Dynamic Dependence and diversification in Corporate Credit
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Discussion

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Summary

- Propose an econometric 'dynamic copula' model to capture general time-varying (tail) dependence in log cds and equity-price changes.
- Estimate the model using weekly log-differences in cds and equity prices of 215 firms constituents of the CDX-NA-IG index from 2001 to 2012.
- Find that
  - Copula correlation in CDS vary substantially over time and tend to increase during the crisis.
  - Copula correlation in equities also increase but not as much.
  - There are fat tails and multi-variate non-normality in both credit and equity returns.
  - Credit dependence is more persistent than equity dependence.
  - VIX is an important driver of Copula correlations over time.
  - Dependence measures have predictive power for credit spread changes (controlling for known predictors).

Marginal distributions

- Specify log changes in equity prices or CDS (R_t) as an ARMA(2,2) where the residuals ϵ_t follow an Engle and Ng (1993) NGARCH(1,1) with "fundamental" shocks z_t assumed to be drawn from an **asymmetric standardized t distribution** (Hansen (1994)):

$$
\begin{aligned}
R_t &= \mu + \phi_1 R_{t-1} + \phi_2 R_{t-2} + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \epsilon_t && (1)\\
\epsilon_t &= \sigma_t z_t && (2)\\
\sigma_t^2 &= (1-\alpha-\beta)\overline{\sigma}^2 + \alpha(\epsilon_{t-1} - \gamma\sigma_{t-1})^2 + \beta\sigma_{t-1}^2 && (3)\\
z_t &\sim ast_{\lambda,\nu}(z) && (4)
\end{aligned}
$$

$\Rightarrow$ This gives the marginal distribution of returns for firm i: $F_{i,t}(r) = P(R_{i,t} \leq r)$

- The key is how to model the correlation between the $z_{i,t}$ $i = 1, \ldots n$.

Q? A simple approach would be to assume that they are drawn from a multivariate skewed t distribution: $t_{\lambda,\nu}(z_{1,t}, \ldots, z_{n,t}; \Psi_t)$, where Ψ_t would be the (possibly dynamic) covariance matrix

- Instead, use multivariate skewed t-copula approach applied to the returns (and not to the residual shocks $z_{i,t}$?) to model dependence between returns.

How to generate correlated returns with their approach

- To simulate n correlated returns at time t (given past history of returns):
  (a) Draw n r.v. from a skewed multi-variate t-distribution: $t_{\lambda_C,\nu_{C,t}}(z_1^*,\ldots,z_n^*;\Psi_t)$, where Ψ_t is the (dynamic) covariance matrix, and $\nu_{C,t}$ is the time-varying degree of freedom parameter.
  (b) Generate n correlated uniform random variables: $u_i = t_{\lambda_C,\nu_{C,t}}(z_i^*)\ \ \forall i = 1,\ldots n$
  (c) Compute $F_{i,t}^{-1}(u_i) = R_{i,t}\ \ \forall i = 1,\ldots n$ to obtain n correlated returns

Q? Why not replace the third step by:
  (c') Compute $ast_{\lambda,\nu}^{-1}(u_i) = z_{i,t}$ to obtain return innovations to plug into equation (1)

$\Rightarrow$ Advantage would be that differences in marginals and copula distribution only arise from different degrees of freedom and asymmetry parameters. Disadvantages?

- Difference relative to drawing the return residuals $z_{i,t}$ directly from a multi-variate skewed t-distribution (which would avoid the copula step) seems to be to allow for different asymmetry and degree-of-freedom parameters in the skewed t-distributions used for the copula than in the asymmetric t-distribution used for the marginals.

$\Rightarrow$ Might be nice to compare model performance relative to simply having the $z_{i,t}$ drawn from a multi-variate skewed t-distribution with possibly time-varying degree of freedom to show benefit of Copula approch.

Questions on the Methodology

- Hardcore econometrics with many references that seem necessary to read to fully understand the econometrics (not my case!)
- The **composite likelihood** function that is maximized is the sum of the log pair-wise correlation (following Engle, Shephard and Sheppard (2008)).

$\Rightarrow$ Are there conditions under which this converges to the true full likelihood?

