Investor Activism and the Green Transition[*]

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Abstract

We develop a tractable model of investor activism with endogenous activist entry, dynamic trading, and contracting with the firm's management. We use our framework to investigate the conditions under which investor activism facilitates the green transition and explore its responsiveness to environmental regulations. In our model, a firm's transition rate depends on the incentives and efforts of both its management and the activist. Activism raises the green transition rate under first best but can hamper transition under moral hazard. Through trading the firm's stock, an activist may intensify its engagement or gradually exit over time. The free-rider problem linked to activist entry can either be mitigated or aggravated by the activist's post-entry trading and engagement. Carbon taxation enhances transition conditional on activist entry, but overly high taxes may deter entry. Green investment subsidies raise firm-level transition efforts, but crowd out activist efforts and generally hamper green transition.

Keywords: Investors activism, limited commitment, contracting, agency conflicts, sustainable finance, environmental regulation

There is widespread consensus that a green transition in production technologies is necessary to address climate change (Acemoglu, Akcigit, Hanley, and Kerr, 2016; Besley and Persson, 2023) and that the rate of green transition significantly influences the long-term environmental impact of economic activity (Popp, 2002; Acemoglu, Aghion, Bursztyn, and Hemous, 2012). Over the past few years, financial markets have sought to accelerate the pace of the green transition by directing companies toward environmental objectives through both passive and active investment strategies. Passive strategies involve investing in "clean" firms and divesting from "dirty" firms so as to influence their cost of capital to incentivize investment in green transition. In active strategies, investors exercise their control rights to impact firms' decisions, such as through board representation, management oversight, strategy development, or voting on proposals. Recent research suggests that passive strategies, despite their popularity, may have little impact on firm behavior (Heath, Macciocchi, Michaely, and Ringgenberg, 2021; Berk and Van Binsbergen, 2022; Saint-Jean, 2023) and could even have adverse environmental effects (Hartzmark and Shue, 2023). Investor activism is thus increasingly being advocated as the preferred and more effective approach to sustainable finance (Krueger, Sautner, and Starks, 2020; Broccardo, Hart, and Zingales, 2022).

In this paper, we develop a unifying model of investor activism with endogenous activist entry, post-entry trading, and activist engagement, in which the activist affects firm outcomes both by providing effort and by contracting with the firm's management. We then use this model to understand when investor activism can facilitate the green transition and how activism responds to environmental regulation and policies. In our model, activism increases the green transition rate under first best, but three frictions may limit its effectiveness. First, the green transition rate depends on the efforts of both the firm's management and the activist, which are costly and subject to moral hazard. The activist's incentives to exert effort increase with its stake in the firm, but may crowd out management's effort incentives and vice versa. Second, activist investors cannot fully capture the gains of activism since existing shareholders free-ride on the activist's efforts. This free-rider problem reduces the activist's incentives to enter and may even deter entry when sufficiently severe. Third, when given the opportunity, the activist discretionally trades in the firm's stock and may get more invested or gradually exit the firm. Therefore, an activist cannot commit to holding a particular stake in the firm and thus to a specific level of effort. Consequently, an activist's impact is determined by interconnected entry, effort, trading, and exit decisions. As we

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show, environmental policies such as carbon taxes or green investment subsidies may distort activist decisions on each of these margins. Our model allows us to examine how activists respond to environmental regulations and which type of regulations facilitate an activist's impact on a green transition. Importantly, our model of investor activism is tractable and can accommodate pecuniary and non-pecuniary preferences and objectives. It applies broadly to various types of activism, including green activism, and activist investors, such as private equity funds, hedge funds, large asset managers, or wealthy individuals.

To capture the key determinants of environmental activism, we develop a dynamic model in which a firm with a dirty/polluting production technology invests to transition toward a clean/green production technology. Transitioning to the clean technology may generate both non-pecuniary and pecuniary benefits to firm owners.[1] However, the transition process is both costly and uncertain. Without loss of generality, one can interpret a successful transition as a general improvement in firm value or performance (not necessarily limited to environmental aspects), such as cash flow enhancement. Thus, the model can describe various forms of investor activism, including but not limited to green activism. Importantly, the rate of successful green transition increases with the effort of the firm's manager, who broadly represents the firm's key personnel and executives that affect firm outcomes. As effort is unobservable, costly, and subject to moral hazard, firm owners provide management with incentives to exert effort by making compensation sensitive to the transition process.

While the firm is initially owned by passive investors, an activist may enter the firm by buying an initial ownership stake. The activist and passive investors differ in two dimensions. First, unlike passive investors, the activist exerts private and costly effort which, in addition to managerial effort, facilitates the green transition. The activist's effort captures its engagement with the firm, for instance, by monitoring management, appointing key personnel and board members, developing strategies, or voting on proposals. Second, the activist incurs a holding (disutility) cost per unit of firm ownership. This holding cost may reflect an activist's financial and capital constraints, which increase the cost of capital and reduce the benefits of investment, or its sustainability preferences and (green) investment mandate, which make it costly to invest in a polluting firm.

We first show that while activism is valuable and speeds up green transition in first best,

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[1]These benefits could emerge from a range of factors. This includes mechanisms like carbon taxation and pricing, which decrease the value of polluting firms. Additionally, sustainability preferences among stock market participants may result in higher (or lower) valuations for more (or less) sustainable firms. Consumer preferences also play a role through higher demand for the goods produced by sustainable firms.

it introduces a double-agency problem that distorts effort incentives in the presence of moral hazard. Notably, the activist's incentives to exert effort are reduced relative to first best as the activist only captures part of the gains from effort while incurring the full cost. This is for two reasons. First, the activist only owns part of the firm. Second, part of the transition surplus accrues to management as part of the incentive contract. In effect, effort incentives provided to management reduce the activist's effort. Thus, the efforts of the activist and management endogenously arise as substitutes and lie below first-best efforts. Due to the double-agency problem, activism can increase or decrease the green transition rate (relative to passive investors owning the firm). In particular, "good" or skilled activists, characterized by a low effort cost and high effort level, facilitate the green transition, whereas "bad" or less skilled activists, characterized by a high effort cost, hamper it.

After entry, the activist may trade discretionally in the firm's stock, allowing it to increase or decrease its stake in the firm. To account for potential trading frictions/costs in reduced form, we assume that the activist cannot trade the firm's stock over a period of time after entry, while trading is frictionless afterward. This assumption also allows us to model both private and public firms, respectively characterized by infrequent and frequent trading opportunities. Once the activist can trade, its trading rate is determined by two opposing forces and, hence, can be positive or negative. First, there are gains from buying a larger stake, as this makes contracting with the manager more efficient. Second, there are gains from selling the firm's stock because of the activist's holding cost. Our analysis reveals that after entering the firm, less skilled activists tend to gradually exit, whereas more skilled activists tend to gain more control by acquiring a larger stake over time. Importantly, this larger stake translates into higher effort levels, as the activist captures a larger fraction of the benefits associated with transition. As a result, conditional on entry, the possibility for the skilled activist to acquire a larger stake over time fosters the transition to a clean technology.

In the model, the activist improves firm value through private and costly effort. However, if this value creation is fully reflected in the price at which it acquires a stake in the firm, the activist cannot capture the gains from activism and thus has no incentive to invest in the first place, causing a free-rider problem (Grossman and Hart, 1980; Shleifer and Vishny, 1986). Two key assumptions regarding entry determine the severity of the free-rider problem. First, the activist must initially acquire a minimum ownership stake to be able to influence firm outcomes and exert control. Second, when entering the firm, the activist can buy a fraction

of the firm at the pre-entry price (i.e., the firm's stock price under passive ownership), while acquiring the remaining shares at a price that reflects the gains achieved through activism. The activist then enters the firm as long as the payoff from engaging with the firm exceeds its reservation utility. As we argue and formally show in an Online Appendix, this reservation utility is lower if the activist has an intrinsic motivation to transform the firm, capturing its preferences for impact or sustainability. We also show that it may be optimal for the activist to enter the firm by acquiring an initial ownership stake (at a relatively low price), while gradually exiting by selling its ownership stake (at a relatively high price) post-entry.

Better trading opportunities exacerbate the free-rider problem for skilled activists by allowing them to acquire a larger share of the firm over time, increasing firm value and the initial acquisition price. In contrast, for less skilled activists, better trading opportunities speed up exit and reduce the initial acquisition price. Better trading opportunities therefore discourage entry by good activists, while encouraging entry by bad activists. Thus, our findings suggest that private markets, characterized by less frequent trading opportunities, feature more impactful activists (e.g., private equity owners) and less (premature) exit.

Without regulation, the manager and the activist underinvest in green transition. The presence of a negative externality thus calls for regulation aimed at fostering the transition. Equipped with our general model of investor activism, we evaluate the effects of two common environmental policies, namely carbon taxes and green investment subsidies, on the green transition rate. In essence, carbon taxes reduce the value of a firm with a dirty technology, thereby increasing the financial gains from transitioning to a green technology. Carbon taxes lead to more effort from the activist and the manager, while increasing the activist's stake post-entry and discouraging activist exit. Hence, conditional on activist entry, carbon taxes facilitate the green transition. However, carbon taxation also potentially reduces the activist's entry incentives. In particular, carbon taxes increase the activist's post-entry impact and hence the value created through activism and the free-rider problem that disincentivizes entry. Whether carbon taxation encourages or discourages activist entry crucially depends on the quality of the activist and the severity of the free-rider problem, as captured by the fraction of the firm that the activist can initially buy at the pre-entry price.

Strikingly, when the free-rider problem is severe, high carbon taxes hamper the green transition by discouraging good (skilled) activists and encouraging bad (less skilled) activists to enter. Thus, to leverage the effects of activism on green transition, the carbon tax or,

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similarly, the price of carbon within a cap and trade scheme must not be too high. That is, the optimal carbon tax exceeds the Pigouvian level when the activist has strong incentives to enter, e.g., due to impact preferences or financial profits from activism. In this case, the carbon tax addresses the double-agency problem without affecting the activist's entry decision. Otherwise, when activism is beneficial for green transition but activist entry is hard to incentivize, the optimal carbon tax can lie below the Pigouvian tax.

We also examine the effects of firm-level investment subsidies on the green transition rate. Such investment subsidies, as for instance stipulated in the Inflation Reduction Act in the U.S.. Firm-level investment subsidies, by reducing the cost of investment, make it optimal to incentivize higher managerial effort. However, this requires a larger incentive compensation for management, thereby reducing the activist's effort incentives and crowding out activist effort. Importantly, we show that precisely when activism is valuable, investment subsidies hamper green transition. This is for two reasons. First, subsidies reduce overall effort by crowding out activist effort. Second, subsidies discourage entry and increase exit incentives of good activists, reducing the extent of "good" activism. Our analysis, therefore, suggests that the regulator should not rely on investment subsidies to boost the green transition, in contrast with the findings in Acemoglu et al. (2012) and Acemoglu et al. (2016) that, absent moral hazard and activism, subsidies are effective at fostering the green transition.

Our main contribution is to develop a tractable model of investor activism, which applies broadly to the study of blockholders and activist investors. This model can be used to assess their impact on firm performance, encompassing a range of objectives including but not restricted to environmental ones. Therefore, our paper primarily relates to the literature on shareholder activism and blockholders (see, e.g., Maug (1998); Noe (2002); Edmans and Manso (2011); Dasgupta and Piacentino (2015); Marinovic and Varas (2021)). Admati and Pfleiderer (2009) and Edmans (2009) analyze the role of exit as a governance mechanism. Most closely related to our paper, Admati, Pfleiderer, and Zechner (1994); DeMarzo and Urošević (2006); Back, Collin-Dufresne, Fos, Li, and Ljungqvist (2018) study the trading of a large shareholder that affects firm performance via costly effort. Our paper differs from these by allowing the activist to affect firm performance not only via its own effort but also by contracting with management to induce managerial effort. A higher activist stake makes contracting more efficient and generates endogenous gains from trade, leading to the novel result that the activist may dynamically buy a larger stake despite holding costs. Another key

innovation is that we endogenize entry, show how entry interacts with dynamic trading and engagement, and highlight a complementarity between entry and exit. Methodologically, we advance the literature on investor activism by jointly and tractably modeling activist entry, dynamic trading, and the interactions between the incentives of the firm's management and the activist. Notably, our modeling allows us to capture the key economic mechanisms and to analytically characterize the equilibrium in closed form.

We solve for the Markov-perfect equilibrium trading strategy using a methodology similar to DeMarzo and Urošević (2006), which is used in other contexts too. For instance, Daley and Green (2020) study dynamic bargaining with adverse selection. Hu and Varas (2022), DeMarzo and He (2021), and DeMarzo, He, and Tourre (2023) respectively study loan sales, corporate leverage, and the dynamics of government debt under limited commitment.

We apply our framework to study the role of activist investors in the green transition and to assess how environmental regulation affects green activism. Thus, our paper also relates to the growing theoretical literature on sustainable finance (see, e.g., Heinkel, Kraus, and Zechner (2001), Albuquerque, Kroskinen, and Zhang (2019), Green and Roth (2021), Hong, Wang, and Yang (2023), Gupta, Kopytov, and Starmans (2022), Edmans, Levit, and Schneemeier (2023), Huang and Kopytov (2023), Landier and Lovo (2023), Allen, Barbalau, and Zeni (2023)). We contribute to this literature by studying how investor activism can foster the green transition in a dynamic model with moral hazard and endogenous activist entry, trading, and exit. In this literature, our paper is most closely related to Broccardo et al. (2022), Jagannathan, Kim, McDonald, and Xia (2022), and Oehmke and Opp (2022). The first two papers study the effectiveness of exit and voice strategies in reducing firms' negative externalities, whereby voice and exit are examined separately. In our model, activists endogenously decide to invest in a firm to reduce externalities under moral hazard and can trade to increase impact or exit, leading to interlinked voice and exit. In Oehmke and Opp (2022), an entrepreneur raises capital from financial or socially responsible investors under moral hazard. Our model differs because (i) the efforts of both management and the activist are crucial for the green transition, causing a double-agency problem that hampers impact, and (ii) the activist's post-entry trading and exit interact with incentives, with novel implications for optimal regulation.

Our paper's application to sustainable finance is motivated by growing empirical evidence that shareholder engagement and environmental activism can facilitate the green transition

(Dimson, Karakaş, and Li, 2015; Kölbel, Heeb, Paetzold, and Busch, 2020; Wiedemann, 2023). According to a recent survey by Krueger et al. (2020), institutional investors consider engagement rather than divestment as a more effective approach to address climate risks. Akey and Appel (2020), Naaraayanan, Sachdeva, and Sharma (2023), and Azar, Duro, Kadach, and Ormazabal (2021) show that engagements by hedge funds, pension funds, and large asset managers respectively cause targeted firms to reduce their emissions. Bellon (2022); Kumar (2023); Cole, Jeng, Lerner, Rigol, and Roth (2023) study the effects on private equity on green transition and sustainable investing in private markets.

The remainder of the paper is structured as follows. Section 1 presents the model of investor activism which we solve in Section 2. Section 3 provides an analysis of the model, with particular focus on applying the model study green activism. Section 4 and Section 5 analyze how investor activism interacts with carbon taxation and green investment subsidies respectively. The heuristic proofs are contained in the main text; additional details for the proofs, tedious arguments, and extensions are deferred to the Online Appendix.

1 A Model of Investor Activism

This section presents a model of investor activism, in which an activist invests in a firm to transform its production technology. The activist does so through its own private effort and by incentivizing the firm's manager with an optimal contract. The manager or, alternatively, management more broadly represents the firm's key personnel and executives who are able to influence firm outcomes. The activist's private effort captures its engagement with the firm, for instance, by monitoring management, appointing key personnel and board members, developing strategies and proposals, providing industry connections, or by voting on proposals. Our model applies to both private and public firms, while the activist may represent a hedge fund, a pension fund, a private equity fund or other types of active investors, such as wealthy individuals or philanthropists.[2]

With no loss in generality, one can also (re-)interpret a successful green transformation as a general improvement in firm value or performance (not necessarily limited to environmental

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[2]Hedge funds and private equity funds often actively engage with their portfolio companies to influence outcomes (Brav, Jiang, Partnoy, and Thomas, 2008; Kaplan and Strömberg, 2009). Bellon (2022); Kumar (2023) study private equity owners' impact on green transition. Akey and Appel (2020); Naaraayanan et al. (2023); Azar et al. (2021) respectively show how the engagement of hedge funds, pension funds, and asset managers causes targeted firms to reduce emissions.

aspects), such as cash flow or productivity enhancement or innovative breakthrough. Under this more general interpretation, the model applies broadly and can describe various forms of investor activism, including but not limited to green activism. Given that in this paper our model is primarily applied to green activism, we will henceforth refer to the improvement in firm performance as *green transition.*

Technology and Preferences. Time $t \in [0, \infty)$ is continuous and infinite. There are three risk-neutral agents with common discount rate $\rho > 0$: An activist investor, a (representative) passive investor, and a manager. We consider a single firm run by the manager that produces cash flows at a constant rate $\mu > 0$. The firm is all equity-financed with a fraction $\theta_t \in [0, 1]$ of its equity held by an activist investor and the remaining $1 - \theta_t$ held by the passive investor. The number of outstanding shares is normalized to one. The manager effectively holds a stake in the firm via its incentive contract and compensation set by the firm's shareholders; this stake may be equity-like too. The firm pays out all cash flows net of managerial compensation as dividends. Its endogenous stock price (i.e., equity value) is denoted by P_t at time t.

At the onset at time $t = 0$, the firm has a polluting (or dirty) production technology, but its owners aim to transition to a clean or green production technology. We model the transition process as an innovation project that is completed at a random time T arriving with intensity $\Lambda > 0$, whereby the outcome of the transition process—success or failure—depends on the activist's and manager's efforts $a_t \in [0, \bar{a}]$ and $m_t \in [0, \bar{m}]$. Specifically, over a short time interval $[t, t + dt)$, the process is completed with exogenous probability Λdt. Upon completion, a transition is successful with probability $a_t + m_t$ and fails otherwise. Thus, over $[t, t + dt)$, the instantaneous probability of successful transition equals $\lambda_t dt := \Lambda(a_t + m_t)dt$. We refer to λ_t as the (green) transition rate. We focus on parameters that lead to optimal interior efforts, i.e., $a_t \in (0, \bar{a})$ $m_t \in (0, \bar{m})$, and a well-defined probability $a_t + m_t \in (0, 1)$. Figure 1 illustrates the transition process over an instant $[t, t + dt)$. We purposefully model a simple transition process, which allows us to analytically characterize the model solution and dynamic effects of activism.[3] As the transition process is stationary, any time dynamics that arise in optimum are endogenous and attributable to activism.

The completion and outcome of the transition process are publicly observable and contractible. In case of success, the firm becomes clean "C" and its owners realize a terminal

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[3]Board and Meyer-ter Vehn (2013), Hu and Varas (2022), and Mayer (2022) among others, employ similar modeling of uncertainty. Similar results would arise if we assumed that transition occurred at random time T arriving with Poisson intensity $\lambda_t = \Lambda(a_t + m_t)$ as in, e.g., Acemoglu et al. (2016).
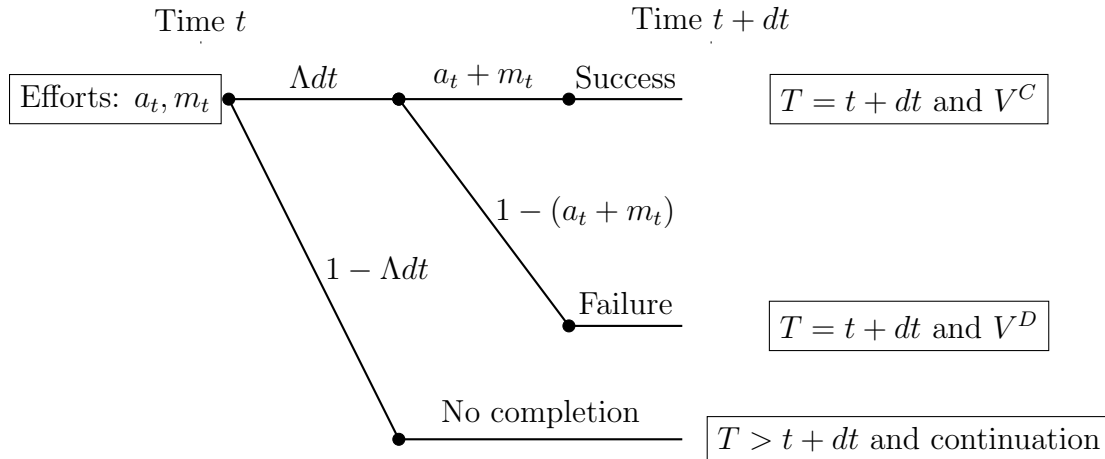
Figure 1: Description of the transformation process and heuristic timing over $[t, t+dt)$. The branches of the tree contain the probabilities of the respective random event over $[t, t+dt)$.

payoff V^C per unit of ownership. In case of failure, the firm remains dirty "D" and generates a terminal payoff V^D per unit of ownership. For simplicity, passive investors and activists derive equal terminal payoffs when the transition process ends. The surplus $\Delta := V^C - V^D \geq 0$ from a successful transition (relative to failure) is positive and broadly captures the financial and non-pecuniary gains from green transition. As we argue later, carbon taxation or investor preferences for green firms imply a higher Δ. Abstracting from non-pecuniary gains of green transition, one can interpret Δ as the difference between a green firm's value (stock price) V^C and a dirty firm's value V^D. Note that even though $\Delta \geq 0$, exerting high effort for green transition can have a negative net present value, due to the cost of effort.

Transition and Moral Hazard. Efforts a_t and m_t are chosen before it is known whether the project is completed over the next instant $[t, t + dt)$. Efforts a_t and m_t are hidden and come at private flow costs $\frac{\kappa a_t^2}{2}$ and $\frac{\phi m_t^2}{2}$ to the activist and the manager respectively, giving rise to a double-agency problem.[4] To deal with this agency problem, the controlling shareholder—which is either the activist or the passive investor—writes at each time t, a short-term contract that stipulates state-contingent compensation to the manager over the next instant $[t, t + dt)$. The short-term contract (B_t, c_t) stipulates a payout B_t in case the firm successfully transitions over $[t, t + dt)$ in addition to a base salary $c_t dt$. The payout B_t corresponds to a lumpy payment and therefore is "large" relative to the base salary which is of order dt, i.e., infinitesimal. With no loss in generality, we normalize the manager's

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[4]Equivalently, we could assume that efforts are observable but not non-contractible.

reservation utility to zero, so that its expected payoff from the contract must be positive. The formulation of the short-term contracting problem in continuous time is similar to He and Krishnamurthy (2011) and is discussed in greater detail in Online Appendix A.1, which also shows that the optimal short-term contract takes the aforementioned form.

Activist Stake and Trading: Public versus Private Firms. The activist acquires an endogenous ownership stake θ_0 in the firm at time $t = 0$. After entry, the activist cannot trade over a time period $(0, T^\beta)$, where T^β is a stochastic time that arrives with intensity $\beta \geq 0$. Thus, after entry, the activist must wait on average $1/\beta$ units of time before being able to adjust its stake. When $\beta = 0$, the activist cannot trade after entry and maintains constant ownership up to time T. In the limit $\beta \to \infty$, trading post-entry is without friction. One may associate a publicly traded firm with high or infinite β, whereas private firms are characterized by lower levels of β and less frequent trading opportunities for investors. For private firms, T^β may also be interpreted as the time of the IPO after which the firm's stock can be traded publicly.[5] In the case of a public firm (high β), the activist may represent a hedge fund, while in the case of a private firm (low β) the activist may represent a private equity sponsor. The assumption of no-trading over $(0, T^\beta)$ can be interpreted as the activist committing to an investment horizon over which it does not adjust its stake and, specifically, does not exit. Under this interpretation, β is inversely related to the activist's commitment power; in fact, $\beta = 0$ implements the full-commitment solution.[6]

When the activist can trade, i.e., after time T^β, the heuristic timing over $[t, t + dt)$ is as follows. First, given an activist stake θ_t, the firm's controlling shareholders (i.e., the activist when $\theta_t > 0$) and the manager sign a contract (B_t, c_t) lasting over $[t, t+dt)$. Second, uncertainty as to whether the transition process is completed and succeeds or fails is resolved. Payments are made by the firm and the contract ends: The manager receives B_t in case of success, zero in case of failure, and $c_t dt$ in case of no completion. In case of completion, active and passive investors realize terminal payoffs, and the model ends. Third, in case the transition process is not completed, the activist trades the firm's stock and chooses $d\theta_t$, determining next-period stake $\theta_{t+dt} = \theta_t + d\theta_t$. Then, at time $t + dt$ with activist stake θ_{t+dt}, controlling shareholders and the manager sign a contract over $[t + dt, t + 2dt)$, and so on.

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[5]Unlike public equity markets, private equity markets feature only infrequent and lumpy trading. Private equity funds typically hold a constant stake for an extended period of time (e.g., the duration of the fund).

[6]Because before time T our setting is essentially stationary and firm fundamentals do not change, there is no motive for dynamic trading under full commitment. As such, under full commitment and conditional on entry, the activist would choose constant ownership stake, which is akin to assuming $\beta = 0$.

Payoffs. Given a contract (B_t, c_t), the manager's expected payoff is $w_t dt$ with

$$w_t := \max_{m_t \geq 0} \left(\Lambda(\hat{a}_t + m_t)B_t - \frac{\phi m_t^2}{2} + c_t \right), \tag{1}$$

where $\hat{a}_t$ is the level of (hidden) effort by the activist that the manager anticipates. In equilibrium, $\hat{a}_t$ coincides with actual effort a_t. The manager chooses at time t its private effort m_t over $[t, t+dt)$ against the cost $\frac{\phi m_t^2}{2}dt$. Over $[t, t+dt)$, the transition process completes with probability Λdt, resulting in success and payment B_t with probability $a_t + m_t$ and failure and zero payment otherwise. See Online Appendix A.1 for more details.

