Is Credit Event Risk Priced?
Modeling Contagion via the Updating of Beliefs

Pierre Collin-Dufresne Robert Goldstein Jean Helwege

Background

- Investment-grade (IG) firms rarely default.
- Further, recovery rates are substantial:

  $\Rightarrow$ expected losses are low...

Moody's 2005 Report: Exhibit 18 - Cumulative Default Rates 1970-2004.

Years	1	2	3	4	5	6	7	8	9	10
Aaa	0.00	0.00	0.00	**0.04**	0.12	0.21	0.30	0.41	0.52	0.63
Baa	0.19	0.54	0.98	**1.55**	2.08	2.59	3.12	3.65	4.25	4.89

Exhibit 27 - Average Recovery Rates by Seniority Class, 1982-2004

Year	Sr. Secured	Sr. Unsecured	Sr. Subordinated	Subordinated Jr.	Subordinated	All bonds
Mean	0.574	**0.449**	0.391	0.320	0.289	0.422

$$\begin{aligned} \text{Expected loss on 4Y-Baa per year} &= (0.0155)(1-0.449)/4 \\ &\approx 21bp \end{aligned}$$

Background

Historical spreads are quite large: (Huang and Huang (2003))

years	4	10
Baa - Treasury	158	194
Aaa - Treasury	55	63
Baa - Aaa	103	131

Thus, only 21bp of the 158 (or 103+) are due to expected losses.

Q? Can these large credit spreads be explained in terms of 'fair compensation' for risk?

- The (Aaa-Treasury) spread seen as convenience yield, removed from credit risk.
  - Blanco, Brennan and Marsh (2006), Feldhutter and Lando (2007)

- Defaults occur in bad states of nature. If agents are sufficiently risk-averse in these states, then at least the (Baa-Aaa) spread can be explained.
  - Chen, Collin-Dufresne and Goldstein (2006), Chen (2007), Bhamra, Kuehn, Strebulaev (2007)

Open Questions Regarding Credit Spreads

- Can we explain credit spreads at short maturities?

- What are the sources of risk that corporate bonds face?

- Can we decompose expected excess returns into these components?

- Why do credit spreads exhibit large co-movements?

- What drives credit risk correlation?
  $\Rightarrow$ crucial for pricing CDO tranches

Two Frameworks for Default Risk

1) Structural Models of default (without jumps)

- Model asset value and liabilities (and their dynamics).
- Default occurs when value of assets reaches 'default boundary' (fct. of liabilities).
- Risky debt equals risk-free debt minus a put option on underlying firm value.

$\Rightarrow$ Correlation across default events driven by correlations in asset returns.

$\Rightarrow$ Credit risk-premium due solely to the 'market' risk of the firm's assets.

$\Rightarrow$ Credit spreads should vanish as maturity goes to zero.

Two Frameworks for Default Risk

2) Reduced-Form Models of Default

- Abstract from the firm value process.
- Model default as a unpredictable stopping time τ_i with intensity λ_i.
- Consistent with structural model with incomplete information (Duffie/Lando).

Correlation across default events determined by:

- correlations between intensities $\{\lambda_i(t)\}$
- correlation between default events $\mathbf{1}_{\{\tau_i \leq t\}}$.

$\Rightarrow$ Risk-premium due to systematic intensity risk as well as event risk ($\lambda^Q \neq \lambda^P$).

– The possibility that jump-to-default event risk is priced provides a new channel, not found in (diffusive) structural models, for capturing risk premia

Standard 'doubly-stochastic' reduced-form framework

- Assumes a risk-neutral measure under which default occurs with intensity $\lambda^Q(t)$:

$$\mathrm{E}_t^Q[d\mathbf{1}_{\{\tau \geq t\}}] = \lambda^Q(t)dt$$

- Under 'doubly-stochastic' model assumption survival probability simply:

$$\mathrm{E}^Q[\mathbf{1}_{\{\tau > T\}}] = \mathrm{E}^Q[e^{-\int_0^T \lambda^Q(s)s}]$$

- Further, risky cash-flows can be valued by discounting at the risk-adjusted rate:

$$\mathrm{E}^Q[e^{-\int_0^T r(s)ds}\mathbf{1}_{\{\tau > T\}}] = \mathrm{E}^Q[e^{-\int_0^T (r(s)+\lambda^Q(s))ds}]$$

$\Rightarrow$ Can readily use risk-free term structure machinery (e.g., affine ts models...).

- $\lambda^Q(t)$ need not be related to actual time series of observed defaults driven by $\lambda^P(t)$
- How can we account for the difference $\lambda^Q(t) \neq \lambda^P(t)$?
  - Risk-premium
  - Liquidity
  - Taxes
  - ...

$\Rightarrow$ Should affect how we hedge and invest in corporate bonds.