$\Rightarrow$ How can this approach pick up "higher order" dependence that leave bi-variate densities identical but generate different probabilities of multiple (> 2) defaults (e.g., frailty models or Hawkes processes)?

- The deterministic time-trend and quadratic step function used (following Engle and Rangel (2008)) to model dynamics of the copula degree of freedom "process" ($\lambda_{C,t}$) seem non-stationary and sample dependent.
- The full (in) sample covariance matrix is used for the copula covariance (Ψ_t) "dynamics" (following Engle and Mezrich (1996)).
  - $\Rightarrow$ Risk of in-sample overfitting?
  - $\Rightarrow$ Out of sample performance?
  - $\Rightarrow$ Could it affect the "predictive" regression of spreads on dependence measures?

General Questions and Comments

- One of the findings is that copula dependence of log-changes in CDS and equity are different. What are the economic implications?
  - In a structural model, a different volatility implied from bond/CDS than from equity prices implies an arbitrage opportunity (or a mis-specified model).
  - Here, are there implications for "capital structure arbitrage"?
- What should we expect to find? The paper focuses on the correlation structure of weekly log-changes in **Investment Grade** CDS.
  - What is a tail event for weekly log-changes in CDS? (e.g., a +10% CDS change?)
  - What is a tail event for weekly log-changes in equity prices (e.g., a -30% return?).

  $\Rightarrow$ Would be interesting to plot the tail events used to estimate "tail dependence" for equity and CDS changes (are they varying over time?).
- Related, it seems difficult to use this model for CDO (tranche) pricing or for risk-management.
  - Both depend on the long-term (say 5-year) default loss distribution.
  - Given short history and few default events among the CDX-IG constituents, the approach seems unlikely to give an accurate estimate of the default distribution.

  $\Rightarrow$ Would be interesting to compare with CDX-tranche implied loss distributions.

The Quant Meltdown and Credit Dependence?

"The most important shocks to credit dependence occur in August 2007 and August 2011, but interestingly these dates are not associated with significant changes to median credit spreads"
"Credit copula correlations show a pronounced and persistent uptick in 2007 around the quant meltdown"
"Credit dependence is more persistent than equity persistence and this greatly affects how major events such as the Quant Meltdown, [. . .], affect subsequent dependence in equity and cds markets."

- Not clear how to extrapolate from an econometric model that presumes some stationarity to assess the future impact of "major" events such as the quant meltdown.
- It may be useful to be more precise:*"The Bear stearns subprime fund collapse and quant meltdown. July/August 2007, henceforth referred to as the quant meltdown"*.
- While these two events may be related in some way, they are very different (types of) events.

The Quant Meltdown and Credit Dependence?

- News of Bear Stearns two subprime hedge funds having lost all their value becomes public July 16-17 coinciding with the dramatic collapse in ABX (HEL subprime mortage) prices.
- IG Credit spreads also widen July 16 to July 30.
- Early August credit and correlation markets seemed to have stabilized.
- Equity markets also seemed stable.
- Interbank markets (LIBOR) seemed stable until August 8.
- The Quant Meltdown really occured in the US on August 1 through 9 (worst days 8-9) with a rebound already on August 10.

$\Rightarrow$ Would be interesting to see if indeed, their copula dependence measures reveal something on the days of the Quant Meltdown (August 1 to 9).

- Interestingly actual correlation markets (CDX tranches) did not reveal anything anomalous during these days.

Conclusion

- Impressive econometric model of dependence.
- Lots of degrees of freedom (pun intended).

⇒ Would be useful to better explain why they are useful and what they capture.

- Not clear how this model can be used to model CDO tranches, which have payoff dependent on loss distribution.
- Estimates of correlation matrix based on weekly IG spread changes may not adequately capture the 5-year default distribution and dependence, especially given the short history and few defaults among the CDX-IG constituents.
- Would be useful to try to calibrate the model to CDX tranche data and see how the calibrated copula parameters compare to the time-series estimates.
- I would be careful of the "causal" interpretation of the impact for credit markets of the "Quant meltdown" which was really a "minor" (and fairly short lived) event .