When the activist does not enter, the passive investor is the controlling shareholder and dynamically chooses contracts (B_t, c_t) to maximize its private valuation of the firm:

$$P_t^0 = \max_{(c_u, B_u)_{u \geq t}} \mathbb{E}_t \left[\int_t^T e^{-\rho(u-t)}(\mu - c_u)du + e^{-\rho(T-t)}\left[V^D + m_T(\Delta - B_T) \right] \right], \tag{2}$$

subject to the participation constraint $w_u \geq 0$ for all $u \geq t$. The passive investor's valuation is the discounted sum of the firm's expected future dividends (i.e., cash flows net of managerial compensation) plus the terminal payoff at time T when the transformation process ends. The terminal payoff is V^D in case of failure and $V^D + \Delta - B_T$ in case of success. The probability of success at time T is m_T because the passive investor does not exert any effort and $a_T = 0$. If the activist does not enter, then P_t^0 is also the firm's stock price.

When the activist enters and $\theta_0 > 0$, it becomes the controlling shareholder (as discussed below) and chooses contracts (B_t, c_t), its effort a_t, and trading $d\theta_t$ to maximize:

$$V_t = \max_{(a_u, c_u, B_u, d\theta_u)_{u \geq t}} \mathbb{E}_t \left[\int_t^T e^{-\rho(u-t)}\left(\theta_u(\mu - \pi - c_u)du - \frac{\kappa a_u^2}{2}du - (P_u + dP_u)d\theta_u \right) \right.$$
$$\left. + e^{-\rho(T-t)}\theta_T\left[V^D + (a_T + m_T)(\Delta - B_T) \right] \right] \tag{3}$$

subject to $w_u \geq 0$ for all $u \geq t$. The activist has sustainability or "green" preferences, in that it derives a disutility flow $\pi\theta_u$ from owning a fraction θ_u of the dirty firm.[7] The disutility flow—akin to a flow holding cost $\pi \geq 0$ per unit of stock— may also reflect (in reduced form) the activist's financial or capital constraints or higher cost of capital, common in models of activist investors (DeMarzo and Urošević, 2006; Marinovic and Varas, 2021). Because the

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[7]We could also model a difference in financial/capital constraints by assuming that the activist applies a higher discount rate. This would lead to qualitatively similar outcomes.

activist only owns a fraction θ_u of the firm, it only collects a fraction θ_u of dividends and terminal payoff (net of managerial compensation), while fully incurring the private cost of effort. The term $-d\theta_u(P_u + dP_u)$ captures the payoff that the activist collects from trading over a short time period $(u, u + du)$; when $u < T^\beta$, we have $d\theta_u = 0$. The activist has price impact and trades over $(u, u + du)$ at "end-of-period" price $P_{u+du} = P_u + dP_u$.

Due to the activist's private cost of effort and green preferences, the value of the firm under activist ownership *from passive investor perspective* differs from V_t and equals

$$P_t = \mathbb{E}_t\left[\int_t^T e^{-\rho(u-t)}(\mu - c_u)du + e^{-\rho(T-t)}\left[V^D + (a_T + m_T)(\Delta - B_T)\right]\right]. \qquad (4)$$

The value of the firm from passive investors' perspective P_t reflects the value generated through activism. Observe that the activist and passive investors differ in two dimensions. First, the activist exerts private effort, while passive investors do not. Second, the activist incurs a flow holding cost (disutility) $\pi \geq 0$ per unit of firm ownership. As such, one can view passive investors as "activists" with $\kappa \to \infty$ and $\pi = 0$.

At time T, the model essentially ends, and we can treat V^D and V^C as terminal payoffs in case of failure and success respectively.[8] Before time T, the activist's value function, the firm's stock price, and other model quantities will be functions of both (i) the activist's stake θ and (ii) whether time T^β has arrived (i.e., whether $t < T^\beta$ or $t \geq T^\beta$). In what follows, we denote the activist's value function and stock price *after* time T^β by $V(\theta)$ and $P(\theta)$. We denote the activist's value function and stock price *before* time T^β by $V^\beta(\theta)$ and $P^\beta(\theta)$. More generally, the superscript "β" will indicate quantities before time T^β.

Entry and the Free-Rider Problem. The activist increases firm value through private and costly effort. However, if this value creation is fully reflected in the price at which it acquires an initial stake in the firm, the activist cannot capture the gains from activism and thus has no incentive to invest in the first place, causing a free-rider problem. Two key assumptions regarding activist entry determine the severity of the free-rider problem. First, the activist must acquire a minimum stake $\tilde{\theta}_0 \in [0, 1]$ to be able to exert control and influence firm outcomes, e.g., via monitoring or voice, in that $\theta_0 \geq \tilde{\theta}_0$. Second, the activist can acquire a fraction $1 - \eta \in [0, 1]$ of the minimum stake $\tilde{\theta}_0$ at the price P_0^0, defined in (2), that would

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[8]After the transition has succeeded or failed at time T, there is no more effort or contracting with management. At time T, the firm's (passive or active) owners realize a payoff $V_T \in \{V^D, V^C\}$ by either continuing to hold the company or by selling it to other equity investors at price V_T.

prevail under passive ownership. The remaining fraction η is bought at a price $P_0 = P^\beta(\theta_0)$, defined in (4), that reflects the gains from activism.[9] The weight η thus captures the severity of the free-rider problem regarding activist entry. Under this assumption, the activist pays

$$K(\theta_0) := (1-\eta)\tilde{\theta}_0 P_0^0 + \left[\theta_0 - (1-\eta)\tilde{\theta}_0\right] P^\beta(\theta_0) \tag{5}$$

to acquire ownership stake $\theta_0 \geq \tilde{\theta}_0$ and enters if and only if

$$E(\theta_0) := \max_{\theta_0 \in [\tilde{\theta}_0, 1]} \left[V^\beta(\theta_0) - K(\theta_0)\right] \geq R, \tag{6}$$

where R is the activist's reservation utility. The activist either acquires (at least) the required minimum stake of $\tilde{\theta}_0$ at time $t = 0$ or never enters. We refer to the case of $\theta_t > 0$ as *active ownership* and to $\theta_t = 0$ as *passive ownership*. We could assume that the activist must keep a minimum stake $\hat{\theta} \leq \tilde{\theta}_0$ for all $t \geq 0$ to be able to have impact on firm outcomes, without changing our model's key implications (see Online Appendix F.4). As we show in Proposition 2, there exists a strictly positive (endogenous) level $\theta_{\min} > 0$ so that the activist's stake never takes any value in $(0, \theta_{\min})$. Online Appendix F.4 shows that our findings remain qualitatively unchanged when imposing a minimum stake $\hat{\theta} > \theta_{\min}$.[10]

Sustainability Preferences. Our model can capture preferences for sustainability on the part of the activist investor via the flow holding cost π, the reservation utility R, and the surplus generated upon successful green transition Δ. For our analysis only the difference between active and passive investors' sustainability-related utility matters, so we do not explicitly model any sustainability preferences for passive investors. We emphasize that the stipulation of non-pecuniary preferences is *not necessary* to motivate the model elements π, R, and Δ, i.e., we present a model of investor activism which *also* applies to impact investing.

First, in addition to capital constraints, an activist's holding costs or (warm-glow) disutil-

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[9]In practice, an activist can indeed accumulate a sizable ownership share before disclosing it, and noise traders may obfuscate large trades by the activist (Kyle and Vila, 1991; Collin-Dufresne and Fos, 2016). In the U.S. for example Section 13(d) of the 1934 Act and Regulation 13D requires owners of more than 5% of the equity of a public firm to file a report with the SEC, at which point the identity of an activist gets revealed. Collin-Dufresne and Fos (2015) report that the average activist holds 7.51% of the target's outstanding shares when making its first public disclosure through a Schedule 13D filing.

[10]We also view this focus as plausible as the activist may need to initially have sufficient ownership and control rights to implement catalyzing changes that influence firm outcomes and policies, such as replacing management or board members or changing the firm's strategic plan. However, after these changes have taken place, the activist may have continued influence with a smaller stake.

ity from owning a polluting firm may also reflect value-alignment preferences (Pástor, Stambaugh, and Taylor, 2021; Green and Roth, 2021; Landier and Lovo, 2023; Dangl, Halling, Wu, and Zechner, 2023), arising from (impure) altruism (Andreoni, 1990), or a green investment mandate (Hong et al., 2023). Second, the activist may have a low reservation utility due to its preferences for impact. The reservation utility R can even be negative when the activist follows a broad impact mandate as in Oehmke and Opp (2022) and has an intrinsic motivation to transform the firm; Online Appendix F.5 provides a micro-foundation for $R < 0$. Third, in addition to a financial payoff component, the surplus generated upon successful green transition Δ may capture a non-pecuniary payoff that is realized by the firm's (passive or active) owners upon successful green transition.[11]

2 Model Solution

2.1 First-Best Benchmark

We start by characterizing the first-best benchmark that is obtained when efforts maximize total surplus. In our model, first best efforts are determined according to[12]

$$(a^{FB}, m^{FB}) := \arg \max_{a,m \geq 0} \left\{ \Lambda(1 - a - m)V^D + \Lambda(a + m)(V^D + \Delta) - \left(\frac{\kappa a^2 + \phi m^2}{2} \right) \right\}. \quad (7)$$

First-best efforts equal $a^{FB} = \frac{\Lambda\Delta}{\kappa}$ and $m^{FB} = \frac{\Lambda\Delta}{\phi}$, increase with the expected benefits of effort $\Lambda\Delta$, and decrease with effort costs. First-best total surplus and efforts do not depend on π, because the firm is owned by passive investors (i.e., the first-best owners) to avoid the deadweight loss stemming from the disutility flow incurred by the activist owning the firm. To ensure optimal interior efforts and $a_t + m_t \in [0, 1]$, we assume that: $\Delta < \overline{\Delta} := \frac{\kappa\phi}{\Lambda(\kappa+\phi)}$.

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[11]To illustrate this idea, consider the extreme case that there is no financial payoff to green transition, but that firm owners derive a lump-sum utility payoff of Δ per unit of ownership from green transition. Owning fraction θ_T of the firm, an activist derives a total utility $\theta_T\Delta$ upon green transition at T, as stipulated in (3). More generally, Δ could consist of both a pecuniary (financial) and such a non-pecuniary payoff.

[12]Total surplus satisfies $\mathcal{S} = \mathbb{E}\left[e^{-\rho T}\left(V^D + (a_T + m_T)\Delta\right) - \int_0^T e^{-\rho t}\left(\frac{\kappa a_t^2 + \phi m_t^2}{2} \right) dt \right]$. As our setting is stationary prior to time T, surplus-maximizing efforts are constant over time, $a_t = a, m_t = m$. Thus, $\mathcal{S} = \frac{1}{\Lambda+\rho}\left[\Lambda(1 - a - m)V^D + \Lambda(a + m)(V^D + \Delta) - \left(\frac{\kappa a^2 + \phi m^2}{2} \right) \right]$, so (a^{FB}, m^{FB}) satisfies (7). In first best, efforts (a^{FB}, m^{FB}) could be incentivized via a contract between the firm's passive owners and the activist (manager), whereby efforts are contractible and the activist (manager) is compensated for the effort cost.

2.2 Effort Incentives and Optimal Contract

Suppose now that effort levels are unobservable (and, thus, not contractible). At any point in time $t \geq 0$, the manager chooses m_t to solve the optimization problem in (1), leading to optimal managerial effort

$$m_t = \frac{\Lambda B_t}{\phi}. \tag{8}$$

As expected, a larger payment B_t incentivizes higher effort m_t, while a higher cost ϕ reduces effort. Next, consider the activist with ownership stake θ_t at time t. With probability Λdt, the transformation process is completed. In case of success, with probability $a_t + m_t$, the activist's payoff is $V^D + \Delta - B_t$. Otherwise, in case of failure, it is V^D. Note that because a_t is hidden, any unobserved change in a_t does not change contracted payouts to the manager. As such, the activist takes the contracted payouts c_t and B_t as given when choosing a_t. That is, the activist solves $\max_{a_t \geq 0} \left(\theta_t \Lambda a_t (\Delta - B_t) - \frac{\kappa a_t^2}{2} \right)$, leading to optimal effort

$$a_t = \frac{\theta_t \Lambda (\Delta - B_t)}{\kappa}. \tag{9}$$

Higher effort incentives provided to the manager through larger payment B_t limits the activist's payoff upon transformation, thus curbing the activist's effort a_t.

To minimize agency costs and maximize its own payoff, the controlling shareholder designs the manager's contract such that the participation constraint $w_t \geq 0$ is tight at any time $t < T$. This implies the manager's flow compensation

$$c_t = \frac{\phi m_t^2}{2} - \Lambda(a_t + m_t)B_t, \tag{10}$$

upon setting $w_t = 0$ in (1). A larger payment B_t is associated with a lower flow compensation so that the manager's total expected compensation remains constant. That is, an increase in B_t increases the performance sensitivity but not the level of managerial compensation.[13]

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[13]In our model, the contract (B_t, c_t) only describes the part of managerial compensation that is related and sensitive to transformation. In practice, the manager's compensation depends on other firm outcomes too. Thus, one can interpret $c_t < 0$ as a salary reduction *relative* to an unmodeled base. One could capture other components of managerial compensation by assuming an outside option $\underline{w} > 0$. Then, payouts in (10) would become $c_t = \underline{w} + \frac{\phi m_t^2}{2} - \Lambda(a_t + m_t)B_t$ with a salary component $\underline{w}$ unrelated to transformation.

2.3 Solution to the Activist's Problem

As in, e.g., DeMarzo and Urošević (2006), Hu and Varas (2022), or DeMarzo et al. (2023), we restrict attention to Markov Perfect Equilibria. Our model features two state variables after time 0: (i) the activist's stake $\theta_t = \theta$ and (ii) whether time T^β has arrived yet (which one could denote by $\mathbb{I}\{t < T^\beta\}$). That is, provided $\theta_t > 0$, the activist takes the state $(\theta, \mathbb{I}\{t < T^\beta\})$ as given, when optimizing its continuation payoff according to (3), while the firm's stock price satisfies (4) given the state variables. All relevant equilibrium quantities and controls will depend on these two state variables; specifically, efforts (a_t, m_t), $(\theta_t)_{t \geq 0}$, and the activist's trading strategy $(d\theta_t)_{t \geq 0}$ are Markov processes chosen according to (3).

We solve the activist's dynamic optimization problem in the unique Markov perfect equilibrium in three steps. First, we characterize optimal trading and efforts after time T^β. Second, we solve for optimal efforts and derive the activist's value function for $t \in (0, T^\beta)$. Third, we characterize the activist's decision to enter at time $t = 0$. Model quantities depend on the activist's stake $\theta_t = \theta$ and on whether time T^β has arrived, but not on calendar time t as such. In what follows, we therefore omit time subscripts and write $a_t = a$, $m_t = m$, $B_t = B$. The model structure is such that we can solve for most quantities in closed form, some of which are relegated to Online Appendix A.3.

2.3.1 Optimal Dynamic Trading and Effort After Time T^β

As we show in greater detail in Online Appendix A.3, there exists an endogenous (possibly empty) smooth trading region, in which the activist optimally trades smoothly according to $d\theta = \dot\theta dt$. By (3) and the dynamic programming principle, the activist's value function $V_t = V(\theta_t)$ then solves

$$(\rho + \Lambda)V(\theta) = \max_{B \geq 0, \dot\theta} \left\{ \theta\left(\mu - \pi - c\right) - \frac{\kappa a^2}{2} \right.$$
$$\left. + \Lambda\theta\left[V^D + (a+m)(\Delta - B)\right] + \dot\theta\left[V'(\theta) - P(\theta)\right] \right\}, \qquad (11)$$

subject to incentive constraints (8) and (9), i.e., $m = \frac{\Lambda B}{\phi}$ and $a = \frac{\theta\Lambda(\Delta - B)}{\kappa}$, and managerial wage (10), i.e., $c = \frac{\phi m^2}{2} - \Lambda(a+m)B$. The term $\dot\theta\left[V'(\theta) - P(\theta)\right]$ captures the gains associated

with (smooth) trading. For an interior solution $\dot{\theta} \in (-\infty, \infty)$, it must be that

$$P(\theta) = V'(\theta). \tag{12}$$

Indeed, the activist is willing to pay $V'(\theta)$ dollars for an additional unit of stock. The cost of purchasing such a unit equals the market price of stock $P(\theta)$, i.e., passive investors' valuation of the firm's stock. In equilibrium, the marginal benefit of buying equals the marginal cost. Importantly, the activist is indifferent between trading smoothly and not trading at all, and cannot capture any gains from trade. Therefore, the activist's value function $V(\theta)$ is determined "as if" it could not trade at all and hence coincides with its payoff $\hat{V}(\theta)$ that would prevail absent any trading opportunities.

Plugging (12) into (11), we can solve for $V(\theta) = \hat{V}(\theta)$ and get optimal efforts in closed form. By (12), we then also obtain $P(\theta)$ in closed-form (see (A.6)). The following proposition summarizes our findings, with additional details provided in the Online Appendix A.3.

Proposition 1. *Denote by $\tilde{\kappa} = \kappa/\theta$ the activist's cost of effort per unit of ownership. In the smooth trading region, the value function satisfies $V(\theta) = \hat{V}(\theta)$ with*

$$\hat{V}(\theta) = \theta \left(\frac{\mu - \pi + \Lambda V^D}{\rho + \Lambda} + \frac{\Delta^2 \Lambda^2 \left(\tilde{\kappa}^2 + \phi\tilde{\kappa} + \phi^2 \right)}{2\phi\tilde{\kappa} \left(\tilde{\kappa} + \phi \right) \left(\rho + \Lambda \right)} \right), \tag{13}$$

while the stock price satisfies $P(\theta) = \hat{V}'(\theta)$, and admits closed-form expression (A.6). Optimal efforts satisfy

$$m = m(\theta) = \frac{\tilde{\kappa}}{\phi} \left(\frac{\Lambda\Delta}{\phi + \tilde{\kappa}} \right) < m^{FB} \quad and \quad a = a(\theta) = \frac{\phi}{\tilde{\kappa}} \left(\frac{\Lambda\Delta}{\phi + \tilde{\kappa}} \right) < a^{FB}. \tag{14}$$

Under passive ownership, passive investors' value function satisfies

$$P^0 = \frac{\mu + \Lambda V^D}{\rho + \Lambda} + \frac{\Delta^2 \Lambda^2}{2\phi(\rho + \Lambda)}. \tag{15}$$

Note that κ and θ affect optimal efforts only via $\tilde{\kappa} = \kappa/\theta$. A lower ownership share θ effectively implies a lower benefit of effort for the activist, reducing effort a. Under passive ownership (i.e. as $\theta \to 0$), we have $\lim_{\theta \to 0} a(\theta) = 0$ and $\lim_{\theta \to 0} m(\theta) = \frac{\Lambda\Delta}{\phi}$.

To obtain the activist's valuation of an additional unit of stock, we can differentiate both

sides of (11) and use $P(\theta) = V'(\theta)$. This yields:

$$(\rho + \Lambda)P(\theta) = \mu - \pi - c + \Lambda\big[V^D + (a+m)(\Delta - B)\big] - \theta\left(\frac{\partial c}{\partial a}\frac{\partial a}{\partial \theta}\right). \tag{16}$$

The term $\left(\frac{\partial c}{\partial a}\frac{\partial a}{\partial \theta}\right)$ captures how an additional unit of firm ownership affects managerial flow payouts through effort incentives a. Crucially, $-\theta\left(\frac{\partial c}{\partial a}\frac{\partial a}{\partial \theta}\right) = \frac{\phi m(\Lambda\Delta - \phi m)}{\tilde{\kappa}} = \frac{\Lambda^2\Delta^2\phi}{(\phi+\tilde{\kappa})^2}$ is positive.[14] As θ increases, the activist exerts more effort and increases the chance of successful transition and of payment of the manager's compensation B. Thus, an increase in θ and a, all else equal, lowers the required flow payouts c, which are set such that the manager breaks even. Intuitively, a higher activist stake θ implies more efficient contracting with the manager, which increases the activist's valuation of an additional unit of stock $V'(\theta)$ in (16).

In addition to satisfying (16), $P(\theta)$ satisfies the pricing equation of passive investors

$$(\rho + \Lambda)P(\theta) = \mu - c + \Lambda\big[V^D + (a+m)(\Delta - B)\big] + P'(\theta)\dot{\theta}. \tag{17}$$

where $P(\theta)$ increases with θ, i.e., $P'(\theta) > 0$. Combining (16) and (17) yields

$$\dot{\theta} = \frac{1}{P'(\theta)}\left[-\pi - \theta\left(\frac{\partial c}{\partial a}\frac{\partial a}{\partial \theta}\right)\right]. \tag{18}$$

The trading rate in (18) is determined by two opposing effects and can be positive or negative. First, there are gains from buying a larger stake, as this makes contracting with the manager more efficient. Second, there are gains from selling shares, because the activist values the firm less due to the flow holding cost π. Expression (A.19) presents a closed-form expression for the trading rate. As we show, provided $\sqrt{\phi\pi} < \Delta\Lambda$, there exists an endogenous level[15]

$$\theta^C = \frac{\kappa\sqrt{\pi}}{\sqrt{\phi}(\Delta\Lambda - \sqrt{\phi\pi})}. \tag{19}$$

at which $\dot{\theta} = 0$ so that $\dot{\theta} < 0$ ($\dot{\theta} > 0$) whenever θ lies in the smooth trading region and $\theta < \theta^C$ ($\theta > \theta^C$). The trading rate $\dot{\theta}$ decreases with the activist's disutility from investing in a dirty firm, as captured by π, and is negative for larger values of π.

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[14]Combining (8) and (10) yields $c = -\frac{\phi m^2}{2} - \phi am$, so $\frac{\partial c}{\partial a} = -\phi m$. From (9), we obtain $\frac{\partial a}{\partial \theta} = \frac{\Lambda(\Delta - B)}{\kappa} = \frac{\Lambda(\Delta - \phi m/\Lambda)}{\kappa}$, and $\theta \cdot \frac{\partial a}{\partial \theta} = \frac{\Lambda(\Delta - \phi m/\Lambda)}{\tilde{\kappa}}$. Using (14), we get $-\theta\left(\frac{\partial c}{\partial a}\frac{\partial a}{\partial \theta}\right) = \left(\frac{\Lambda\Delta}{\phi+\tilde{\kappa}}\right)\left(\Lambda\Delta - \frac{\Lambda\tilde{\kappa}\Delta}{\phi+\tilde{\kappa}}\right) = \frac{\Lambda^2\Delta^2\phi}{(\phi+\tilde{\kappa})^2}$.

[15]When $\sqrt{\phi\pi} \geq \Delta\Lambda$, θ^C is not well-defined from (19), and $\dot{\theta} < 0$ in the entire smooth trading region. When $\theta^C \leq 0$ (respectively $\theta^C \geq 1$), $\dot{\theta} > 0$ (respectively $\dot{\theta} < 0$) in the entire smooth trading region.

To avoid studying degenerate cases, we assume in what follows that the smooth trading region is non-empty; Online Appendix F.3 characterizes the activist's optimal trading when the smooth trading region is empty. The smooth trading region $(\underline{\theta}, \overline{\theta})$ is characterized by two thresholds $\underline{\theta} < \overline{\theta}$, available in closed-form in Online Appendix A.3.2. To solve for $\underline{\theta}$ and $\overline{\theta}$, let P^y denote the price of the firm if the activist's ownership stake θ perpetually equals $y \in \{0, 1\}$. One can show that once θ reaches $y \in \{0, 1\}$ after time $t = 0$, the activist stops trading, so the price indeed equals P^y.[16]

Once θ reaches $\overline{\theta}$ (from below), the activist is indifferent and randomizes between not trading at all (i.e., $\dot{\theta} = 0$) and buying the entire firm, i.e., the remaining $1 - \overline{\theta}$ units of stock, at price P^1. The threshold $\overline{\theta}$ satisfies $\overline{\theta} = \inf\{\theta \in [0, 1] : \hat{V}(\theta) = V(1) - (1 - \overline{\theta})P^1\}$ and the trading intensity $\overline{\xi}$ at which the activist buys the entire firm is determined such that the price level equals $P(\overline{\theta}) = \hat{V}'(\overline{\theta})$. Then, θ remains constant at $\overline{\theta}$ until it jumps to one. For $\theta \in (\overline{\theta}, 1)$, the activist immediately buys the entire firm, i.e., $d\theta = 1 - \theta$.[17]

Once θ reaches $\underline{\theta}$ (from above), the activist is indifferent and randomizes between not trading at all and selling its entire stake at price P^0. Then, θ remains constant at $\underline{\theta}$ until it jumps to zero. The threshold $\underline{\theta}$ satisfies $\underline{\theta} = \sup\{\theta \in [0, 1] : \hat{V}(\theta) = \theta P^0\}$ and the rate $\underline{\xi}$ at which the activist exits is such that $P(\underline{\theta}) = \hat{V}'(\underline{\theta})$; Appendix A.3.5 presents closed-form expressions for $\underline{\xi}$ and $\overline{\xi}$. For $\theta \in (0, \underline{\theta})$, the activist immediately exits, i.e., $d\theta = -\theta$.

