How integrated are Credit and Equity markets? Evidence from index options

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Motivation

The CDX option market

Quote data

A structural credit risk model

The relative pricing of CDX and SPX options

What explains 'mispricing'?

Structural models and the credit spread level

- Structural model of default risk views bonds (CDSs) and equity as contingent claims on the **same** underlying firm value (Black-Scholes-Merton).
- **First-generation** structural models underpredict the level of credit spreads when calibrated to low historical default rates:

→ The **credit spread puzzle**
(Jones, Mason, and Rosenfeld (1984), Huang and Huang (2003))

- **Second-generation** structural models calibrated to match equity risk premia and equity option implied volatilities improve significantly at matching the **level** of credit spreads
(Cremers, Driessen, and Maenhout (2008), Chen, Collin-Dufresne, and Goldstein (2009), Du, Elkamhi, and Ericsson (2019))

→ "a good deal of integration between corporate bond and options markets" (Culp, Nozawa, and Veronesi (2018)).

Structural models and credit spread changes

- Mixed evidence when looking at corporate bond **returns**:
  - Common factors in credit spread changes unexplained by standard structural model factors
    (Collin-Dufresne, Goldstein, and Martin (2001), He, Khorrami, and Song (2020))
  - Equity factor bond betas do not explain cross-section of bond returns
    (Fama and French (1993), Choi and Kim (2018), Bai, Bali, and Wen (2019))
  - CDS and bond returns seem integrated with equity returns
    (Ericsson, Jacobs, and Oviedo (2009), Koijen, Lustig, and Van Nieuwerburgh (2017))

$\Rightarrow$ This paper studies relation between corporate bond and equity return **volatilities** using data from a market on credit index options that has become very active since 2012

Summary: Model setup

- Prices of credit options are quoted using a Black model, which assumes that the forward *credit spread* follows a log-normal model.
  - $\rightarrow$ Can be rationalized using *reduced-form* model.
- Prices of equity index options are quoted using a Black-Scholes model, which assumes that the underlying *stock price* index follows a log-normal model.
  - $\rightarrow$ Both models are inconsistent.
- We propose a **consistent structural model** with closed-form prices for equity & debt and for options on equity & debt both for single-names and indexes:
  - Firm asset values follow correlated jump-diffusion processes.
  - Firms issue short- and long-term debt.
  - $\rightarrow$ CDS & equity $\sim$ put & call (Merton (1974)).
  - $\rightarrow$ CDS & equity options $\sim$ **compound options** (Geske (1979)).
  - $\rightarrow$ CDS & equity index distribution obtained using **large homogeneous pool** approximation (Vasicek (1987)).
  - $\rightarrow$ CDS & equity index option prices obtained in **closed form**.

Summary: Model results

- We calibrate the model parameters to match the SPX and CDX index as well as the SPX volatility surface and price CDX options out-of-sample.
- Main results:
  - Jump-diffusion model matches well joint dynamics of SPX-CDX implied vol surfaces (**leverage effects and skews**).
  - But, it significantly **underpredicts the level of credit volatilities** (across all strikes).
  - Trading strategy selling CDX vol generates significantly higher Sharpe ratio than SPX vol.
- We discuss several hypothesis for the apparent discrepancy between equity and credit index volatility levels including:
  - Model misspecification
  - Options span different economic states
  - Differences in index constituents
  - Investor clienteles
  - Bond-market-specific risk factors (e.g., stochastic bankruptcy costs).

Related literature

- Credit spreads and structural models:
  Cremers, Driessen, and Maenhout (2008), Chen, Collin-Dufresne, and Goldstein (2009), Zhang, Zhou, and Zhu (2009), Chen (2010), Bhamra, Kuehn, and Strebulaev (2010), Huang and Huang (2012), Chen, Cui, He, and Milbradt (2017), Du, Elkamhi, and Ericsson (2018), Culp, Nozawa, and Veronesi (2018)

- Determinants of credit spread changes (including SPX implied volatility):
  Collin-Dufresne, Goldstein, and Martin (2001), Hull, Nelken, and White (2004), Ericsson, Jacobs, and Oviedo (2009), Cremers, Driessen, Maenhout, and Weinbaum (2008)

- Factors in corporate bond and equity returns:
  Fama and French (1993), Bai, Bali, and Wen (2019), Israel, Palhares, and Richardson (2018), Ericsson, Jacobs, and Oviedo (2009), Koijen, Lustig, and Van Nieuwerburgh (2017), Lewis (2019))

Related literature

- Bond and equity market segmentation:
  Kapadia and Pu (2012), Bao and Pan (2013), Driessen and Van Zundert (2017)

- The relative pricing of tranche swaps and SPX options:
  Coval, Jurek, and Stafford (2009), Collin-Dufresne, Goldstein, and Yang (2012), Seo and Wachter (2018)

- The basket-index spread and the leverage effect:
  Kelly, Lustig, and Van Nieuwerburgh (2016), Bai, Goldstein, and Yang (2019)

- The out-of-the-money put premium:
  Bates (2001), Pan (2002)

- CDX swaption pricing and credit volatility risk-premium:
  White (2014), Chen, Doshi, Seo (2020), Amman and Moerke (2019)

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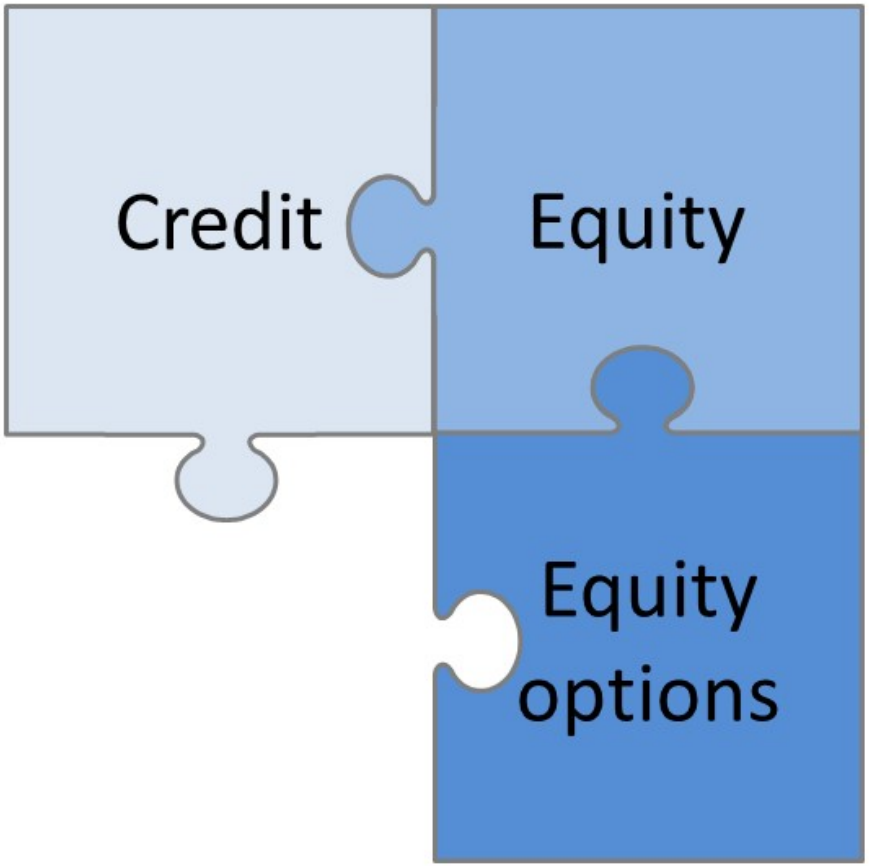
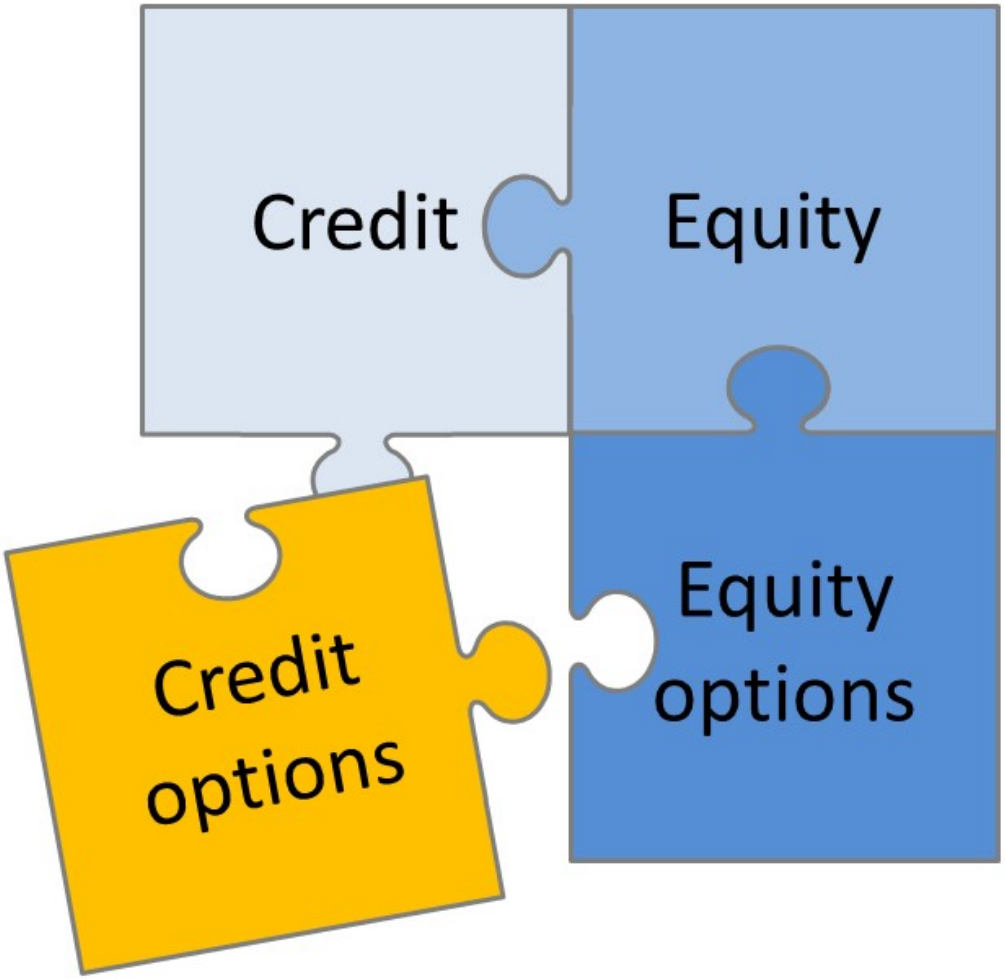
How integrated are credit and equity markets?



How integrated are credit and equity markets?