λ^Q vs. λ^P

Example:

- Constant interest rate (r),
- Constant default probability (λ^P); hence, no 'diffusion risk premium'
- Zero recovery
- Pricing kernel: $\frac{d\Lambda(t)}{\Lambda(t)} = -r dt + \Gamma\left(d\mathbf{1}_{\{\tau \le t\}} - \lambda^P \mathbf{1}_{\{\tau > t\}} dt\right)$

$$\iff \qquad \frac{\Lambda(T)}{\Lambda(t)} = e^{-(r+\lambda^P\Gamma)(T-t)} \left(1+\Gamma\right)^{\mathbf{1}_{\{\tau \in [t,T]\}}}.$$

Risky bond price:

$$\begin{aligned} P(t,T) &= \mathrm{E}_t[\frac{\Lambda(T)}{\Lambda(t)}\mathbf{1}_{\{\tau > T\}}] \\ &= e^{-(r+\lambda^P\Gamma)(T-t)}\mathrm{E}_t[\mathbf{1}_{\{\tau > T\}}] \\ &= e^{-(r+\lambda^Q)(T-t)}\mathbf{1}_{\{\tau > t\}}, \end{aligned}$$

where we have defined the Q-intensity $\lambda^Q \equiv (1+\Gamma)\lambda^P$

λ^Q vs. λ^P

$$P(t,T) \;=\; e^{-(r+\lambda^Q)(T-t)}\mathbf{1}_{\{\tau>t\}}$$

$\Rightarrow$ Note that the credit spread for all (i.e., even short) maturities equals λ^Q.

Risk-premium:

$$\begin{aligned}\left(\frac{1}{dt}\mathrm{E}\left[\frac{dP(t,T)}{P(t,T)}\right] - r\,dt\right) \;&=\; -\frac{1}{dt}\mathrm{E}\left[\frac{d\Lambda}{\Lambda}\frac{dP}{P}\right]\\ &=\; \Gamma\lambda^P\\ &=\; \left(\lambda^Q - \lambda^P\right).\end{aligned}$$

$\Rightarrow$ $\lambda^Q \neq \lambda^P$ only if $\Gamma \neq 0$;
$\Rightarrow$ that is, only if $d\mathbf{1}_{\{\tau\leq t\}}$ shows up in pricing kernel.

$\Rightarrow$ Systematic jump event risk ($d\Lambda(t)\,d\mathbf{1}_{\{\tau\leq t\}} > 0$) can **potentially** explain high credit spreads even at short maturities.

$\Rightarrow$ Diffusion risk ($d\Lambda(t)\,d\lambda(t)$) not sufficient, at least at short end.

How large is the average jump-event premium?

- Cross-section estimate: Driessen (2006) estimates λ^Q from corporate bond data, and λ^P from historical default rates. Finds $\lambda^Q/\lambda^P \approx [2, 6]$.

- Berndt, Douglas, Duffie, Ferguson and Schranz (2007) estimate default risk premium from data on CDS and KMV EDFs. Find $\lambda^Q/\lambda^P \approx 2$ (but highly time varying) .

- What does this imply for the pricing kernel?

$$\frac{d\Lambda(t)}{\Lambda(t)} = -rdt + \sum_{i=1}^{N} \Gamma_i \left(d\mathbf{1}_{\{\tau_i \leq t\}} - \lambda_i^P \mathbf{1}_{\{\tau_i > t\}} dt \right)$$

  If firms are symmetric, implies variance of the pricing kernel (assume $N = 1000$): $\Gamma_{,i} \approx [1, 5]$ and $V[\frac{d\Lambda_t}{\Lambda_t}] = \Sigma_i \lambda_i \Gamma_{,i}^2 \approx 1000 * 0.001 * (1^2, 5^2) = (100\%, 2500\%)$!

- Q? When should jump-to-default risk be priced (i.e., default events affect the pricing kernel)?

  A! When it cannot be diversified away.

  A1) When many firms jump to default simultaneously.

  A2) When default of one firm correlated with losses in bond portfolio

Why Should Jump-Risk be Priced?

- Models of default event correlation:
  - $d\lambda_i(t)\, d\lambda_j(t) \neq 0$: Correlated default intensities (possibly with common jumps).
    $\Rightarrow$ Jumps are 'conditionally diversifiable,' (not priced) in Cox-process framework (Jarrow, Lando, Yu (01), Das, Freed, Geng, Kapadia (2006)).

  - $d\mathbf{1}_{\{\tau_i \leq t\}}\, d\mathbf{1}_{\{\tau_j \leq t\}} \neq 0$: Systemic Risk (multiple firms default at same time).
    (Davis and Lo (01), Driessen (2006), Guo, Miao, Morellec (2005))
    $\Rightarrow$ Little empirical evidence (Peso problem?)