Consequently, starting from a stake $\theta_0 = \theta_{T^\beta}$, the activist's stake remains constant if $\theta_0 = \theta^C$, the activist eventually exits the firm in that $\lim_{t \to \infty} \theta_t = 0$ if $\theta_0 < \max\{\theta^C, \underline{\theta}\}$, and the activist eventually acquires the entire firm in that $\lim_{t \to \infty} \theta_t = 1$ if $\theta_0 > \min\{\theta^C, \overline{\theta}\}$. Proposition 2 summarizes our findings so far and the model solution after T^β; Online Appendix A.3 provides additional details for the proof.

Proposition 2 (Trading, Effort, and Firm Value). *After time T^β, the following holds:*

1. *The activist's value function satisfies $V(\theta) = \theta P^0$ for $\theta \in [0, \underline{\theta})$, $V(\theta) = \hat{V}(\theta)$ for $\theta \in [\underline{\theta}, \overline{\theta}]$, and $V(\theta) = V(1) - (1 - \theta)P^1$ for $\theta \in (\overline{\theta}, 1]$. Efforts are characterized in (14).*

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[16]Importantly, the state $\theta = 0$ is absorbing if reached after time $t = 0$, i.e., post-entry. At the onset at time $t = 0$, the activist starts out with zero ownership but may buy an initial ownership stake $\theta_0 > 0$, because the initial trade is at favorable terms and therefore fundamentally different from post-entry trading after time T^β. When the activist chooses its initial stake θ_0 at $t = 0$, it can acquire part of this initial stake at a discount. In contrast, after time T^β (i.e., post-entry), the activist always trades at the fair market price.

[17]This is reminiscent of the trading strategies of Michael Dell with Dell Inc or Elon Musk with Twitter, where both investors were holding a large stake in the target firms before buying the whole equity and taking the firms private to implement significant strategic changes.

2. *The stock price satisfies $P(\theta) = V'(\theta)$ in all states θ where $V(\theta)$ is differentiable. Thus, $P(\theta) = \hat{V}'(\theta)$ for $\theta \in [\underline{\theta}, \overline{\theta}]$, $P(\theta) = P^0$ for $\theta \in [0, \underline{\theta})$, and $P(\theta) = P^1$ for $\theta \in (\overline{\theta}, 1]$.*

3. *For $\theta \in (\underline{\theta}, \overline{\theta})$, the activist trades smoothly, in that $d\theta = \dot{\theta} dt$ with $\dot{\theta}$ from (18). At $\theta = \overline{\theta}$, when $\dot{\theta} > 0$ in a left-neighbourhood of $\overline{\theta}$, we have $d\theta_t = (1 - \overline{\theta}) d\overline{N}_t$, where $d\overline{N}_t \in \{0, 1\}$ is a jump process with intensity $\overline{\xi}$. At $\theta = \underline{\theta}$, when $\dot{\theta} < 0$ in a right-neighbourhood of $\underline{\theta}$, we have $d\theta_t = -\underline{\theta} d\underline{N}_t$, where $d\underline{N}_t \in \{0, 1\}$ is a jump process with intensity $\underline{\xi}$.[18] For $\theta \in [0, \underline{\theta})$, $d\theta = -\theta$. For $\theta \in (\overline{\theta}, 1]$, $d\theta = (1 - \theta)$. The activist stops trading once $\theta = 0$ or $\theta = 1$. Finally, there exists $\theta_{\min} > 0$ such that $\theta_t \notin (0, \theta_{\min})$ for all $t \geq 0$.*

2.3.2 Solution for $t \in (0, T^\beta)$

Before time T^β, the activist cannot trade and its value function is a weighted average of the payoff absent any trading opportunities $\hat{V}(\theta)$ and with trading opportunities $V(\theta)$, in that

$$V^\beta(\theta) = \frac{(\Lambda + \rho)\hat{V}(\theta) + \beta V(\theta)}{\Lambda + \rho + \beta}. \tag{20}$$

For $\theta \in [\underline{\theta}, \overline{\theta}]$, we have $\hat{V}(\theta) = V(\theta)$, so $V^\beta(\theta) = V(\theta)$. However, for $\theta < \underline{\theta}$ or $\theta > \underline{\theta}$, the activist would strictly prefer to trade if it could, so that $V(\theta) > V^\beta(\theta)$. Likewise, the stock price satisfies

$$P^\beta(\theta) = \frac{\mu - c + \Lambda\big[V^D + (a + m)(\Delta - B)\big] + \beta P(\theta)}{\Lambda + \rho + \beta}. \tag{21}$$

Importantly, efforts $a(\theta)$ and $m(\theta)$ only depend on θ and not on whether the activist can trade and are characterized in (14). We conclude with the following proposition.

Proposition 3. *Over $(0, T^\beta)$, the activist's value function satisfies (20), the stock price satisfies (21), and efforts are given in (14).*

2.3.3 Activist Entry and the Free-Rider Problem

The activist improves firm value through private and costly effort. However, if this value creation is fully reflected in the price at which it can acquire a stake, the activist cannot capture the gains from activism and thus has no incentive to acquire a stake in the first place. The severity of this free-rider problem is captured by the difference between the activist's

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[18]When $\dot{\theta} < 0$ ($\dot{\theta} > 0$) in a left-neighbourhood of $\overline{\theta}$ (right-neighbourhood of $\underline{\theta}$), no time is spent in state $\overline{\theta}$ ($\underline{\theta}$) and θ moves immediately back into the smooth trading region $(\underline{\theta}, \overline{\theta})$ from $\overline{\theta}$ ($\underline{\theta}$).

valuation of a unit of firm ownership, $V^\beta(\theta_0)/\theta_0$, and the passive investors' valuation of a unit of firm ownership, $P^\beta(\theta_0)$. To gain some intuition, suppose that the activist acquires the minimum required stake $\theta_0 = \tilde{\theta}_0$. In this case, entry is profitable (relative to not entering) if and only if $V^\beta(\theta_0) - K(\theta_0) \geq R$ which we can rewrite using (5) as

$$(1-\eta)\big[V^\beta(\theta_0)/\theta_0 - P^0\big] + \eta\big[V^\beta(\theta_0)/\theta_0 - P^\beta(\theta_0)\big] \geq R/\theta_0,$$

The term $V^\beta(\theta_0)/\theta_0 - P^0$ captures value creation from activism of which the activist can capture a fraction $1 - \eta$. The term $\eta\big[V^\beta(\theta_0)/\theta_0 - P^\beta(\theta_0)\big]$ quantifies the free rider problem, which gets more severe as η increases. Proposition 4 follows by direct calculation and characterizes activist entry; Online Appendix C provides additional details.

Proposition 4 (Activist Entry). *The activist enters if and only if (6) holds, i.e. $E(\theta_0) \geq R$, with an initial stake given by $\theta_0 = \arg\max_{\theta \in [\tilde{\theta}_0, 1]} \big[V^\beta(\theta) - K(\theta)\big]$. When the free-rider problem is sufficiently severe in that*

$$\eta \geq \frac{\kappa(\kappa + \phi)}{\phi^2 + 3\kappa\phi + \kappa}, \tag{22}$$

the activist buys the minimum stake, so that $\theta_0 = \tilde{\theta}_0$. For $\theta_0 \leq \bar{\theta}$, the entry condition can be written as

$$\underbrace{F}_{\substack{Financial \\ payoff}} + \underbrace{U}_{\substack{Non\text{-}pecuniary \\ payoff}} \geq 0. \tag{23}$$

1. *When $\theta_0 = \theta \in [\underline{\theta}, \bar{\theta}]$, then with $\tilde{R} = R/\theta$ and $\tilde{\eta} = \frac{\theta_0 - \tilde{\theta}_0 + \eta\tilde{\theta}_0}{\theta_0}$ we have that $F = \hat{F}(\theta; \beta) \equiv \chi\Delta^2$ for*

$$\chi := \frac{\phi\Lambda^2}{2(\rho + \Lambda)\tilde{\kappa}(\phi + \tilde{\kappa})^2}\left(\tilde{\kappa}(1 - \tilde{\eta}) + \phi(1 - 2\tilde{\eta}) - \frac{2\beta\tilde{\kappa}\tilde{\eta}}{\Lambda + \rho + \beta}\right), \tag{24}$$

and

$$U = \hat{U}(\theta; \beta) := -\tilde{R} - \frac{\pi}{\Lambda + \rho}\left(1 - \frac{\beta\tilde{\eta}}{\Lambda + \rho + \beta}\right). \tag{25}$$

2. *When $\theta_0 = \theta \in [0, \underline{\theta})$, then $F = \frac{(\Lambda + \rho)\hat{F}(\theta, 0)}{\Lambda + \rho + \beta}$ and $U = -\tilde{R} - \frac{\pi}{\Lambda + \rho + \beta}$.*

Condition (23) states that the activist enters at $t = 0$ as long as the sum of financial payoff F and the sustainability preferences-related payoff U is positive.[19] We refer to the

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[19]The entry condition for $\theta_0 > \bar{\theta}$ is in closed form too, but less intuitive and thus omitted.

sustainability preferences-related payoff component U as *non-pecuniary payoff*, because the activist's (non-pecuniary) sustainability preferences—as quantified by π and R—only affect the activist's payoff and incentives to enter via U.[20] The non-pecuniary payoff U consists of two components: (i) the effective net "utility" $-R$ from entering at $t = 0$, reflecting the activist's broad impact preferences, and (ii) the activist's disutility from owning the (dirty) firm over $[0, T]$, which may reflect sustainability preferences in a narrow sense ("value-alignment preferences"). Narrow sustainability preferences, as captured by higher π, discourage entry by decreasing U, but broad impact preferences as captured by lower R encourage entry by increasing U. We say that the activist follows a *broad impact mandate* when $U > 0$, in which case it may be willing to invest in the firm despite a financial loss. Online Appendix F.5 provides a micro-foundation for $R < 0$ and thus for $U > 0$.

3 Model Analysis

Three core economic mechanisms shape the dynamics of activism and green transition within our framework. First, in any state θ, incentive provision and optimal efforts are subject to a double-agency problem. Second, the activist may dynamically trade and adjust its stake after entry, affecting effort and the transition rate. Third, a free-rider problem determines the activist's incentives to invest in the firm. In the following, we discuss how these mechanisms interact and affect activism and the green transition. Unless otherwise mentioned, we consider that $\theta_0 \in (\underline{\theta}, \overline{\theta})$ with $0 < \theta_0 < 1$, leading to non-trivial trading dynamics after T^β.

3.1 Does Activism Foster the Green Transition?

While activism fosters the green transition under first best, it introduces a double-agency problem that distorts effort incentives under moral hazard. Notably, the activist's incentives to exert effort are reduced relative to the first-best case as the activist only captures part of the gains from effort while incurring the full private cost. As shown by (9), this is for two reasons. First, the activist only owns a fraction $\theta \le 1$ of the firm. Second, part of the transition surplus accrues to the manager as part of the incentive contract. As a result, effort incentives provided to the manager reduce the activist's effort. Due to the double agency problem, efforts a and m endogenously arise as substitutes, and lie below first-best efforts.

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[20]That is, π and R affect the activist's payoff only via U in the entry condition.

Increasing m requires a higher compensation B, thus lowering a, and vice versa.

Interestingly, this double-agency problem can be so severe that more activism, i.e., higher θ and lower $\tilde{\kappa}$, can reduce overall effort $a+m$. In particular, under passive ownership, that is, $\theta \to 0$ and $\lim_{\theta \to 0} \lambda = \lambda^P = \frac{\Lambda^2 \Delta}{\phi}$, the transition rate λ is larger than under active ownership with $\theta > 0$, when κ is large. Thus, activism increases the green transition rate if and only if the activist's stake θ is large and its cost of effort κ is small relative to that of the manager. Likewise, the transition rate increases in the extent of activism, in that $\frac{\partial \lambda}{\partial \theta} > 0$, if θ is large and κ is small relative to ϕ. A sufficient condition for $\frac{\partial \lambda}{\partial \theta} > 0$ is $\theta > \frac{\kappa}{\phi}$.

The following proposition formalizes these results.

Proposition 5 (Double-agency and effort choice). *The following holds:*

1. *The manager's effort m increases in κ and decreases in θ while the activist's effort a decreases in κ and increases in θ.*

2. *Activism increases the transition rate relative to passive ownership, i.e., $\lambda(\theta) > \lambda^P$, if and only if $\theta > \frac{\kappa}{\phi}$.*

3. *The transition rate $\lambda(\theta)$ increases in the stake of the activist if and only if $\theta > \frac{\kappa(\sqrt{2}-1)}{\phi}$.*

The following corollary generalizes our results by characterizing the effect of activism on the *average* transition rate $\overline{\lambda}_0$ which accounts for dynamics of activist stake and is defined via

$$\overline{\lambda}_t = \mathbb{E}_t^{T^\beta} \left[\int_t^\infty e^{-\Lambda(u-t)} \Lambda \lambda_u du \right], \tag{26}$$

where we integrated out the random time T and the expectation $\mathbb{E}_t^{T^\beta}$ is taken at time t with respect to the random time T^β. The details on how to compute $\overline{\lambda}_t$ are deferred to Online Appendix F.1. For $\beta = 0$, we have $\overline{\lambda}_0 = \lambda(\theta_0)$. Under passive ownership $\theta_t = 0$ for all $t \geq 0$ and we have $\overline{\lambda}_0 = \lambda^P = \frac{\Lambda^2 \Delta}{\phi}$.

Corollary 1 (Good Activism and Bad Activism). *Suppose that the activist enters at time $t = 0$. The following holds:*

1. *Activism improves the average transition rate (relative to passive ownership), in that $\overline{\lambda}_0 > \lim_{\tilde{\kappa} \to \infty} \overline{\lambda}_0 = \lambda^P$, if (i) $\kappa < \phi$ and (ii) $\theta_t > \frac{\kappa}{\phi}$ whenever $\theta_t > 0$.*

2. *Activism reduces the average transition rate, in that $\overline{\lambda}_0 < \lambda^P$, if $\phi \leq \kappa$.*

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The key takeaway is that skilled or good activists (characterized by low κ) stimulate green transition, whereas less skilled or bad activists (characterized by large κ) hamper green transition.

3.2 The Dynamics of Activism

After time T^β, the activist dynamically trades the firm's stock, so that θ, and efforts a and m vary over time. Crucially, the activist may buy or sell the firm's stock and the size of the activist's initial stake θ_0 relative to the thresholds θ^C from (19) and $\underline{\theta}$ from (A.10) determines whether the activist accumulates a larger stake or gradually exits after entry. In particular, the activist divests its stake after entry and eventually exits the firm if $\theta_0 < \max\{\theta^C, \underline{\theta}\}$. By contrast, it accumulates a larger stake over time and eventually acquires the entire firm if $\theta_0 > \min\{\theta^C, \underline{\theta}\}$. As θ^C and $\underline{\theta}$ increase with κ and π, the activist exhibits a higher propensity to exit when its cost of effort κ or its disutility flow π are large, whereas it accumulates a larger stake when κ and π are small.

As a result, when κ is low, the transition rate λ increases in θ (Proposition 5) and the activist accumulates a larger stake after entry, causing its effort and the overall transition rate to increase over time. When κ is high, the activist divests its stake over time and exits, while activism is detrimental to green transition. In this case, the transition rate tends to decrease with θ and increases over time, as the activist gradually divests.

Importantly, κ also affects the activist's entry decision. An increase in κ implies lower activist effort, which alleviates the free-rider problem. However, higher κ also limits value creation from activism. For instance, the entry condition (23) implies that when β is sufficiently small and U is either positive or not too negative, an increase in κ encourages entry.[21] Under these circumstances, less skilled activists exhibit both a higher propensity to enter and to exit. Skilled activists, on the other hand, are less willing to enter but, conditional on entry, stay invested longer in the firm, take more control, and exert more effort for impact.

Why does an activist enter in the first place, if it finds it optimal to exit after entry? The main reason is that at the time of entry, the activist can acquire an ownership stake at favorable terms, specifically at a price lower than the firm's stock price post-entry. Therefore,

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[21]When $\theta_0 = \tilde{\theta}_0 \in (\underline{\theta}, \overline{\theta})$ (so $\eta = \tilde{\eta}$) and $\hat{\chi} := \tilde{\kappa}_0(1 - \eta) + \phi(1 - 2\eta) - \frac{2\beta\tilde{\kappa}_0\eta}{\Lambda + \rho + \beta} > 0$, the activist enters if $U \geq 0$ or $U < 0$ is sufficiently close to zero. The claims follows from $\frac{\partial \hat{\chi}}{\partial \tilde{\kappa}_0} > 0$ if and only if $\eta < \frac{\Lambda + \rho + \beta}{\Lambda + \rho + 3\beta}$, or equivalently $\beta(3\eta - 1) < (\Lambda + \rho)(1 - \eta)$. The claim can be generalized for case $\theta_0 \neq \tilde{\theta}_0$.

it can be optimal for the activist to initially enter the firm by buying the firm's stock (at a relatively low price) and to gradually exit the firm by selling its stock right after entry (at a relatively high price), effectively "buying low and selling high."

3.3 The Effects of Dynamic Trading on Green Transition

Dynamic trading not only changes optimal efforts but also shapes the activist's entry decision by affecting the post-entry stock price $P^\beta(\theta_0)$, as the following corollary shows.

Corollary 2. *Suppose $\theta_0 \in (\underline{\theta}, \overline{\theta})$. An increase in β tightens (loosens) the entry condition (23) if $\theta_0 > \theta^C$ (if $\theta_0 < \theta^C$), in that $\mathrm{sgn}\left(\frac{\partial E(\theta_0)}{\partial \beta}\right) = \mathrm{sgn}(\theta^C - \theta_0)$.*

When $\theta_0 > \theta^C$, the activist gradually accumulates a larger stake after T^β, which implies that the stock price appreciates over time. This leads to a higher initial (post-entry) price and to a more severe free-rider problem that hampers activist entry (by tightening (23)). As a result, when $\theta_0 > \theta^C$, more frequent trading discourages activist entry. When $\theta_0 < \theta^C$, the activist gradually divests after T^β, causing the stock price to decrease over time. This implies a lower post-entry price and alleviates the free-rider problem at entry. Overall, entry and exit are complementary. By allowing the activist to divest and exit the firm, trading opportunities provide incentives to the activist to enter in the first place.

Better trading opportunities, i.e., an increase in β, thus curb the entry incentives of skilled activists characterized by low κ and θ^C, while stimulating entry by less skilled activists characterized by larger κ and θ^C. According to our previous findings, less skilled activists also exhibit higher propensity to exit, whereas relatively more skilled activists acquire a larger stake and more control over time. Taken together, these findings suggest that activists in (low β) private markets tend to (i) exert more effort, (ii) hold larger stakes, (iii) have more skill, and (iv) are more likely to affect firm outcomes.

Figure 2 shows that depending on whether κ is large relative to ϕ, better trading opportunities may increase or decrease the extent of activism and the rate of transition. In the upper two panels, activism is beneficial for green transition, i.e., $\overline{\lambda}_0 > \lambda^P$ and κ is small relative to ϕ. Better trading opportunities then allow the activist to accumulate a larger stake after entry, which boosts transition, but also reduce the activist's incentives to enter. In particular, $F + U$ decreases with β (see Panel A) and the activist does not enter for $\beta > \beta^*$, leading to a drop in the transition rate at β^*. In the lower two panels activism
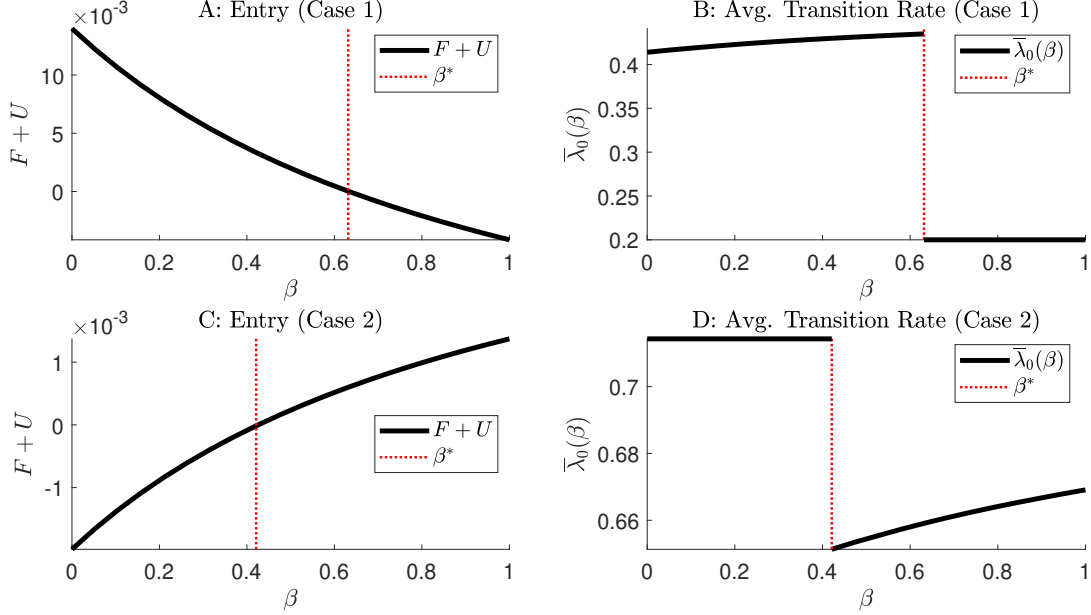
Figure 2: **Comparative statics with respect to β.** The relevant parameters are $\Delta = 1$, $\Lambda = 1$, $\kappa = 1$, $\tilde{\beta} = 1$, $\rho = 0.05$, and $\tilde{\theta}_0 = 0.5$. The upper two panels A and B use $\phi = 5$, $\eta = 0.75$, $\pi = 0.05$, and $U = 0.08$. The lower two panels C and D use $\phi = 1.4$, $\eta = 0.25$, $\pi = 0.15$, and $U = -0.048$.

hampers transition, i.e., $\overline{\lambda}_0 < \lambda^P$. Here, κ is large relative to ϕ and the activist gradually exits after entry. Better trading opportunities speed up exit, which raises transition rate but also encourages entry, in that the activist enters when $\beta > \beta^*$ (Panel C). Raising β beyond β^* then reduces transition rate, as it incentivizes the high-κ activist to enter (Panel D).

4 Carbon Taxation and the Green Transition

4.1 Carbon Taxation and Incentives

On an intuitive level, the surplus Δ from a successful transition should reflect both (i) the green preferences of investors and consumers and (ii) the effects of carbon taxation or cap and trade schemes. To see this, suppose that the firm produces gross cash flows at rate μ. However, unlike a clean firm, a polluting firm incurs additional costs of production via carbon taxes or emission certificates of $\rho\tau^C$ per unit of time, where τ^C is the regulator's choice and can be seen as the (scaled) tax rate. Then, the net cash flow of the firm is $\mu - \rho\tau^C$ when it operates a polluting technology. Cash flows equal dividends in our setting, and all investors have discount rate ρ. Thus, after time T, the (purely financial) market values of clean and

dirty firms are respectively $V^C = \frac{\mu}{\rho}$ and $V^D = \frac{\mu - \rho\tau^C - \rho\pi^C}{\rho} = V^C - \tau^C - \pi^C$,[22] where $\pi^C\rho \geq 0$ is the disutility flow that equity investors derive from holding the stock of a dirty firm. As a result, we can write $\Delta = \tau^C + \pi^C$, which is the sum of carbon tax and a greenium π^C. Likewise, a preference for clean products by consumers would raise V^C and Δ. In sum, we can interpret carbon taxes, carbon price (in a cap and trade scheme), and preferences for green firms as increasing Δ. In particular, when setting the tax rate τ^C, the regulator pins down Δ, in that Δ is a monotonic transformation of the tax rate τ^C.

Raising the carbon tax and thus Δ affects (i) effort incentives, (ii) dynamic trading, and (iii) entry, and as a result the average transition rate $\overline{\lambda}_0 = \overline{\lambda}_0(\Delta)$. First, for any level of the activist stake θ, higher Δ increases the incentives to transition to a clean technology, raising both the activist's and manager's efforts and transition rate; see (14). Second, as the following Corollary shows, higher carbon taxes and Δ increase the activist's trading rate and reduce its incentives to exit after entry, thereby increasing the extent of activism.

Corollary 3. *When $\pi > 0$, $\frac{\partial\dot{\theta}}{\partial\Delta} > 0$ (in the smooth trading region), $\frac{\partial\theta^C}{\partial\Delta} < 0$, and $\frac{\partial\underline{\theta}}{\partial\Delta} < 0$ (provided that $\underline{\theta} \in (0,1)$).*

Third, to understand the effect of Δ on activist entry, we write $F = \chi\Delta^2$ in (23) where χ—defined in equation (24)—decreases with η and captures the severity of the free-rider problem.[23] Higher Δ implies higher effort by the activist, raising both the value created from activism and the severity of the free rider problem. When $\chi > 0$ and activism generates a financial gain, an increase in Δ raises the comparative advantage of the activist and the value created through activism, thus stimulating entry. On the other hand, when $\chi < 0$, an increase in Δ raises the free-rider problem more than the value created through activism, thereby discouraging activist entry. In this case, regulation aiming at stimulating green transition has to resolve the trade-off between addressing the double agency problem by raising taxes and mitigating the free rider problem by lowering taxes.

Because an increase in Δ increases the transition rate $\lambda(\theta)$ and the trading rate for any given stake θ, we expect the average transition rate $\overline{\lambda}_0 = \overline{\lambda}_0(\Delta)$ to increase with Δ (a sufficient condition is that κ is sufficiently low).[24] Likewise, under passive ownership, the

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[22]π^C is the disutility of equity investors in general (not necessarily the activist from holding dirty stocks), whereas π is the difference in activist's and passive investors' disutility from holding dirty stocks. In that regard, the activist's disutility is $\pi^C + \pi$, while other equity investors' disutility is π^C.

