert$	RVol	$\lvert$OI$\rvert$	σ(OI)
s	-0.17	0.22	0.18	0.40	-0.06	-0.00
τ		0.24	0.23	0.32	0.59	0.78
σ			0.49	0.47	0.10	0.12
$\lvert r\rvert$				0.41	0.13	0.14
RVol					0.12	0.17
$\lvert$OI$\rvert$						0.60

Large caps

	τ	σ	$\lvert r\rvert$	RVol	$\lvert$OI$\rvert$	σ(OI)
s	0.15	0.34	0.22	0.51	0.15	0.30
τ		0.41	0.32	0.48	0.40	0.72
σ			0.50	0.61	0.14	0.22
$\lvert r\rvert$				0.41	0.13	0.19
RVol					0.14	0.26
$\lvert$OI$\rvert$						0.48

How Does Order Imbalance Volatility Affect Other Liquidity Measures?

- **Price impact**
  - In the line of Amihud (2002):
    $\mathrm{ILLIQ}_{it} = \frac{1}{\#\text{traded intervals}} \sum_{k \epsilon \{j | \mathrm{DVOL}_{itj} > 0\}} \frac{|r_{itk}|}{\mathrm{DVOL}_{itk}}$
  - Alternative: $r_{itk} = \delta_{it} + \lambda_{it} \sqrt{|\mathrm{OI}^{\$}_{itk}|} \mathrm{sign}(\mathrm{OI}^{\$}_{itk}) + e_{it}$ (Hasbrouck (2009))
- **Depth**
  - Time-weighted share depth at the best bid and best ask (as a fraction of shares outstanding)

Price Impact (Amihud)

Year	β_τ	β_{RVol}	$\beta_{\sigma(\text{OI})}$
2002	-1.10*** (-54.89)	0.90*** (40.95)	0.24*** (18.72)
2003	-1.21*** (-56.32)	0.88*** (81.51)	0.29*** (27.03)
2004	-1.20*** (-100.27)	0.88*** (65.68)	0.27*** (37.15)
2005	-1.17*** (-103.29)	0.90*** (44.62)	0.24*** (39.38)
2006	-1.15*** (-98.54)	0.92*** (87.78)	0.21*** (35.00)
2007	-1.12*** (-101.99)	0.98*** (72.96)	0.16*** (29.75)
2008	-1.10*** (-82.55)	0.96*** (58.25)	0.10*** (18.51)
2009	-1.07*** (-141.59)	0.92*** (41.61)	0.10*** (15.21)
2010	-1.10*** (-72.77)	0.92*** (26.01)	0.11*** (21.17)
2011	-1.12*** (-107.44)	0.96*** (45.96)	0.11*** (23.25)
2012	-1.09*** (-101.16)	0.83*** (72.48)	0.12*** (19.40)
2013	-1.14*** (-67.34)	0.89*** (30.81)	0.14*** (23.18)
2014	-1.14*** (-148.76)	0.88*** (86.04)	0.15*** (36.93)
2015	-1.15*** (-123.23)	0.89*** (66.42)	0.15*** (32.22)
2016	-1.14*** (-108.79)	0.88*** (52.77)	0.14*** (32.52)
2017	-1.11*** (-135.75)	0.79*** (67.02)	0.15*** (43.56)
$\bar{R}^2$(%)		77.05	

Depth ▸ back

Year	β_τ	β_{RVol}	$\beta_{\sigma(\text{OI})}$	β_s
2002	0.35*** (20.58)	-0.22*** (-9.94)	-0.00 (-0.32)	-0.19*** (-15.34)
2003	0.43*** (22.47)	-0.31*** (-30.46)	-0.04*** (-5.18)	-0.09*** (-16.94)
2004	0.47*** (31.15)	-0.42*** (-14.13)	-0.04*** (-7.23)	-0.10*** (-15.55)
2005	0.46*** (30.23)	-0.44*** (-15.49)	-0.05*** (-10.80)	-0.07*** (-13.95)
2006	0.44*** (30.65)	-0.51*** (-18.67)	-0.06*** (-11.93)	-0.07*** (-12.71)
2007	0.41*** (25.41)	-0.56*** (-22.82)	-0.02*** (-4.58)	-0.04*** (-6.93)
2008	0.40*** (18.33)	-0.69*** (-17.47)	-0.01*** (-2.70)	0.02*** (2.78)
2009	0.38*** (23.72)	-0.66*** (-22.94)	-0.00 (-0.12)	-0.03*** (-3.54)
2010	0.39*** (16.04)	-0.66*** (-14.65)	-0.01 (-1.07)	-0.02** (-2.39)
2011	0.38*** (19.13)	-0.65*** (-17.34)	-0.02*** (-3.06)	0.03*** (4.03)
2012	0.35*** (29.22)	-0.40*** (-22.19)	-0.03*** (-6.01)	-0.00 (-0.22)
2013	0.40*** (18.79)	-0.48*** (-10.24)	-0.05*** (-9.43)	0.02** (2.47)
2014	0.31*** (34.06)	-0.39*** (-23.40)	-0.01 (-1.56)	-0.01*** (-2.92)
2015	0.30*** (21.97)	-0.34*** (-15.81)	-0.02*** (-4.05)	0.01*** (2.77)
2016	0.30*** (15.37)	-0.37*** (-11.26)	-0.02*** (-4.91)	0.03*** (4.61)
2017	0.28*** (26.71)	-0.27*** (-14.35)	-0.03*** (-10.16)	0.02*** (4.23)
$\bar{R}^2(\%)$		41.70		

