Discussion of "Hedging in Fixed income markets" by Malkhozov, Mueller, Vedolin and Venter

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Summary

- Propose a theoretical model of the term structure based on
  - Exogenous interest rate dynamics
  - Net Supply of bonds driven by Mortgage hedging motivated by the (classic) story:
    - If rates $\downarrow$ then prepayment $\uparrow$ which implies MBS duration $\downarrow$.
    - Mortgage hedgers (agencies) then buy long term bonds, which leads to further rates $\downarrow$.
  - Risk-averse arbitrageurs
- The model can explain many stylized facts of the term structure
  - Predictability of long term bonds driven both by Fama-Bliss-slope and by Cochrane-Piazzesi tent-shaped factor.
  - Hump-shaped bond volatility term structure
  - Time-varying bond volatility and negative volatility risk-premia
- In addition find empirical results consistent with model predictions:
  - Regress Treasury bond yields and excess returns on MBS duration (from Barclays) and find positive relation
  - Regress Treasury bond volatility on MBS convexity (from Barclays) measures and find strong positive relation
  - Regress volatility risk-premia measures on MBS convexity and find strong positive relation

Term Structure 101 refresher

- Suppose the short rate model is exogenously given by:

$$dr_t = \kappa(\theta - r)dt + \sigma_r dz_t$$

- Assume zero-coupon bond $P(t, T_i) = P_i(t, r_t)$. Construct a portfolio $V(t) = n_1 P^1(t) + n_2 P^2(t)$, that is self financing:

$$\begin{aligned} dV_t &= n_1(t)dP^1(t) + n_2(t)dP^2(t) \\ &= \left[n_1 \left(\frac{1}{2} P^1_{rr}\sigma_r^2 + P^1_r \kappa(\theta - r) + P^1_t \right) + n_2 \left(\frac{1}{2} P^2_{rr}\sigma_r^2 + P^2_r \kappa(\theta - r) + P^2_t \right) \right] dt \\ &\quad + \left[n_1 P^1_r + n_2 P^2_r \right] \sigma_r dz_t \end{aligned}$$

- If we choose $\{n_1, n_2\}$ such that the portfolio is locally risk-free: (A)

$$(n_1 P^1_r + n_2 P^2_r)\sigma_r = 0$$

- then it should earn the risk-free rate: (B)

$$n_1 \left(\frac{1}{2} P^1_{rr}\sigma_r^2 + P^1_r \kappa(\theta - r) + P^1_t \right) + n_2 \left(\frac{1}{2} P^2_{rr}\sigma_r^2 + P^2_r \kappa(\theta - r) + P^2_t \right) = r \left(n_1 P^1 + n_2 P^2 \right)$$

Term Structure 101 refresher

- Combining equations (A) and (B), we find

$$\frac{\left(\frac{1}{2}P^1_{rr}\sigma_r^2 + P^1_r\kappa(\theta - r) + P^1_t - rP^1\right)}{P^1_r\sigma_r} = \frac{\left(\frac{1}{2}P^2_{rr}\sigma_r^2 + P^2_r\kappa(\theta - r) + P^2_t - rP^2\right)}{P^2_r\sigma_r}$$

$\Rightarrow$ There exists a **market price of risk** process λ such that the instantaneous sharpe ratio is equalized across all bonds (in a one-factor model).

$$\lambda_t = \frac{\left(\frac{1}{2}P_{rr}\sigma_r^2 + P_r\kappa(\theta - r) + P_t - rP^1\right)}{P_r\sigma_r} \equiv \frac{\mu_P(t,T) - r_t}{\sigma_P(t,T)}$$

$\Rightarrow$ The prices of all bonds (and European path-independent derivatives) solve

$$0 = \frac{1}{2}P_{rr}\sigma_r^2 + P_r(\kappa(\theta - r) - \lambda\sigma_r) + P_t - rP^1$$

subject to appropriate boundary conditions.

- How do we select the market price of risk **process** λ?

Finance 101 refresher

Q? How do we select the market price of risk λ_t

A! To get tractable bond prices:

- First generation: constant λ so that $\kappa(\theta - r) - \lambda\sigma_r = \kappa(\theta^Q - r)$ (Vasicek)
- Second generation: $\lambda = \lambda_0 + \lambda_r r$ so that $\kappa(\theta - r) - \lambda\sigma_r = \kappa^Q(\theta^Q - r)$ (Duffee)

- Why are 'essentially affine' second generation models useful?

$$\mu_P(t, T) - r = \lambda(r)\sigma_P(t, T)$$

- First generation risk-premia impose that compensation for risk is a fixed multiple of short-rate volatility. In particular, risk-premia are constant (Gaussian) or cannot switch sign over time (CIR).
- More general structure of risk-premia breaks this link, which is necessary to capture evidence on predictability (Duffee (2002), Dai-Singleton (2002)).
- N.B.: $\sigma_P(t, T) = -\sigma_r \frac{P_r(t,T)}{P(t,T)} < 0$ so need $\kappa^Q < 0$ (i.e., $\lambda_r < 0$) to generate negative (positive) relation between level (slope) and expected return
(recall that in one factor model level and slope perfectly negatively correlated).

Intuition for their model

- Risk-averse arbitrageurs can freely trade bonds of all maturities.

→ No-arbitrage holds and Sharpe ratios across all bonds are equalized.