  - $d\lambda_i(t)\, d\mathbf{1}_{\{\tau_j \leq t\}} \neq 0$: Contagion Risk

    1) Jarrow, Yu (2001): Counterparty Risk due to direct ties between firms
    2) Our paper: Contagion between firms with no 'direct' economic ties:
       - Enron default led to significant increase in spread of other firms (e.g., GE, Tyco) with no direct economic ties to Enron
       - RJR LBO demonstrated that no firm was too large for LBO

Focus of paper

- This paper proposes simple reduced-form model of counterparty risk based on information contagion in credit spreads where:
  - Common jumps due to changes in the perception of risk (updating of beliefs).
  - Consistent with simple structural model with imperfect information.
  - Tractable for arbitrary number of firms.
  - Can generate substantial time variation in λ^Q/λ^P.

Results

- Theoretical results
  - Reduced-form model of contagion due to updating of beliefs shows credit spread of firm k affected by:
    1. Common jump between firm k and pricing kernel ($\Gamma_{\Lambda,k}$).
    2. Product of contagion jump between k and each firm i ($\Gamma_{k,i}$) and each $\Gamma_{\Lambda,i}$.
  - Jump to default risk is priced only if there is contagion risk. λ^Q/λ^P is determined by the average contagion jump.
- Empirical results
  - Identify individual firm credit events ('surprises')
  - Find evidence that in the event months:
    1. Corporate bond credit spreads widen,
    2. Treasury return exhibits abnormal positive return ('flight to quality'),
    3. Equity market drop on average (although not statistically significant).

    $\Rightarrow$ Suggests there is scope for contagion risk and systematic event risk premia.
- Calibration shows that:
  - Systematic jump risk alone cannot contribute significantly to spreads ($\approx$ 3bp, for reasonable volatility of the pricing kernel).
  - Contagion risk could, in principle, explain $\approx$ 20bp for investment grade debt.

Related Literature

- Learning:
  - Detemple (1989), Gennotte (1986)
- Regime Switching
  - Hamilton (1989), Veronesi (2000), Guo, Miao, and Morellec (2002)
- Contagion (theoretical)
  - King and Wadhwani (1990), Kodres and Pritsker (2002)
- Contagion (empirical)
  - Lang and Stulz (1992), Bae, Karolyi and Stulz (2000)
- Default Risk
  - Jarrow, Lando and Yu (2001), Duffie and Lando (2001), Driessen (2002)
- Correlated Default risk (theoretical)
  - Jarrow and Yu (2001), Giesecke (2001), Schönbucher and Shubert (2002), Davis and Lo (2001)
- Correlated Default risk (empirical/numerical)
  - Duffie, Singleton (2002), Das, Freed, Geng and Kapadia (2000), Yu (2003), Berndt, Douglas, Duffie, Ferguson, Schranz (2005), Duffie, Eckner, Horel, Saita (2007), Das, Duffie, Kapadia, Saita (2007), Jorion, Zhang (2007).

Simple example of 'information contagion' model

- N firms with default intensities that are all either in high (λ_i^H) or low (λ_i^L) state.
- Investors form prior that $p^H(t) = \Pr(H \mid \mathcal{F}_t)$, thus $\mathcal{F}_t$-default intensity is:

$$\overline{\lambda}_i(t) = p^H(t)\lambda_i^H + (1 - p^H(t))\lambda_i^L$$

- Investors continuously update $p^H(t)$ conditional on discrete arrival of (default) events.

$\Rightarrow$ Using Bayes rule we find:

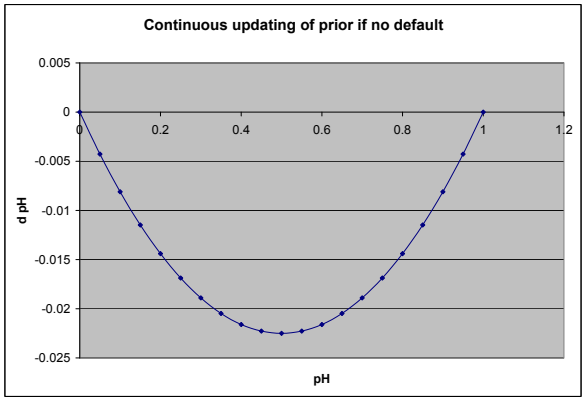
$$dp^H(t) = p^H(t)\left(1 - p^H(t)\right) \sum_{i=1}^{N} \frac{\lambda_i^H - \lambda_i^L}{\overline{\lambda}_i(t)} \mathbf{1}_{\{\tau_i > t\}} \left(d\mathbf{1}_{\{\tau_i \leq t\}} - \overline{\lambda}_i(t)dt \right).$$