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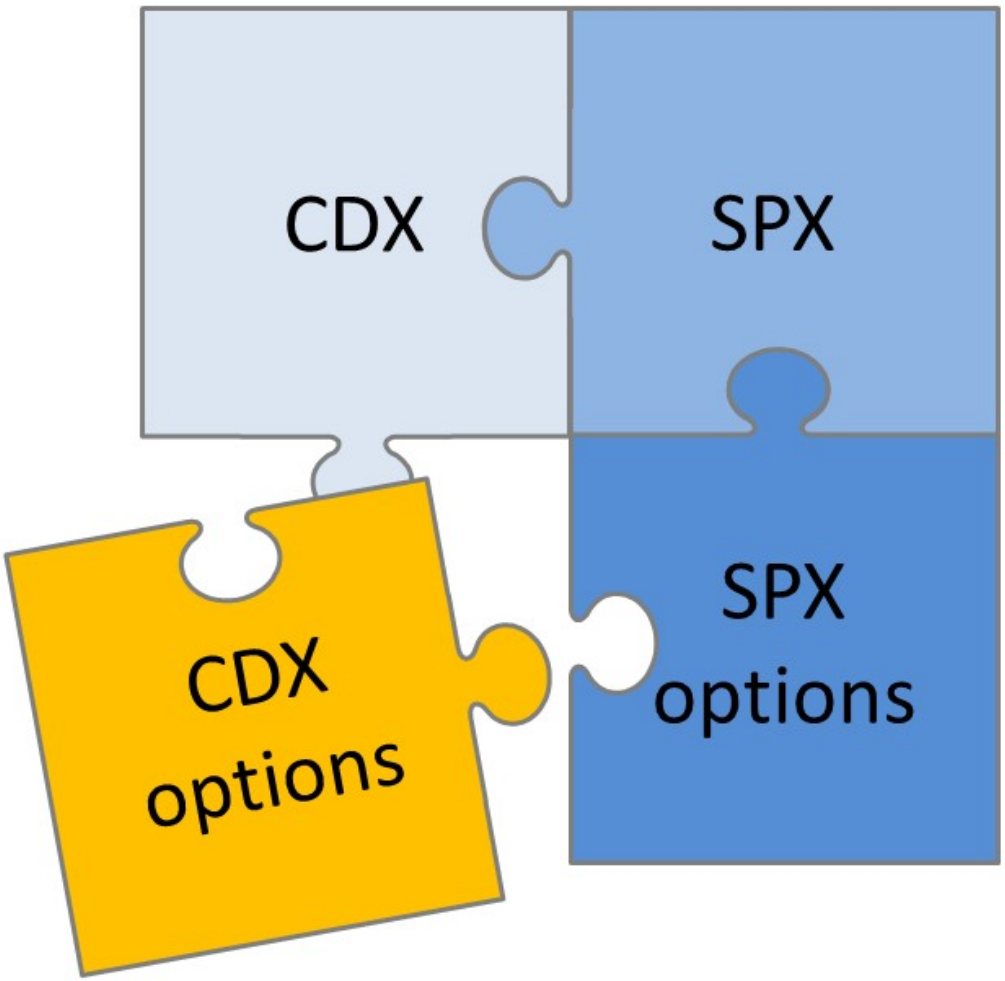


CDX credit default swap index

- CDS contract to insure against default risk on a broad portfolio of firms
- Main indices in North America:
  - CDX.IG: Default protection on 125 investment-grade names
  - CDX.HY: Default protection on 100 high-yield names
- Maturities from 1Y to 10Y
  - 5Y most liquid
- Every 6 months, new index (new series) is launched
  - Set of index constituents revised according to rating and liquidity criteria
  - On-the-run index most liquid
- Focus on CDX.IG, 5 year, *on-the-run* contract.

→ very liquid (Collin-Dufresne, Junge, and Trolle (2018))

CDX pricing

- Actual cash flows to buy protection:
  - pay
    (i) upfront amount (PUF) and
    (ii) annual fixed coupon of 100bps,paid quarterly on remaining non-defaulted principal.
  - receive full notional net of recovery $(1 - R)$ on any default in the index.

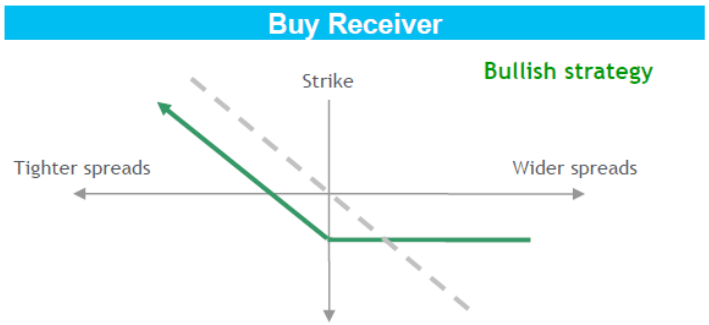
$$PUF = PV(prot) - PV(coupons)$$

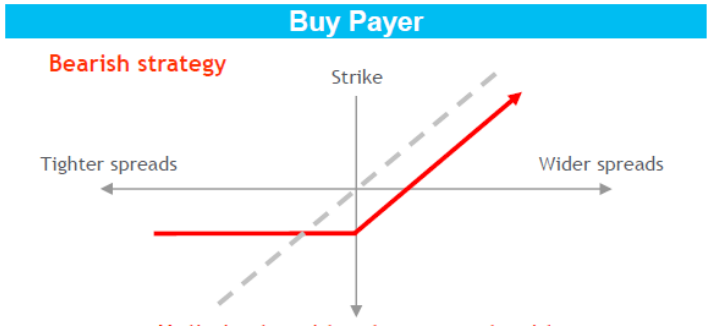
- Typically quoted in terms of par spread defined as the coupon level s.t. PUF=0 (using *ISDA CDS Standard Model*).

- Par spread $\approx$ annual insurance premium.

CDX options

- Option to buy/sell CDX protection at a future date at an agreed spread
  - Payer swaption: Right to buy protection
    $\rightarrow$ call option on spread
  - Receiver swaption: Right to sell protection
    $\rightarrow$ put option on spread





- Main features:
  - Underlying: 5Y, on-the-run index
  - European style
  - Expiration on the 3rd Wednesday of the month
  - Wide strike range

Summary statistics of transactions

	CDX	CDX options
Trades per day	202	18
Median trade size (in million USD)	50	100
Average daily volume (in million USD)	11,133	1,442
Five-year tenor (% of trades)	96.1	98.1
On-the-run series (% of trades)	88.9	94.0
Bespoke contract terms (% of trades)	1.2	8.0
Cleared (% of trades)	90.4	17.6
Block size (% of trades)	24.9	64.3
Capped trade size (% of trades)	22.3	66.5
On-SEF execution (% of trades)	83.9	3.8
Payer (% of trades)	—	63.1

Table 1: Descriptive statistics for CDX and CDX option trades

The table shows descriptive statistics for CDX and CDX option trades. Tenor is the initial time to expiration of the CDX contract (the underlying CDX contract in case of CDX options). The on-the-run series is the most recently launched CDX contract. A trade is block-sized if the notional amount traded exceeds a certain minimum block size. Typically, reported trade sizes are capped when the notional amount traded exceeds USD 100 million or USD 110 million. The sample period is from December 31, 2012 to April 30, 2020. The sample comprises 371,693 CDX trades and 32,669 CDX option trades.

- Source: Swap data repositories
- On main interdealer SEF (GFI), capped CDX option trades have an average size of 354 mln USD for CDX

→ used to estimate the actual trading volume.

Market size

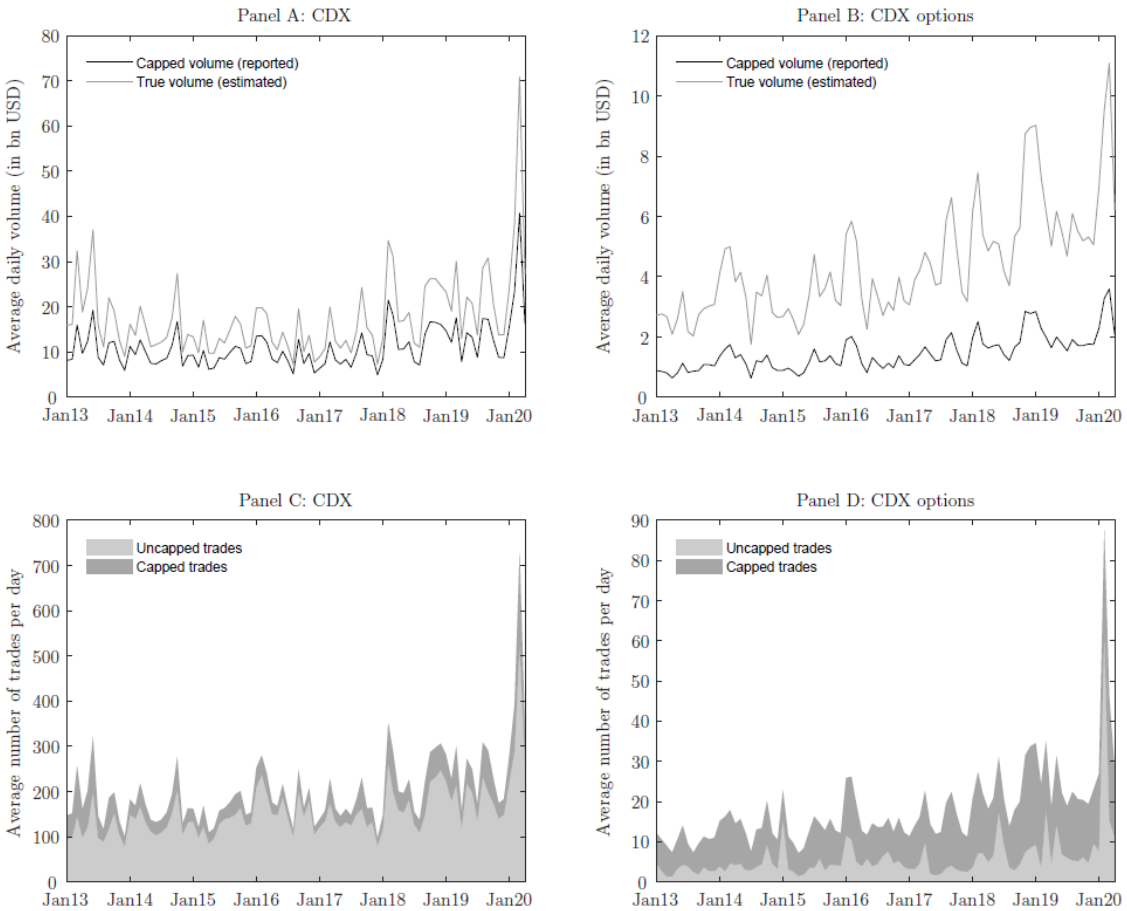


Figure 1: Trading activity for CDX and CDX options

Panels A and B show the average daily trading volume for CDX and CDX options. Panels C and D show the average number of trades per day for CDX and CDX options. Daily market activity reports from the GFI SEF are used to compute the average amount by which the actual notionals of capped trades on the GFI SEF exceed the reported notionals. This is done separately for CDX and CDX options (see Footnote 13 for details). The estimated true volume in Panels A and B is obtained by adding the average amount to the reported notionals for all capped trades. The frequency of observations is monthly. The sample period is December 31, 2012 to April 30, 2020 (88 observations).