→ Arbitrageurs are indifferent to maturity mix (one bond and cash completes market).

- Consider portfolio choice with one bond (maturity T) and risk-free rate.

$$dW(t) = r(t)W(t) + X(t)(\frac{dP(t)}{P(t)} - r(t))dt \quad (1)$$

$$= r(t)W(t) + X(t)\sigma_P(t)\left(\lambda_t dt + dz(t)\right) \quad (2)$$

- Arbitrageurs are instantaneous mean-variance optimizers choose dollar position X:

$$\max_X E[dW(t)] - \frac{a}{2}V[dW(t)]$$

So first order condition is

$$X(t) = \frac{\lambda}{a\sigma_P(t)} \equiv \frac{\lambda}{a\sigma_r D}$$

where bond duration is $D = -\frac{P_r}{P}$

Intuition for their model

- In equilibrium, net dollar supply of bond $S(t)$ absorbed by arbitrageur:

$$X(t) = S(t)$$

- Equivalently

$$\frac{\lambda}{a\sigma_r D} = S(t)$$

- Now posit that net dollar **duration** supply is proportional to MBS duration:

$$S(t)D = \frac{\partial MBS}{\partial r}$$

- Further posit two functional forms for the mortgage duration:
  - Case I: $\frac{\partial MBS}{\partial r} = \phi_0 + \phi_1 r$ leads to $\lambda = a\sigma_r(\phi_0 + \phi_1 r)$
  - Case II: $\frac{\partial MBS}{\partial r} = \phi_0 + \phi_1 r + \phi_2 r^2$ leads to $\lambda = a\sigma_r(\phi_0 + \phi_1 r + \phi_2 r^2)$

- Case I is an essentially affine Vasicek model.

- Case II is ($\sim$) to a quadratic model.

MBS Duration should be endogenous

- If we think of a MBS as a callable bonds, then MBS duration is a non-linear function of interest rates and interest rate volatility.
- If in addition we want to think of the market price of risk as a function of MBS duration, then we (should) have an interesting fixed-point problem:
  - $\Rightarrow$ The valuation of the callable bond depends on the risk-neutral dynamics of the short rate, which depends on the (derivative of) the callable bond price (the MBS Duration), which depends on the risk-neutral dynamics of the short rate...
- This is not the route the authors follow. Instead they exogenously specify a linear, or quadratic relation between MBS duration and the short rate.
- For their calibration they exogenously fit that relation using historical data.
- Instead it would seem more interesting to seek a fixed point for this relation, explicitly valuing the embedded call option.

Should the level of the short rate be endogenous?

- All the intuitive argument talk about price pressure:
  "lower yields reduce MBS duration that in turn puts pressure on long-term yields as hedgers reduce the net supply of bonds in the market"

$\Rightarrow$ Might expect this to feedback to the level of (short term) rates.

- In theory, long term yields can move because of changes in the market price of risk (risk-premia) without affecting the short rate.
- But in the model there is no dissociation between changes in long and short term yields, since they are both deterministic functions of each other (one-factor model).
- Would greatly benefit from an extension to multiple factors, possibly driving separately risk-premia and level of rates.

$\rightarrow$ A converse to the recent models of *unspanned* risk-premium factors: Duffee (2009), Joselin, Priebsch and Singleton (2010).

To gauge the Empirical results we need a proper Null Hypothesis

- Regressing bond yields, or bond volatilities on measures of MBS duration or convexity, is not a good test of the model, *without a proper null hypothesis.*
- MBS are callable bonds, so one would expect a contemporaneous relation (and indeed even a predictive relation for excess returns under some assumption on the 'leverage' effect).
- Need to specify a proper null hypothesis and explain why findings are more in line with existing model.
- In particular, one would like to find a test that will separate this 'theory' from alternative 'theories' that are observationally equivalent from pure term structure pricing implications (e.g., Duffee (2002)).
- Bring to bear specific data to discriminate between MBS hedging pressure and alternative stories.
  - International comparisons with countries where MBS markets (or prepayment options) are less important.
  - Comparisons across time and regimes in the US. Over time more MBS hedging done with LIBOR swaps. (could perhaps instrument with LIBOR-swap duration publishing by dealers).

Propose more specific tests of the theory

- Is is plausible that MBS hedging pressure is the source of the predictability of bond returns (by slope or by Cochrane-Piazzesi factor)?
- We see similar 'puzzles' in other markets where high yields predict high capital gains (e.g., equity markets and currencies).
- Makes it unlikely that MBS hedging demand is **solely** responsible for the predictability results in the term structure.

$\Rightarrow$ Thus theory and tests should allow for alternative sources of predictability.

- The CP factor cannot be 'explained' by the present model:
  - CP's tent shaped factor 'beats' level-slope-curvature by using information in the fifth principal component of yields, whereas in the model all the predictability comes from the level itself.
  - CP argue that a **unique** tent-shaped combination of **specific** maturity yields best predicts bond returns, which (if taken at face value) is difficult to generate in a standard n-dimensional (finite) affine model, where **any** combination of *n* yields will do.
- Is it plausible that the channel by which QE1 (if it) had an effect on yield **levels** was via its impact on MBS hedging pressure by agencies?

Conclusion

- Ambitious paper.
- Tries to put legs under a well-known story of MBS hedging pressure and its impact on Treasury yield curve.
- The theory should be pushed a bit more.
- Empirical tests should be more focused.