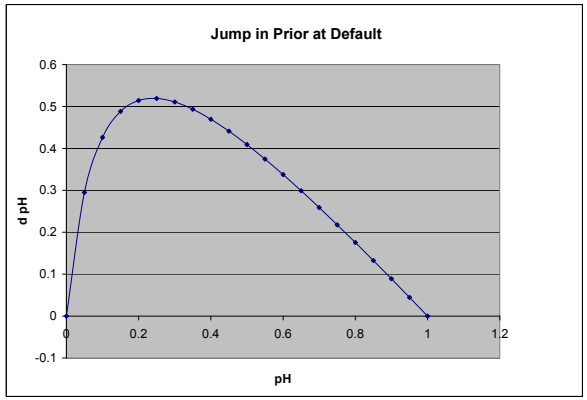
- Intuition:
  - If p^H is either 0 or 1, no updating.
  - On average $p^H(t)$ does not change (it is a martingale).
  - When no default, revise downward the probability of being in high state.
  - At default revise upward the probability.
- Survival probability easily computed (Bayes). On $\{\tau > t\}$:

$$\mathrm{E}_t[\mathbf{1}_{\{\tau_i > T\}}] = p^H(t)e^{-\lambda_i^H(T-t)} + (1 - p^H(t))e^{-\lambda_i^L(T-t)} \quad \left(\neq \mathrm{E}_t\left[e^{-\int_t^T \overline{\lambda}_i(s)ds}\right] \right)$$

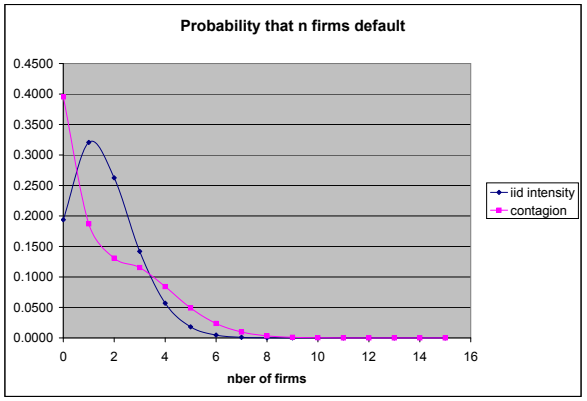
Illustration: updating for portfolio

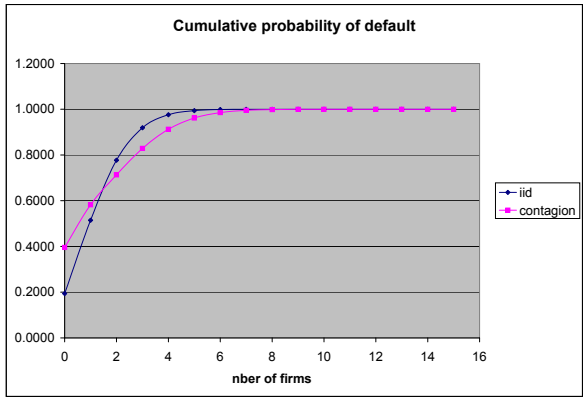
- Consider portfolio of 100 risky zero-coupon bonds with 30 year maturity.
- Two states (J=2). ($\lambda^H = 0.01$) and ($\lambda^L = 0.001$) are constants.
- N= 1000 Firms share the contagion risk.





1. As long as no default is observed, priors revised down.
2. Observing no-default is more informative the larger the remaining pool size,
3. Updating is maximum at $p(t) = 0.5$, i.e. when investor is more 'confused.'
4. Prior revised upward at time of occurrence of a default (Maximum below $p^H = 0.5$).





- The risk-neutral probability density of observing $n = 1, \ldots 25$ defaults at a thirty year horizon with this portfolio, vs.
- The probability density estimated if one were to assume firms had constant risk-neutral intensity $\widehat{\lambda}$ estimated to match the same average default frequency.

  $\Rightarrow$ The figure shows that the contagion-model puts more weights into the tails of the distribution than the iid model.

  $\Rightarrow$ Impacts values of tranches in typical CDO deals. (Contagion model tends to decrease price of top tranches and increase value of residual tranches relative to i.i.d.).

General Reduced-Form Framework

- Assume there are J states with 'prior' $p^j(t)$.
- In each state the firms $i = (1, N)$ have default intensity $\lambda_{ij}(t)$ with $\lambda_{i1} < \ldots < \lambda_{iJ}$.
- The 'conditional' intensities λ_{ij} can be stochastic, for example $\lambda_{ij}(t) = a_{ij}\, x(t)$ with $a_{i1} < \ldots < a_{iJ}$ and $dx_t = \kappa(\theta - x_t)dt + \sigma dz_t + \gamma dq_t$.