Trading across the volatility surface

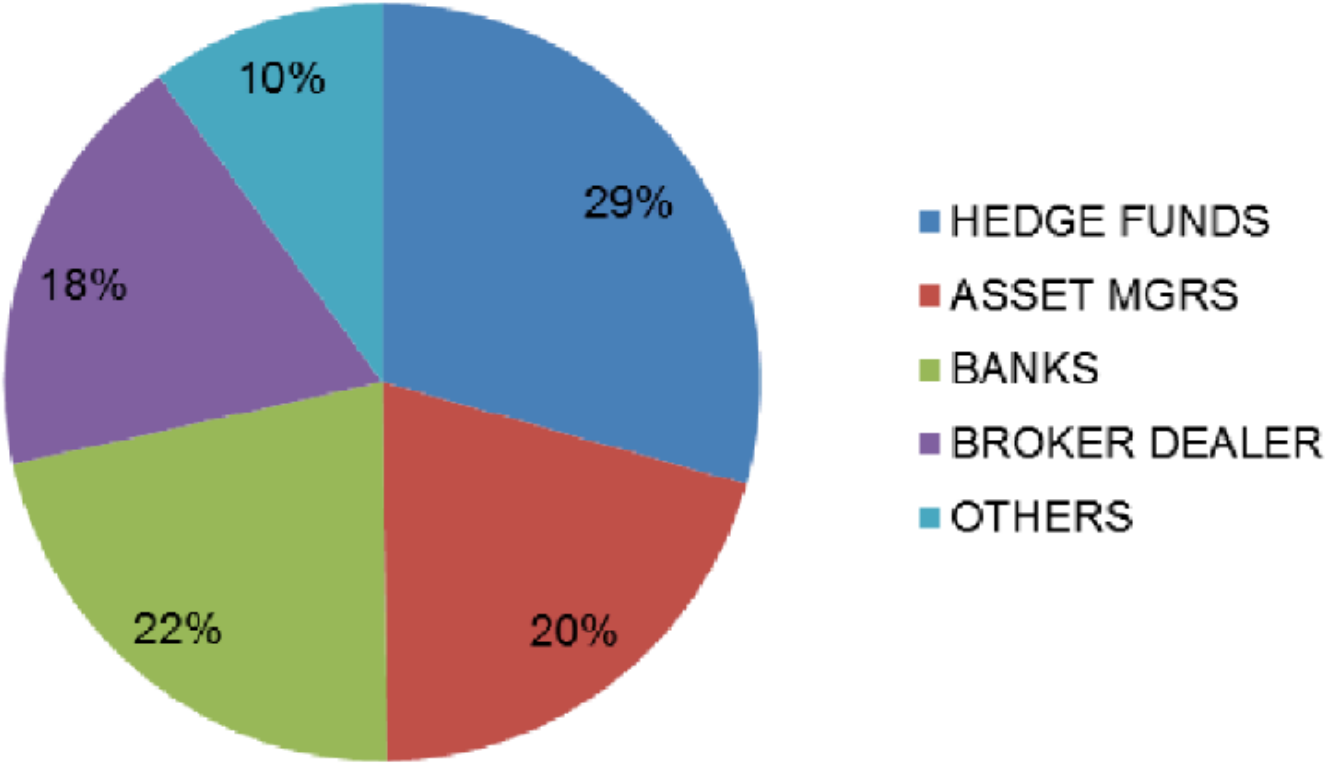
	Days to expiration						
Moneyness	< 15	15–44	45–74	75–104	105–134	≥ 135	Total
$m < -1.5$	0.20	0.24	0.06	0.01	0.00	0.00	0.52
$-1.5 \leq m < -0.5$	0.94	3.99	2.94	1.62	0.54	0.39	10.42
$\lvert m \rvert \leq 0.5$	2.12	15.28	9.61	6.48	2.27	1.28	37.03
$0.5 < m \leq 1.5$	1.53	9.01	10.34	8.75	4.17	2.30	36.10
$m > 1.5$	1.42	5.96	4.45	2.65	0.97	0.47	15.92
Total	6.21	34.47	27.40	19.51	7.95	4.45	

Table 2: Distribution of trading volume across the volatility surface

The table shows the percentage of CDX option volume across the volatility surface. Moneyness is defined as $m = \log(K/F(\tau))/(\sigma\sqrt{\tau})$, where K is the strike, $F(\tau)$ is the front-end-protected τ-forward spread, σ is at-the-money implied volatility, and $\tau = d/365$ is time to expiration, and d is days to expiration. The underlying of all options is the five-year on-the-run index. The sample period is from December 31, 2012 to April 30, 2020. The sample comprises 28,409 CDX option trades.

- Trading is concentrated in short-term (<4M), OTM payer swaptions (calls on spread)

Diverse set of market participants



Source: J.P. Morgan

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Quoted maturities

- CDX options: End-of-day quotes from Markit
  - Composites of "dealer runs" sent to clients
- Most quoting activity for short-term options (<4M)
- Quotations quickly dissipate as time-to-maturity drops below two weeks

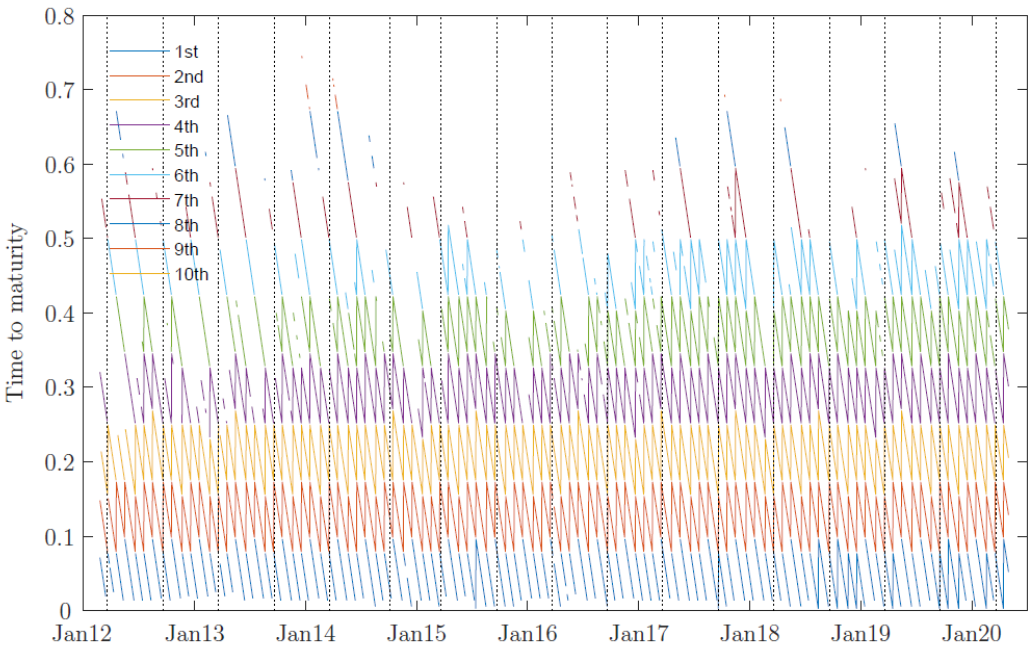

Figure IA1: Time series of quoted CDX option maturities

Option expiries are 3rd Wednesday of the month. Vertical dotted lines mark roll dates. Daily data from February 24, 2012 until April 30, 2020.

Standardized data set

- Focus on CDX vs. SPX
  - Similar characteristics of firm constituents (ratings, leverage)
- Maturity dimension:
  - Consider the first three option maturities with $\tau > 14$ days
  - Denote these 1M, 2M, and 3M options
- Strike dimension:
  - Compute moneyness as

$$m = \frac{\log(K/F)}{\sigma_{ATM}\sqrt{\tau}},$$

    where K = strike, F = forward, σ_{ATM} = ATM implied volatility, and τ = option maturity
  - Consider 13 moneyness intervals: $-3.25 < m \leq -2.75$, $-2.75 < m \leq -2.25$, ... , $2.75 < m \leq 3.25$...
  - ...and find option that is closest to the mid of the interval.

Estimate implied ATM volatility and skew

- On each date and for each option maturity, run cross-sectional regression
$$\sigma^{IV}(m) = \beta_0 + \beta_1 m + \beta_2 m^2 + \epsilon,$$
where m is moneyness
- β_0 captures the ATM implied volatility
- β_1 captures the skewness of the implied volatility smile
- Same procedure for SPX options (using end-of-day CBOE quotes)

Index volatility during the COVID-19 crisis

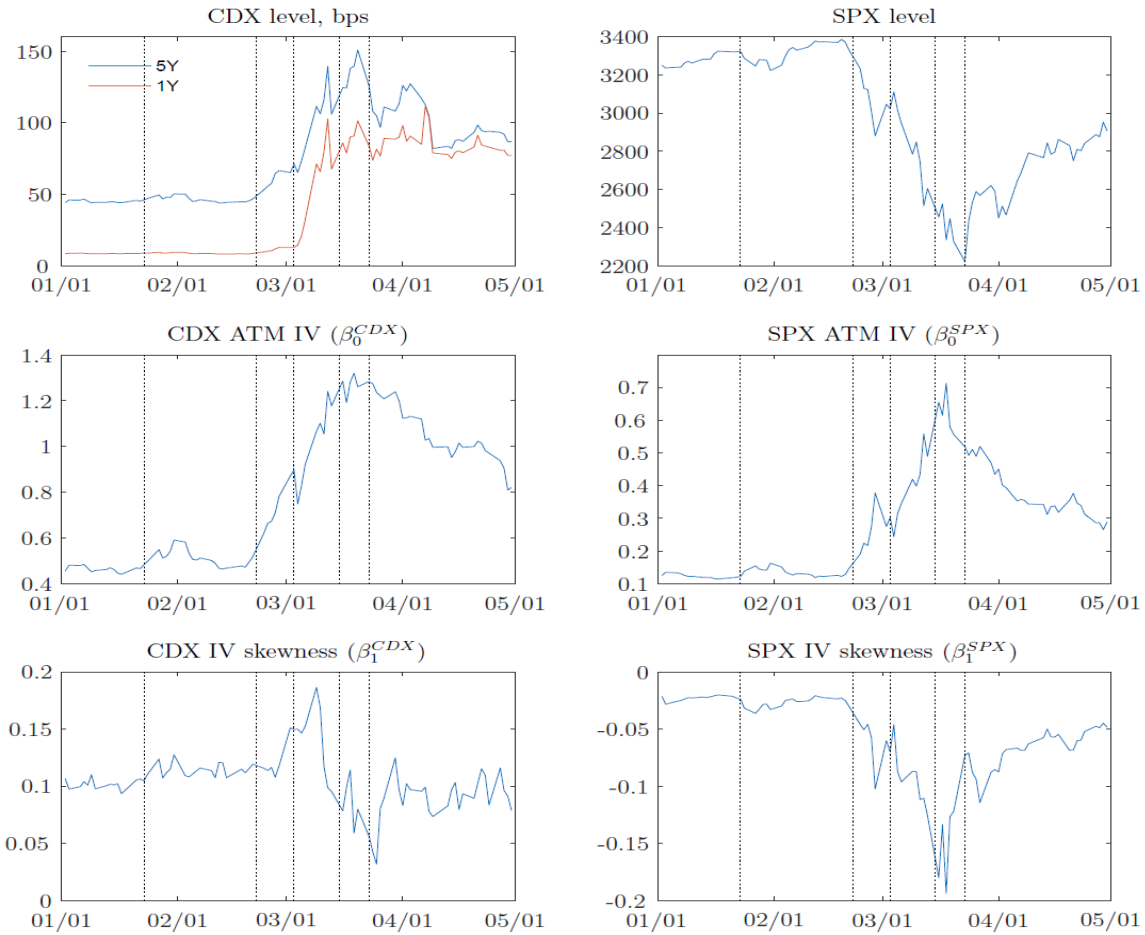

Figure IA6: Summary of CDX and SPX options markets during Covid-19 crisis

The top left (right) panel shows time series of the CDX (SPX) level. The middle left (right) panel shows time series of the at-the-money CDX (SPX) implied volatility proxied by the β_0-estimate for the M2 option. The bottom left (right) panel shows time series of the skewness of the CDX (SPX) implied volatility smile proxied by the β_1-estimate for the M2 option. The vertical dotted lines mark the Wuhan lockdown on January 23, the Italy quarantine on February 22, the 50 bps rate cut by the Federal Reserve on March 3, 2020, the 100 bps rate cut and credit market support by the Federal Reserve on March 15, 2020, and the expansion of credit market support by Federal Reserve on March 23, 2020. Daily data from January 2, 2020 until April 30, 2020.