[23]One can use the entry condition (23) evaluated at $\theta = \theta_0$ to get a closed-form expression for $\chi = \chi(\theta)$,

[24]A sufficient condition for $\overline{\lambda}_0'(\Delta) > 0$ is that $\theta_0 \geq \theta^C \geq \underline{\theta}$ and $\theta_0 > \frac{\kappa(\sqrt{2}-1)}{\phi}$. Then, the activist gradually

average transition rate $\lambda^P = \frac{\Lambda^2 \Delta}{\phi}$ increases with Δ.

However, an increase in Δ can actually decrease the average transition rate when it affects the activist's entry decision. For this sake, note that the activist never enters if both χ and U are (strictly) negative and always enters when $\chi, U \geq 0$. The following Proposition shows that carbon taxation can reduce the green transition rate by discouraging entry by good (low κ) activists or encouraging entry by bad (high κ) activists.

Proposition 6 (Carbon taxes). *Suppose $\theta_0 = \tilde{\theta}_0 \in (\underline{\theta}, \overline{\theta})$. For $\chi < 0 \leq U$ or $U \leq 0 < \chi$, the entry condition* (23) *binds for $\Delta = \Delta^E := \sqrt{\frac{-U}{\chi}} \geq 0$. Suppose $\Delta^E \in (0, \overline{\Delta})$. The average transition rate $\overline{\lambda}_0 = \overline{\lambda}_0(\Delta)$ exhibits a downward jump at $\Delta = \Delta^E$, in that $\lim_{\Delta \uparrow \Delta^E} \overline{\lambda}_0(\Delta) > \lim_{\Delta \downarrow \Delta^E} \overline{\lambda}_0(\Delta)$, in the following two scenarios.*

1. *Carbon taxation hampers activist entry, in that $\chi < 0 < U$, and activism benefits green transition, in that $\lim_{\Delta \uparrow \Delta^E} \overline{\lambda}_0(\Delta) > \lambda^P$.*

2. *Carbon taxation encourages activist entry, in that $\chi > 0 > U$, and activism hampers green transition, in that $\lim_{\Delta \uparrow \Delta^E} \overline{\lambda}_0(\Delta) < \lambda^P$.*

Provided that $\overline{\lambda}_0(\Delta)$ increases with Δ for $\Delta \neq \Delta^E$, cases 1. and 2. from Proposition 6 describe the only circumstances in which an increase in Δ reduces green transition rate $\overline{\lambda}_0(\Delta)$. That is, an increase in carbon tax generally boosts green transition, *unless* it affects activist entry and discourages (encourages) good (bad) activists to enter. Sufficient conditions for $\overline{\lambda}_0(\Delta) \gtrless \lambda^P$ are provided in Corollary 1. Also note that in the limit case $\beta = 0$, the above statements simplify, as $\overline{\lambda}_0(\Delta) > \lambda^P$ if and only if $\tilde{\kappa} < \phi$. Importantly, the sign of χ is related to whether κ is large relative to ϕ, i.e., whether activism fosters green transition. To see this, note that when $\eta > 1/2$ and $\beta(3\eta - 1) < (\Lambda + \rho)(1 - \eta)$, then χ is negative for low κ values and positive for high κ values.[25]

Figure 3 provides a graphical illustration of the findings of Proposition 6, with Panels A and B depicting case 1 and Panels C and D depicting case 2. In case 1, activism is beneficial for green transition (i.e., κ is low relative to ϕ) and an increase in Δ reduces entry incentives

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acquires a larger stake in the firm after entry and a higher stake increases $\lambda(\theta)$ (see Proposition 5). These conditions are met when κ is sufficiently small and the activist is "good."

[25]When $\theta_0 = \tilde{\theta}_0 \in (\underline{\theta}, \overline{\theta})$ (so $\eta = \tilde{\eta}$), the sign of χ is determined by $\hat{\chi} := \tilde{\kappa}_0(1 - \eta) + \phi(1 - 2\eta) - \frac{2\beta\tilde{\kappa}_0\eta}{\Lambda + \rho + \beta}$. We have $\frac{\partial \hat{\chi}}{\partial \tilde{\kappa}_0} > 0$ if and only if $\eta < \frac{\Lambda + \rho + \beta}{\Lambda + \rho + 3\beta}$, or equivalently $\beta(3\eta - 1) < (\Lambda + \rho)(1 - \eta)$. As $\eta > 1/2$, we have $\hat{\chi} < 0$ if $\tilde{\kappa}_0 \propto \kappa = 0$. As $\hat{\chi}$ increases with $\tilde{\kappa}_0$ and thus κ, it follows that $\hat{\chi} < 0$ if κ is sufficiently small, and $\hat{\chi} > 0$ if κ is sufficiently large.
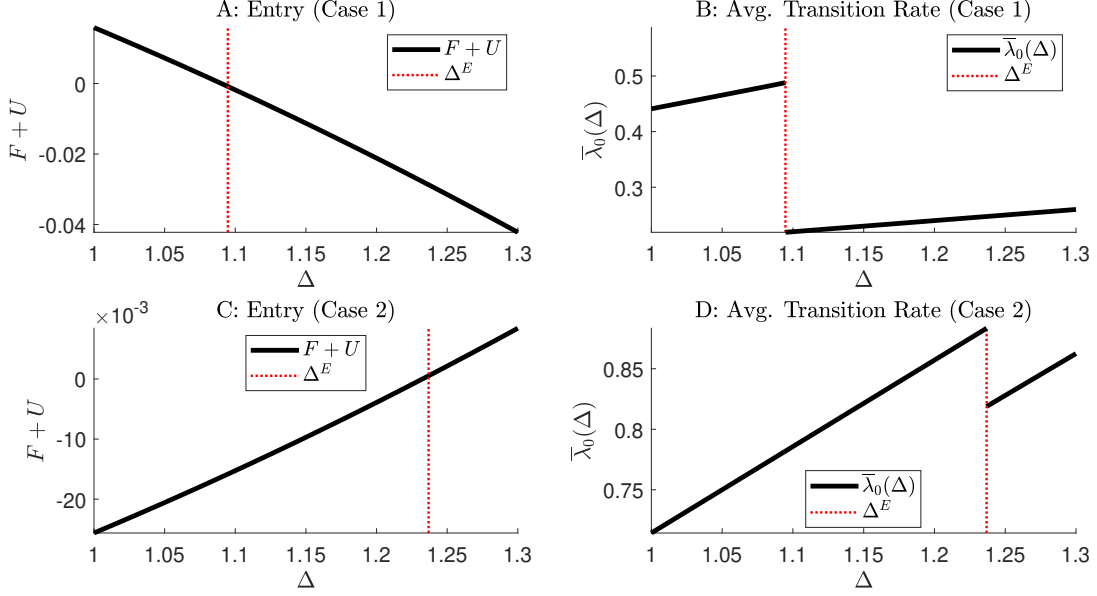
Figure 3: **Carbon taxes and clean transformation.** Parameters are set to $\Lambda = 1$, $\kappa = 1$, $\beta = 1$, $\rho = 0.05$, $\pi = 0.05$, and $\tilde{\theta}_0 = 0.5$. Panels A and B illustrate case 1 of Proposition 6 with $\phi = 5$, $\eta = 0.75$, and $U = 0.1$. Panels C and D illustrate case 2 of Proposition 6 with $\phi = 1.4$, $\eta = 0.25$ and $U = -0.075$.

since $\chi < 0$. Thus, raising Δ beyond Δ^E precludes activist entry, causing a downward jump in the transition rate at Δ^E (see Panel B). In case 2 and Panel C, activism is detrimental to green transition (i.e., κ is large relative to ϕ) and an increase in Δ encourages entry, i.e., $\chi > 0$. As shown in Panel D, raising Δ beyond Δ^E induces activist entry and, because activism reduces transition rate relative to passive ownership, leads to a downward jump of transition rate at Δ^E.

4.2 Optimal Taxation and Pigouvian Tax

To study optimal taxation, we add more structure to the baseline model by assuming that a dirty firm generates a flow social cost of $\pi^S \rho > 0$ (e.g., via its production), while the social cost of a clean firm is normalized to zero. Unlike the manager and investors, the regulator/policymaker/government internalizes this social cost and sets taxes (i.e., determines Δ) to minimize the sum of social cost and the cost of transformation given by

$$\mathcal{G} := \mathbb{E} \left[\int_0^\infty e^{-\rho t} \left(\pi^S \rho \mathcal{I}_t + \frac{\kappa a_t^2 + \phi m_t^2}{2} \right) dt \right]. \tag{27}$$

In (27), $\mathcal{I}_t$ is an indicator that equals one if the firm is dirty and zero if the firm is clean at time t. For symmetry, the regulator discounts at rate ρ. Online Appendix F.2 shows how to compute $\mathcal{G}$. Recall that, as argued in Section 4.1, choosing Δ is akin to setting carbon tax rate.[26] We now solve for the optimal level of $\Delta \in [0, \overline{\Delta}]$ which minimizes $\mathcal{G}$, i.e., $\Delta^G := \min_{\Delta \in [0,\overline{\Delta}]} \mathcal{G}$. We assume that parameters are such that Δ^G is interior, i.e., $\Delta^G \in (0, \overline{\Delta})$. With some slight abuse of notation, when the activist's entry condition (23) binds and the activist is indifferent between entering and not entering, we break ties in favor of and pick the outcome (i.e., activist entry or not) that leads to lower $\mathcal{G}$ (given Δ).[27]

The following proposition analyzes the "Pigouvian" benchmark under first best.

Proposition 7. *In first-best with optimal efforts $a^{FB} = \frac{\Lambda \Delta}{\kappa}$ and $m^{FB} = \frac{\Lambda \Delta}{\phi}$, $\Delta^G = \pi^S$.*

From time T onward, the firm's (discounted) cumulative social cost equals π^S if the firm remains dirty and 0 if the firm becomes green ("success") at time T. Thus, the reduction in total social cost and gain in social surplus is π^S if transition succeeds at time T relative to failure. Proposition 7 establishes that in first-best, investors should fully internalize the reduction in social cost associated with successful transition, leading to $\Delta^G = \Delta = \pi^S$. Setting $\Delta^G = \pi^S$ is akin to setting a Pigouvian tax that makes investors fully internalize the firm's social cost and, as such, the social surplus gain associated with a successful transition.

We now analyze optimal taxation with double agency and endogenous activism. For this sake, we focus on the most interesting and practically relevant case, namely, that activism is sufficiently beneficial in that κ is low. As such, we consider that the regulator minimizes $\mathcal{G}$ while incentivizing the activist to enter, in that it solves $\Delta^G := \min_{\Delta \in [0,\overline{\Delta}]} \mathcal{G}$ subject to (6). The following proposition shows that when the activist's (intrinsic) entry incentives are strong and (6) does not bind, for instance, because the activist has a broad impact mandate (i.e., internalizes part of the social cost as captured in reduced form by large U), then the tax rate lies *above* the Pigouvian tax (for $\beta = 0$).

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[26]According to Section 4.1, one could micro-found $\Delta = \tau^C + \pi^C$, where $\rho \tau^C$ is the carbon tax (in dollars) and π^C reflects investors' preferences ("greenium"). Thus, Δ then pins down the level of the carbon tax, in that Δ is a monotonic transformation of the carbon tax (and vice versa).

[27]This assumption is needed to ensure the existence of a solution to the minimization problem. To see why, take the scenario of Figure 3 Panel D and suppose we are looking for the level of Δ that maximizes $\overline{\lambda}_0(\Delta)$. In Panel D, the average transition rate has no maximum when the activist enters when $\Delta \geq \Delta^E$. On the other hand, the average transition rate is maximized for $\Delta = \Delta^E$ if we assumed that the activist enters only if $\Delta > \Delta^E$. This argument extends to the minimization of $\mathcal{G}$: To ensure that this minimization has a solution, we "pick" the equilibrium ownership (passive vs. active) yielding lower $\mathcal{G}$ when $\Delta = \Delta^E$.
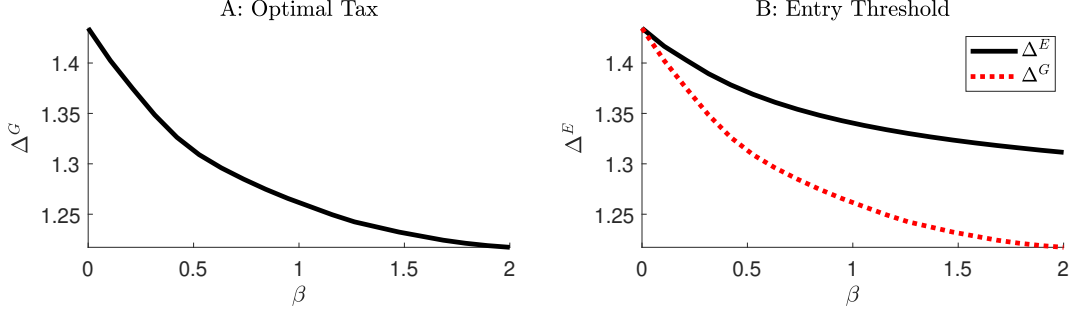
Figure 4: **Optimal Tax and Trading.** Parameters are set to $\Lambda = 1$, $\kappa = 1$, $\beta = 1$, $\rho = 0.05$, $\pi = 0.05$, $\tilde{\theta} = 0.5$, $\phi = 5$, $\eta = 0.75$, $U = 0.1$, and $\pi^S = 0.5$. Panel A plots the optimal tax Δ^G that minimizes $\mathcal{G}$ and Panel B plots the threshold Δ^E above which the activist does not enter. As argued in Section 4.1, Δ is a montonic transformation of the carbon tax, so increasing Δ is akin to raising carbon tax. The parameter choice follows Figure 3; the qualitative patterns are robust to the choice of these parameters.

Proposition 8 (Incentivizing effort through carbon taxation). *Suppose $\theta_0 = \theta = \tilde{\theta}$ and $\beta = 0$. Then $\Delta^G = \Delta_*^G := \pi^S \left(\frac{(\phi^2 + \tilde{\kappa}^2)(\phi + \tilde{\kappa})}{\theta\phi^3 + \tilde{\kappa}^3} \right) > \pi^S$ when* (6) *holds for $\Delta = \Delta_*^G$.*

By continuity, the proposition's statements and the ones of the proposition below also apply when β is sufficiently small. Thus, when taxation is not constrained by the activist's entry condition and $\beta \approx 0$, taxation solely needs to deal with the double agency problem. As the double agency problem leads to efforts that are lower than first best efforts, the regulator raises the tax rate above the Pigouvian benchmark to increase efforts.

Next, recall that while an increase in carbon taxes raises the transition rate conditional on activist entry, it also discourages activist entry. In particular, Proposition 6 shows that when activism is associated with financial losses (i.e., $\chi < 0 \leq U$), the activist enters only if $\Delta \leq \Delta^E = \sqrt{\frac{U}{-\chi}}$. That is, to incentivize beneficial activism, the carbon tax must not be too high. The following proposition demonstrates that under these circumstances, the optimal tax lies below the Pigouvian tax when $\Delta^E < \pi^S$, i.e., for low U or large $-\chi$.

Proposition 9 (Endogenous entry and optimal carbon taxation). *Suppose $\beta = 0$ and $\theta_0 = \tilde{\theta}_0 \in (\underline{\theta}, \overline{\theta})$. When $\chi < 0 \leq U$, $\Delta^G = \min\{\Delta^E, \Delta_*^G\}$.*

Next, we show that when activism is beneficial for the green transition (i.e., κ is low) but subject to financial losses (i.e., $\chi < 0 \leq U$), the optimal carbon tax should decrease with the ease of trading (β). When κ is low, the activist gradually acquires a larger stake after entry. Conditional on entry, more frequent trading opportunities increase the activist's stake dynamically, alleviating the double agency problem. As the regulator raises the tax above

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the Pigouvian benchmark to address double agency, more frequent trading opportunities and thus less severe double agency reduce the optimal tax. In addition, because activism is subject to a financial loss, an increase in Δ discourages entry and the activist enters if and only if $\Delta \leq \Delta^E$. Because Δ^E decreases with β, more frequent trading opportunities discourage entry too. As a result, better trading opportunities may require lower taxes to incentivize activist entry in the first place. The following proposition formalizes these insights for $\pi = 0$, bearing in mind that the results also apply when π is not too large.

Proposition 10. *Suppose that $\pi = 0$, that $\Lambda > 0$ is sufficiently small, and $\chi < 0 \leq U$. When $\theta_0 = \tilde{\theta}_0 \in (\underline{\theta}, \overline{\theta})$ and $\theta_0 > \kappa/\phi$, then Δ^G and $\Delta^E = \sqrt{\frac{U}{-\chi}}$ decrease with β.*

Figure 4 graphically illustrates that the optimal tax rate Δ^G and the threshold Δ^E decrease with the frequency of trading opportunities β, when activism is beneficial (i.e., κ is low relative to ϕ) but associated with financial losses (i.e., $\chi < 0 < U$). In the numerical example of Figure 4, we have $\Delta^G > \pi^S$, i.e., the optimal carbon tax is above the Pigouvian tax and the constraint (6) only binds for $\beta \approx 0$. Thus, for larger values of β, the regulator can set the tax without having to consider the activist's entry incentives.

5 The Pitfalls of Green Investment Subsidies

Policymakers often subsidize green capital investment; for instance, a firm may receive direct subsidies or a tax advantage for investing in transformation. An illustrative instance of this is the Investment Tax Credit (ITC), which is offered under the Inflation Reduction Act to encourage green investments in the United States.[28] As utility is in monetary terms and there are no capital constraints, we can, without loss of generality, interpret the managerial costs of effort as monetary investment costs at the firm level, with effort representing investment. A firm-level subsidy s will be based on the anticipated (or contracted) effort $\hat{m}_t$, which may differ from actual effort m_t upon deviation. In optimum, we have $m_t = \hat{m}_t$ and a subsidy implies a transfer to the firm proportional to the cost of effort, in that the firm-level subsidy raises firm cash flows by $\frac{s\phi\hat{m}_t^2}{2}$. That is, a fraction $s \in [0, \overline{s}]$ of the investment costs are subsidized; for simplicity, the fraction s is constant over time. To ensure optimal interior effort, we stipulate $\overline{s} < \min\{1, \frac{\phi}{\kappa}\}$.

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[28]Likewise, in the European Union and, notably Germany, firms may receive tax credits or subsidies for transforming their production toward sustainability, for instance, by reducing carbon emissions.

With the investment subsidy, the activist's value function becomes

$$V_t = \max_{(a_u, c_u, B_u, d\theta_u)_{u \geq t}} \mathbb{E}_t \left[\int_t^T e^{-\rho(u-t)} \left(\theta_u \left(\mu - \pi + \frac{\phi s \hat{m}_u^2}{2} - c_u \right) du - \frac{\kappa a_u^2}{2} du \right. \right.$$
$$\left. \left. - d\theta_u (P_u + dP_u) \right) + \theta_T \big[V^D + (a_T + m_T)(\Delta - B_T) \big] \right], \quad (28)$$

subject to $w_u \geq 0$ for all $u \geq t$ (the manager's payoff in the contract must be positive). The firm's stock price under active ownership becomes

$$P_t = \mathbb{E}_t \left[\int_t^T e^{-\rho(u-t)} \left(\mu + \frac{\phi s \hat{m}_u^2}{2} - c_u \right) du + e^{-\rho T} \big[V^D + (a_T + m_T)(\Delta - B_T) \big] \right]. \quad (29)$$

The value function/firm value under passive ownership then satisfies $P_t^0 = \lim_{\tilde{\kappa} \to \infty, \pi \to 0} V_t / \theta_t$. All other model elements, optimization problems, and notation remain unchanged relative to the baseline, and thus are not repeated for the sake of brevity. Online Appendix E presents the solution to this model variant.

A subsidy based on anticipated effort $\hat{m}_t$ does not change the incentive constraints (8) and (9). That is, the activist and the manager take the subsidy as given when choosing actual efforts a_t and m_t. The incentive constraints (8) and (9) then ensure $\hat{a}_t = a_t$ and $\hat{m}_t = m_t$. Contracted payouts to the manager take the form given in (10). However, the subsidies affect optimal contracting with the manager as well as the activist's dynamic trading and entry. The solution is similar to the baseline solution (modulo a redefinition of the value function and stock price to account for subsidies) and is presented in Online Appendix E. The following proposition summarizes the outcomes relevant to the analysis.

Proposition 11. *With investment subsidies $s \in [0, \overline{s}]$ for $\overline{s} < \min\{1, \frac{\phi}{\kappa}\}$, optimal effort is*

$$a = a(\theta) = \frac{\Delta \Lambda (\phi - \tilde{\kappa} s)}{\tilde{\kappa} (\tilde{\kappa}(1 - s) + \phi)} \quad and \quad m = m(\theta) = \frac{\Delta \Lambda \tilde{\kappa}}{\phi(\tilde{\kappa}(1 - s) + \phi)}. \quad (30)$$

The transition rate $\lambda = \lambda(\theta) = \Lambda(a + m)$ satisfies $\mathrm{sgn}\left(\frac{\partial \lambda}{\partial s}\right) = \mathrm{sgn}(\tilde{\kappa} - \phi)$. Activism increases λ, i.e., $\lambda(\theta) > \lambda^P = \lim_{\theta \to 0} \lambda(\theta)$, if and only if $\phi(1 - s) > \tilde{\kappa}(1 + s - s^2)$, and otherwise reduces λ. The activist enters at $t = 0$ if and only (6) holds. In case, $\theta_0 = \theta \leq \overline{\theta}$, the entry condition simplifies to $F + U \geq 0$ where

1. When $\theta_0 = \theta \in [\underline{\theta}, \overline{\theta}]$, then with $\tilde{R} = R/\theta$ and $\tilde{\eta} = \frac{\theta_0 - \tilde{\theta}_0 + \eta\tilde{\theta}_0}{\theta_0}$, we have $F = \hat{F}(\tilde{\theta}; \beta)$ for

$$\hat{F}(\tilde{\theta}; \beta) := \frac{\Delta^2 \Lambda^2}{2(\rho + \Lambda)\tilde{\kappa}\left[\phi + \tilde{\kappa}(1-s)\right]^2} \Big[\phi\tilde{\kappa}\left(1 + 2(s-2)s - \left(4s^2 - 6s + 1\right)\tilde{\eta}\right)$$
$$+ (s-1)s\tilde{\kappa}^2\left(2(s-1)\tilde{\eta} - s + 2\right) + (s-1)\phi^2\left(2\tilde{\eta} - 1\right) - T(\tilde{\theta}; \beta)\Big];$$
$$T(\theta; \beta) := \frac{2\beta\tilde{\eta}\tilde{\kappa}\left(\phi - s\tilde{\kappa}\right)(1-s)}{\beta + \Lambda + \rho}; \quad U = \hat{U}(\tilde{\theta}_0; \beta) := -\tilde{R} - \frac{\pi}{\Lambda + \rho}\left(1 - \frac{\beta\tilde{\eta}}{\Lambda + \rho + \beta}\right).$$

2. When $\theta_0 = \theta \in (0, \underline{\theta})$, then $F = \frac{(\Lambda + \rho)\hat{F}(\tilde{\theta}, 0)}{\Lambda + \rho + \beta}$ and $U = -\tilde{R} - \frac{\pi}{\Lambda + \rho + \beta}$.

Intuitively, a firm-level subsidy makes it optimal to increase the manager's effort m, which requires a higher payment B_t for the manager to provide the necessary incentives. This, in turn, decreases the activist's effort incentives a. Hence, a firm-level subsidy raises firm-level investment but crowds out efforts by the activist, potentially reducing the overall transition rate. Indeed, given an activist stake θ, an increase in the firm-level subsidy s actually *decreases* transition rate λ, i.e., $\frac{\partial\lambda}{\partial s} < 0$, if $\tilde{\kappa} < \phi$, that is, if the activist's cost of effort is low or its stake is large. Otherwise, when κ is large relative to ϕ or θ is small, the firm-level subsidy stimulates green transition and specifically does so under passive ownership (i.e., $\theta \to 0, \tilde{\kappa} \to \infty$). In addition, when $\tilde{\kappa}$ is large relative to ϕ so that $\phi(1-s) < \tilde{\kappa})(1 + s - s^2)$, activism hampers the green transition rate.

We next characterize the level of green investment subsidy $s^* = \arg\max_{s \in [0, \overline{s}]} \overline{\lambda}_0$ that maximizes the rate of green transition. Proposition 11 implies that when $\tilde{\kappa}$ is low, higher s reduces the transition rate under active ownership. Conditional on activist entry, a firm-level subsidy $s > 0$ is then detrimental to the transition rate, so $s^* = 0$. On the other hand, when $\tilde{\kappa}$ is large, raising s improves the transition rate under active (and passive) ownership. In this case, investment subsidies can foster transition and $s^* > 0$. In a nutshell, firm-level subsidies hamper green transition, precisely when activism is valuable for green transition while fostering transition otherwise. The following corollary formalizes these insights for $\beta = 0$. By continuity, its statements apply as long as β is not too large.