Evidence from Intraday Patterns

The degree of informed trading and liquidity trading is likely not constant over the day

1. Informational advantage of trading on overnight information is likely short-lived (Foster and Viswanathan (1990))
2. Liquidity traders cluster their trades to reduce adverse selection (Admati-Pfleiderer (1980))

Evidence from Intraday Patterns

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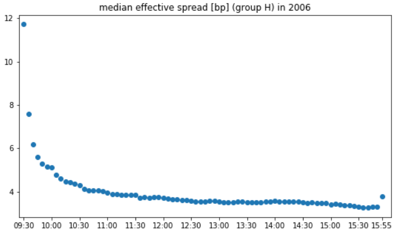
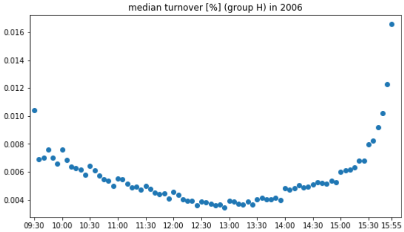
Informative to examine intraday patterns of elasticities

- Split the day into five-minute intervals and focus on large stocks
- We are *not* looking at levels but at sensitivities
  - Control for interval-stock fixed effects

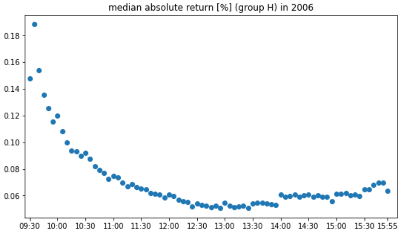
Intraday Median Values - 2006



Spread



Turnover



Volatility (absolute return)

Intraday Evidence

- Volume elasticity of spread is higher at the end of the day, when inventory risk or market power may be high
  - Consistent with evidence from intraday order imbalances
Appendix
- The intraday elasticity pattern does not 'mechanically' reflect intraday variations in spread, volume, and volatility
  - Spreads may be lower around the close but are more sensitive to trading volume

This evidence supports adverse selection effects and competition/inventory effects

- More competitive liquidity provision in recent years?

Volume in the continuous-time Kyle model

- $VOL = \frac{1}{2}(|dX_t^i| + |dX_t^u| + |dX_t^i + dX_t^u|)$
- Insider trade in absolutely continuous fashion: $dX_t^i = \mu_i dt$
- Whereas $dX_t^u = \sigma_u dZ_t$ for some Brownian motion Z_t
- $E[VOL]^2 = 2/\pi \sigma_u^2 dt$
- Total cumulative order flow is $Y_t = X_t^u + X_t^i$ and $Var[dY_t] = \sigma_u^2 dt$

Inventory Shocks and Endogenous Entry

Allow for entry of liquidity providers at a fixed cost in the model of Campbell, Grossman, and Wang (1993)

- Stationary OLG economy with exogenous risk-free rate and a risky asset that pays dividends every date
- Liquidity providers with exponential utility absorb volatile supply shocks every date
- In equilibrium, we show that an increase in the volatility of supply shocks *decreases* price impact, in contrast to the original model
- The inventory explanation requires some barriers to entry

Gallant-Rossi-Tauchen (1992) Methodology ▸ back

For each stock regress the spread and turnover series on a set of control variables x:

$$y = x'\beta + u.$$

The residuals are used to construct the following variance equation:

$$\log(u^2) = x'\gamma + v.$$

The adjusted y series is then given by:

$$y_{\text{adj}} = a + b(\hat{u}/\exp(x'\gamma/2)),$$

where the parameters a and b are chosen such that the mean and standard deviation of y_{adj} are the same as that of y.
Control variables x: day-of-the-week dummies; month-of-the-year dummies; a dummy for trading days around holidays when the stock market is closed; a dummy for trading days on federal holidays when the stock market is open; linear and quadratic trend variables. For the turnover series, we also include a cubic trend variable.