Stylized facts 1: Within- and across-market correlations

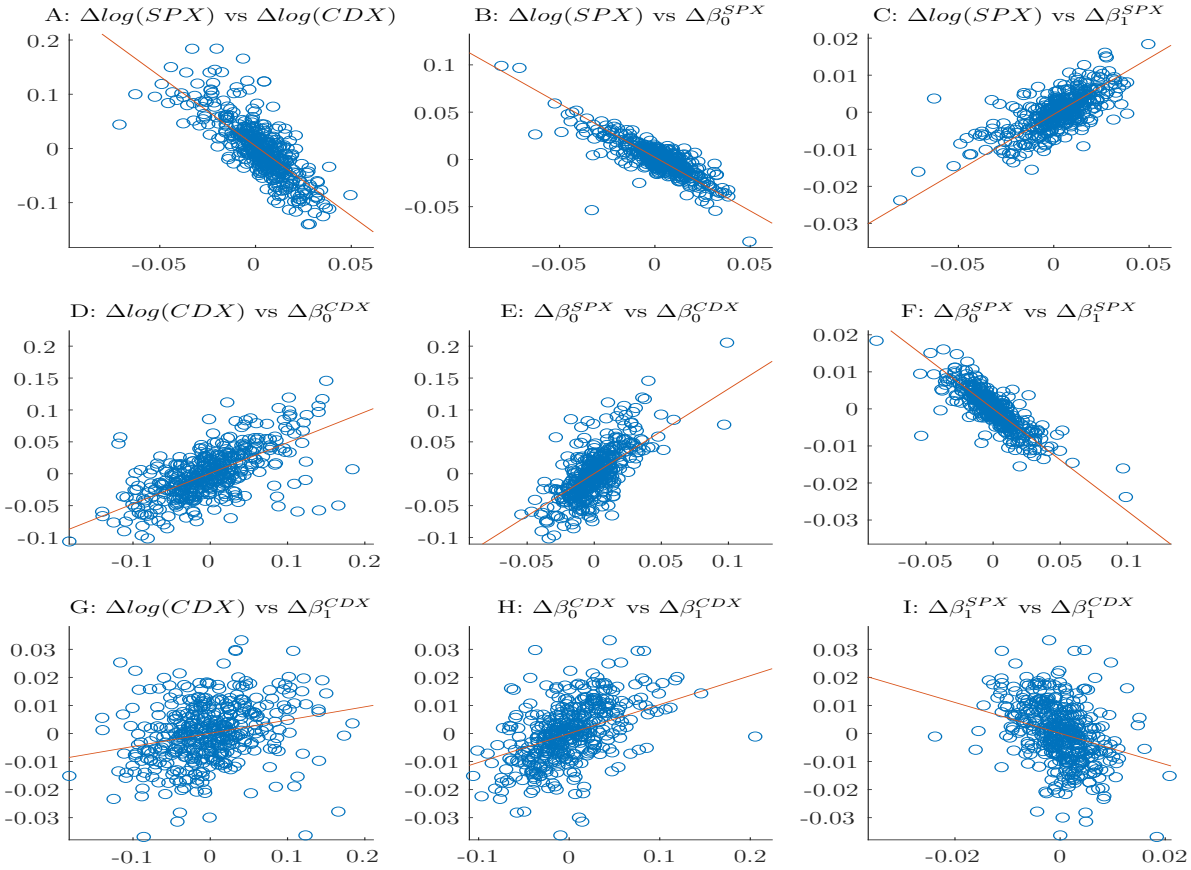

Figure 4: Within- and cross-market interactions

The scatterplots along the diagonal show the cross-market interactions: Weekly SPX returns ($\Delta log(SPX)$) vs. log CDX spread changes ($\Delta log(CDX)$) in Panel A; weekly SPX volatility changes ($\Delta\beta_0^{SPX}$) vs. CDX volatility changes ($\Delta\beta_0^{CDX}$) in Panel E; and weekly SPX skewness changes ($\Delta\beta_1^{SPX}$) vs. CDX skewness changes ($\Delta\beta_1^{CDX}$) in Panel I. Scatterplots below the diagonal show the CDX-market interactions: Weekly log CDX spread changes vs. CDX volatility changes in Panel D; Weekly log CDX spread changes vs. CDX skewness changes in Panel G; and Weekly CDX volatility changes vs. CDX skewness changes in Panel H. Scatterplots above the diagonal show the SPX-market interactions: Weekly SPX returns vs. SPX volatility changes in Panel B; Weekly SPX returns vs. SPX skewness changes in Panel C; and Weekly SPX volatility changes vs. SPX skewness changes in Panel F. We only display observations that fall within the 0.5th and 99.5th percentile of the univariate distributions. The red lines show the fits of linear regressions applied to the data. The sample period is February 29, 2012 to April 29, 2020 (426 weekly observations).

Stylized fact 2: Positively skewed vol smiles, CDX

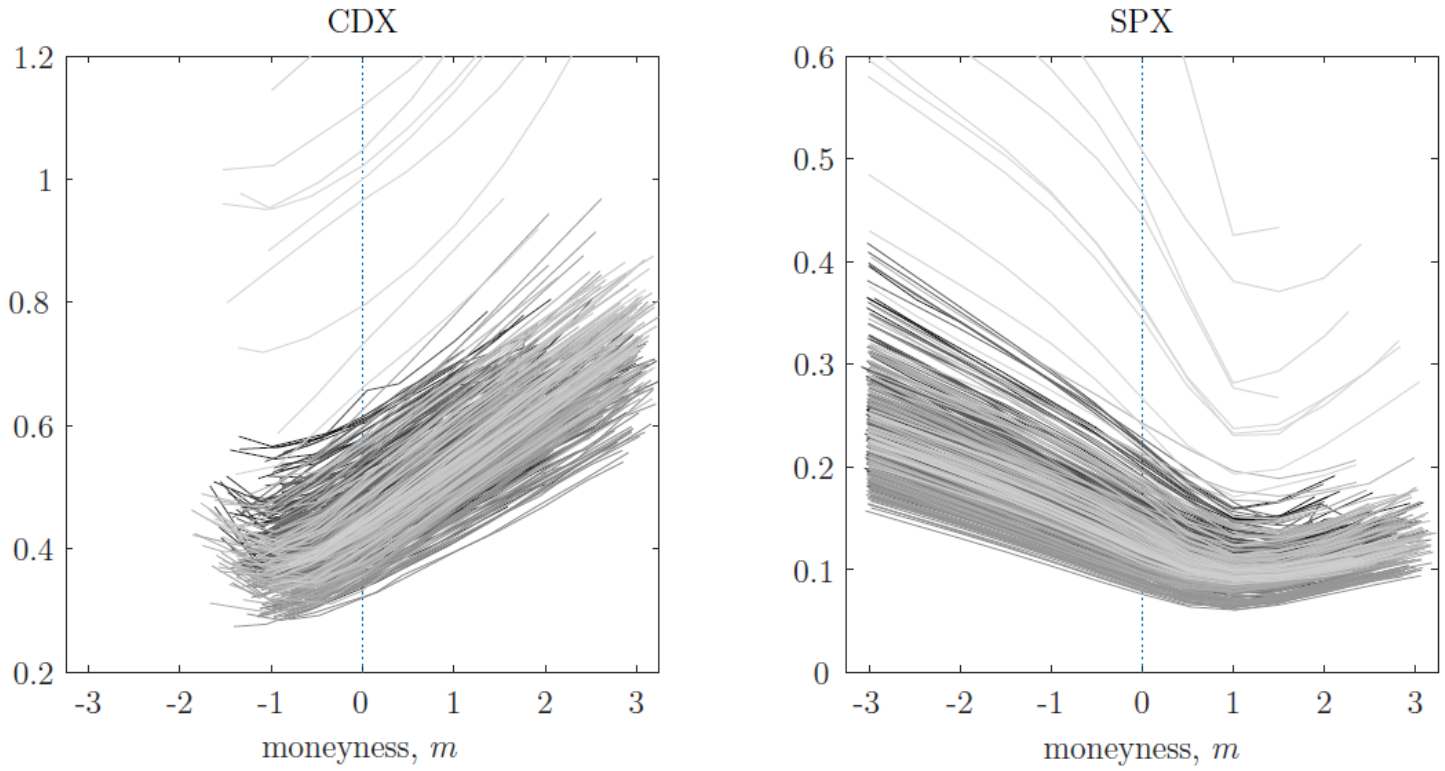

Figure 2: CDX and SPX implied volatility smiles

The figure shows weekly (Wednesday) two-month implied volatility smiles for CDX and SPX. CDX data is displayed in the left panel and SPX data is displayed in the right panel. Moneyness is defined as $m = \log(K/F(\tau))/(\sigma\sqrt{\tau})$, where K is the strike, $F(\tau)$ is the forward (front-end-protected) spread in case of CDX options and the forward price in case of SPX options), σ is the at-the-money implied volatility, and τ is the maturity. Sample period is from February 29, 2012 until April 29, 2020 (426 observations).

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Structural credit risk model à la Merton (1974)

- Risk-neutral asset dynamics of (ex-ante identical) firms

$$\frac{dA_t^i}{A_t^i} = \frac{dA_t}{A_t} + \sqrt{1-\rho}\sigma dW_t^i + (e^{\gamma^i} - 1)dN_t^i - \lambda^i \nu^i dt.$$
$$\frac{dA_t}{A_t} = (r-\delta)dt + \sqrt{\rho}\sigma dW_t + (e^{\gamma} - 1)dN_t - \lambda \nu dt.$$

W_t^i are independent Brownian motions, N^i are Poisson counting processes with intensity λ^i, γ^i are independent $\sim N(m^i, v^i)$ and $\nu^i = \mathrm{E}[e^{\gamma^i} - 1] = e^{m^i + \frac{v^i}{2}} - 1$.

- Payouts (at rate of δ) go to equity holders
- Firms issue
  - short-term debt with notional D_1 and maturity $T_1 = 1$ and
  - long-term debt with notional D_2 and maturity $T_2 = 5$
- Default occurs at T_1 or T_2 if continuation value is less than payment owed to debt holders.

$\rightarrow$ Defines endogenous default threshold Φ at T_1 (and D_2 at T_2)

- In case of default at T_1 a fraction $1-\alpha$ of asset value is lost in bankruptcy and residual is paid out to debt holders in proportion to outstanding principal.

Debt and equity

- The value of the short-term bond is

$$B_1^i(T_0) = e^{-r(T_1-T_0)}\Big(D_1\mathbb{E}_{T_0}[\mathbf{1}_{\{A^i_{T_1}\geq\Phi\}}] + \mathbb{E}_{T_0}[R_1 A^i_{T_1}\mathbf{1}_{\{A^i_{T_1}<\Phi\}}]\Big)$$

- The value of the long-term bond is

$$\begin{aligned} B_2^i(T_0) = {} & e^{-r(T_2-T_0)}\Big(D_2\mathbb{E}_{T_0}[\mathbf{1}_{\{A^i_{T_1}\geq\Phi, A^i_{T_2}\geq D_2\}}] + \mathbb{E}_{T_0}[\alpha A^i_{T_2}\mathbf{1}_{\{A^i_{T_1}\geq\Phi, A^i_{T_2}<D_2\}}]\Big) \\ & + e^{-r(T_1-T_0)}\mathbb{E}_{T_0}[R_2 A^i_{T_1}\mathbf{1}_{\{A^i_{T_1}<\Phi\}}] \end{aligned}$$

- The equity value is equal to the asset value less debt and bankruptcy costs:

$$\begin{aligned} S^i_{T_0}(A^i_{T_0}) = {} & A^i_{T_0} - e^{-r(T_1-T_0)}\Big(D_1\mathbb{E}_{T_0}[\mathbf{1}_{\{A^i_{T_1}\geq\Phi\}}] + \mathbb{E}_{T_0}[A^i_{T_1}\mathbf{1}_{\{A^i_{T_1}<\Phi\}}]\Big) \\ & - e^{-r(T_2-T_0)}\Big(D_2\mathbb{E}_{T_0}[\mathbf{1}_{\{A^i_{T_1}\geq\Phi, A^i_{T_2}\geq D_2\}}] + \mathbb{E}_{T_0}[A^i_{T_2}\mathbf{1}_{\{A^i_{T_1}\geq\Phi, A^i_{T_2}<D_2\}}]\Big) \end{aligned}$$

CDS

- Consider a CDS contract from T_0 to T_2 with unit notional.
- The value of the protection leg is

$$\begin{aligned}\mathrm{Prot}_2^i(T_0) =& e^{-r(T_1-T_0)}\mathbb{E}_{T_0}[(1-\frac{\alpha A^i_{T_1}}{D_1+D_2})\mathbf{1}_{\{A^i_{T_1}<\Phi\}}] \\ &+ e^{-r(T_2-T_0)}\mathbb{E}_{T_0}[(1-\frac{\alpha A^i_{T_2}}{D_2})\mathbf{1}_{\{A^i_{T_1}\geq\Phi, A^i_{T_2}<D_2\}}].\end{aligned}$$

- The risky annuity (for continuous coupons) is $\mathcal{A}_2^i(T_0) = \int_{T_0}^{T_1} e^{-r(t-T_0)}dt + \int_{T_1}^{T_2} e^{-r(t-T_0)}dt\mathbb{E}_{T_0}[\mathbf{1}_{\{A^i_{T_1}\geq\Phi\}}]$.
- With a coupon rate of C, the PUF of the CDS contract is $U_2^i(T_0) = \mathrm{Prot}_2^i(T_0) - C \times \mathcal{A}_2^i(T_0)$.