Corollary 4. *Suppose that $\beta = 0$ and $\theta_0 = \tilde{\theta}_0 = \theta$. Then, the following holds:*

1. *When $\tilde{\kappa} < \phi$, $F + U \geq 0$ for $s = 0$, and $\overline{s}$ is sufficiently small, then it is optimal not to subsidize green investment, in that $s^* = 0$.*
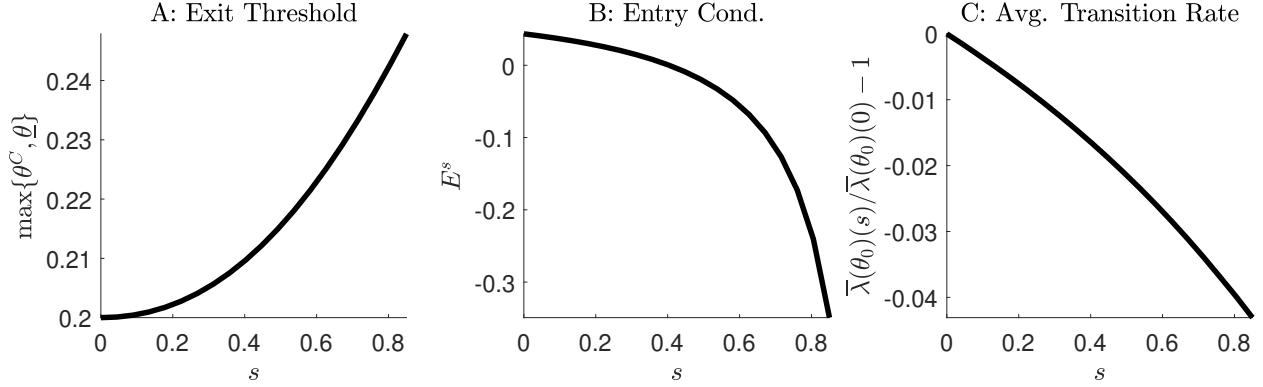
Figure 5: **The Effects of Investment Subsidy.** This figure illustrates the model dynamics. The relevant parameters are $\Delta = 1$, $\Lambda = 1$, $\kappa = 1$, $\beta = 1$, $\rho = 0.05$, $\pi = 0.05$, $\eta = 0.25$, $\tilde{\theta}_0 = 0.5$, and $\phi = 5$. Qualitative outcomes are robust to the choice of these parameters.

2. When $\tilde{\kappa} > \phi$, then $s^* \geq 0$, where the inequality is strict if $F + U \neq 0$ and the entry condition does not bind for $s = 0$.

Figure 5 illustrates the effects of investment subsidies when $\beta > 0$ and κ is small relative to ϕ, i.e., activism is beneficial. Panel A shows that the exit threshold $\max\{\underline{\theta}, \theta^C\}$ increases with s, illustrating that firm-level investment subsidies increase the activist's propensity to exit. Panel B plots $E^s = F + U$ for $\tilde{R} = 0$; it thus depicts the *maximum* value of $\tilde{R}$ that satisfies (23). Because E^s decreases with s, investment subsidy tightens the entry condition and discourages entry as shown in Corollary 4. The right panel C plots the change in average transition rate relative to the case without subsidies $s = 0$. In line with Corollary 4, it shows that the investment subsidy hampers transition, in that the transition rate decreases with s and $s = 0$ maximizes the transition rate. Overall, in Figure 5, the firm-level subsidy decreases the extent of activism in three ways, (i) by reducing activist effort (for a given stake θ), (ii) by incentivizing exit, and (iii) by discouraging entry.

6   Conclusion

We develop a model of investor activism with endogenous activist entry, post-entry trading, and contracting with the firm's management. We apply our framework study the effects of green activism and how it responds to environmental regulation and policies. In this model, efforts by the activist and the firm's manager improve the firm's green transition rate, but are subject to moral hazard. Activism raises the green transition rate under first best, but

introduces a double-agency problem and therefore causes underinvestment in efforts under moral hazard. Due to this double-agency, activist engagement crowds out managerial efforts and only facilitates the green transition when the activist is sufficiently skilled. Depending on its efficacy, the activist accumulates a larger stake or eventually exits after entry. Post-entry trading opportunities have an ambiguous impact on investor activism and the green transition: They encourage entry and subsequent exit of low-skill activists and discourage entry and exit of high-skill activists.

Our model has implications for optimal environmental regulation via carbon taxes or carbon pricing within a cap-and-trade scheme and green investment subsidies. Carbon taxation improves green transition rates conditional on activist entry, but can deter activists from entering. Consequently, optimal carbon tax can be below or above the Pigouvian level. When the activist has strong incentives to enter, e.g., due to impact preferences or financial profits from activism, optimal carbon tax can address the double-agency problem without affecting the activist's entry decision and, therefore, lies above the Pigouvian level. Otherwise, when socially responsible activism is hard to incentivize and associated with financial losses, optimal carbon tax lies below the Pigouvian level to encourage activist entry. Finally, we show that green investment subsidies, as, for instance, offered under the Inflation Reduction Act, increase firm-level investments in green transition but crowd out impact activism and generally have adverse effects on the green transition in the presence of moral hazard.

One can also (re-)interpret a successful green transformation as a general improvement in firm value or performance (not necessarily limited to environmental aspects), such as cash flow or productivity enhancement or innovative breakthrough. Under this more general interpretation, the model applies broadly and can describe various forms of investor activism, including but not limited to green activism. Methodologically, we advance the literature on investor activism by jointly and tractably modeling activist entry, dynamic trading, and the interactions between the incentives of the firm's management and the activist.

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Online Appendix

A   Activist Optimization and Trading

A.1   Optimal Short-Term Contracting

We characterize the optimal short-term contract over $[t, t + dt]$, which is signed at time t between the controlling shareholder (i.e., the activist or the passive investor) and the manager. This contract stipulates state-contingent transfers dC_t^S to the manager at time $t + dt$. Recall the heuristic timing of actions. First, given stake θ_t, the activist and manager sign a contract over $[t, t + dt)$. Second, uncertainty as to whether the transition process is completed is resolved. Payments are made, and the contract ends. Third, in case of no completion, the activist trades and chooses $d\theta_t$, pinning down the next-period stake $\theta_{t+dt} = \theta_t + d\theta_t$. Then, at time $t + dt$ with activist stake θ_{t+dt}, the activist and the manager sign a contract over $[t + dt, t + 2dt)$, and so on. Given this timing of events, the contract can only condition on the outcomes of the completion process, but not on the activist's trading as the contract ends before the activist chooses $d\theta_t$.

There are three possible states S at time $t + dt$, denoted $S = 1, 2, 3$. First, with probability $1 - \Lambda dt$, the transition is not completed yet—which we denote by $S = 1$. Second, with probability $\Lambda dt(1 - a_t - m_t)$, the transition process fails—which we denote by $S = 2$. Third, with probability $\Lambda dt(a_t + m_t)$, the transition process succeeds—which we denote by $S = 3$. When the passive investor is the controlling shareholder, then $a_t = 0$ which coincides with the effort level $\hat{a}_t$ that the manager anticipates. If the activist is the controlling shareholder, then $a_t > 0$ and the manager anticipates effort level $\hat{a}_t$ (which coincides with a_t in optimum).

Given the state-contingent payments dC_t^S and anticipating effort $\hat{a}_t$ from the controlling shareholder, the manager chooses m_t to maximize its payoff (for $t < T$):

$$\hat{W}_t = \max_{m_t \geq 0} \left\{ e^{-\rho dt} \left((1 - \Lambda dt)dC_t^1 + \Lambda dt(1 - a_t - m_t)dC_t^2 + \Lambda dt(\hat{a}_t + m_t)dC_t^3 \right) - \frac{\phi m_t^2}{2} dt \right\}.$$

Optimal interior effort m_t solves then the first-order condition

$$\Lambda dt(dC_t^3 - dC_t^2) = e^{\rho dt}\phi m_t dt \quad \Longleftrightarrow \quad dC_t^3 - dC_t^2 = \frac{\phi m_t}{\Lambda}. \tag{A.1}$$

We used that in the continuous time limit $dt \to 0$, it follows $e^{\rho dt}\phi m_t dt = (1 + \rho dt)\phi m_t dt = \phi m_t dt$ due to $(dt)^2 = 0$, and divided both sides by dt.

The game ends at time T, and the manager's payoff at time $t = T$ equals dC_t^2 in case of failure and dC_t^3 in case of success. The manager's expected payoff must be positive at any point in time, including at times $t \geq T$. That is, $dC_t^2, dC_t^3 \geq 0$ (to ensure positive payoff at time T) as well as $\hat{W}_t \geq 0$ (to ensure positive payoff at times $t \leq T$).

The controlling shareholder's value function at time $t < T$ is V_t, and its ownership stake

equals $\theta_t = \theta$. The controlling shareholder chooses payments dC_t^S to maximize

$$\max_{dC_t^S, a_t} \left\{ -\frac{\kappa a_t^2}{2} + \theta e^{-\rho dt} \Big((1 - \Lambda dt)(V_{t+dt} - dC_t^1) \right.$$

$$\left. + \Lambda dt(1 - a_t - m_t)(V^D - dC_t^2) + \Lambda dt(a_t + m_t)(V^D + \Delta - dC_t^3) \Big) \right\}$$

subject to (A.1), $\hat{W}_t \geq 0$, and $dC_t^2, dC_t^3 \geq 0$.

It is clear that setting $dC_t^2 = 0$ is optimal. We write now $B_t = dC_t^3$. It follows from the incentive compatibility constraint (A.1) that B_t is not infinitesimal, i.e., not of order dt.

Further, it is clear that to maximize its own payoff, the controlling shareholder minimizes agency rents by designing the contract such that $\hat{W}_t = 0$ and the manager breaks even. Thus,

$$e^{-\rho dt} \left((1 - \Lambda dt)dC_t^1 + \Lambda dt(\hat{a}_t + m_t)B_t \right) - \frac{\phi m_t^2}{2} dt = 0. \tag{A.2}$$

For (A.2) to hold, it must be that dC_t^1 is infinitesimal and of order dt, in that we can write $dC_t^1 = c_t dt$. In the continuous time limit $dt \to 0$, it follows $e^{\rho dt} \phi m_t dt = (1 + \rho dt)\phi m_t dt = \phi m_t dt$ due to $(dt)^2 = 0$ as well as $(c_t dt)(\Lambda dt) = 0$. Thus, (A.2) simplifies to $c_t = \frac{\phi m_t^2}{2} - \Lambda(\hat{a}_t + m_t)B_t$, which is (10). This concludes the argument.

A.2 Details for the Proof of Proposition 1

Consider that optimal trading is smooth, i.e., $d\theta = \dot{\theta}dt$. Then, by the dynamic programming principle and the integral expression (3) for activist payoff, the activist's value function $V_t = V(\theta_t)$ satisfies the HJB equation (11). In addition, the optimality condition (12) applies, i.e., $V'(\theta) = P(\theta)$. Inserting (12) into (11), we obtain

$$(\rho + \Lambda)V(\theta) = \max_{B \geq 0} \left\{ \theta(\mu - \pi - c) - \frac{\kappa a^2}{2} + \Lambda\theta\Big[V^D + (a + m)(\Delta - B)\Big] \right\}, \tag{A.3}$$

subject to incentive constraints (8) and (9), i.e., $m = \frac{\Lambda B}{\phi}$ and $a = \frac{\theta\Lambda(\Delta - B)}{\kappa}$, and managerial wage (10), i.e., $c = \frac{\phi m^2}{2} - \Lambda(a + m)B$. Inserting (10) into (A.3), we can simplify (A.3) to

$$(\rho + \Lambda)V(\theta) = \max_{m \geq 0} \left\{ \theta\left(\mu - \pi - \frac{\phi m^2}{2}\right) - \frac{\kappa a^2}{2} + \Lambda\theta\Big[V^D + (a + m)\Delta\Big] \right\}, \tag{A.4}$$

subject to incentive constraints (8) and (9), i.e., $m = \frac{\Lambda B}{\phi}$ and $a = \frac{\theta\Lambda(\Delta - B)}{\kappa}$.

The first-order condition with respect to m yields (with $\frac{\partial B}{\partial m} = \frac{\phi}{\Lambda}$ and $\frac{\partial a}{\partial m} = -\frac{\theta\phi}{\kappa}$):

$$\theta\left(-\phi m + \Lambda\left(1 - \frac{\phi}{\kappa}\right)\Delta\right) + \theta\phi a = 0. \tag{A.5}$$

Observe that $a = \frac{\theta\Lambda(\Delta - B)}{\kappa} = \frac{\theta(\Lambda\Delta - \phi m)}{\kappa}$, so the first-order condition (A.5) simplifies (for $\theta > 0$)

to $-\phi m + \Lambda\left(1 - \frac{\phi}{\kappa}\right)\Delta + \frac{\theta(\Lambda\Delta - \phi m)}{\kappa} = 0$. Denoting $\tilde{\kappa} = \kappa/\theta$, we can solve $m = m(\theta) = \frac{\tilde{\kappa}}{\phi}\left(\frac{\Lambda\Delta}{\phi + \tilde{\kappa}}\right)$.
Thus, $a = a(\theta) = \frac{\theta(\Lambda\Delta - \phi m)}{\kappa} = \frac{\phi}{\tilde{\kappa}}\left(\frac{\Lambda\Delta}{\phi + \tilde{\kappa}}\right)$. These expressions for a and m coincide with those
presented in (14).

The manager's bonus is $B = B(\theta) = \frac{\phi m(\theta)}{\Lambda}$ and its wage then becomes

$$c = c(\theta) = \frac{\phi m(\theta)^2}{2} - \Lambda(a(\theta) + m(\theta))B(\theta) = -\frac{\phi m(\theta)^2}{2} - \phi a(\theta)m(\theta).$$

Inserting optimal m and a and $a + m = \frac{\Delta\Lambda\left(\tilde{\kappa}^2 + \phi^2\right)}{\tilde{\kappa}\,\phi\,(\tilde{\kappa} + \phi)}$ into the HJB equation (A.4), we obtain

$$(\rho + \Lambda)V(\theta) = \theta\left(\mu - \pi - \left(\frac{\tilde{\kappa}^2}{2\phi}\right)\left(\frac{\Lambda\Delta}{\phi + \tilde{\kappa}}\right)^2\right) - \left(\frac{\phi^2\kappa\theta^2}{2\kappa}\left(\frac{\Lambda\Delta}{\phi + \tilde{\kappa}}\right)^2\right) + \Lambda\theta\left[V^D + \frac{\Delta^2\Lambda\left(\tilde{\kappa}^2 + \phi^2\right)}{\tilde{\kappa}\,\phi\,(\tilde{\kappa} + \phi)}\right].$$

This can be simplified to obtain (13). Observe that $\hat{V}(\theta)$ solves (A.4), and represents the activist's payoff "as if" it did not trade at all and θ remained constant up to transformation. Taking the limit $P^0 = \lim_{\pi \to 0, \tilde{\kappa} \to \infty} \frac{\hat{V}(\theta)}{\theta}$, we obtain (15).

A.3 Details for the Proof of Proposition 2

The proof proceeds in several parts. Part I establishes the convexity of function $\hat{V}(\theta)$ characterized in Proposition 1 and derives an upper and lower bound for the stock price $P(\theta)$. Part II characterizes the endogenous thresholds $\underline{\theta}$ and $\overline{\theta}$ at which the activist is indifferent between not trading at all and selling its entire stake or buying the entire firm respectively; as we show, these thresholds pin down the smooth trading region $(\underline{\theta}, \overline{\theta})$. Part III presents general results regarding the activist's optimal trading. Part IV characterizes the different trading regions and, in particular, establishes that (i) smooth trading within $(\underline{\theta}, \overline{\theta})$ is optimal, (ii) the activist stops trading once $\theta \in \{0, 1\}$, (iii) the activist finds it strictly optimal to trade toward 0 (1) when $\theta \in (0, \underline{\theta})$ ($\theta \in (\overline{\theta}, 1)$), and (iv) the activist randomizes between not trading at all and buying the entire firm (selling the entire firm) when $\theta = \overline{\theta}$ ($\theta = \underline{\theta}$). Part V solves for the endogenous trading rate in the different regions of the state space $[0, 1]$. Part VI shows that whenever $\theta > 0$, then θ is bounded away from zero, in that there exits $\theta_{\min} > 0$ such that $\theta_t \geq \theta_{\min}\mathbb{I}\{\theta_t > 0\}$, with indicator function $\mathbb{I}\{\cdot\}$.

To avoid the study of degenerate cases, we assume (throughout the paper and in the following proof) that the smooth trading region is non-empty. This turns out equivalent to $\underline{\theta} < \overline{\theta}$. Appendix F.3 characterizes the activist's optimal trading when the smooth trading region is empty.

In the proof, we distinguish several cases and analyze different endogenous regions in the state space $\theta \in [0, 1]$ separately—some of these regions might be empty. In what follows, we adopt the convention that whenever we analyze a certain interval/region within the state space, we implicitly assume that this region is non-empty (without explicitly spelling it out); otherwise, the analysis does not apply, and the argument can be skipped.

A.3.1 Part I: Convexity of $\hat{V}(\theta)$ and Bounds for Stock Price

We prove that $\hat{V}(\theta)$ is strictly convex on $(0, 1)$. For this purpose, we first take the derivative of $\hat{V}(\theta)$ to obtain

$$\hat{V}'(\theta) = \frac{\mu - \pi + \Lambda V^D}{\rho + \Lambda} + \frac{\Delta^2 \Lambda^2 \left(2\theta^3 \phi^3 + 4\theta^2 \kappa \phi^2 + 2\theta \kappa^2 \phi + \kappa^3\right)}{2\phi\kappa \left(\kappa + \phi\theta\right)^2 (\rho + \Lambda)}. \tag{A.6}$$

In the smooth trading region, $P(\theta) = \hat{V}'(\theta)$. We then calculate

$$\hat{V}''(\theta) = \frac{\Delta^2 \, \Lambda^2 \, \phi \, \theta \, \left(3\,\kappa^2 + 3\,\kappa\,\phi\,\theta + \phi^2\,\theta^2\right)}{\kappa \, (\kappa + \phi\,\theta)^3 \, (\rho + \Lambda)} > 0, \tag{A.7}$$

which is positive as desired.

Next, we show that in any state θ, we have $P^0 \le P(\theta) \le P^1$. The value P^1 is defined as

$$P^1 = \frac{\mu - c(1) + \Lambda\left[V^D + (a(1) + m(1))(\Delta - B(1))\right]}{\Lambda + \rho} = \frac{\mu - \frac{\phi m(1)^2}{2} + \Lambda\left[V^D + (a(1) + m(1))\Delta\right]}{\Lambda + \rho}$$

$$= \mathbb{E}_t\left[\int_t^\infty e^{-(\Lambda+\rho)(u-t)}\left(\mu - \frac{\phi m(1)^2}{2} + \Lambda\left[V^D + (a(1) + m(1))\Delta\right]\right) du\right], \tag{A.8}$$

which is the (hypothetical) stock price if the activist owns the entire firm in perpetuity, leading to efforts $a(1)$ and $m(1)$. To prove $P(\theta) \le P^1$, note that we can express the price $P_t = P(\theta)$ at time $t < T$ in state $\theta_t = \theta$ as follows:

$$P(\theta) = \mathbb{E}_t\left[\int_t^\infty e^{-(\Lambda+\rho)(u-t)}\left(\mu - \frac{\phi m(\theta_u)^2}{2} + \Lambda\left[V^D + (a(\theta_u) + m(\theta_u))\Delta\right]\right) du\right],$$

where, by incentive compatibility,

$$a(\theta) = \frac{\theta(\Lambda\Delta - \phi m(\theta))}{\kappa} = \frac{\Lambda\Delta - \phi m(\theta)}{\tilde{\kappa}} \quad \text{with} \quad \tilde{\kappa} = \frac{\kappa}{\theta}.$$

The activist's optimization over effort in state θ is

$$(a(\theta), m(\theta)) \in \arg\max_{a,m}\left(\Lambda\Delta(a + m) - \frac{\kappa a^2}{2\theta} - \frac{\phi m^2}{2}\right) \quad \text{s.t.} \quad a = \frac{\theta(\Lambda\Delta - \phi m)}{\kappa}.$$

Thus, $-\frac{\phi m(\theta_u)^2}{2} + \Lambda\Delta(a(\theta_u) + m(\theta_u))$ (strictly) increases in θ. As a result,

$$P^0 = \mathbb{E}_t\left[\int_t^\infty e^{-(\Lambda+\rho)(u-t)}\left(\mu - \frac{\phi m(0)^2}{2} + \Lambda\left[V^D + (a(0) + m(0))\Delta\right]\right) du\right]$$

$$\le \mathbb{E}_t\left[\int_t^\infty e^{-(\Lambda+\rho)(u-t)}\left(\mu - \frac{\phi m(\theta_u)^2}{2} + \Lambda\left[V^D + (a(\theta_u) + m(\theta_u))\Delta\right]\right) du\right] = P_t \le P^1,$$

which is what we wanted to show.

Finally, note that in state $\theta_t = \theta \in (0, 1)$ with $P(\theta_t) = P_t$, the above inequalities are strict, whenever θ stays within $(0, 1)$ with positive probability over a non-empty time interval.

A.3.2 Part II: Threshold Determination

Define the function

$$K_L(\theta) = \theta P^0 - \hat{V}(\theta).$$

Clearly, $K_L(0) = 0$ and $K'_L(\theta) = P^0 - \hat{V}'(\theta)$, so $K''_L(\theta) = -\hat{V}''(\theta) < 0$. Thus, $K_L(\theta)$ has maximally one root on $(0,1)$. Provided $K_L(\theta)$ has a root on $(0,1)$, denoted $\underline{\theta}_0$, then one can calculate

$$\underline{\theta}_0 = \frac{\kappa\left(\sqrt{\phi\pi\,(2\,\Delta^2\,\Lambda^2 + \phi\,\pi)} + \phi\pi\right)}{\Delta^2\,\Lambda^2\,\phi}. \tag{A.9}$$

It is clear that $\underline{\theta}_0 \geq 0$ and strictly so if $\pi > 0$. The lower threshold $\underline{\theta}$, defined as $\underline{\theta} = \sup\{\theta \in [0,1] : \hat{V}(\theta) = \theta P^0\}$, then satisfies

$$\underline{\theta} = \min\{\underline{\theta}_0, 1\} \in [0,1]. \tag{A.10}$$

Thus, $P^0\theta > \hat{V}(\theta)$ for any $\theta \in (0,\underline{\theta})$, $P^0\theta = \hat{V}(\theta)$ for $\theta = \underline{\theta}$ if $\underline{\theta} < 1$, and $P^0\theta < \hat{V}(\theta)$ for $\theta > \underline{\theta}$.

Next, define

$$K_H(\theta) = \hat{V}(1) - (1-\theta)P^1 - \hat{V}(\theta),$$

where P^1 is from (A.8). Note that $K'_H(\theta) = P^1 - \hat{V}'(\theta)$ and $K''_H(\theta) = -\hat{V}''(\theta) < 0$. Since $K_H(1) = 0$, it follows that $K_H(\theta)$ has maximally one root on $(0,1)$. Provided $K_H(\theta)$ has a root on $(0,1)$, denoted $\bar{\theta}_0$, then one can calculate $\bar{\theta}_0 = N/D$ with

$$\begin{aligned}
N :=& \Delta^2\Lambda^2\phi\left(-\kappa^2 - \kappa\phi + \phi^2\right) + 2\pi\kappa\phi(\kappa+\phi)^2 + \sqrt{\phi}\Big(\Delta^4\Lambda^4\phi\left(\kappa^4 + 6\kappa^3\phi + 7\kappa^2\phi^2 + 2\kappa\phi^3 + \phi^4\right) \\
&+ 4\pi\Delta^2\kappa\Lambda^2(\kappa+\phi)^2\left(2\kappa^3 + 3\kappa^2\phi + \kappa\phi^2 + \phi^3\right) + 4\pi^2\kappa^2\phi(\kappa+\phi)^4\Big)^{1/2}
\end{aligned}$$

and

$$D := 2\,\Delta^2\,\Lambda^2\,\phi\,(\kappa+\phi)^2$$

The upper threshold $\bar{\theta}$, defined as $\bar{\theta} = \inf\{\theta \in [0,1] : \hat{V}(\theta) = \hat{V}(1) - (1-\theta)P^1\}$, then satisfies

$$\bar{\theta} = \min\{[\bar{\theta}_0]^+, 1\}, \tag{A.11}$$

where $[x]^+ = \max\{x, 0\}$. Thus, $\hat{V}(1) - (1-\theta)P^1 > \hat{V}(\theta)$ for $\theta > \bar{\theta}$, $\hat{V}(1) - (1-\theta)P^1 = \hat{V}(\theta)$ for $\theta = \bar{\theta}$, and $\hat{V}(1) - (1-\theta)P^1 < \hat{V}(\theta)$ for $\theta < \bar{\theta}$.

Note that the regions $(0,\underline{\theta})$ or $(\bar{\theta},1)$ might be empty. Further, recall that it is assumed that the smooth trading region is non-empty, which, as will be shown, is equivalent to $\bar{\theta} > \underline{\theta}$. See Appendix F.3 for the characterization of the activist's optimal trading when the smooth trading region is empty.

A.3.3 Part III: Optimality of Trading Strategy (Preliminaries)

We allow for continuous and lumpy trading for times $t \geq T^\beta$ by specifying the dynamics of the activist's stake as

$$d\theta_t = \dot{\theta}_t dt + dI_t, \tag{A.12}$$

where $\dot{\theta}_t$ is the drift of $d\theta_t$ and dI_t captures solely lumpy trading, in that $I_t = \int_0^t dI_s$ is constant except for a countable number of times t.