- The default intensity for firm i is $\overline{\lambda}_i(t) = \Sigma_{j=1}^{J} \lambda_{ij}(t) p^j(t)$, where p^j follows:

$$dp^j(t) = \sum_{i=1}^{N} \left[p^j(t^-) \left(\frac{\lambda_{ij}(t^-)}{\overline{\lambda}_i(t^-)} - 1 \right) \right] \left(d\mathbf{1}_{\{\tau_i \le t\}} - \overline{\lambda}_i(t) \mathbf{1}_{\{\tau_i > t\}} dt \right)$$

- We show that if (1) $p^j(t_0) > 0$, and (2) $\Sigma_{j=1}^{J} p^j(t_0) = 1$, then the p^j can be interpreted as 'conditional probabilities' in the sense that:
  1. $1 > p^j(t) > 0$ and $\Sigma_{j=1}^{J} p^j(t) = 1 \; \forall \, (t, \omega)$
  2. $p^j(t)\mathbf{1}_{\{\tau_i \le t\}} - \int_0^t p^j(s)\lambda_{ij}(s)\mathbf{1}_{\{\tau_i > s\}} ds$ is a martingale.

- N.B.: Counterparty risk arises through the jump in $\{p^j\}$, i.e. $dp^j(t) d\mathbf{1}_{\{\tau_i \le t\}} \neq 0$, which implies $d\overline{\lambda}_k(t) d\mathbf{1}_{\{\tau_i \le t\}} \neq 0$

- As a result survival probability have simple form:

$$\mathrm{E}_t\Big[\mathbf{1}_{\{\tau_i>T\}}\Big] = \mathrm{E}_t\Big[\sum_{j=1}^{J} p^j(T)\mathbf{1}_{\{\tau_i>T\}}\Big] = \sum_{j=1}^{J} p^j(t)\,\mathrm{E}_t\Big[e^{-\int_t^T \lambda_{ij}(s)ds}\Big]\,\mathbf{1}_{\{\tau_i>t\}}.$$

- In particular, if the $\lambda_{ij}(t)$ follow an affine process, then the historical measure survival probability is a weighted average of exponential affine functions.

- N.B.: The model is outside the 'doubly-stochastic' diffusion framework of Lando (1998) and Duffie and Singleton (1999), because of cross-contagion risk.

- As a result the survival probability $\mathrm{E}_t[\mathbf{1}_{\{\tau_i>T\}}]$ is **not** equal to $\mathrm{E}_t[e^{-\int_t^T \overline{\lambda}_i(s)ds}]\mathbf{1}_{\{\tau_i>t\}}$

- However, model is tractable for arbitrary number of firms.

- Further, the model is consistent with simple multi-firm extension of structural model of Duffie-Lando (2001).

A Structural Model of Contagion based on Duffie and Lando (2001)

- Each firm asset value follows geometric Brownian motion:

$$dX_i(t) = \mu_i X_i(t)\, dt + \sigma_i X_i(t)\, dz_i(t).$$

- Investors receive continuous noisy information $y_i(t) = \log X_i(t - \ell_i)$
  $\Rightarrow$ current lagged estimate is a 'sufficient statistic'

- Length of time lag (i.e., quality of accounting information) is uncertain, but shared across firms: $\ell_i = \ell_i^H$ or ℓ_i^L for all firms.

  $\Rightarrow$ A default by 'Enron' changes perception of accounting quality for other firms

- Conditional on a lag ℓ default is unpredictable stopping time with intensity:

$$\lambda(t) = \frac{\frac{1}{\sqrt{2\pi\sigma^2\ell}}\left(\frac{y(t)}{\ell}\right) e^{-\frac{(m\ell+y(t))^2}{2\sigma^2\ell}}}{\mathrm{N}\left(\frac{y(t)+m\ell}{\sigma\sqrt{\ell}}\right) - e^{\frac{-2y(t)\,m}{\sigma^2}}\ \mathrm{N}\left(\frac{-y(t)+m\,\ell}{\sigma\sqrt{\ell}}\right)}. \tag{1}$$

- When information is uncertain default intensity of firm i is:

$$\overline{\lambda}_i(t) = p^H(t)\lambda_i^H(t) + (1 - p^H(t))\lambda_i^L(t) \quad \text{where } p^H(t) \text{ is prior that } \ell_i = \ell^H.$$

$\Rightarrow$ This is a special case of the reduced form model presented before!

