

## Discussion of “Decomposing the Yield Curve” by John H Cochrane and Monika Piazzesi

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- Summary
- Affine model?
- Observable factors?
- 'One-factor' risk-premium
- Final thoughts

## Summary

- ▶ **Objective:** Decompose term premia (US Treasury curve) into expected future interest rate changes and risk-premia
- ▶ **Motivation:** Low long term yield 'Conundrum' of 2004-06: if due to low future interest rates or low risk-premia has potentially different policy implications.
- ▶ **How?** Use a four-factor Gaussian affine framework written in terms of 'observable' factors with parsimonious risk-premium structure that accommodates major findings (revisited) of Cochrane and Piazzesi (2005):
  - ▶ A unique factor (a 'tent-shaped' combination of all forward rates) drives excess returns of all bonds ('at least the economically interesting component')
  - ▶ This factor is not spanned by standard level, slope, curvature state variables (even though these account for  $> 99\%$  of the variation of yields)
- ▶ Estimate/calibrate the model using simple three-ish-step procedure
- ▶ **Findings:** Conundrum due to decline in risk-premia: *'markets believe inflation is conquered, and you're welcome'*

## Model specification and estimation procedure

- ▶ The model has four observable (as opposed to latent) state variables:
  - ▶ The return-forecasting factor ( $x_t$ ) is the CP(2005) factor obtained as the first PC of expected excess returns (fitted values in a regression of excess returns on all forwards).
  - ▶ In addition  $L, S, C$  are the first-three PC of yield changes, orthogonalized with respect to  $x_t$  (i.e, 3 PC of residuals from regression of forwards on  $x_t$ ).
  - ▶ Assume observable  $X_t = (x, L, S, C)$  follows a vector AR1 process under both P and Q measure:

$$dX_t = (\kappa_0^P + \kappa_1^P X_t)dt + \Sigma dZ_t^P \quad (1)$$

$$dX_t = (\kappa_0^Q + \kappa_1^Q X_t)dt + \Sigma dZ_t^Q \quad (2)$$

- ▶ The market price of risk is restricted to generate a one-factor structure in excess returns consistent with CP(2005):

$$dZ_t^Q = dZ_t^P + (\lambda_0 + \lambda_1 X_t)dt$$

$$\lambda_0 = \begin{bmatrix} 0 \\ \lambda_{0L} \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad \lambda_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \lambda_{1L} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

## Model specification and estimation procedure

- ▶ With these assumptions yields (and forwards) are affine in the '*observable*' state variables:

$$Y(t, T) = A(T - t) - B(T - t)X_t$$

- ▶ Note that the *observable* state variables  $X_t$  are also linear combinations of yields:

$$X_t = \alpha \cdot \mathbf{Y}(t)$$

(with  $\alpha$  loadings of various eigenfactors and  $\mathbf{Y}(t)$  vector of yields with different maturities).

- ▶ The estimation technique then consists in:
  - ▶ Estimating the variance covariance matrix of  $X_t$  from time series.
  - ▶ Estimating  $\kappa^Q$  parameters from minimization of SSQ differences between model yields and actual yields (ignoring time-series information).
  - ▶ Lastly, estimate  $\lambda_0, \lambda_1$  from two time series moments to obtain  $\kappa^P$ .

## Why use an Affine model?

- ▶ Alternative (naive?) approach:
  - ▶ Forecast future yields or Forward rates (VAR).
  - ▶ Attribute difference between quoted prices and forecasts to risk-premium
- ▶ Why use an affine model instead?
  - ▶ Idea is to use affine model because it gives 'precise' estimates of the Q-dynamics using only cross-sectional information (the criteria is the small RMSE).
  - ▶ Hope that by restricting the risk-premium to two parameters, obtain better estimates of the P-dynamics than what one usually obtains from time series estimation. (essentially force P and Q to be 'close').
- ▶ Potential problems: this approach is very dependent on
  - ▶ the Q-measure dynamics specification and estimation, or alternatively stated, on how informative the cross-section of bond prices is for Q-dynamics of interest rates.
  - ▶ Accuracy of the change of measure restrictions which rely on the 'one-factor in level only' risk-premium assumption (based on CP(2005) results).