More specifically, we consider that at an endogenous (state-dependent) intensity $\xi \in [0, \infty]$, the activist conducts a lumpy trade toward state $\hat{\theta} \in [0, 1]$, where $\hat{\theta}$ is optimally chosen by the activist and thus endogenous. With a slight abuse of notation, $\xi = +\infty$ corresponds to a lumpy trade that occurs with some atom of probability (possibly with probability one). That is, we can write

$$d\theta = \dot{\theta}dt + (\hat{\theta} - \theta)dN,$$

where $dN \in \{0, 1\}$ is a jump process with $\mathbb{E}[dN] = \xi dt$. The activist's value function then satisfies the HJB equation

$$(\rho + \Lambda)V(\theta) = \max_{B \geq 0, \dot{\theta}, \xi, \hat{\theta}} \left\{ \theta\left(\mu - \pi - c\right) - \frac{\kappa a^2}{2} + \Lambda\theta\left[V^D + (a + m)(\Delta - B)\right] \right.$$
$$\left. + \dot{\theta}\left[V'(\theta) - P(\theta)\right] + \xi\left[V(\hat{\theta}) - V(\theta) - (\hat{\theta} - \theta)P(\hat{\theta})\right] \right\}, \quad \text{(A.13)}$$

subject to incentive constraints (8) and (9), i.e., $m = \frac{\Lambda B}{\phi}$ and $a = \frac{\theta\Lambda(\Delta - B)}{\kappa}$, and managerial wage (10), i.e., $c = \frac{\phi m^2}{2} - \Lambda(a + m)B$. For an interior solution $\dot{\theta} \in (-\infty, +\infty)$ and $\xi \in [0, \infty)$ to be optimal, it must be

$$P(\theta) = V'(\theta) \quad \text{and} \quad \max_{\hat{\theta} \in [0,1]} \left[V(\hat{\theta}) - V(\theta) - (\hat{\theta} - \theta)P(\hat{\theta})\right] \leq 0. \quad \text{(A.14)}$$

As setting $\xi = 0$ is optimal whenever $\max_{\hat{\theta} \in [0,1]} \left[V(\hat{\theta}) - V(\theta) - (\hat{\theta} - \theta)P(\hat{\theta})\right] < 0$, we obtain $\xi\left[V(\hat{\theta}) - V(\theta) - (\hat{\theta} - \theta)P(\hat{\theta})\right] = 0$ for any $\hat{\theta}$ whenever (A.14) holds. Plugging these conditions back into (A.13), we obtain

$$(\rho + \Lambda)V(\theta) = \max_{B \geq 0} \left\{ \theta\left(\mu - \pi - c\right) - \frac{\kappa a^2}{2} + \Lambda\theta\left[V^D + (a + m)(\Delta - B)\right] \right\},$$

which is akin to (A.4) and admits closed-form solution $V(\theta) = \hat{V}(\theta)$. Thus, the activist's value function is $V(\theta) = \hat{V}(\theta)$ whenever there is no lumpy trade with an atom of probability, i.e., whenever $\dot{\theta} \in (-\infty, \infty)$ and $\xi \in [0, \infty)$.

We define the endogenous region

$$\mathcal{S} = \{\theta \in [0, 1] : \dot{\theta} \in (-\infty, \infty) \text{ and } \xi \in [0, \infty)\}$$

That is, for $\theta \in \mathcal{S}$, we have $\hat{V}(\theta)$ and $P(\theta) = \hat{V}'(\theta)$. As $\hat{V}''(\theta) > 0$, we have that price $P(\theta)$ is increasing in the interior of $\mathcal{S}$, i.e., in $int(\mathcal{S})$, in that $P'(\theta) = \hat{V}''(\theta) > 0$.

Consider that $\theta \in int(\mathcal{S})$, i.e., θ lies in the interior of $\mathcal{S}$. If the activist trades from state θ toward state $\hat{\theta} \in int(\mathcal{S})$, its payoff changes by

$$G(\hat{\theta}; \theta) := \hat{V}(\hat{\theta}) - V(\theta) - (\hat{\theta} - \theta)P(\hat{\theta}).$$

Clearly, $G(\theta; \theta) = 0$ and

$$\frac{\partial G(\hat{\theta}; \theta)}{\partial \hat{\theta}} = V'(\hat{\theta}) - P(\hat{\theta}) - (\hat{\theta} - \theta)P'(\hat{\theta}) = -(\hat{\theta} - \theta)P'(\hat{\theta}).$$

As $P'(\hat{\theta}) > 0$ and $G(\theta; \theta) = 0$, we obtain that $G(\hat{\theta}; \theta) < 0$ for any $\theta \neq \hat{\theta} \in int(\mathcal{S})$. Thus, in state $\theta \in int(\mathcal{S})$, any lumpy trade toward state $\hat{\theta} \in int(\mathcal{S})$ is strictly sub-optimal.

Consider state $\theta \notin \mathcal{S}$, so the activist conducts a lumpy trade with an atom of probability. It is without loss of generality to assume that this trade brings its stake θ, i.e., the state variable, into the set $\mathcal{S}$.[29] The activist's value function therefore satisfies $V(\theta) = \max_{\hat{\theta} \in \mathcal{S}} \left[\hat{V}(\hat{\theta}) - (\hat{\theta} - \theta)P(\hat{\theta}) \right]$. Suppose to the contrary that $\hat{\theta} \in int(\mathcal{S})$. Then, the first-order condition for the optimal choice of $\hat{\theta}$, that is

$$\hat{V}'(\hat{\theta}) - P(\hat{\theta}) - (\hat{\theta} - \theta)P'(\hat{\theta}) = -(\hat{\theta} - \theta)P'(\hat{\theta}) = 0,$$

must hold, where we used that $\hat{V}'(\hat{\theta}) = P(\hat{\theta})$ for $\hat{\theta} \in int(\mathcal{S})$. However, due to $P'(\hat{\theta}) > 0$, this first-order condition cannot hold. Thus, $\hat{\theta}$ must lie on the edge of the set $\mathcal{S}$, i.e., any lumpy trade brings θ onto the edge of $\mathcal{S}$.

The above arguments also generalize to the case that $\theta \in \mathcal{S} - int(\mathcal{S})$. When $\theta \in \mathcal{S} - int(\mathcal{S})$, then any lumpy trade toward $\hat{\theta} \in int(\mathcal{S})$ is strictly suboptimal.

A.3.4   Part IV: Optimality of the Trading Strategy

We now conjecture and verify that $\mathcal{S} = [\underline{\theta}, \overline{\theta}] \cup \{0, 1\}$, implying that $V(\underline{\theta}) = \hat{V}(\underline{\theta})$ and $V(\overline{\theta}) = \hat{V}(\overline{\theta})$. We further conjecture and verify that states 0 and 1 are absorbing (i.e., the activist stops trading once its stake reaches 0 or 1), so that $P(1) = P^1$ and $P(0) = P^0$.[30] Because of $P(0) = P^0$ and $P(1) = P^1$ as well as $V(\underline{\theta}) = \hat{V}(\underline{\theta})$ and $V(\overline{\theta}) = \hat{V}(\overline{\theta})$, the activist is indifferent between not trading at all and trading toward 0 (1) when in state $\underline{\theta}$ ($\overline{\theta}$).

Moreover, given the conjectures, we obtain for any $\theta \in \mathcal{S} \cap (0, 1)$ that $P^0 < P(\theta) < P^1$, because—starting within $\mathcal{S} \cap (0, 1)$—θ stays for non-trivial amount of time within the interval $(0, 1)$. In that case, according to Part I, the inequalities $P(\theta_t) \geq P^0$ and $P(\theta_t) \leq P^1$ hold as strict inequalities.

Given our previous findings, it is strictly suboptimal in state $\theta \in (\underline{\theta}, \overline{\theta})$ to conduct a lumpy trade toward state $\hat{\theta} \in (\underline{\theta}, \overline{\theta})$. It is easy to see that by continuity of price and value function on $[\underline{\theta}, \overline{\theta}]$, this statement extends to $\theta, \hat{\theta} \in [\underline{\theta}, \overline{\theta}]$, in that trading from $\theta \in [\underline{\theta}, \overline{\theta}]$ toward $\hat{\theta} \in [\underline{\theta}, \overline{\theta}]$ is strictly suboptimal.

For the remainder of the proof, we proceed in several steps and distinguish several cases.

State $\theta \in (0, \underline{\theta})$. First, suppose to the contrary that $\dot{\theta} \in (-\infty, \infty)$ and $\xi \in [0, \infty)$ is optimal. Then, $\theta \in \mathcal{S}$ and $V(\theta) = \hat{V}(\theta)$. But, as $\theta \in (0, \underline{\theta})$, we have $\hat{V}(\theta) < \theta P^0$ and the activist

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[29]Otherwise, the lumpy trade would be followed immediately by another lumpy trade. Two consecutive lumpy trades are equivalent to one trade. Consolidating (a finite number of) consecutive lumpy trades into one lumpy trade, we obtain one lumpy trade toward $\hat{\theta} \in \mathcal{S}$.

[30]Note that by Part I we have $P(0) \geq P^0$ and $P(1) \leq P^1$. These inequalities hold in equalities when states 0 and 1 are absorbing.

could attain strictly higher payoff, namely, $\theta P(0) \geq \theta P^0$, through a lumpy trade $d\theta = -\theta$ toward state 0, a contradiction. As a consequence, $\theta \notin \mathcal{S}$.

Thus, the activist optimally conducts a lumpy trade in state $\theta \in (0, \underline{\theta})$. According to our previous results from Part III, this lumpy trade brings θ onto the edge of $\mathcal{S}$, so the activist trades toward $\hat{\theta} \in \{0, 1, \underline{\theta}, \overline{\theta}\}$. Trading toward $\hat{\theta} = 0$ yields payoff $\theta P(0) \geq \theta P^0$. Trading toward $\hat{\theta} = \underline{\theta}$ yields payoff

$$\hat{V}(\underline{\theta}) - (\underline{\theta} - \theta)P(\underline{\theta}) = \underline{\theta}P^0 - (\underline{\theta} - \theta)P(\underline{\theta}) < \underline{\theta}P^0 - (\underline{\theta} - \theta)P^0 = \theta P^0.$$

where we used that $\hat{V}(\underline{\theta}) = \underline{\theta}P^0$ and $P^0 < P(\underline{\theta})$ for $\underline{\theta} > 0$. Thus, trading toward $\hat{\theta} = 0$ strictly dominates trading toward $\hat{\theta} = \underline{\theta}$.

A trade toward $\overline{\theta} < 1$ with $\overline{\theta} > \underline{\theta}$ yields lower payoff than a trade toward $\underline{\theta}$ at price $P(\underline{\theta})$ immediately followed by a trade from $\underline{\theta}$ toward $\overline{\theta}$ at price $P(\overline{\theta})$, since $P(\overline{\theta}) \geq P(\underline{\theta})$. As we have shown, it is strictly sub-optimal in state $\underline{\theta}$ to trade toward $\overline{\theta}$. In addition, it is strictly suboptimal to trade from θ toward $\underline{\theta}$. Thus, it must be strictly suboptimal to trade from 0 toward $\overline{\theta}$. Because, by the definition of $\overline{\theta}$, the activist is indifferent in state $\overline{\theta}$ to trade toward $\hat{\theta} = 1$ or not trading at all, while it is strictly suboptimal to trade toward $\overline{\theta}$, it must also be strictly suboptimal to trade toward $\hat{\theta} = 1$. Taken together, in state $\theta \in (0, \underline{\theta})$, immediately trading toward zero, i.e., $\hat{\theta} = 0$, is strictly optimal.

State $\theta = 0$. Consider next $\theta = 0$. Importantly, notice that we consider $t \geq T^\beta$, i.e., the scenario that the state $\theta = 0$ is reached *after* time $T^\beta \geq 0$ and in particular after the activist has entered.[31] If the activist conducts a lumpy trade, then this trade is toward state $\hat{\theta} \in \{\underline{\theta}, \overline{\theta}, 1\}$ on the edge of $\mathcal{S}$. A trade toward $\hat{\theta} = \underline{\theta} \in (0, 1)$ yields

$$\hat{V}(\underline{\theta}) - \underline{\theta}P(\underline{\theta}) < \hat{V}(\underline{\theta}) - \underline{\theta}P^0 = 0,$$

where the first inequality follows from $P(\underline{\theta}) > P^0$ (for $\underline{\theta} > 0$) and the equality from $\hat{V}(\underline{\theta}) - \underline{\theta}P^0 = 0$. A trade toward $\overline{\theta} < 1$ with $\overline{\theta} > \underline{\theta}$ yields a lower payoff than a trade toward $\underline{\theta}$ at price $P(\underline{\theta})$ immediately followed by a trade from $\underline{\theta}$ toward $\overline{\theta}$ at price $P(\overline{\theta})$, since $P(\overline{\theta}) \geq P(\underline{\theta})$. As we have shown, it is strictly sub-optimal in state $\underline{\theta}$ to trade toward $\overline{\theta}$. In addition, it is strictly suboptimal to trade from 0 toward $\underline{\theta}$. Thus, it must be strictly suboptimal to trade from 0 toward $\overline{\theta}$. Because, by the definition of $\overline{\theta}$, the activist is indifferent at $\overline{\theta}$ to trade toward $\hat{\theta} = 1$ or not trading at all (yielding payoff $\hat{V}(\hat{\theta})$), while it is strictly suboptimal to trade toward $\overline{\theta}$, it is also strictly suboptimal to trade toward $\hat{\theta} = 1$.

Next, suppose to the contrary that smooth trading in state $\theta = 0$ is optimal. Thus, $\dot{\theta} > 0$. But, as we have shown, in any state $\theta \in (0, \underline{\theta})$, it is strictly optimal to (immediately) trade toward zero. As such, smooth trading $\dot{\theta} > 0$ in state $\theta = 0$ cannot be, because it would be immediately followed by a lumpy trade toward zero.

As a result, state $\theta = 0$, if reached at time $t > 0$, is absorbing, i.e., the activist stops trading once θ reaches zero.

Finally, recall that the activist's initial trade at time $t = 0$ is fundamentally different from post-entry trading after time T^β. When the activist chooses its initial stake θ_0 at $t = 0$, it can acquire part of this initial stake at a discount (i.e., below post-entry market price). In

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[31] As long as $\beta < \infty$, we have $T^\beta > 0$ almost surely.

contrast, after time T^β (i.e., post-entry), the activist always trades at the fair market price.

State $\theta \in (\bar{\theta}, 1)$. Note that $\hat{V}(1) \leq V(1)$, as the activist always has the option not to trade at all in state $\theta = 1$ yielding a payoff $\hat{V}(1)$.

Next, suppose to the contrary that $\dot{\theta} \in (-\infty, \infty)$ and $\xi \in [0, \infty)$ is optimal. Then, $\theta \in \mathcal{S}$ and $V(\theta) = \hat{V}(\theta)$. But, as $\theta \in (\bar{\theta}, 1)$, we have $\hat{V}(\theta) < \hat{V}(1) + (1 - \theta)P^1 \leq V(1) + (1 - \theta)P^1$ and the activist could attain strictly higher payoff through a lumpy trade toward 1, i.e., $d\theta = 1 - \theta$, a contradiction. As a consequence, $\theta \notin \mathcal{S}$.

Thus, the activist conducts a lumpy trade. According to our findings in Part III, this lumpy trade brings θ onto the edge of $\mathcal{S}$, so the activist trades toward $\hat{\theta} \in \{0, 1, \underline{\theta}, \bar{\theta}\}$. Trading toward $\hat{\theta} = 1$ yields $V(1) - (1 - \theta)P^1 \geq \hat{V}(1) + (1 - \theta)P^1$. Trading toward $\hat{\theta} = \bar{\theta}$ yields

$$\hat{V}(\bar{\theta}) + (\theta - \bar{\theta})P(\bar{\theta}) = \hat{V}(1) - (1 - \bar{\theta})P^1 + (\theta - \bar{\theta})P(\bar{\theta})$$
$$< \hat{V}(1) - (1 - \bar{\theta})P^1 + (\theta - \bar{\theta})P^1 \leq V(1) - (1 - \theta)P^1,$$

where we have used that $\hat{V}(\bar{\theta}) = \hat{V}(1) - (1 - \bar{\theta})P^1$ and $P(\bar{\theta}) < P^1$ for $\bar{\theta} < 1$. Thus, trading toward $\hat{\theta} = 1$ strictly dominates trading toward $\hat{\theta} = \bar{\theta}$.

A trade toward $\underline{\theta}$ with $\bar{\theta} > \underline{\theta}$ at price $P(\underline{\theta})$ yields lower payoff than a trade toward $\bar{\theta}$ at price $P(\bar{\theta})$ immediately followed by a trade from $\bar{\theta}$ toward $\underline{\theta}$ at price $P(\underline{\theta})$, since $P(\bar{\theta}) \geq P(\underline{\theta})$. As we have shown, it is strictly sub-optimal in state $\bar{\theta}$ to trade toward $\underline{\theta}$. In addition, it is strictly suboptimal to trade from 1 toward $\bar{\theta}$. Thus, it must be strictly suboptimal to trade from 1 toward $\underline{\theta}$. Because, by the definition of $\underline{\theta}$, the activist is indifferent at $\underline{\theta}$ between trading toward $\hat{\theta} = 0$ and not trading at all, while it is strictly suboptimal to trade toward $\underline{\theta}$, it is also strictly suboptimal to trade toward $\hat{\theta} = 0$.

In state $\theta \in (\bar{\theta}, 1)$, immediately trading toward $\hat{\theta} = 1$, i.e., $d\theta = (1 - \theta)$, is thus strictly optimal.

State $\theta = 1$. Consider $\theta = 1$. If the activist conducts a lumpy trade, then this trade is toward state $\hat{\theta} \in \{\underline{\theta}, \bar{\theta}, 0\}$ on the edges of $\mathcal{S}$. Relative to not trading and collecting payoff $\hat{V}(1)$, a trade toward $\bar{\theta} \in (0, 1)$ changes payoff by

$$\hat{V}(\bar{\theta}) + (1 - \bar{\theta})P(\bar{\theta}) - \hat{V}(1) < \hat{V}(\bar{\theta}) + (1 - \bar{\theta})P^1 - \hat{V}(1) = 0,$$

where we have used that $P(\bar{\theta}) < P^1$ (for $\bar{\theta} < 1$) and that, by definition of $\bar{\theta}$, $\hat{V}(\bar{\theta}) + (1 - \bar{\theta})P^1 - \hat{V}(1) = 0$ for $\bar{\theta} \in (0, 1)$.

A trade toward $\underline{\theta} \in (0, 1)$ with $\bar{\theta} > \underline{\theta}$ at price $P(\underline{\theta})$ yields lower payoff than a trade toward $\bar{\theta}$ at price $P(\bar{\theta})$ immediately followed by a trade from $\bar{\theta}$ toward $\underline{\theta}$ at price $P(\underline{\theta})$, since $P(\bar{\theta}) \geq P(\underline{\theta})$. As we have shown, it is strictly sub-optimal in state $\bar{\theta}$ to trade toward $\underline{\theta}$. In addition, it is strictly suboptimal to trade from 1 toward $\bar{\theta}$. Thus, it must be strictly suboptimal to trade from 1 toward $\underline{\theta}$. Because, by the definition of $\underline{\theta}$, the activist is indifferent at $\underline{\theta}$ between trading toward zero and not trading at all, while it is strictly suboptimal to trade toward $\underline{\theta}$, it is also strictly suboptimal to trade toward 0.

Next, suppose to the contrary that smooth trading in state $\theta = 1$ is optimal. Thus, $\dot{\theta} < 0$. But, as we have shown, in any state $\theta \in (\bar{\theta}, 1)$, it is strictly optimal to trade toward one. As such, smooth trading $\dot{\theta} < 0$ in state $\theta = 1$ cannot be, because it would be immediately followed by a lumpy trade toward one.

As a result, state $\theta = 1$ is absorbing.

State $\theta \in (\underline{\theta}, \overline{\theta})$. As argued before, it is strictly suboptimal in state $\theta \in (\underline{\theta}, \overline{\theta})$ to conduct a lumpy trade toward state $\hat{\theta} \in (\underline{\theta}, \overline{\theta})$. It is easy to see that by continuity of price and value function on $[\underline{\theta}, \overline{\theta}]$, this statement extends to $\theta, \hat{\theta} \in [\underline{\theta}, \overline{\theta}]$, in that trading from $\theta \in [\underline{\theta}, \overline{\theta}]$ toward $\hat{\theta} \in [\underline{\theta}, \overline{\theta}]$ is strictly suboptimal. Thus, in state θ any lumpy trade must be toward state $\hat{\theta} \notin [\underline{\theta}, \overline{\theta}]$. As it is optimal to trade immediately from state $\theta \in (0, \underline{\theta})$ toward 0 and from state $\theta \in (\overline{\theta}, 1)$ toward 1, it is without loss of generality to consider that $\hat{\theta} \in \{0, 1\}$. In state $\theta \in [\underline{\theta}, \overline{\theta}]$, the activist's value function satisfies $V(\theta) \geq \hat{V}(\theta)$, as not trading at all is always an option. By definition of $\underline{\theta}$, we have $\hat{V}(\theta) > \theta P^0$, hence a lumpy trade toward zero is strictly suboptimal. By definition of $\overline{\theta}$, we have $\hat{V}(\theta) > \hat{V}(1) - (1 - \theta)P^1$, hence a lumpy trade toward one is strictly suboptimal too. This verifies that any lumpy trade in state $\theta \in (\underline{\theta}, \overline{\theta})$ is strictly suboptimal, so that $\theta \in int(\mathcal{S})$.

State $\theta = \underline{\theta}$. Suppose $\underline{\theta} > 0$. In state $\theta = \underline{\theta}$, the activist is indifferent between not trading at all, yielding payoff $\hat{V}(\underline{\theta})$, or selling the entire stake at once, yielding payoff $\underline{\theta} P^0$. Thus, it is weakly optimal to randomize between these two options. This implies that it is optimal to set $\dot{\theta} \geq 0$, as $\dot{\theta} < 0$ is akin to an immediate lumpy trade. A strictly negative trading rate $\dot{\theta} < 0$ would bring θ into the region $(0, \underline{\theta})$ which would trigger an immediate lumpy trade toward zero. Thus, in state $\theta = \underline{\theta}$, it is optimal for the activist to either (i) randomize between selling the entire stake and not trading (i.e., $\dot{\theta} = 0$ and $\xi \in [0, \infty)$ with $\hat{\theta} = 0$) or (ii) trade smoothly at positive (finite) rate $\dot{\theta} > 0$.

State $\theta = \overline{\theta}$. Suppose $\overline{\theta} < 1$. In state $\theta = \overline{\theta}$, the activist is indifferent between not trading at all, yielding payoff $\hat{V}(\overline{\theta})$, or buying the entire firm at once, yielding payoff $\hat{V}(1) - (1 - \overline{\theta})P^1$. Thus, it is weakly optimal to randomize between these two options. This implies that it is optimal to set $\dot{\theta} \leq 0$, as $\dot{\theta} > 0$ is akin to an immediate lumpy trade. A strictly positive trading rate $\dot{\theta} > 0$ would bring θ into the region $(\overline{\theta}, 1)$ which would trigger an immediate lumpy trade toward one. Thus, in state $\theta = \overline{\theta}$, it is optimal for the activist to either (i) randomize between buying the entire firm and not trading (i.e., $\dot{\theta} = 0$ and $\xi \in [0, \infty)$ with $\hat{\theta} = 1$) or (ii) trade smoothly at negative (finite) rate $\dot{\theta} < 0$.

A.3.5   Part V: Solving for Trading Rate

We now solve for the optimal trading rate. Recall that the activist stops trading in states 0 and 1, as shown in the previous part. Further, recall that when $\theta \in (0, \underline{\theta})$, it is strictly optimal to trade towards 0, while when $\theta \in (\overline{\theta}, 1)$, it is strictly optimal to trade towards 1.

It turns out convenient to define

$$\hat{P}(\theta) := \frac{\mu - \frac{1}{2}\phi m(\theta)^2 + \Lambda\left[V^D + (a(\theta) + m(\theta))\Delta\right]}{\Lambda + \rho} \tag{A.15}$$

which is the hypothetical price of the firm under the scenario that θ remains constant up to T. It is straightforward to see that $\hat{P}(\theta) > \hat{V}'(\theta) = P(\theta)$ ($\hat{P}(\theta) < \hat{V}'(\theta) = P(\theta)$) if $\dot{\theta} < 0$ ($\dot{\theta} > 0$), with equality at $\theta = \theta^C$. Also note $P^0 = \hat{P}(0)$ and $P^1 = \hat{P}(1)$.

For the remainder of this part of the proof, we distinguish several cases.

State $\theta \in (\underline{\theta}, \overline{\theta})$. For $\theta \in (\underline{\theta}, \overline{\theta})$, i.e., $\theta \in int(\mathcal{S})$, the optimality condition for trading (A.14)

implies $V(\theta) = \hat{V}(\theta)$ and $P(\theta) = \hat{V}'(\theta)$; see Part III of the proof for details. Furthermore, it is optimal to trade smoothly, i.e., $\xi = 0$.

Differentiating the closed-form expression for $\hat{V}(\theta)$ with respect to θ and using $\hat{V}'(\theta) = P(\theta)$, we obtain

$$(\rho + \Lambda)P(\theta) = \mu - \pi - \frac{\phi m(\theta)^2}{2} + \Lambda\left[V^D + (a(\theta) + m(\theta))\Delta\right] + \frac{\Lambda^2\Delta^2\phi}{(\phi + \tilde{\kappa})^2}. \tag{A.16}$$

In addition to satisfying (A.16), $P(\theta)$ satisfies the pricing equation of passive investors

$$(\rho + \Lambda)P(\theta) = \mu - \frac{\phi m(\theta)^2}{2} + \Lambda\left[V^D + (a(\theta) + m(\theta))\Delta\right] + P'(\theta)\dot{\theta}. \tag{A.17}$$

where $P(\theta)$ increases with θ, i.e., $P'(\theta) > 0$. Combining (A.16) and (A.17) yields

$$\dot{\theta} = \frac{1}{P'(\theta)}\left[-\pi + \frac{\Lambda^2\Delta^2\phi}{(\phi + \tilde{\kappa})^2}\right], \tag{A.18}$$

with $P'(\theta) = \hat{V}''(\theta) > 0$.