Measure of Volatility: Realized Volatility

What about a more sophisticated measure of volatility?

- *Realized variance:* $\text{RVol}(k)_t^2 = \sqrt{\sum_{k=1}^{K} r_{t,k}^2}$, where $r_{t,k}$ is the intraday return over interval k
- But what should we expect?

Using log returns, it can be shown that:

$$\text{RVol}(k)_t^2 = r_t^2 + \Pi_t,$$

where $\Pi_t = \sum_{k=2}^{K}(-2\sum_{j=1}^{k-1} r_j)r_{t,k} \Rightarrow$ intraday reversal strategy

$\text{corr}(s_t, \Pi_t) > 0$?

Large Stocks' Elasticities with Realized Volatility

$$\log s_{i,t} = \alpha_i + \beta_{\tau,C}\tau^C_{i,t} + \beta_{\tau,I}\tau^I_{i,t} + \beta_{\text{RVol}}\text{RVol}_{i,t} + \text{controls} + \epsilon_{i,t}$$

Year	$\beta_{\tau,C}$	$\beta_{\tau,I}$	β_{RVol}
2002	0.12** (2.46)	0.02** (2.47)	0.42*** (13.22)
2003	-0.05 (-1.05)	0.08*** (11.45)	0.45*** (42.76)
2004	0.01 (0.29)	0.07*** (11.23)	0.38*** (39.58)
2005	0.16*** (3.18)	0.07*** (11.79)	0.34*** (28.41)
2006	0.11*** (2.77)	0.08*** (11.05)	0.30*** (29.47)
2007	0.25*** (5.45)	0.09*** (8.98)	0.33*** (16.58)
2008	0.12*** (2.64)	0.00 (0.18)	0.42*** (17.93)
2009	0.09** (1.99)	0.03*** (3.28)	0.24*** (11.07)
2010	0.10*** (2.70)	0.03*** (3.40)	0.27*** (11.77)
2011	0.06** (1.96)	0.02* (1.73)	0.30*** (17.04)
2012	0.27*** (3.12)	0.03*** (3.23)	0.27*** (16.69)
2013	0.13*** (2.68)	0.02 (1.52)	0.31*** (16.16)
2014	0.08 (1.19)	-0.06*** (-4.30)	0.34*** (17.54)
2015	-0.00 (-0.01)	-0.11*** (-9.76)	0.41*** (19.61)
2016	-0.07* (-1.96)	-0.11*** (-8.53)	0.39*** (18.30)
2017	0.11 (1.52)	-0.10*** (-7.36)	0.40*** (20.47)
$\bar{R}^2$(%)		20.59	

Large Stocks' Elasticities with Realized Volatility

back

$$\Delta s_{i,t} = \alpha_i + \beta_{\tau,C}\Delta\tau^C_{i,t} + \beta_{\tau,I}\Delta\tau^I_{i,t} + \beta_{\mathrm{RVol}}\Delta\mathrm{RVol}_{i,t} + \text{controls} + \epsilon_{i,t}$$

Year	$\beta_{\tau,C}$	$\beta_{\tau,I}$	β_{RVol}
2002	0.18*** (2.68)	0.06*** (6.10)	0.35*** (10.06)
2003	0.00 (0.02)	0.14*** (16.32)	0.41*** (35.94)
2004	0.15*** (3.10)	0.13*** (18.16)	0.36*** (39.17)
2005	0.29*** (4.30)	0.16*** (19.68)	0.31*** (27.00)
2006	0.23*** (4.96)	0.16*** (18.66)	0.26*** (25.61)
2007	0.52*** (6.81)	0.22*** (14.90)	0.25*** (13.52)
2008	0.37*** (4.75)	0.10*** (9.23)	0.31*** (15.01)
2009	0.28*** (3.64)	0.12*** (9.35)	0.19*** (9.09)
2010	0.29*** (5.14)	0.13*** (10.37)	0.21*** (9.33)
2011	0.19*** (4.51)	0.10*** (9.62)	0.23*** (16.73)
2012	0.43*** (3.28)	0.13*** (9.07)	0.19*** (10.59)
2013	0.24*** (3.94)	0.11*** (8.10)	0.25*** (16.15)
2014	0.32*** (3.30)	0.03* (1.79)	0.28*** (15.30)
2015	0.20*** (3.22)	-0.02 (-1.24)	0.32*** (14.16)
2016	0.16** (2.39)	-0.03*** (-2.64)	0.34*** (20.53)
2017	0.39*** (3.58)	-0.01 (-0.44)	0.32*** (18.53)
$\bar{R}^2$(%)		8.84	