Pricing defaultable claims in contagion model with systematic jump risk

- Assume there exists pricing kernel $\Lambda_t = e^{-\int_0^t r_s ds}\xi^c(t)\,\Sigma_{j=1}^J p^j(t)\xi^j(t)$ with dynamics:

$$d\xi^c(t) = -\xi^c(t)\theta_t^\top\, dz_t \qquad \text{and} \qquad d\xi^j(t) = \xi^j(t^-)\,\Sigma_{i=1}^N \left(\gamma_{ij} d\mathbf{1}_{\{\tau_i \le t\}} - \lambda_{ij}(t)\widehat{\gamma}_{ij}\mathbf{1}_{\{\tau_i > t\}} dt\right)$$

$$\frac{d\Lambda(t)}{\Lambda(t^-)} = -r_t dt - \theta_t^\top dz_t + \sum_{i=1}^N \left(\Gamma_{\xi,i} d\mathbf{1}_{\{\tau_i \le t\}} - \overline{\lambda}_i(t)\widehat{\Gamma}_{\xi,i}\mathbf{1}_{\{\tau_i > t\}} dt\right)$$

- The risk-neutral probability of state j and corresponding conditional intensities are:

$$q^j(t) = p^j(t)\frac{\xi^j(t)}{\Sigma_j p^j(t)\xi^j(t)} \qquad \text{and} \qquad \lambda_{kj}^Q(t) = \lambda_{kj}(t)(1+\widehat{\gamma}_{kj})$$

- The price of a risky bond with zero recovery is:

$$B_k(t) := \mathrm{E}_t^Q\left[e^{-\int_t^T r_s ds}\mathbf{1}_{\{\tau_k > T\}}\right] = \sum_{j=1}^J q^j(t)\mathrm{E}_t^Q\left[e^{-\int_t^T \left(r(s)+\lambda_{kj}^Q(s)\right)ds}\right]\mathbf{1}_{\{\tau_k > t\}}$$

- Further the instantaneous premium for credit risk is given by:

$$\frac{1}{dt}\mathrm{E}^P\left[\frac{dB_k(t)}{B_k(t)}\right] - r_t = \sigma_B(t)^\top\theta(t) + \sum_{i\neq k}\overline{\lambda}_i(t)\mathrm{E}[\Gamma_{k,i}\Gamma_{\xi,i}] + \overline{\lambda}_k(t)\mathrm{E}[\Gamma_k\,\Gamma_{\xi,k}]$$

$\Rightarrow$ Three components in credit risk-premium:

- Continuous (intensity) co-variation term.
- Individual firm systematic jump risk premium ($\Gamma_{\xi,k}$).
- Interaction of contagion ($\Gamma_{k,i}$) and each firm i's jump risk ($\Gamma_{\xi,i}$).

Relation between Contagion and Jump-to-default risk

- Suppose kernel is perfectly correlated with bond market portfolio $M_t = \Sigma_{i=1}^{N} B_i(t)$:

$$\frac{d\Lambda(t)}{\Lambda(t)} = \alpha_t dt - \beta_t \frac{dM(t)}{M(t)}$$

- If all firms are symmetric ($\frac{B_k}{\Sigma_i B_i} \approx \frac{1}{N}$) then with $\Gamma_{i,k} \equiv \Gamma_c$ and $\Gamma_k \equiv \Gamma_d$ obtain:

$$\frac{d\Lambda_t}{\Lambda_t} = -r(t)dt - \beta\sigma_B dz_t + \beta \sum_{i=1}^{N} \left(\frac{\Gamma_d}{N} + \frac{(N-1)}{N}\Gamma_c \right) (d\mathbf{1}_{\{\tau_i \le t\}} - \lambda^P dt).$$

- This implies

$$\frac{1}{dt}\mathrm{E}\left[\frac{dB_k(t)}{B_k(t)}\right] - r = \beta\sigma_B^2 + \frac{\beta\lambda}{N}\left(\Gamma_d + (N-1)\Gamma_c\right)^2.$$

$\Rightarrow$ If $N \to \infty$ jump-to-default risk priced (i.e., $\lambda^Q > \lambda^P$) only if contagion risk $\Gamma_c > 0$.

$\Rightarrow$ Jump-to-default credit risk premium $\frac{\beta\lambda}{N}\left(\Gamma_d + (N-1)\Gamma_c\right)\Gamma_d$

$\Rightarrow$ Contagion risk premium $\frac{(N-1)\beta\lambda}{N}\left(\Gamma_d + (N-1)\Gamma_c\right)\Gamma_c$

$\Rightarrow$ As $N \Rightarrow \infty$, need $\Gamma_C \sim \frac{1}{\sqrt{N}}$ for risk premia to be finite.

$\Rightarrow$ Expect $\left(\Gamma_d \gg \Gamma_c\right)$ but $\left(\Gamma_d \ll (N-1)\Gamma_c\right)$

Empirical Analysis

- We investigate whether jump in individual firm credit spread can have market wide impact.
- Warga Fixed Income Database 1973-1998.
- Define event to be a change in credit spread larger than 200bp.
- Check each month/event for 'surprise news' (Lexis,-Nexus, S&P credit week, WSJ).
  - Out of 264,099 Δ CS identify 112 'events' of Δ CS $>$ 200 bp experienced by 40 firms.
  - 25 out of 298 months exhibit an event.
- Compare monthly excess returns on Lehman corporate bond index, Treasury index and Equity index in months with and without event (at most, bonds subject to event represent 27 out of 3811 bonds in Lehman index).
- Control for size of firm experiencing the event two ways: (1) total outstanding bonds, (2) total assets.
- Attempt to control for other explanatory variables (causality), by running regressions of changes in excess corporate bond excess returns, Treasury returns and NYSE stock returns on various predictors and dummy for event month.