Observe that $-\theta\left(\frac{\partial c}{\partial a}\frac{\partial a}{\partial \theta}\right) = \frac{\phi m(\Lambda\Delta - \phi m)}{\tilde{\kappa}} = \frac{\Lambda^2\Delta^2\phi}{(\phi+\tilde{\kappa})^2}$, so (A.18) is akin to (18). To see this, combine (8) and (10) to get $c = -\frac{\phi m^2}{2} - \phi am$, so $\frac{\partial c}{\partial a} = -\phi m$. From (9), we obtain $\frac{\partial a}{\partial \theta} = \frac{\Lambda(\Delta - B)}{\kappa} = \frac{\Lambda(\Delta - \phi m/\Lambda)}{\kappa}$, and $\theta \cdot \frac{\partial a}{\partial \theta} = \frac{\Lambda(\Delta - \phi m/\Lambda)}{\tilde{\kappa}}$. Using (14), we get $-\theta\left(\frac{\partial c}{\partial a}\frac{\partial a}{\partial \theta}\right) = \left(\frac{\Lambda\Delta}{\phi+\tilde{\kappa}}\right)\left(\Lambda\Delta - \frac{\Lambda\tilde{\kappa}\Delta}{\phi+\tilde{\kappa}}\right) = \frac{\Lambda^2\Delta^2\phi}{(\phi+\tilde{\kappa})^2}$, as desired.

One can expand above expression (A.18) and plug in the closed-form solution for $\hat{V}''(\theta)$ from (A.7) to obtain

$$\dot{\theta} = \frac{\kappa(\Lambda + \rho)(\kappa + \theta\phi)\left[\Lambda^2\Delta^2\theta^2\phi - \pi(\kappa + \theta\phi)^2\right]}{\Lambda^2\Delta^2\theta\phi\left(\theta^2\phi^2 + 3\theta\kappa\phi + 3\kappa^2\right)}. \tag{A.19}$$

Note that in state $\theta \in (\underline{\theta}, \overline{\theta})$, we have $\dot{\theta} = 0$ for $\Lambda^2\Delta^2\theta^2\phi - \pi(\kappa + \theta\phi)^2 = 0$. This quadratic equation has two roots

$$\theta_{\pm}^C = \frac{\kappa\sqrt{\phi\pi}\left(\sqrt{\phi\pi} \pm \Delta\,\Lambda\right)}{\phi(\Delta^2\,\Lambda^2 - \phi\,\pi)} = \frac{\kappa\sqrt{\phi\pi}\left(\sqrt{\phi\pi} \pm \Delta\,\Lambda\right)}{\phi(\Delta\Lambda - \sqrt{\phi\pi})(\Delta\Lambda + \sqrt{\phi\pi})}$$

When $\phi\pi \geq \Delta^2\Lambda^2 \iff \sqrt{\phi\pi} \geq \Delta\Lambda$, then $\theta_{\pm}^C < 0$ or $\theta_{\pm}^C$ is not finite, i.e., there exists no positive and finite solution. Under these circumstances, we have $\dot{\theta} < 0$. When $\phi\pi < \Delta^2\Lambda^2 \iff \sqrt{\phi\pi} < \Delta\Lambda$, we discard the negative solution to obtain (19). Under these circumstances, $\dot{\theta} > 0$ for $\theta > \theta^C$ and $\dot{\theta} < 0$ for $\theta < \theta^C$.

State $\theta = \underline{\theta}$.

Unless otherwise mentioned, consider $\underline{\theta} > 0$. When $\underline{\theta} > \theta^C$, then $\dot{\theta} > 0$ in a right-neighbourhood of $\underline{\theta}$. Then, θ drifts into the interior of $[\underline{\theta}, \overline{\theta}]$ and no time is spent in state $\underline{\theta}$. In this case, the price satisfies (A.17) and $\dot{\theta}$ satisfies (A.19); there is no randomization over lumpy trading in that $\xi = 0$. In the knife-edge case $\underline{\theta} = \theta^C$, we have $\dot{\theta} = \xi = 0$.

Next, consider that $\underline{\theta} < \theta^C$, so $\dot{\theta} < 0$, $P(\theta) = \hat{V}'(\theta)$, and $\hat{P}(\theta) > P(\theta)$ in a right-neighbourhood of $\underline{\theta}$. Because we have in addition that $\dot{\theta} \geq 0$ at $\underline{\theta}$, it must be that $\dot{\theta} = 0$ at $\underline{\theta}$. By definition of $\underline{\theta}$, the activist is indifferent between not trading at all and selling the entire stake at once at $\underline{\theta}$. Once θ reaches zero, the endogenous stock price becomes $P(0)$. As the activist stops trading once θ reaches zero, we have $P(0) = P^0$. Because $P(\theta) = \hat{V}'(\theta)$ on $(\underline{\theta}, \overline{\theta})$, we have $P(\underline{\theta}) = \hat{V}'(\underline{\theta})$.

At $\theta = \underline{\theta}$ with $\dot{\theta} = 0$, the randomization rate $\underline{\xi}$ is such that $P(\underline{\theta}) = \hat{V}'(\underline{\theta})$ satisfies the pricing equation

$$(\rho + \Lambda)P(\underline{\theta}) = \mu - c(\underline{\theta}) + \Lambda V^D + \Lambda[a(\underline{\theta}) + m(\underline{\theta})](\Delta - B(\underline{\theta})) + \underline{\xi}(P(0) - P(\underline{\theta}))$$
$$= \mu - \frac{\phi m(\underline{\theta})^2}{2} + \Lambda V^D + \Lambda[a(\underline{\theta}) + m(\underline{\theta})]\Delta + \underline{\xi}(P(0) - P(\underline{\theta})),$$

which can be rewritten as $P(\underline{\theta}) = \frac{(\Lambda + \rho)\hat{P}(\underline{\theta}) + \underline{\xi}P^0}{\Lambda + \rho + \underline{\xi}}$. As a consequence,

$$\underline{\xi} = \frac{(\Lambda + \rho)[\hat{P}(\underline{\theta}) - P(\underline{\theta})]}{P(\underline{\theta}) - P^0}. \tag{A.20}$$

with $\hat{P}(\theta)$ for (A.15). Note that $\dot{\theta} < 0$ and $\hat{P}(\theta) > P(\theta)$ in a right-neighbourhood of $\underline{\theta}$, so $\hat{P}(\underline{\theta}) > P(\underline{\theta})$. Because $\hat{V}(\theta)$ is strictly convex and $\hat{V}(\underline{\theta}) = \underline{\theta}P^0$, we have $P(\underline{\theta}) > P^0$ (as long as $\underline{\theta} > 0$). As a result, when $0 < \underline{\theta} < \theta^C$, the randomization rate $\underline{\xi}$ from (A.20) is well-defined and strictly positive.

State $\theta = \overline{\theta}$. Consider $1 > \overline{\theta} > \underline{\theta} \geq 0$. When $\overline{\theta} < \theta^C$, then $\dot{\theta} < 0$ in a left-neighbourhood of $\overline{\theta}$. Then, θ drifts into the interior of $[\underline{\theta}, \overline{\theta}]$ and no time is spent in state $\overline{\theta}$. In this case, the price satisfies (A.17) and $\dot{\theta}$ satisfies (A.19); there is no randomization over lumpy trading in that $\overline{\xi} = 0$. In the knife-edge case $\overline{\theta} = \theta^C$, we have $\dot{\theta} = \overline{\xi} = 0$.

Next, consider that $\overline{\theta} > \theta^C$, so $\dot{\theta} > 0$, $P(\theta) = \hat{V}'(\theta)$, and $\hat{P}(\theta) < P(\theta)$ in a left-neighbourhood of $\overline{\theta}$. Because we have in addition that $\dot{\theta} \leq 0$ at $\overline{\theta}$, it must be that $\dot{\theta} = 0$ at $\overline{\theta}$. By definition of $\overline{\theta}$, the activist is indifferent between not trading at all and buying the entire firm at once in state $\overline{\theta}$. Once θ reaches one, the endogenous stock price becomes $P(1)$. As the activist stops trading once θ reaches one, we have $P(1) = P^1$ with P^1 characterized in (A.8). Because $P(\theta) = \hat{V}'(\theta)$ on $(\underline{\theta}, \overline{\theta})$, we have $P(\overline{\theta}) = \hat{V}'(\overline{\theta})$.

At $\theta = \overline{\theta}$ with $\dot{\theta} = 0$, the randomization rate $\overline{\xi}$ is such that $P(\overline{\theta}) = \hat{V}'(\overline{\theta})$ satisfies the pricing equation

$$(\rho + \Lambda)P(\overline{\theta}) = \mu - c(\overline{\theta}) + \Lambda V^D + \Lambda[a(\overline{\theta}) + m(\overline{\theta})](\Delta - B(\theta)) + \overline{\xi}(P(1) - P(\overline{\theta}))$$
$$= \mu - \frac{\phi m(\theta)^2}{2} + \Lambda V^D + \Lambda[a(\overline{\theta}) + m(\overline{\theta})]\Delta + \overline{\xi}(P(1) - P(\overline{\theta}))$$

which can be rewritten as $P(\overline{\theta}) = \frac{(\Lambda + \rho)\hat{P}(\overline{\theta}) + \overline{\xi}P^1}{\Lambda + \rho + \overline{\xi}}$ so that

$$\overline{\xi} = \frac{(\Lambda + \rho)[\hat{P}(\overline{\theta}) - P(\overline{\theta})]}{P(\overline{\theta}) - P^1} = \frac{(\Lambda + \rho)[P(\overline{\theta}) - \hat{P}(\overline{\theta})]}{P^1 - P(\overline{\theta})}. \tag{A.21}$$

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with $\hat{P}(\theta)$ for (A.15). Note that $\dot{\theta} > 0$ and $\hat{P}(\theta) < P(\theta)$ in a left-neighbourhood of $\bar{\theta}$, so $\hat{P}(\bar{\theta}) < P(\bar{\theta})$. Because $\hat{V}(\theta)$ is strictly convex and $\hat{V}(\bar{\theta}) = \hat{V}(1) - (1 - \bar{\theta})P^1$, we have $P(\bar{\theta}) < P^1$. As a result, when $1 > \bar{\theta} > \theta^C$, the randomization rate $\bar{\xi}$ from (A.21) is well-defined and strictly positive.

A.3.6 Part VI: Lower Bound of θ

Given the trading behavior we have solved for, we can characterize a lower bound $\theta_{\min} := \inf\{\theta_t : \theta_t > 0\}$ on θ, in that $\theta_t \geq \theta_{\min}\mathbb{I}\{\theta_t > 0\}$ at all times $t \geq 0$ where $\mathbb{I}\{\cdot\}$ is the indicator function. First, consider $\tilde{\theta}_0 \geq \theta^C$. Then, due to $\theta_0 \geq \tilde{\theta}_0 \geq \theta^C$, the state variable does not take any value within the interval $(0, \theta^C)$, so $\theta_{\min} = \tilde{\theta}_0$. Second, consider $\tilde{\theta}_0 < \theta^C$. Then, the state variable does not take any value in $(0, \min\{\underline{\theta}, \tilde{\theta}_0\})$, and $\theta_{\min} = \min\{\underline{\theta}, \tilde{\theta}_0\}$. Overall,

$$\theta_{\min} := \inf\{\theta_t : \theta_t > 0\} = \min\{\tilde{\theta}_0, \underline{\theta}\}\mathbb{I}\{\tilde{\theta}_0 < \theta^C\} + \tilde{\theta}_0\mathbb{I}\{\tilde{\theta}_0 \geq \theta^C\}. \tag{A.22}$$

Next, note that $\tilde{\theta}_0 < \theta^C$ requires $\theta^C > 0$ and as such $\pi > 0$. Observe that $\pi > 0$ implies $\underline{\theta} > 0$. Overall, we have that $\theta_{\min} > 0$.

B Details for the Proof of Proposition 3

The claims follow immediately from the arguments presented in the main text. Notably, for times $t \in (0, T^\beta)$, the state variable $\theta_t = \theta$ remains constant. The activist's value function $V^\beta(\theta)$ therefore solves the HJB equation

$$(\rho + \Lambda + \beta)V^\beta(\theta) = \max_{B \geq 0}\left\{\theta\left(\mu - \pi - \frac{\phi m^2}{2}\right) - \frac{\kappa a^2}{2} + \Lambda\theta\left[V^D + (a + m)\Delta\right] + \beta V(\theta)\right\},$$

subject to (9), (8), and (10). Recall that $\hat{V}(\theta)$ is the solution to

$$(\rho + \Lambda)V(\theta) = \max_{B \geq 0}\left\{\theta\left(\mu - \pi - \frac{\phi m^2}{2}\right) - \frac{\kappa a^2}{2} + \Lambda\theta\left[V^D + (a + m)\Delta\right]\right\},$$

subject to (9), (8), and (10).
    This readily implies

$$(\rho + \Lambda + \beta)V^\beta(\theta) = (\Lambda + \rho)\hat{V}(\theta) + \beta V(\theta) \iff V^\beta(\theta) = \frac{(\Lambda + \rho)\hat{V}(\theta) + \beta V(\theta)}{\Lambda + \rho + \beta}.$$

Furthermore, it follows that both before and after time T^β optimal efforts $(a(\theta), m(\theta))$ satisfy

$$(a(\theta), m(\theta)) \in \arg\max_{a,m}\left(\Lambda\Delta(a + m) - \frac{\kappa a^2}{2\theta} - \frac{\phi m^2}{2}\right) \quad \text{s.t.} \quad a = \frac{\theta(\Lambda\Delta - \phi m)}{\kappa}.$$

Thus, optimal efforts (a, m) are characterized in (14).

With $\hat{P}(\theta)$ from (A.15), the price $P^\beta(\theta)$ satisfies the pricing equation

$$P^\beta(\theta) = \frac{\mu - c + \Lambda\big[V^D + (a+m)(\Delta - B) + \beta P(\theta)\big]}{\Lambda + \rho + \beta} = \frac{(\Lambda + \rho)\hat{P}(\theta) + \beta P(\theta)}{\Lambda + \rho + \beta}. \qquad \text{(B.1)}$$

C   Details for the Proof of Proposition 4

C.1   Part I

The optimal level of $\theta_0 \geq \tilde{\theta}_0 > 0$ solves (6).

Case $\theta_0 \in [\underline{\theta}, \overline{\theta}]$. Suppose now $\theta_0 = \theta \in [\underline{\theta}, \overline{\theta}]$, so $V(\theta) = \hat{V}(\theta)$ and $P(\theta) = \hat{V}'(\theta)$. Let $1 - \tilde{\eta} = (1 - \eta)\tilde{\theta}_0/\theta_0 \leq 1 - \eta$ denote the fraction of the initial stake θ_0 which is acquired at the (pre-entry) price $P^\beta(0) = P^0$. Thus

$$\tilde{\eta} = 1 - (1 - \eta)\tilde{\theta}_0/\theta_0 = \frac{\theta_0 - \tilde{\theta}_0 + \eta\tilde{\theta}_0}{\theta_0}. \qquad \text{(C.1)}$$

and $(1 - \eta)\tilde{\theta}_0 = \theta_0\big[(1 - \eta)\tilde{\theta}_0/\theta_0\big] = \theta_0(1 - \tilde{\eta})$. Then,

$$\begin{aligned}
V^\beta(\theta_0) - K(\theta_0) &= V^\beta(\theta_0) - (1 - \eta)\tilde{\theta}_0 P^0 - \big[\theta_0 - (1 - \eta)\tilde{\theta}_0\big]P^\beta(\theta_0) \\
&= V^\beta(\theta_0) - (1 - \tilde{\eta})\theta_0 P^0 - \theta_0\tilde{\eta}P^\beta(\theta_0) \\
&= \hat{V}(\theta_0) - (1 - \tilde{\eta})\theta_0 P^0 - \theta_0\tilde{\eta}\left(\frac{(\Lambda + \rho)\hat{P}(\theta_0) + \beta P(\theta_0)}{\rho + \Lambda + \beta}\right).
\end{aligned}$$

Note that the activist enters if and only if $V^\beta(\theta_0) - K(\theta_0) \geq R$. Dividing both sides of this inequality by $\theta_0 > 0$ and defining $\tilde{R} = R/\theta_0$, we obtain that the activist enters if and only if

$$\frac{\hat{V}(\theta_0)}{\theta_0} - (1 - \tilde{\eta})P^0 - \tilde{\eta}\left(\frac{(\Lambda + \rho)\hat{P}(\theta_0) + \beta P(\theta_0)}{\rho + \Lambda + \beta}\right) \geq \tilde{R},$$

whereby (13) implies $\frac{\hat{V}(\theta)}{\theta} = \frac{\mu - \pi + \Lambda V^D}{\rho + \Lambda} + \frac{\Delta^2\Lambda^2\big(\tilde{\kappa}^2 + \phi\tilde{\kappa} + \phi^2\big)}{2\phi\tilde{\kappa}(\tilde{\kappa} + \phi)(\rho + \Lambda)}$. Straightforward but tedious calculations yield that for $\theta = \theta_0$, the activist enters as long as $\hat{F}(\theta; \beta) + \hat{U}(\theta; \beta) \geq 0$, with

$$\begin{aligned}
\hat{F}(\theta; \tilde{\eta}) &= \Delta^2\left(\frac{\phi\Lambda^2}{2(\rho + \Lambda)\tilde{\kappa}(\phi + \tilde{\kappa})^2}\right)\left(\tilde{\kappa}(1 - \tilde{\eta}) + \phi(1 - 2\tilde{\eta}) - \frac{2\beta\tilde{\kappa}\tilde{\eta}}{\Lambda + \rho + \beta}\right) \\
\hat{U}(\theta; \beta) &:= -\tilde{R} - \frac{\pi}{\Lambda + \rho}\left(1 - \frac{\beta\eta}{\Lambda + \rho + \beta}\right).
\end{aligned}$$

Note that $\tilde{\kappa} = \kappa/\theta = \kappa/\theta_0$.

Case $\theta_0 \in [0, \underline{\theta})$. Suppose $\theta = \theta_0 \in [0, \underline{\theta})$. In this case, $P(\theta) = P^0$ and $V(\theta) = \theta P^0$, as well

as $V^\beta(\theta) = \frac{(\Lambda+\rho)\hat{V}(\theta)+\beta P^0}{\Lambda+\rho+\beta}$. The activist enters if and only if

$$\frac{\hat{V}^\beta(\theta_0)}{\theta_0} - (1-\tilde{\eta})P^0 - \tilde{\eta}\left(\frac{(\Lambda+\rho)\hat{P}(\theta_0)+\beta P(\theta_0)}{\rho+\Lambda+\beta}\right) \geq \tilde{R}.$$

With $\omega := \frac{\Lambda+\rho}{\Lambda+\rho+\beta}$, this simplifies to

$$\frac{\omega\hat{V}(\theta_0)}{\theta_0} - \eta\omega\hat{P}(\theta_0) + (1-\omega)P^0 - (1-\tilde{\eta})P^0 - \tilde{\eta}(1-\omega)P^0 \geq \tilde{R}.$$

This is equivalent to $\omega\left(\frac{\hat{V}(\theta_0)}{\theta_0} - (1-\tilde{\eta})P^0 - \tilde{\eta}\hat{P}(\theta_0)\right) \geq \tilde{R}. \iff \frac{\hat{V}(\theta_0)}{\theta_0} - (1-\tilde{\eta})P^0 - \tilde{\eta}\hat{P}(\theta_0) \geq \frac{\tilde{R}}{\omega}$. It follows that the activist enters so long as $\omega\hat{F}(\theta_0;0) - \frac{\omega\pi}{\Lambda+\rho} \geq \tilde{R}$.

C.2 Part II

To begin, note that the objective function in (6), that is,

$$O(\theta_0) := V^\beta(\theta_0) - \left\{(1-\eta)\tilde{\theta}_0 P^\beta(0) + \left[\theta_0 - (1-\eta)\tilde{\theta}_0\right]P^\beta(\theta_0)\right\} \tag{C.2}$$

is differentiable on $(0,1)$ except at points $\underline{\theta}$ and $\bar{\theta}$. At these points, it exhibits a downward jump, in that $\lim_{\theta_0\uparrow\theta'} O(\theta_0) > \lim_{\theta_0\downarrow\theta'} O(\theta_0)$ for $\theta' = \underline{\theta}, \bar{\theta}$. Here, $V^\beta(\theta)$ is from (20) and $P^\beta(\theta)$ from (21), with $\hat{V}(\theta)$ from (13) and $\hat{P}(\theta)$ from (A.15).

For $\theta_0 > \tilde{\theta}_0$ to be optimal, it must be for θ in a left-neighbourhood of θ_0 that

$$O'(\theta) = (V^\beta)'(\theta) - P^\beta(\theta_0) - \left[\theta - (1-\eta)\tilde{\theta}\right](P^\beta)'(\theta) \geq 0,$$

i.e., $\lim_{\theta\uparrow\theta_0} O'(\theta) \geq 0$, with equality at θ_0 if $\theta_0 \in (\tilde{\theta}_0, 1) - \{\underline{\theta}, \bar{\theta}\}$.

In particular, if $O'(\theta) < 0$ for all $\theta \in (\tilde{\theta}_0, 1) - \{\underline{\theta}, \bar{\theta}\}$, then $\theta_0 = \tilde{\theta}_0$ is optimal, conditional on activist entry. In the remainder of the proof, we provide a sufficient parameter condition for $O'(\theta) < 0$ for all $\theta \in (\tilde{\theta}_0, 1) - \{\underline{\theta}, \bar{\theta}\}$ and therefore for optimality of $\theta_0 = \tilde{\theta}_0$.

Consider $\theta \in (\tilde{\theta}_0) - \{\underline{\theta}, \bar{\theta}\}$ and recall that $V'(\theta) = P(\theta)$. We obtain

$$O'(\theta) = \frac{(\Lambda+\rho)(\hat{V}'(\theta)-\hat{P}(\theta))}{\Lambda+\rho+\beta} - \left[\theta - (1-\eta)\tilde{\theta}\right]\frac{(\Lambda+\rho)\hat{P}'(\theta)+\beta P'(\theta)}{\Lambda+\rho+\beta}$$
$$\propto (\Lambda+\rho)\left[-\pi + \theta\left(\frac{\Lambda^2\Delta^2\phi}{(\phi+\tilde{\kappa})^2}\right)\right] - \left[\theta - (1-\eta)\tilde{\theta}\right]\left[(\Lambda+\rho)\hat{P}'(\theta)+\beta P'(\theta)\right],$$

where the proportionality sign considers a multiple of the expression by $\Lambda + \rho + \beta$. We also used (for $\tilde{\kappa} = \kappa/\theta$) that

$$\hat{V}'(\theta) - \hat{P}(\theta) = \left[-\pi + \theta\left(\frac{\Lambda^2\Delta^2\phi}{(\phi+\tilde{\kappa})^2}\right)\right],$$

which follows from (13) and (A.15) via direct calculation.

Next, calculate using (A.15):

$$\hat{P}'(\theta) = \frac{\Delta^2 \, \Lambda^2 \, \phi \, \theta \, (\kappa^2 + 3 \, \kappa \, \phi \, \theta + \phi^2 \, \theta^2)}{\kappa \, (\kappa + \phi \, \theta)^3 \, (\Lambda + \rho)}.$$

Accordingly, we obtain

$$O'(\theta) \propto -\frac{\Delta^2 \, \Lambda^2 \, \phi^2 \, (2 \, \tilde{\kappa} + \phi)}{\tilde{\kappa} \, (\tilde{\kappa} + \phi)^3} + (1 - \eta)\tilde{\theta}_0/\theta_0 \left(\frac{\Delta^2 \, \Lambda^2 \, \phi \, (\tilde{\kappa}^2 + 3 \, \tilde{\kappa} \, \phi + \phi^2)}{\tilde{\kappa} \, (\tilde{\kappa} + \phi)^3} \right)$$
$$- (\Lambda + \rho)\pi - \beta \big[\theta_0 - (1 - \eta)\tilde{\theta}_0\big] P'(\theta).$$

With $\tilde{\eta} = \frac{\theta - \tilde{\theta}_0 + \eta \tilde{\theta}}{\theta}$, we can simplify this expression to

$$O'(\theta) \propto \frac{\Delta^2 \Lambda^2 \phi}{\tilde{\kappa} \, (\tilde{\kappa} + \phi)^3} \left((1 - \tilde{\eta})(\tilde{\kappa}^2 + 3\tilde{\kappa}\phi + \phi^2) - 2\tilde{\kappa}\phi - \phi^2 \right) - (\Lambda + \rho)\pi - \beta \theta_0 \tilde{\eta} P'(\theta).$$

As $1 - \tilde{\eta} \leq 1 - \eta$ and $\tilde{\eta} \geq \eta$, we obtain that $O'(\theta) < 0$ when

$$\frac{\Delta^2 \Lambda^2 \phi}{\tilde{\kappa} \, (\tilde{\kappa} + \phi)^3} \left[\eta \phi^2 + \tilde{\kappa}\phi(3\eta - 1) - (1 - \eta)\tilde{\kappa}^2 \right] - (\Lambda + \phi)\pi - \beta \theta_0 \tilde{\eta} P'(\theta) < 0,$$

that is, if

$$\eta \phi^2 + \tilde{\kappa}\phi(3\eta - 1) - (1 - \eta)\tilde{\kappa}^2 \geq 0 \iff \eta \geq \frac{\tilde{\kappa}(\tilde{\kappa} + \phi)}{\phi^2 + 3\tilde{\kappa}\phi + \tilde{\kappa}^2}.$$

Since

$$\frac{\partial}{\partial \tilde{\kappa}} \left(\frac{\tilde{\kappa}(\tilde{\kappa} + \phi)}{\phi^2 + 3\tilde{\kappa}\phi + \tilde{\kappa}^2} \right) \propto (\phi^2 + 3\tilde{\kappa}\phi + \tilde{\kappa}^2)(2\tilde{\kappa} + \phi) - (\tilde{\kappa}^2 + \tilde{\kappa}\phi)(3\phi + 2\tilde{\kappa}) = \phi^3 + 2\tilde{\kappa}^2\phi + 2\tilde{\kappa}\phi^2 > 0,$$

a sufficient condition for $O'(\theta) < 0$ and thus for $\theta_0 = \tilde{\theta}_0$ being optimal is $\eta \geq \frac{\kappa(\kappa + \phi)}{\phi^2 + 3\kappa\phi + \kappa^2}$.