CDX index

- The PUF of the CDX is a simple average of the PUFs of the $N = 125$ single-name CDSs for the index constituents.
- We use the large homogeneous portfolio approach of Vasicek (1987) and approximate the index PUF by letting $N \to \infty$:

$$
\begin{aligned}
U_{T_0}(A_{T_0}) &= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} U_2^i(T_0) \\
&= \mathbb{E}[U_2^i(T_0)|A_{T_0}] \\
&= e^{-r(T_1-T_0)}\left((1+C_1)\mathbb{E}[\mathbf{1}_{\left\{A^i_{T_1}<\Phi\right\}} \mid A_{T_0}] - \frac{\alpha}{D_1+D_2}\mathbb{E}[A^i_{T_1}\mathbf{1}_{\left\{A^i_{T_1}<\Phi\right\}} \mid A_{T_0}]\right) \\
&\quad + e^{-r(T_2-T_0)}\left(\mathbb{E}[\mathbf{1}_{\left\{A^i_{T_1}\geq\Phi, A^i_{T_2}<D_2\right\}} \mid A_{T_0}] - \frac{\alpha}{D_2}\mathbb{E}[A^i_{T_2}\mathbf{1}_{\left\{A^i_{T_1}\geq\Phi, A^i_{T_2}<D_2\right\}} \mid A_{T_0}]\right) \\
&\quad - C_0 - C_1 e^{-r(T_1-T_0)}.
\end{aligned}
$$

- All of the expectations have closed-form solutions in terms of sums of univariate and bivariate normal cumulative distribution functions.

SPX index

- Similarly, the value of the SPX is given by

$$
\begin{aligned}
S_{T_0}(A_{T_0}) &= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} S^i_{T_0}(A_{T_0}) \\
&= \mathbb{E}[S^i_{T_0} | A_{T_0}] \\
&= A_{T_0} - e^{-r(T_1-T_0)} \Big(D_1 \mathbb{E}[\mathbf{1}_{\left\{A^i_{T_1} \geq \Phi\right\}} \mid A_{T_0}] + \mathbb{E}[A^i_{T_1} \mathbf{1}_{\left\{A^i_{T_1} < \Phi\right\}} \mid A_{T_0}] \Big) \\
&\quad - e^{-r(T_2-T_0)} \Big(D_2 \mathbb{E}[\mathbf{1}_{\left\{A^i_{T_1} \geq \Phi, A^i_{T_2} \geq D_2\right\}} \mid A_{T_0}] + \mathbb{E}[A^i_{T_2} \mathbf{1}_{\left\{A^i_{T_1} \geq \Phi, A^i_{T_2} < D_2\right\}}\} \mid A_{T_0}] \Big)
\end{aligned}
$$

- Also obtained in closed form.

CDX options

- The time-0 value of a CDX call option with strike K and expiration at T_0 is

$$
\begin{aligned}
C_0^{CDX} &= e^{-rT_0}\mathbb{E}_0[\max(U_{T_0}(A_{T_0}) - K, 0)] \\
&= e^{-rT_1}\Big((1+C_1)\mathbb{E}_0[\mathbf{1}_{\{A_{T_0}<\overline{A}, A^i_{T_1}<\Phi\}}] - \frac{\alpha}{D_1+D_2}\mathbb{E}_0[A^i_{T_1}\mathbf{1}_{\{A_{T_0}<\overline{A}, A^i_{T_1}<\Phi\}}]\Big) \\
&\quad + e^{-rT_2}\Big(\mathbb{E}_0[\mathbf{1}_{\{A_{T_0}<\overline{A}, A^i_{T_1}\geq\Phi, A^i_{T_2}<D_2\}}] - \frac{\alpha}{D_2}\mathbb{E}_0[A^i_{T_2}\mathbf{1}_{\{A_{T_0}<\overline{A}, A^i_{T_1}\geq\Phi, A^i_{T_2}<D_2\}}]\Big) \\
&\quad - e^{-rT_0}\tilde{K}\mathbb{E}_0[\mathbf{1}_{\{A_{T_0}<\overline{A}\}}],
\end{aligned}
$$

  where

  - $\tilde{K} = K + C_0 + C_1 e^{-r(T_1-T_0)}$,
  - $\overline{A}$ is the unique value such that $U_{T_0}(\overline{A}) = K$.

- Closed-form solutions for all the expectations in the index option formulas in terms of sums of univariate, bivariate, and trivariate normal distributions.

SPX options

- The time-0 value of an SPX call option with strike K and expiration at T_0 is

$$
\begin{aligned}
C_0^{SPX} &= e^{-rT_0}\mathbb{E}_0[\max(S_{T_0}(A_{T_0}) - K, 0)] \\
&= e^{-rT_0}\mathbb{E}_0[A_{T_0}\mathbf{1}_{\left\{A_{T_0} \geq \overline{A}\right\}}] - e^{-rT_1}\left(D_1\mathbb{E}_0[\mathbf{1}_{\left\{A_{T_0} \geq \overline{A}, A^i_{T_1} \geq \Phi\right\}}] + \mathbb{E}_0[A^i_{T_1}\mathbf{1}_{\left\{A_{T_0} \geq \overline{A}, A^i_{T_1} < \Phi\right\}}]\right) \\
&\quad - e^{-rT_2}\left(D_2\mathbb{E}_0[\mathbf{1}_{\left\{A_{T_0} \geq \overline{A}, A^i_{T_1} \geq \Phi, A^i_{T_2} \geq D_2\right\}}] + \mathbb{E}_0[A^i_{T_2}\mathbf{1}_{\left\{A_{T_0} \geq \overline{A}, A^i_{T_1} \geq \Phi, A^i_{T_2} < D_2\right\}}]\right) \\
&\quad - e^{-rT_0}K\mathbb{E}_0[\mathbf{1}_{\left\{A_{T_0} \geq \overline{A}\right\}}],
\end{aligned}
$$

  where

  - $\overline{A}$ is the unique value such that $S_{T_0}(\overline{A}) = K$.

- Also obtained in closed form.

Agenda

Calibration

- Idiosyncratic jump parameters set to $m_i = -2$ and $v_i = 0$ (i.e., jump to default) to only solve for λ_i
- Bankruptcy costs: $1 - \alpha = 0.2$ (Andrade and Kaplan (1998))
- Calibrate 10 parameters $(A_0, D_1, D_2, \delta, \sigma, \rho, \lambda, m, v, \lambda_i)$ weekly to perfectly fit:
  - level and slope of CDX (1Y and 5Y) $\rightarrow \lambda_i, \sigma$
  - level of SPX $\rightarrow A_0$
  - SPX dividend yield $\rightarrow \delta$
  - average short- and long-term leverage of CDX and SPX $\rightarrow D_1, D_2$
- Remaining degrees of freedom (ρ, λ, m, v) used to fit (~ 39) SPX implied volatility surface
- ...and then price CDX options out-of-sample.

Parameter estimates

	A_0	δ	σ	ρ	λ	m	$\sqrt{v}$	λ_i
Main	2893.8	0.0149	0.3405	0.0526	0.9531	-0.0875	0.0770	0.0032
	(408.8)	(0.0023)	(0.0150)	(0.0158)	(0.8756)	(0.0231)	(0.0164)	(0.0017)
$\alpha = 0.5$	2899.2	0.0149	0.3096	0.0608	0.9676	-0.0864	0.0764	0.0026
	(409.1)	(0.0023)	(0.0154)	(0.0192)	(0.8807)	(0.0228)	(0.0162)	(0.0014)
$\lambda_i = 0$	2894.5	0.0149	0.3580	0.0436	0.9832	-0.0877	0.0767	0
	(408.8)	(0.0023)	(0.0122)	(0.0128)	(0.8648)	(0.0223)	(0.0160)	
$\lambda = \lambda_i = 0$	2894.6	0.0149	0.3756	0.0894	0	0	0	0
	(408.8)	(0.0023)	(0.0112)	(0.0324)				

Table 4: Parameter estimates

For each model specification, the table reports the sample mean and sample standard deviation of the calibrated parameters. All specifications have $m_i = -2$ and $v_i = 0$. The first, third, and fourth specification have $\alpha = 0.8$. The sample period is February 29, 2012 to April 29, 2020 (426 weekly observations).

The pure diffusion model fails 'in-sample'

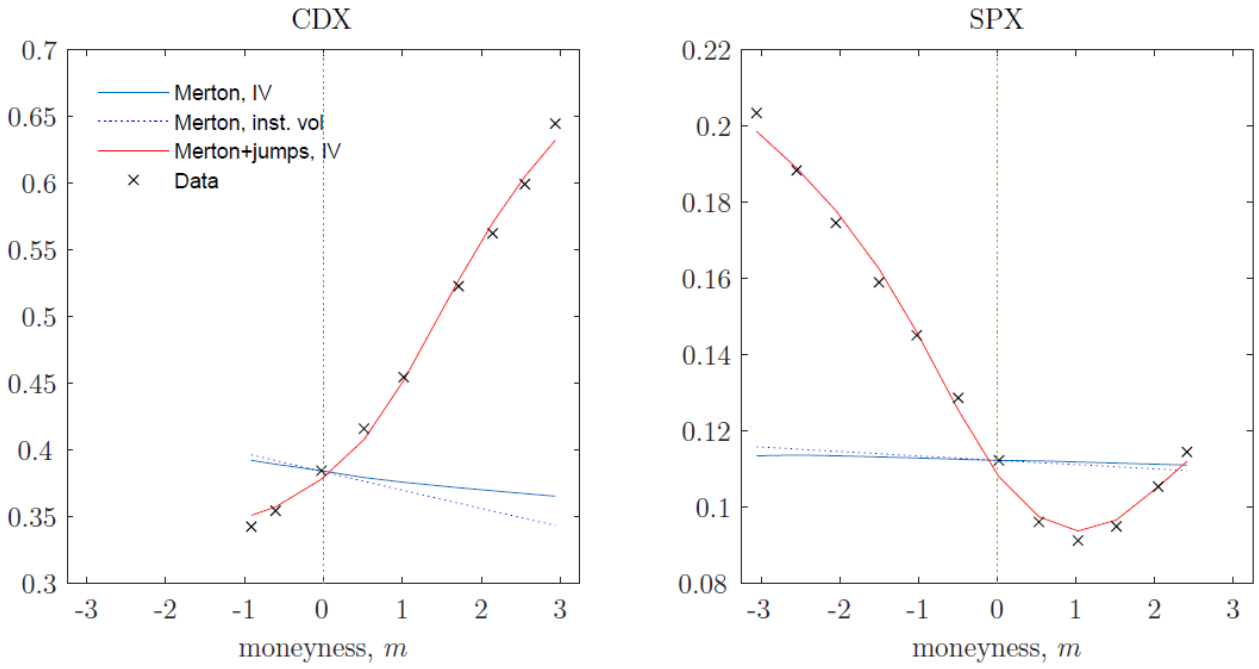


Figure 4: CDX and SPX implied volatility smiles in Merton model

The figure shows actual and fitted two-month implied volatility smiles for CDX and SPX on January 3rd, 2017. CDX data is displayed in the left panel and SPX data is displayed in the right panel. Moneyness is defined as $m = \log(K/F(\tau))/(\sigma\sqrt{\tau})$, where K is the strike, $F(\tau)$ is the forward (front-end-protected) spread in case of CDX options and the forward price in case of SPX options), σ is the at-the-money implied volatility, and τ is the maturity. Crosses show data. The solid blue line shows the implied volatility smile in the Merton model, when the model is fitted to the forward CDX spread, the SPX, firm leverage, and the ATM implied volatility (of either CDX or SPX options). The dotted blue line shows the instantaneous volatility of the underlying (either forward CDX spread or SPX), when its value equals the option strike. The solid red line shows the implied volatility smile in an extended Merton model with systematic jumps in the asset value, when the model is fitted perfectly to the forward CDX spread, the SPX, and firm leverage, and as closely as possible to the implied volatility smile (of either CDX or SPX options).