	Number of observations	Percentage	Average Return	Excess Return	Average Duration
Over 10 percentage points	7	0.01%	-31.33%	-31.37%	2.36%
5 to 10 percentage points	6	0.01%	-17.43%	-16.79%	3.17%
3 to 5 percentage points	32	0.06%	-12.40%	-12.91%	4.26%
2 to 3 percentage points	113	0.21%	-7.74%	-8.93%	4.57%
1.5 to 2 percentage points	146	0.28%	-5.42%	-6.69%	4.76%
1.25 to 1.5 percentage points	131	0.25%	-4.08%	-5.50%	5.10%
1 to 1.25 percentage points	273	0.52%	-2.85%	-4.08%	4.87%
0.75 to 1 percentage points	572	1.08%	-2.00%	-3.63%	5.70%
0.5 to 0.75 percentage points	1919	3.63%	-1.08%	-2.84%	6.30%
0.25 to 0.5 percentage points	5776	10.93%	-0.54%	-1.18%	6.39%
Less than 0.25 percentage points	43853	83.01%	0.08%	-0.23%	7.71%

Table 1: Distribution of Spread Increases

		Size measured by bonds outstanding		Size measured by total assets	
	All Events	Events Involving Large Issuers	Events Involving Small Issuers	Events Involving Large Issuers	Events Involving Small Issuers
Number of events	25	11	14	13	12
Corporate Bond Returns:					
in months with events	-0.33	-1.05	0.24	-0.53	-0.11
in months with no events	0.06	0.07	0.02	0.06	0.04
Difference in returns	0.39	1.12	-0.22	0.58	0.15
T-statistic	1.74	3.42	-1.50	1.91	0.47
p-value	0.084	0.001	0.149	0.057	0.641
Treasury Bond Returns:					
in months with events	1.28	1.22	1.33	1.55	0.30
in months with no event	0.70	0.73	0.72	0.71	0.76
Difference in returns	-0.59	-0.49	-0.62	-0.84	0.46
T-statistic	-1.70	-0.96	-1.37	-3.05	0.72
p-value	0.090	0.336	0.173	0.008	0.471
Stock Market Returns:					
in months with events	0.80	0.05	1.38	0.24	1.40
in months with no event	1.13	1.14	1.09	1.14	1.09
Difference in returns	0.33	-1.09	-0.30	0.90	-0.31
T-statistic	0.36	0.57	-0.25	0.72	-0.24
p-value	0.721	0.578	0.807	0.474	0.811

Table 2: Mean Excess Returns of Corporate Bond, Treasury and Stock Indices When Credit Events Occur

	Corporate Bond Excess Returns			Treasury Bond Returns		Stock Market Excess Returns	
	(1)	(2)	(3)	(1)	(2)	(1)	(2)
Constant	-0.22	0.10	-0.09	1.19	1.19	-1.09	-0.95
	(-1.88)	(1.64)	(-0.84)	(5.20)	(5.18)	(-4.28)	(-3.66)
Credit event month	-0.43	-	-	0.54	-	-0.52	-
	(-2.14)			(1.72)		(-1.61)	
Large firm credit event month	-	-0.39	-0.77	-	0.67	-	-0.79
		(-2.67)	(-3.23)		(2.08)		(-2.52)
Small firm credit event month	-	-0.03	-0.20	-	0.42	-	-0.50
		(-0.38)	(-0.80)		(0.78)		(-0.92)
Change in Fed Funds	-0.22	-	-	-0.32	-0.32	0.29	0.37
	(-1.63)			(-1.64)	(-1.64)	(1.37)	(1.79)
Change in CPI	-	-	-	-1.14	-1.15	1.02	0.67
	-	-	-	(-2.75)	(-2.75)	(2.21)	(0.48)
Change in Taylor rule	-	0.07	-0.27	-	-		-
		(0.54)	(-2.30)				
Change in defaults	14.45	-14.95	16.16	-	-	-	-
	(0.90)	(-2.04)	(1.01)				
Change in upgrade/downgrade ratio	0.02	0.08	0.04	-	-	-	-
	(0.07)	(1.62)	(0.13)				
Slope of the yield curve	0.35	0.008	0.35	-	-	-0.07	-0.06
	(3.26)	(0.17)	(3.28)			(-0.58)	(-0.45)
Change in consumer confidence	-0.79	-	-	-4.94	-4.93	5.01	-
	(-0.80)			(-2.63)	(-2.61)	(2.66)	
Change in current account	0.17	0.06	0.17	0.12	0.12	-0.13	-0.19
	(2.93)	(1.54)	(2.76)	(1.25)	(1.16)	(-1.28)	(-1.68)
Change in payrolls	0.0003	-	-	-0.001	-0.001	0.001	0.001
	(0.77)			(1.33)	(-1.27)	(1.34)	(1.22)
Change in IP	-0.0006	-	-	-	-	-	-
	(-0.006)						
Indicator for 1987 crash	-1.32	0.80	-0.59	2.82	2.81	-3.03	-3.04
	(-5.73)	(1.15)	(-0.85)	(8.37)	(8.32)	(-9.24)	(-8.70)
Flight to quality indicator	0.28	-	-	0.72	0.72	-0.70	-0.75
	(1.55)			(2.43)	(2.43)	(-2.35)	(-2.45)
Shock to institutional money funds	-	-.02	-				-
		(-.22)					
Change in VIX (post 1986)	-	-0.047	-	-	-	-	-
		(-2.52)					
Stock return volatility	-	-	-9.14	-	-	-	-
			(-0.66)				
Adjusted R^2	0.12	0.13	0.13	0.16	0.16	0.16	0.11