## How 'precise' is the estimation of Q-dynamics?

- ▶ Many models can give a good cross-sectional fit and yet imply very different dynamics, **if** no time-series restriction is imposed.
  - ▶ Example 1: USV versus non-USV model. Same cross-sectional fit, totally different dynamics.
  - ▶ Example 2: 6 factor model of 6 yields. Will have a perfect fit, no matter what the assumed Q-dynamics are (if use solely a cross-sectional approach).
- ▶ Low RMSE does not seem adequate criterion to assess quality of Q-measure dynamics implied by the model. Time-series dimension may impose useful discipline on model.
- ▶ Having observable factors alleviates this problem to some extent, since the estimation procedure does not have the luxury of distorting their dynamics to fit yields.

## Q-maximality and Observable factors?

- ▶ Using an affine model written in terms of 'observable' factors as opposed to latent factors is useful because state variables do not need to be inverted from the yield curve. However, the dynamics of the system under the Q-measure are not unrestricted (Duffie-Kan (1996)).
- ▶ To see this, consider an equivalent representation of the 4-factor Gaussian model in terms of the short rate  $r_t$ , its risk-neutral drift ( $\mu_1^Q(t) = E_t^Q[dr_t]$ ), the drift of the drift ( $\mu_2^Q(t) = E_t^Q[d\mu_1^Q(t)]$ ) and  $\mu_3^Q(t) = E_t^Q[d\mu_2^Q(t)]$
- ▶ Clearly, by definition the risk-neutral dynamics of the state  $\tilde{X}_t = [r_t \mu_1^Q(t) \mu_2^Q(t) \mu_3^Q(t)]'$  are:

$$d\tilde{X}_t = \begin{bmatrix} 0 \\ 0 \\ 0 \\ m_0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ m_r & m_1 & m_2 & m_3 \end{bmatrix} \tilde{X}_t + \tilde{\Sigma} dZ_t^Q$$

- ▶ By definition, there are at most 5 free risk-neutral parameters (in addition to the full variance covariance matrix).
- ▶ **Conclusion:** Of the 20 risk-neutral parameters specified by CP for  $\kappa_0^Q, \kappa_1^Q$  only 5 (combinations of parameters) are free!



## Q-maximality and Observable factors?

- ▶ The CP methodology (I believe) ignores this and searches over 16 free parameters.
  - ▶ This may be OK, if the model is perfectly specified and if the data is perfect. In practice, may be better to impose the restrictions.
  - ▶ Note, that with only 5 parameters identified from the Q-measure, the restriction on the market price of risk imposed by CP means that their model has a total of only 7 free parameters as opposed to 25 for the maximal model (in addition to the full covariance matrix).
- ⇒ Heavily tilted tradeoff between efficiency and bias!

## The 'one-factor-in-level' risk-premium assumption

- ▶ Simple experiment:
    - ▶ Estimate a three-factor essentially affine Gaussian model (Duffee (2002), Dai-Singleton (2002)).
    - ▶ Simulate realistic (i.e., small) samples from it and replicate CP (2005) methodology.
  - ▶ What do we find?
    - ▶ First, there are large small-sample (Stambaugh) biases in estimated coefficients. Also, as is well-documented  $R^2$  are overstated.
    - ▶ Second, evidence for a one-factor structure in expected excess returns (in the sense that one PCA captures more than 99%) of the predictability is overwhelming despite the fact that, in population, there is a three-factor structure.
- ⇒ Considerable statistical uncertainty (at least in my mind) about this 'one-factor in level' risk-premium assumption.
- ▶ Can we argue the predictive factor is 'observable' ?
    - ▶ The loadings ( $\alpha$ ) are not available in real-time (look ahead bias since we use all the sample to compute Principal components).
    - ▶ The coefficients are estimated with considerable sampling uncertainty and bias. It is not clear (to me) that the bias is proportional across maturities.
  - ▶ Further, if we take CP (2005) at face value, then it is not clear that the standard four-factor affine framework is the right framework:
    - ▶ CP find that one unique factor drives common expected return of **all** forward rates (in particular, it seems important to use the four minus five year yield spread). In a standard affine model, **any** four distinct forwards (or yields) should do.

## Conclusions

- ▶ Interesting paper. Poses lots of challenges and interesting questions for future research.
- ▶ What is the economic reason for this one-factor structure that seems to put a lot of weight on the difference between the 4 and 5 year yield (fifth principal component)?
- ▶ Why restrict predictability factors to yield based models? There is some evidence that macro-variables may help on top of prices.
- ▶ Forward looking bias in most estimation techniques attribute much of the dynamics to over-parametrized risk-premia, when in fact it was just updating of parameters and models due to non-stationarity (learning, changes in monetary regimes. . . ). (I think this was part of the motivation for this paper).
- ▶ Interesting to see what the model(s) will say about the 'Conundrum' after we add a few more years of data.