The jump-diffusion model 'out-of-sample'

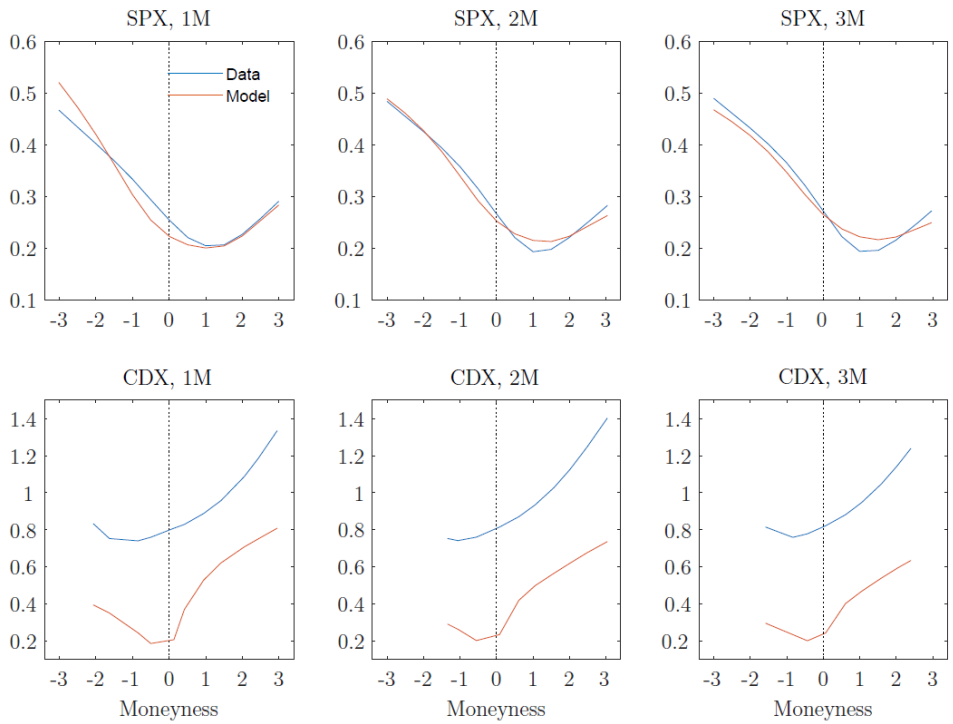


Figure 5: Fit to SPX and CDX implied volatility smiles on last Wednesday in sample
First row shows the fit to 1M, 2M, and 3M SPX implied volatility smiles on April 29, 2020. Second row shows the fit to 1M, 2M, and 3M CDX implied volatility smiles.

- Model captures well the SPX level and smile.
- Model captures CDX skewness well, but predicts too low level of ATM credit volatility, i.e., market $>$ model CDX option value.

$\Rightarrow$ CDX options appear overpriced relative to SPX options.

Fit to CDX options

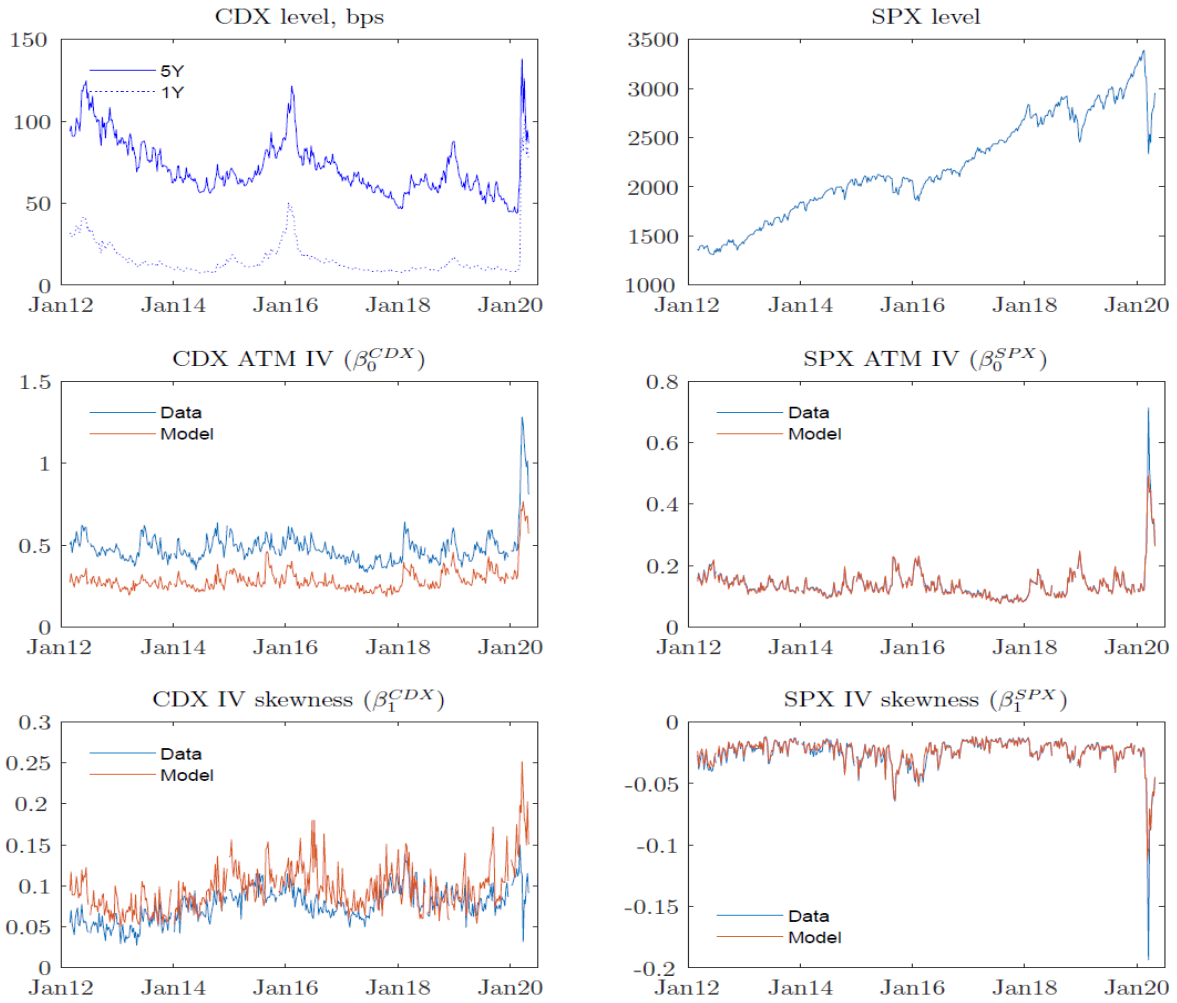


Figure 3: Summary of CDX and SPX options markets

The top left (right) panel shows time series of the CDX (SPX) level. The middle left (right) panel shows time series of the at-the-money CDX (SPX) implied volatility proxied by the β_0-estimate for the M2 option. The bottom left (right) panel shows time series of the skewness of the CDX (SPX) implied volatility smile proxied by the β_1-estimate for the M2 option. Weekly data from February 29, 2012 until April 29, 2020 (426 observations).

Fit of CDX option ATM vols

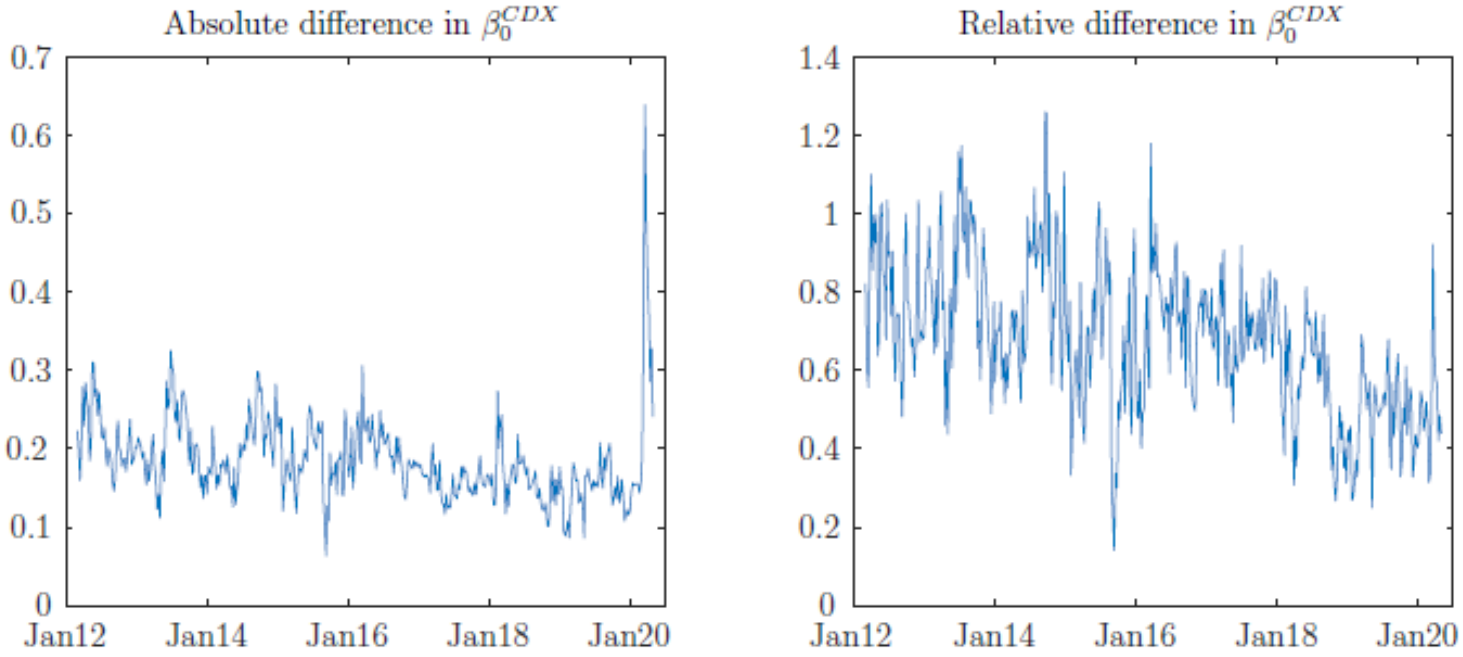


Figure 6: Time series of CDX option pricing errors

The left (right) panel shows the time series of the difference (relative difference) between β_0^{CDX} inferred from the data and the fitted data. Weekly data from February 29, 2012 until April 29, 2020 (426 observations).