Table 3: Regression of Effects of Credit Events on Corporate bonds, Treasuries and Stocks Bond Index. Stock returns are 2 percent. Change in defaults and is industrial production. market. Stock return volatility

- Results:
  1. Significant contagion effect of events triggered by large firms.
  2. Strong evidence of 'flight to quality' both statistically and economically.
  3. Some impact (although not statistically significant) on equity market.
  4. Event month dummy strongly significant and robust to various controls.
  5. While CS increase in event month, Corporate bond return experiences positive return in these months (priced off Treasury).
- Questions:
  - Event definition?
  - Liquidity/Quotes for 3800 bonds in Lehman bond index?
  - Flight to quality? Can rebalancing from Corporate market 'cause' substantial impact in Treasury market?
  - Hedging of Corporate bond index credit spread shock with Treasury?

Contagion and systematic jump risk: A Simple Calibration

- Component of credit spread due to contagion and jump risk is:
$$\Sigma_{i\neq k}\,\overline{\lambda}_i(t)\mathrm{E}[\Gamma_{k,i}\Gamma_{\xi,i}] + \overline{\lambda}_k(t)E[\Gamma_k\Gamma_{\xi,k}]$$
- Table 1 shows that excess return of corporate bond over Treasuries conditional on a 200 bp change is on the order of -10%, which is much smaller than -50% drop associated with a jump to default from par to average recovery rate ($\approx 50\%$)
- Such events occurred 112 times out of 260,000 events.
$\Rightarrow$ Annualized estimate of λ^P is $\frac{112\times 12}{260{,}000} \approx .005$.
$\Rightarrow \lambda^P$ much smaller if interpreted 'literally' as a jump to default (i.e., $\in [0.001, 0.0001]$).
- The average annualized volatility of the pricing kernel is on the order of 0.5 (Sharpe ratio of 'Market Portfolio' $\approx \frac{.09-.01}{.16}$, Cochrane (2000))
- Variance of pricing kernel is $V_\Lambda^2 = \sigma_\theta^2 + \Sigma_{i=1}^N \lambda_i^P \Gamma_{\xi,i}^2$. Implies $\Gamma_{\xi,i} = 0.22$ (with $N = 1000$, $\sigma_\theta = 0$).
- Table 3 gives $\Gamma_{k,i} = 39$ bp.

N	V_Λ	λ^p	$\Gamma_{\xi,i}$	$\Gamma_{k,i}$	Γ_k	Jump risk premium	Contagion premium
1000	0.5	0.0052	0.22	0.0039	0.1	0.00011	0.00443
1000	0.5	0.001	0.5	0.0039	0.5	0.00025	0.00195

Conclusion

- Propose a general reduced form framework that is tractable and consistent with contagion effects in credit spreads.
- Common jumps are generated through updating of beliefs about risk-states.
- Consistent with Duffie Lando 'noisy accounting' information model with multiple firms and correlated accounting procedures.
- Decompose credit spreads into systematic jump component and contagion risk component.
- Jump to default is priced (and $\lambda^Q > \lambda^P$) only if it cannot be diversified away. Thus contagion risk determines the magnitude of λ^Q/λ^P.
- Document empirically the presence of contagion in bond markets and of apparent 'flight to quality.'
- However, the magnitude seems to imply that jump risk premium contribute less than 5bp per year to expected returns on bonds.
- Instead, contagion risk could explain an additional 20bp (with N=1000).
- As long as jump in price due to contagion is not N times less than LGD, contagion risk premium $>$ jump to default premium.