Fit of within- and cross-market correlations

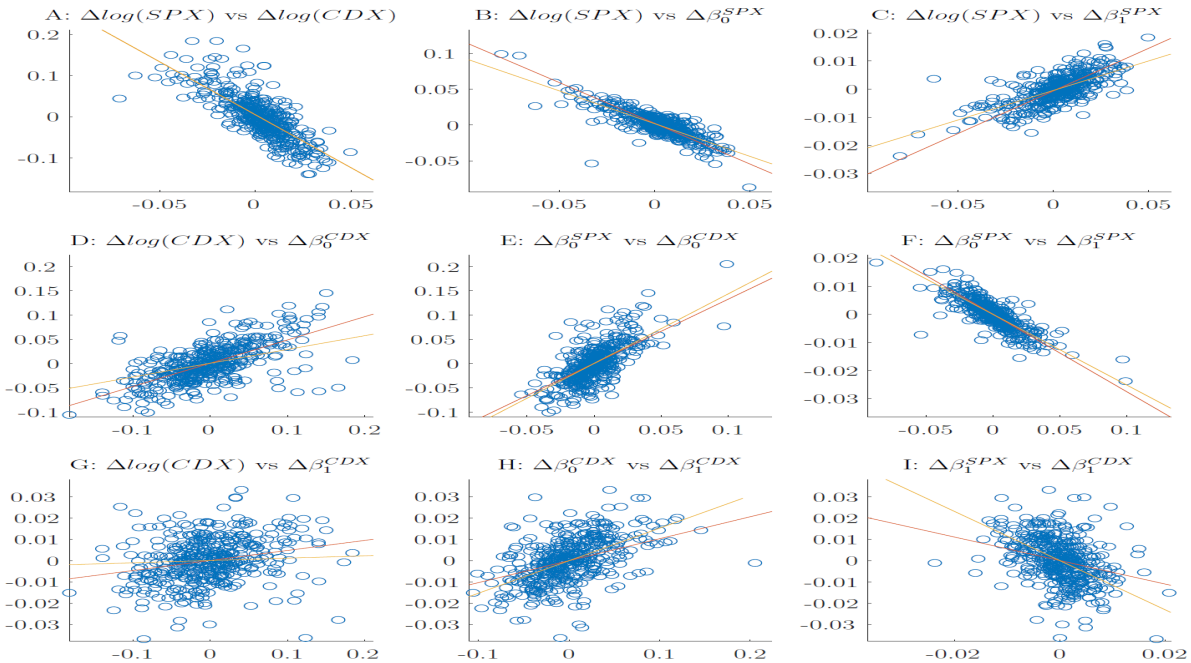


Figure 4: Within- and cross-market interactions

The scatterplots along the diagonal show the cross-market interactions: Weekly SPX returns ($\Delta log(SPX)$) vs. log CDX spread changes ($\Delta log(CDX)$) in Panel A; weekly SPX volatility changes ($\Delta\beta_0^{SPX}$) vs. CDX volatility changes ($\Delta\beta_0^{CDX}$) in Panel E; and weekly SPX skewness changes ($\Delta\beta_1^{SPX}$) vs. CDX skewness changes ($\Delta\beta_1^{CDX}$) in Panel I. Scatterplots below the diagonal show the CDX-market interactions: Weekly log CDX spread changes vs. CDX volatility changes in Panel D; Weekly log CDX spread changes vs. CDX skewness changes in Panel G; and Weekly CDX volatility changes vs. CDX skewness changes in Panel H. Scatterplots above the diagonal show the SPX-market interactions: Weekly SPX returns vs. SPX volatility changes in Panel B; Weekly SPX returns vs. SPX skewness changes in Panel C; and Weekly SPX volatility changes vs. SPX skewness changes in Panel F. We only display observations that fall within the 0.5th and 99.5th percentile of the univariate distributions. The red (yellow) lines show the fits of linear regressions applied to the data (fitted data using the benchmark specification of the model in Section 5). The sample period is February 29, 2012 to April 29, 2020 (426 weekly observations).

Agenda

Motivation

The CDX option market

Quote data

A structural credit risk model

The relative pricing of CDX and SPX options

What explains 'mispricing'?

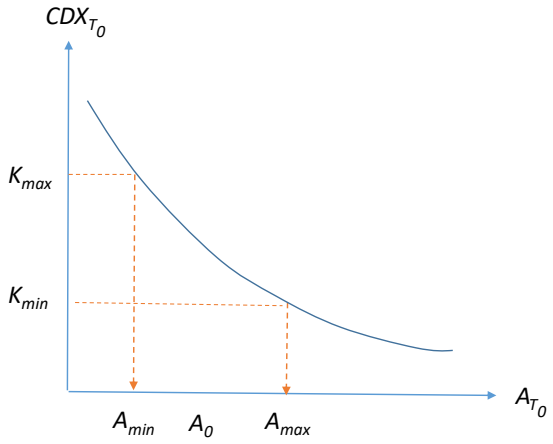
Possible explanations for 'mispricing'

- Model misspecification:
  - Heterogeneity among constituents.
  - $\rightarrow$ allow for heterogeneous leverage.
  - Finite number of constituents
  - $\rightarrow$ simulation analysis.
  - Stochastic volatility
  - $\rightarrow$ model is recalibrated weekly.

- CDX and SPX options are not redundant:
  - They span different economic states
  - The underlying indexes are different
  - There are different investor clienteles
  - There are bond-market-specific risk factors (e.g, stochastic bankruptcy costs)

Economic states spanned by options are very similar

- For CDX, compute
  - A_{min} such that $CDX_{T_0}(A_{min}) = K_{max}$
  - A_{max} such that $CDX_{T_0}(A_{max}) = K_{min}$
- For SPX, compute
  - A_{min} such that $SPX_{T_0}(A_{min}) = K_{min}$
  - A_{max} such that $SPX_{T_0}(A_{max}) = K_{max}$



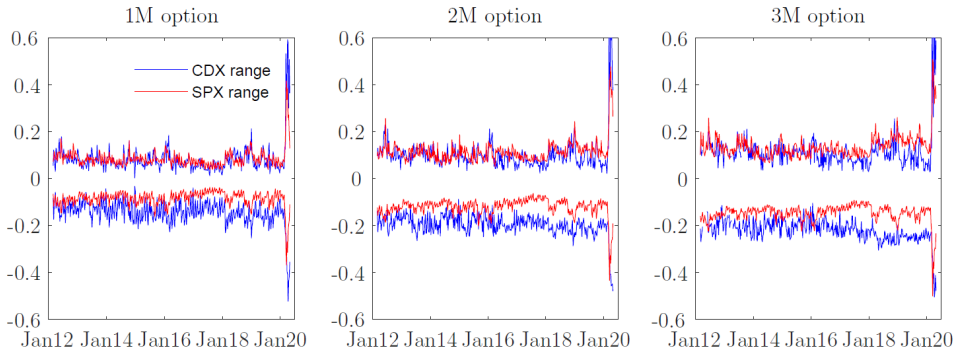


Figure IA5: Range of economic states spanned by options
The figure shows $\frac{A^{min}-A^{fwd}}{A^{fwd}}$ and $\frac{A^{max}-A^{fwd}}{A^{fwd}}$ for CDX and SPX options. A^{fwd} is the forward value of the common factor. For CDX options, A^{min} solves $U_{T_0}(A^{min}) = K^{max}$ and A^{max} solves $U_{T_0}(A^{max}) = K^{min}$. For SPX options, A^{min} solves $S_{T_0}(A^{min}) = K^{min}$ and A^{max} solves $S_{T_0}(A^{max}) = K^{max}$. The sample period is February 29, 2012 to April 29, 2020 (426 weekly observations).

→ Are CDX options redundant?

Index characteristics: leverage and rating distribution

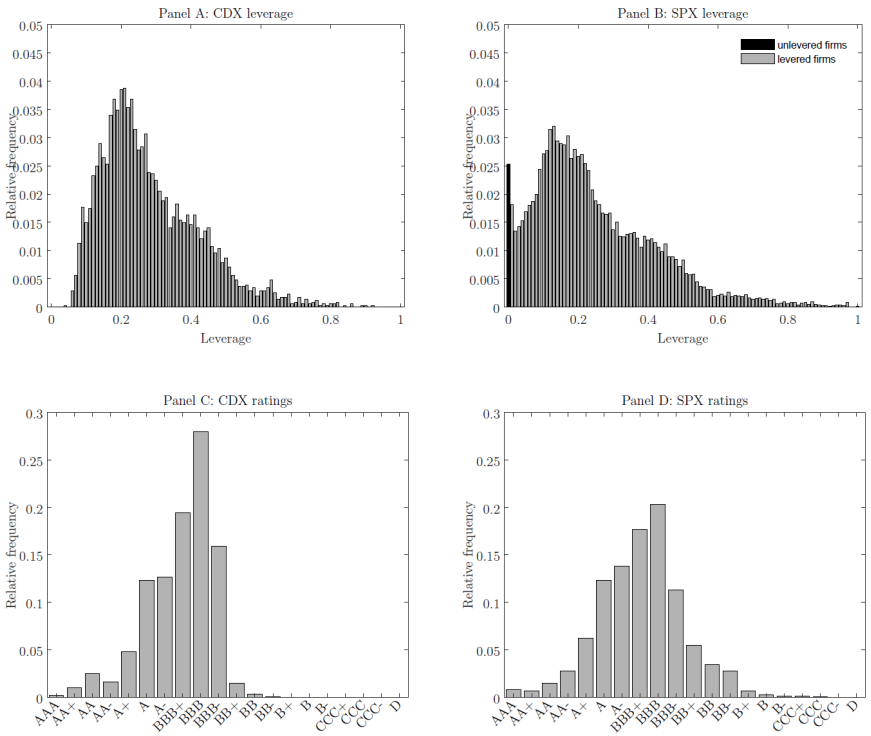


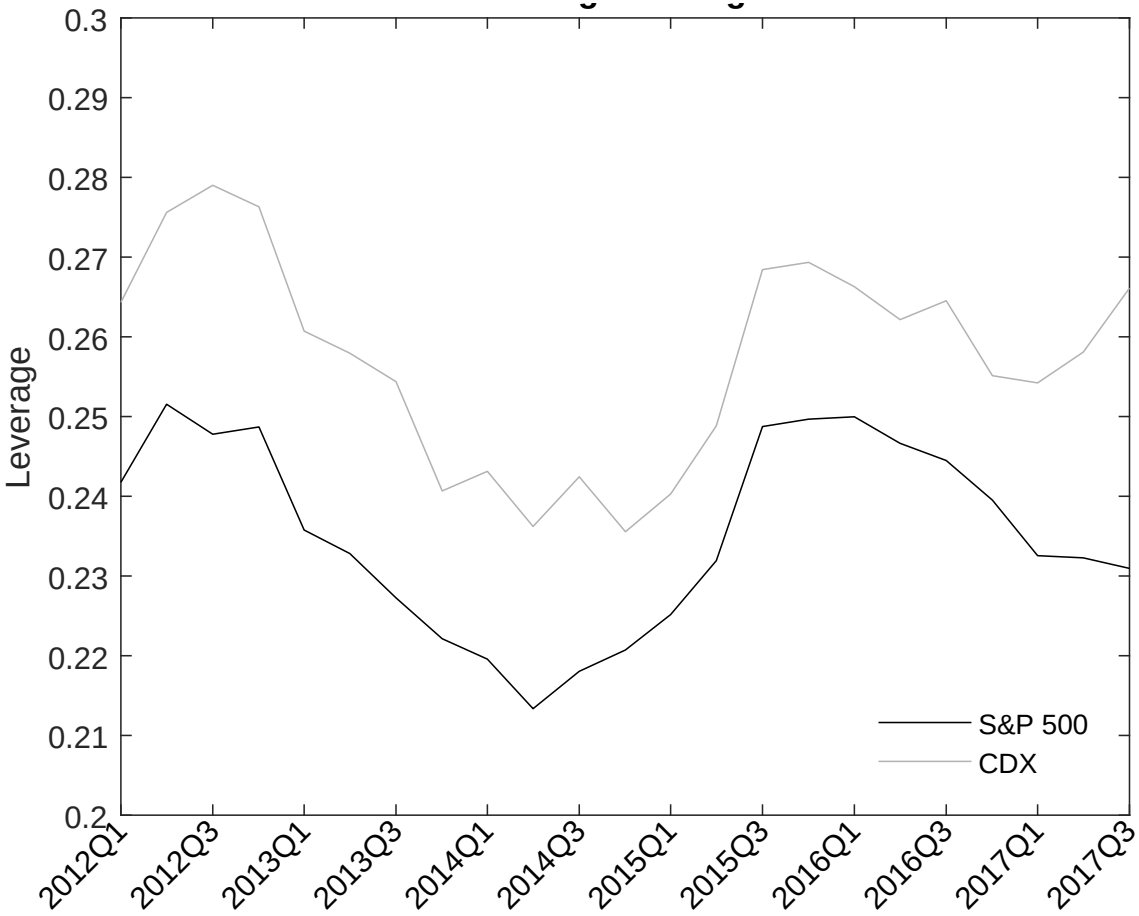
Figure 7: Distributions of leverage and ratings across index constituents

Panels A and B show the distribution of firm-quarter leverage observations for the constituents of the CDX and SPX, respectively, between the first quarter of 2012 and the first quarter of 2020. Leverage is defined as book value of debt over the sum of book value of debt and market value of equity. Panels C and D show the distribution of firm-quarter rating observations for the constituents of the CDX and SPX, respectively, between the first quarter of 2012 and the third quarter of 2017.

- Mean (median) leverage is 0.277 (0.244) for CDX vs. 0.238 (0.200) for SPX, while mean (median) rating is BBB+ for both.
- Dispersion is higher for SPX.

Index characteristics: leverage time series

- Time series of average leverage of CDX and SPX firms:



Index characteristics: Asset Volatilities

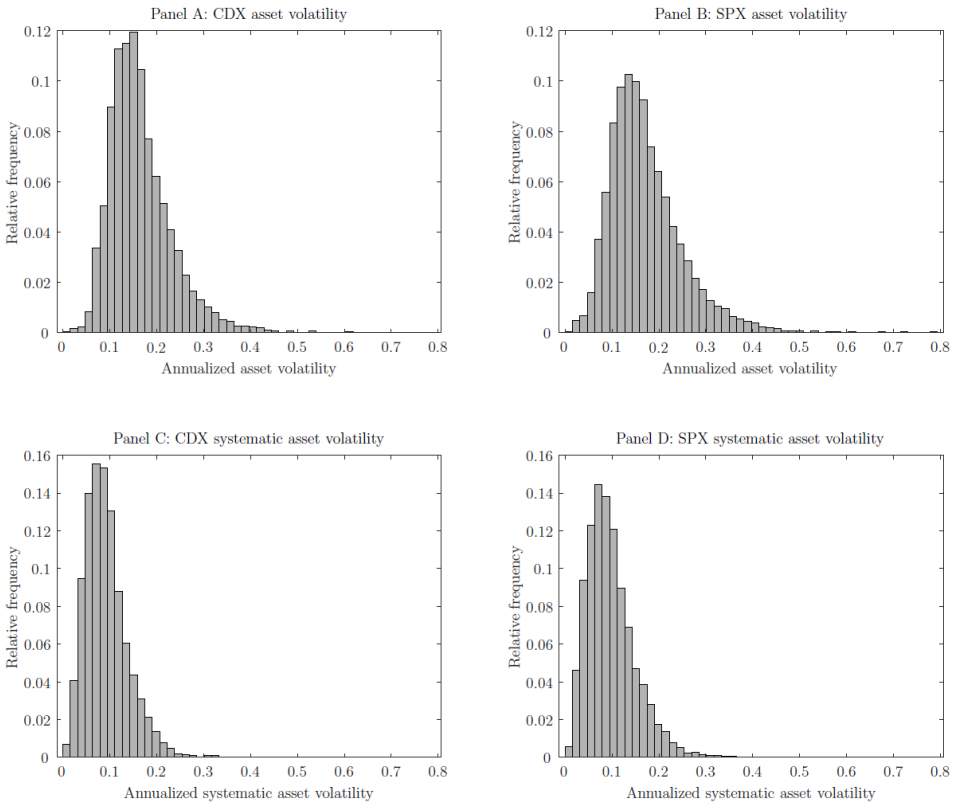


Figure 9: Distributions of asset volatility across index constituents

Panels A and B (C and D) show the distribution of firm-quarter total (systematic) asset return volatility for the constituents of the CDX and SPX, respectively. Asset returns are computed using daily data from January 3, 2012 until December 31, 2019.

- Total asset volatilities: mean (median) [standard deviation] of 0.167 (0.154) [0.068] for CDX vs. 0.173 (0.158) [0.080] for SPX.
- Systematic asset volatilities: mean (median) [standard deviation] of 0.092 (0.085) [0.045] for CDX vs. 0.098 (0.089) [0.052] for SPX.

Investor clienteles demand for options

- Demand for OTM SPX puts:
  - Pre-crisis: Non-dealer demand is positive; e.g., Garleanu, Pedersen, and Poteshman (2009)
  - Post-crisis: Non-dealer demand more balanced; e.g., Chen, Joslin, and Ni (2018)
- Structural non-dealer demand for OTM CDX calls:

> There is a dedicated investor base whose main business seems to be to buy OTM payers for "tail-hedging", due to their low dollar price. This makes implied vol in OTM payers high.

- Do regulators treat CDX and SPX options as substitutes for risk-mitigating hedges?

  "Options on index CDS have risen in popularity as a way of minimising the profit- and-loss impact of hedging the CVA capital charge."

  "The european bank's head of CVA says it uses short-dated credit swaptions instead of traditional index CDS hedges because the only P&L volatility the bank would face is the loss of the option premium."

  source: Risk.net, "CVA hedge losses prompt focus on swaptions and guarantees," 28 Oct, 2014

Can we trade on the 'mispricing'?

	CDX options				SPX options				CDX vs. SPX options			
	M1	M2	M3	EW	M1	M2	M3	EW	M1	M2	M3	EW
Panel A: Full sample												
Mean	0.122	0.154	0.189	0.155	0.052	0.073	0.069	0.064	0.035	0.041	0.060	0.045
t-stat	3.125	3.842	4.742	4.208	1.514	2.052	1.919	1.880	1.865	2.153	3.167	2.623
Std.dev.	0.100	0.100	0.100	0.089	0.100	0.100	0.100	0.098	0.057	0.057	0.057	0.052
SR	1.217	1.539	1.888	1.744	0.515	0.726	0.692	0.659	0.613	0.710	1.051	0.877
Skewness	-2.051	-2.378	-1.493	-2.219	-2.497	-2.277	-2.101	-2.285	-0.236	0.069	0.205	0.142
Kurtosis	14.167	19.322	19.149	17.586	17.593	16.775	15.177	16.125	9.008	10.395	12.045	9.772
Panel B: Ex-Covid-19 sample												
Mean	0.157	0.206	0.238	0.200	0.081	0.119	0.125	0.108	0.038	0.043	0.057	0.046
t-stat	4.105	5.344	6.276	5.757	2.418	3.481	3.635	3.269	1.932	2.163	2.815	2.539
Std.dev.	0.100	0.100	0.100	0.087	0.100	0.100	0.100	0.098	0.059	0.059	0.059	0.053
SR	1.568	2.055	2.383	2.291	0.811	1.187	1.248	1.106	0.647	0.731	0.963	0.868
Skewness	-1.802	-1.781	-1.122	-1.717	-2.598	-2.285	-1.980	-2.314	-0.327	0.020	0.175	0.072
Kurtosis	12.718	14.466	19.333	14.476	19.176	18.696	15.770	17.950	9.399	10.810	12.372	10.226

Table 7: Summary statistics of trading strategies
In each market and for each option maturity category, the strategy sells closest-to-ATM straddles each trading day with a holding period of one day. We assume that the strategy requires an initial amount of capital proportional to the option premium, and we adjust the proportionality factor to achieve a 10% unconditional annualized volatility of realized excess returns for each option maturity. "EW" denotes an equally weighted portfolio of the three option maturities. "CDX vs. SPX options" denotes a short-long strategy that allocates 50% of funds to selling CDX straddles and 50% to buying SPX straddles. Means, standard deviations, and Sharpe ratios ("SR") are annualized. t-statistics are corrected for heteroscedasticity and serial correlation up to four lags using the approach of Newey and West (1987). The full sample consists of 1881 daily returns between February 28, 2012 and April 30, 2020. The ex-Covid-19 sample consists of 1801 daily returns between February 28, 2012 and December 31, 2019.

Can we trade on the 'mispricing'?

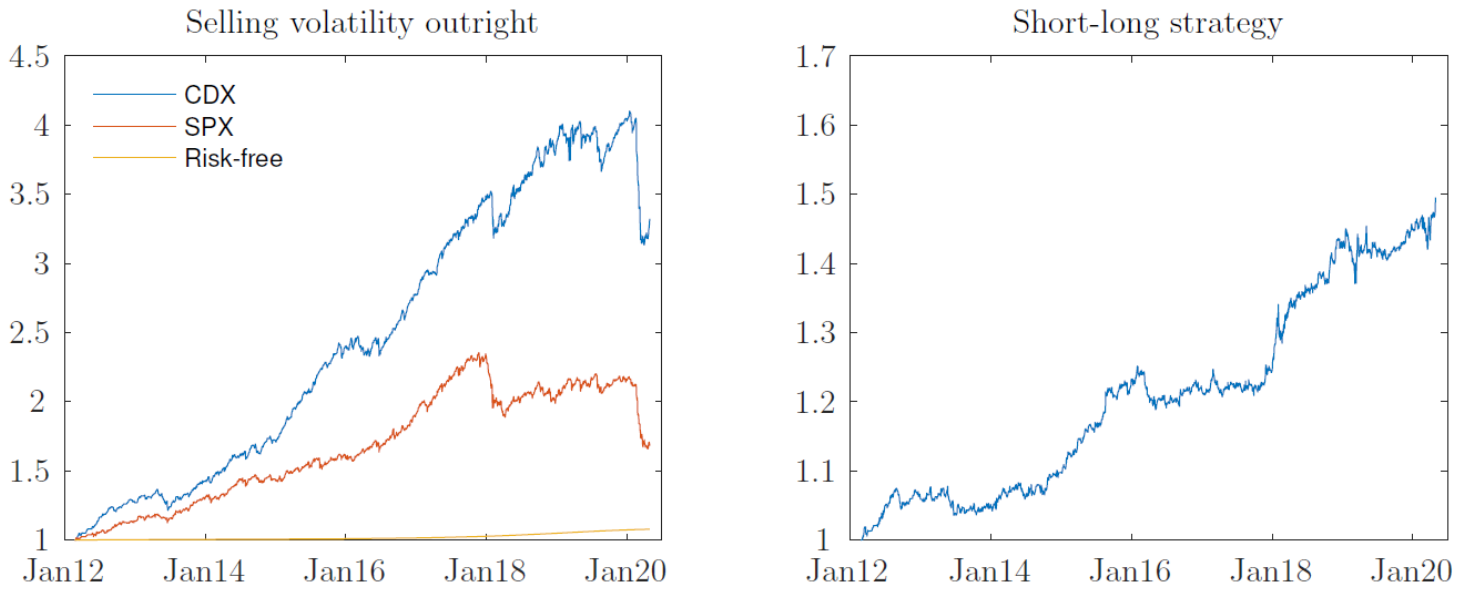


Figure 12: Cumulative performance of trading strategies

The figure shows the evolution of one dollar invested in each of the EW strategies at the beginning of the sample (see Table 7 for details on the trading strategies). The left panel shows the performance of selling CDX and SPX straddles outright. The right panel shows the performance of the short-long strategy that allocates 50% of funds to selling CDX straddles and 50% to buying SPX straddles. On those trading days where options returns on unavailable, we invest at the risk-free rate. The sample period is from February 24, 2012 to April 30, 2020 (2042 daily observations).

Conclusion

- Study relatively new CDX options markets
- Develop consistent structural model for CDX and SPX options
- Calibrated model shows:
  - Skew is consistent across both markets, but ...
  - Credit implied volatility $>$ equity implied volatility.
- Possible explanations:
  - Model misspecification?
  - Differences in the underlying portfolio characteristics?
  - Clientele effect: Demand for bond portfolio hedging?
    (regulators more likely to accept CDX options as a risk-mitigating hedge).
  - Risk factors for bonds might be different than for equity
    (bond-specific liquidity or bankruptcy cost factor might add distinct credit spread volatility component, whereas skew reflects fear for market-wide crash risk).
- Selling CDX vol delivers significantly higher Sharpe ratio than SPX vol. Long-short strategy has high sharpe ratio and fewer drawdowns.
- Implications for 'pseudo-bonds'?

Thank You !