D Other Results in Main Text

D.1 Proof of Proposition 5

The first claim follows from the closed-form expressions for effort (14), noting that $\tilde{\kappa} = \kappa/\theta$ and $\frac{\tilde{\kappa}}{\phi + \tilde{\kappa}}$ increases in $\tilde{\kappa}$ (decreases in θ), while $\frac{1}{\tilde{\kappa}(\phi + \tilde{\kappa})}$ decreases in $\tilde{\kappa}$ (increases in θ).

Next, calculate

$$a + m = \frac{\Delta \, \Lambda \, (\tilde{\kappa}^2 + \phi^2)}{\tilde{\kappa} \, (\tilde{\kappa} + \phi)}.$$

Note that under passive ownership, $\lim_{\tilde{\kappa} \to \infty}(a + m) = \frac{\Delta \Lambda}{\phi}$ and $a + m - \lim_{\tilde{\kappa} \to \infty}(a + m) = \frac{\Delta \Lambda (\phi - \tilde{\kappa})}{\tilde{\kappa} \, (\tilde{\kappa} + \phi)}$. Thus, activism with stake θ strictly increases transition rate $\lambda = \Lambda(a + m)$ relative to passive ownership ($\theta = 0$) if $\tilde{\kappa} < \phi$, i.e., $\theta > \kappa/\phi$ which requires $\kappa < \phi$ since $\theta \leq 1$. Activism reduces transition rate if $\tilde{\kappa} > \phi$, i.e., $\theta < \kappa/\phi$.

Finally, calculate $\text{sgn}\left(\frac{\partial \lambda}{\partial \tilde{\kappa}}\right) = \text{sgn}\left(\tilde{\kappa}^2 - 2\tilde{\kappa}\phi - \phi^2\right)$. Because an increase in θ implies an decrease in $\tilde{\kappa}$, we obtain $\text{sgn}\left(\frac{\partial \lambda}{\partial \theta}\right) = \text{sgn}\left(-\tilde{\kappa}^2 + 2\tilde{\kappa}\phi + \phi^2\right)$ Rewrite $-\tilde{\kappa}^2 + 2\tilde{\kappa}\phi + \phi^2$ as $-\kappa^2 + 2\theta\kappa\phi + \theta^2\phi^2$. Then, solve $\kappa^2 - 2\theta\kappa\phi - \theta^2\phi^2 = 0$ for θ to obtain that the only positive root is $\theta_{Root} = \frac{\kappa(\sqrt{2}-1)}{\phi} < \frac{\kappa}{\phi}$. As such, $\frac{\partial \lambda}{\partial \theta} > 0$, if and only if $\theta > \theta_{Root}$.

D.2 Proof of Lemma 1

Recall that of $\lambda = \lambda(\theta) > \lim_{\theta \to 0} \lambda(\theta) = \lim_{\tilde{\kappa} \to \infty} \lambda$, we require $\theta > \kappa/\phi$. As $\theta \in [0,1]$, this requires $\kappa < \phi$. When $\theta_t \geq \kappa/\phi$ at all times t, then activism increases state-contingent transition rate $\lambda(\theta_t)$ at any point in time t and the claim follows. Clearly, when $\phi \leq \kappa$, then $\tilde{\kappa} \geq \phi$, and activism cannot improve transition rate relative to passive ownership.

D.3 Proof of Corollary 2

Assume that $\theta_0 \in (\underline{\theta}, \overline{\theta})$. Then, the entry condition simplifies to (23). For any $\theta \in [\underline{\theta}, \overline{\theta}]$ and $\tilde{\eta} = \frac{\theta - \tilde{\theta}_0 + \eta\tilde{\theta}_0}{\theta}$, we have (in the smooth trading region)

$$\frac{\partial[\hat{F}(\theta;\beta) + \hat{U}(\theta;\beta)]}{\partial\beta} = \frac{1}{\Lambda + \rho + \beta}\left(2\tilde{\kappa}\tilde{\eta}\Delta^2\left(\frac{\phi\Lambda^2}{2(\rho+\Lambda)\tilde{\kappa}(\phi+\tilde{\kappa})^2}\right) - \frac{\tilde{\eta}\pi}{\Lambda+\rho}\right) = \frac{\Delta^2\Lambda^2\phi}{(\phi+\tilde{\kappa})^2} - \pi = P'(\theta)\dot{\theta},$$

which has the same sign as $\theta - \theta^C$. Thus, $\text{sgn}\left(\frac{\partial[\hat{F}(\theta;\beta)+\hat{U}(\theta;\beta)]}{\partial\beta}\right) = \text{sgn}(\theta - \theta^C)$ holds for any $\theta \in [\underline{\theta}, \overline{\theta}]$. It follows that $\text{sgn}\left(\frac{\partial E(\theta_0)}{\partial\beta}\right) = \text{sgn}(\theta_0 - \theta^C)$.

D.4 Proof of Corollary 3

It follows from (A.19) that

$$\dot{\theta} = \frac{\kappa(\Lambda+\rho)(\kappa+\theta\phi)\left[\Lambda^2\Delta^2\theta^2\phi - \pi(\kappa+\theta\phi)^2\right]}{\Lambda^2\Delta^2\theta\phi\left(\theta^2\phi^2 + 3\theta\kappa\phi + 3\kappa^2\right)} = \frac{\kappa(\Lambda+\rho)(\kappa+\phi\theta)\left[\theta^2\phi - \frac{\pi}{\Delta^2\Lambda^2}(\kappa+\phi\theta)^2\right]}{3\kappa^2\phi\theta + 3\kappa\phi^2\theta^2 + \phi^3\theta^3}.$$

Thus, $\frac{\partial\dot{\theta}}{\partial\Delta} > 0$ when $\pi > 0$; otherwise, $\frac{\partial\dot{\theta}}{\partial\Delta} = 0$.

Next, (19) immediately implies that $\frac{\partial\theta^C}{\partial\Delta} < 0$ for $\pi > 0$; otherwise, when $\pi = 0$, $\frac{\partial\theta^C}{\partial\Delta} = 0$.

Finally, when $\underline{\theta} \in (0,1)$, then according to (A.9):

$$\underline{\theta} = \frac{\kappa\left(\sqrt{\phi\pi\left(2\Delta^2\Lambda^2 + \phi\pi\right)} + \phi\pi\right)}{\Delta^2\Lambda^2\phi} = \frac{\kappa\left(\sqrt{\phi\pi\left(2\Lambda^2/\Delta + \phi\pi/\Delta^4\right)} + \phi\pi/\Delta^2\right)}{\Lambda^2\phi}.$$

Thus, $\frac{\partial\underline{\theta}}{\partial\Delta} < 0$ for $\pi > 0$; otherwise, when $\pi = 0$, $\frac{\partial\underline{\theta}}{\partial\Delta} = 0$.

D.5 Proof of Proposition 6

Consider case 1, i.e., $\lim_{\Delta \uparrow \Delta^E} \overline{\lambda}_0(\Delta) > \lambda^P$ and $\chi < 0 < U$. The activist enters as long as $\Delta \leq \Delta^E$ and does not enter for $\Delta > \Delta^E > 0$. As such, $\lim_{\Delta \uparrow \Delta^E} \overline{\lambda}_0(\Delta) > \lim_{\Delta \downarrow \Delta^E} \overline{\lambda}_0(\Delta) = \lambda^P$, which was to show. Consider case 2, i.e., $\lim_{\Delta \uparrow \Delta^E} \overline{\lambda}_0(\Delta) < \lambda^P$ and $\chi < 0 < U$. and

$\chi > 0 > U$. The activist enters as long as $\Delta \geq \Delta^E$ and does not enter for $\Delta < \Delta^E$; we could break the ties alternatively by assuming that the activist only enters if $\Delta > \Delta^E$. As such, $\lim_{\Delta \uparrow \Delta^E} \overline{\lambda}_0(\Delta) = \lambda^P > \lim_{\Delta \downarrow \Delta^E} \overline{\lambda}_0(\Delta)$, which was to show.

D.6 Proof of Proposition 7

We start by calculating (for $t < T$):

$$
\begin{aligned}
\mathcal{G}_t &= \mathbb{E}_t \left[\int_t^\infty e^{-\rho(u-t)} \left(\pi^S \rho \mathcal{I}_u + \frac{\kappa a_u^2 + \phi m_u^2}{2} \right) du \right] \\
&= \pi^S - \int_t^\infty e^{-(\rho+\Lambda)(u-t)} \left(\lambda_u \pi^S - \frac{\kappa a_u^2 + \phi m_u^2}{2} \right) du.
\end{aligned}
$$

Next, define $\mathcal{E}_t := \pi^S - \mathcal{G}_t$. Minimizing $\mathcal{G} = \mathcal{G}_0$ is equivalent to maximizing $\mathcal{E}_0$.

Consider the regulator's auxiliary optimization $\max_{(a_t, m_t)_{t \geq 0}} \mathcal{E}_0$. It is clear that the solution features time-stationary effort levels (for $t < T$) with $a_t = a^*$ and $m_t = m^*$. In particular,

$$
(a^*, m^*) := \arg \max_{a,m \geq 0} \left\{ \Lambda(a + m)\pi^S - \left(\frac{\kappa a^2 + \phi m^2}{2} \right) \right\}. \tag{D.1}
$$

The solution to (D.1) is $a^* = \frac{\Lambda \pi^S}{\kappa}$ and $m^* = \frac{\Lambda \pi^S}{\phi}$.

In first best, optimal efforts satisfy $a_t = a^{FB} = \frac{\Lambda \Delta}{\kappa}$ and $m_t = m^{FB} = \frac{\Lambda \Delta}{\phi}$. Setting $\pi^S = \Delta$ implements $x^{FB} = x^*$ for $x = a, m$ and thus minimizes $\mathcal{G}$.

D.7 Proof of Proposition 8

Conditional on activist entry, the solution is stationary with constant activist stake θ and constant effort levels characterized in (14). As argued in the proof of Proposition 7, Appendix F.2, and the main text, minimizing $\mathcal{G}$ is equivalent to maximizing

$$
\mathcal{E}_0 = \int_0^\infty e^{-(\rho+\Lambda)t} \left(\lambda_t \pi^S - \frac{\kappa a_t^2 + \phi m_t^2}{2} \right) dt,
$$

which under $\beta = 0$ simplifies to

$$
\mathcal{E}_0 = \frac{1}{\Lambda + \rho} \left\{ \Lambda(a + m)\pi^S - \left(\frac{\kappa a^2 + \phi m^2}{2} \right) \right\}.
$$

Provided that (6) does not bind and holding $\theta = \tilde{\theta}_0$ fixed, optimal $\Delta^G = \Delta$ solves the first order condition

$$
\pi^S \frac{\partial \lambda}{\partial \Delta} - \kappa a \frac{\partial a}{\partial \Delta} - \phi m \frac{\partial m}{\partial \Delta} = 0.
$$

Thus, $(\Lambda \pi^S - \kappa a)\frac{\partial a}{\partial \Delta} + (\Lambda \pi^S - \phi m)\frac{\partial m}{\partial \Delta} = 0$. Using (14), the first-order condition becomes

$$
\left(\pi^S - \frac{\theta \phi \Delta}{\phi + \tilde{\kappa}} \right) \frac{\phi}{\tilde{\kappa}} \left(\frac{1}{\phi + \tilde{\kappa}} \right) + \left(\pi^S - \frac{\tilde{\kappa} \Delta}{\phi + \tilde{\kappa}} \right) \frac{\tilde{\kappa}}{\phi} \left(\frac{1}{\phi + \tilde{\kappa}} \right) = 0.
$$

This is equivalent to

$$\phi^2 \left(\pi^S(\phi + \tilde{\kappa}) - \theta\phi\Delta \right) + \tilde{\kappa}^2 \left(\pi^S(\phi + \tilde{\kappa}) - \tilde{\kappa}\Delta \right) = 0,$$

which we solve for

$$\Delta^G = \Delta = \pi^S \left(\frac{(\phi^2 + \tilde{\kappa}^2)(\phi + \tilde{\kappa})}{\theta\phi^3 + \tilde{\kappa}^3} \right).$$

Clearly, for $\phi, \tilde{\kappa} \in (0, \infty)$, we have $\frac{(\phi^2 + \tilde{\kappa}^2)(\phi + \tilde{\kappa})}{\theta\phi^3 + \tilde{\kappa}^3} > 1$, so that $\pi^S \left(\frac{(\phi^2 + \tilde{\kappa}^2)(\phi + \tilde{\kappa})}{\theta\phi^3 + \tilde{\kappa}^3} \right) > \pi^S$.

D.8   Proof of Proposition 9

Conditional on activist entry, the solution is time-stationary with constant activist stake θ and constant effort levels characterized in (14). As shown in Proposition 8, it is optimal to set $\Delta^G = \Delta_*^G$ when (6) does not bind. When (6), we obtain under the stipulated conditions that $\Delta = \Delta^G = \Delta^E$. As a result, $\Delta^G = \min\{\Delta^E, \Delta_*^G\}$.

D.9   Proof of Proposition 10

As argued in the proof of Proposition 7, Appendix F.2, and the main text, minimizing $\mathcal{G}$ is equivalent to maximizing

$$\mathcal{E}_0 = \int_0^\infty e^{-(\rho + \Lambda)t} \left(\lambda_t \pi^S - \frac{\kappa a_t^2 + \phi m_t^2}{2} \right) dt.$$

with $a_t = a(\theta_t)$ and $m_t = m(\theta_t)$ characterized in (14). We can calculate

$$\mathcal{E}_0 = \frac{\lambda(\theta_0)\pi^S - \frac{\kappa a(\theta_0)^2 + \phi m(\theta_0)^2}{2}}{\Lambda + \rho + \beta} + \frac{\beta}{\Lambda + \rho + \beta} \left(\int_{T^\beta}^\infty e^{-(\rho + \Lambda)(t - T^\beta)} \left(\lambda_t \pi^S - \frac{\kappa a_t^2 + \phi m_t^2}{2} \right) dt \right).$$

Next, we introduce

$$\delta(\theta) := \frac{\partial}{\partial \Delta} \left\{ \Lambda(a + m)\pi^S - \left(\frac{\kappa a^2 + \phi m^2}{2} \right) \right\}.$$

Using (14), we can calculate $\delta(\theta) = \pi^S \frac{\partial \lambda}{\partial \Delta} - \kappa a \frac{\partial a}{\partial \Delta} - \phi m \frac{\partial m}{\partial \Delta} = 0$ and by means of (14):

$$\delta(\theta) = \pi^S \left(\frac{\Lambda \left(\tilde{\kappa}^2 + \phi^2 \right)}{\tilde{\kappa} \left(\tilde{\kappa} + \phi \right)} \right) - \frac{\theta\phi^2}{\tilde{\kappa}} \left(\frac{\Lambda^2 \Delta}{(\phi + \tilde{\kappa})^2} \right) - \frac{\tilde{\kappa}^2}{\phi} \left(\frac{\Lambda^2 \Delta}{(\phi + \tilde{\kappa})^2} \right).$$

The term $\left(\frac{\Lambda \left(\tilde{\kappa}^2 + \phi^2 \right)}{\tilde{\kappa} \left(\tilde{\kappa} + \phi \right)} \right)$ strictly increases in θ (i.e., decreases in $\tilde{\kappa}$) for $\tilde{\kappa} < \phi$, i.e., for $\kappa < \phi$ and $\theta > \kappa/\phi$. Therefore, when Λ is sufficiently small, the first term "dominates" and it follows that $\delta'(\theta) > 0$ for $\tilde{\kappa} < \phi$ (which is assumed).

Suppose that $\Delta^P = \Delta^E$, i.e., (6). Then, U does not depend on β, while χ decreases with β (see Proposition 4). It follows that when $U \geq 0 > \chi$, $\Delta^E = \sqrt{\frac{-U}{\chi}}$ decreases with β and

strictly so for $U > 0$.

Next, suppose that $\Delta^P < \Delta^E$, i.e., (6) does not bind in optimum. As $\pi = 0$, the level of Δ does not affect the activist's trading, in that $\frac{\partial x}{\partial \Delta} = 0$ for $x \in \{\dot{\theta}, \overline{\theta}, \underline{\theta}, \theta^C\}$. Furthermore, $\pi = 0$ implies that after time T^β, $\dot{\theta} > 0$ and the activists stake $\theta_t = \theta$ gradually increases over time. As such, $\delta(\theta_t)$ increases over time after T^β. That is, for any times $t' \geq t$, we obtain $\theta_{t'} \geq \theta_t$ and $\delta(\theta_{t'}) \geq \delta(\theta_t)$, where the inequalities may be strict after T^β.

Because (6) does not bind (and assuming $\Delta^G < \overline{\Delta}$), we obtain that Δ^G solves the first-order condition $\frac{\partial \mathcal{E}_0}{\partial \Delta} = 0$, that is,

$$\frac{1}{\Lambda + \rho + \beta} \left\{ \frac{\partial}{\partial \Delta} \left(\lambda(\theta_0)\pi^S - \frac{\kappa a(\theta_0)^2 + \phi m(\theta_0)^2}{2} \right) \right.$$
$$\left. + \beta \left(\int_{T^\beta}^\infty e^{-(\rho+\Lambda)(t-T^\beta)} \frac{\partial}{\partial \Delta} \left(\lambda_t \pi^S - \frac{\kappa a_t^2 + \phi m_t^2}{2} \right) dt \right) \right\} = 0.$$

where it was used that Δ does not affect trading strategy. We can rewrite the first order condition as

$$\frac{1}{\Lambda + \rho + \beta} \left\{ \delta(\theta_0) + \beta \left(\int_{T^\beta}^\infty e^{-(\rho+\Lambda)(t-T^\beta)} \delta(\theta_t) \right) \right\} = 0.$$

Note that $\delta(\theta_t) \geq \delta(\theta_0)$. As, in addition, there exists an interval of times after time T^β on which $\theta_t > \theta_0$ and $\delta(\theta_t) > \delta(\theta_0)$, it follows that $\frac{1}{\Lambda + \rho + \beta} \left\{ \delta(\theta_0) + \beta \left(\int_{T^\beta}^\infty e^{-(\rho+\Lambda)(t-T^\beta)} \delta(\theta_t) \right) \right\}$ increases in β (given Δ) and therefore Δ^G increases in β.

E   Proofs of Proposition 11 and Corollary 4

E.1   HJB Equation and Efforts after time T^β

Consider that $t \geq T^\beta$, i.e., the activist can trade in the firm's stock. As in the baseline, there exists a smooth trading region $(\underline{\theta}, \overline{\theta})$. To avoid studying degenerate cases, we assume (as in the main text) that this smooth trading region is non-empty in that $0 \leq \underline{\theta} < \overline{\theta} < 1$.

With firm-level investment subsidies, the activist's HJB equation in the smooth trading region changes to

$$(\rho + \Lambda)V(\theta) = \max_{B \geq 0, \dot{\theta}} \left\{ \theta \left(\mu + \frac{\phi m^2 s}{2} - \pi - c \right) - \frac{\kappa a^2}{2} \right. \tag{E.1}$$
$$\left. + \Lambda\theta \left[V^D + (a+m)(\Delta - B) \right] + \dot{\theta} \left[V'(\theta) - P(\theta) \right] \right\},$$

subject to (9), (8), and (10); given the incentive condition (8), we have $\hat{m} = m$. Equation (E.1) differs from the HJB equation in the baseline, i.e., (11), only in that the firm's cash flow rate is now augmented by the firm-level subsidy $\frac{\phi m^2 s}{2}$. As before, we have for an interior solution $\dot{\theta} \in (-\infty, \infty)$ that $V'(\theta) = P(\theta)$, that is, (12) holds in the smooth trading region.

Inserting (10) and using $V'(\theta) = P(\theta)$ simplifies to

$$(\rho + \Lambda)V(\theta) = \max_{B \geq 0} \left\{ \theta \left(\mu - \pi - \frac{\phi m^2 (1 - s)}{2} \right) - \frac{\kappa a^2}{2} + \Lambda \theta \left[V^D + (a + m)\Delta \right] \right\}, \quad \text{(E.2)}$$

subject to (8), i.e., $m = \frac{\Lambda B}{\phi}$.

The first-order condition with respect to m yields (with $\frac{\partial B}{\partial m} = \frac{\phi}{\Lambda}$ and $\frac{\partial a}{\partial m} = -\frac{\theta\phi}{\kappa}$):

$$\theta \left(-\phi(1 - s)m + \Lambda \left(1 - \frac{\phi}{\kappa} \right) \Delta \right) + \theta\phi a = 0. \quad \text{(E.3)}$$

Observe that according to (9): $a = \frac{\theta\Lambda(\Delta - B)}{\kappa} = \frac{\theta(\Lambda\Delta - \phi m)}{\kappa}$. Consequently, the first-order condition (E.3) simplifies (for $\theta > 0$): $-\phi(1 - s)m + \Lambda \left(1 - \frac{\phi}{\kappa} \right) \Delta + \frac{\theta(\Lambda\Delta - \phi m)}{\kappa} = 0$. Denoting $\tilde{\kappa} = \kappa/\theta$, we can solve $a = a(\theta) = \frac{\Delta\Lambda(\phi - \tilde{\kappa} s)}{\tilde{\kappa}(\tilde{\kappa}(1-s)+\phi)}$ and $m = m(\theta) = \frac{\Delta\Lambda\tilde{\kappa}}{\phi(\tilde{\kappa}(1-s)+\phi)}$, which is (30). Using (10), the manager's flow wage is

$$c(\theta) = \frac{\phi m(\theta)^2}{2} - \Lambda(a(\theta) + m(\theta))B(\theta) \quad \text{with} \quad B(\theta) = \frac{\phi m(\theta)}{\Lambda}. \quad \text{(E.4)}$$

Accordingly, the transition rate $\lambda = \Lambda(a + m)$ satisfies

$$\lambda = \lambda(\theta) = \frac{\Delta\Lambda^2 \left(\tilde{\kappa}^2 + \phi^2 - \tilde{\kappa}\phi s \right)}{\phi\tilde{\kappa} \left(\tilde{\kappa}(1-s) + \phi \right)}. \quad \text{(E.5)}$$

Thus, $\frac{\partial\lambda}{\partial s} = \frac{\Delta\Lambda^2\tilde{\kappa}(\tilde{\kappa}-\phi)}{\phi(\tilde{\kappa}(1-s)+\phi)^2}$. It follows that $\text{sgn} \left(\frac{\partial\lambda}{\partial s} \right) = \text{sgn}(\tilde{\kappa} - \phi)$.

Next, calculate $\lambda^P = \lim_{\theta \to 0} \lambda(\theta) = \frac{\Delta\Lambda^2}{\phi(1-s)}$. One can calculate $\lambda(\theta) > \lambda^P$ if and only if $\phi(1 - s) > \tilde{\kappa}(1 + s - s^2)$. That is, activism increases the transition rate if and only if $\phi(1 - s) > \tilde{\kappa}(1 + s - s^2)$. This condition implies $\phi > \tilde{\kappa}$.

Plugging optimal efforts from (30) back into the HJB equation (E.2), we obtain the solution denoted by $\hat{V}(\theta) = V(\theta)$, i.e.,

$$\hat{V}(\theta) = \frac{\theta}{\rho + \Lambda} \left(\mu - \pi + \Lambda V^D + \frac{\Delta^2\Lambda^2 \left(\kappa\phi\theta(1-s) + \kappa^2 + \theta^2\phi^2 \right)}{2\kappa\phi \left(\kappa(1-s) + \phi\theta \right)} \right). \quad \text{(E.6)}$$

Note that $V(\theta) = \hat{V}(\theta)$ in the smooth trading region. Further, $P(\theta) = \hat{V}'(\theta)$ in the smooth trading region. As in the baseline, $\hat{V}(\theta)$ is the activist's value function and payoff that would prevail absent any trading opportunities. That is, in the smooth trading region, the activist cannot capture any gains from trade.

E.2   Further Solution Steps

The further solution steps run as in the baseline. There exists a lower threshold $\underline{\theta}$. We present a closed-form solution in the general case with firm-level subsidy $s \in [0, 1]$. Define

$\tilde{\kappa} = \kappa/\theta$. We have for the activist's value function in the smooth trading region:

$$\hat{V}(\theta) = \frac{\theta}{\rho + \Lambda} \left(\mu - \pi + \Lambda V^D + \frac{\Delta^2 \Lambda^2 \left(\tilde{\kappa}\,\phi(1-s) + \tilde{\kappa}^2 + \phi^2 \right)}{2\tilde{\kappa}\,\phi\left(\tilde{\kappa}(1-s) + \phi \right)} \right) \tag{E.7}$$

and for the stock price under perpetual passive ownership:

$$P^0 = \lim_{\tilde{\kappa} \to \infty, \pi \to 0} \frac{\hat{V}(\theta)}{\theta} = \frac{1}{\rho + \Lambda} \left(\mu + \Lambda V^D + \frac{\Delta^2 \Lambda^2}{2\phi(1-s)} \right). \tag{E.8}$$

The price in the smooth trading region is $P(\theta) = \hat{V}'\theta)$ with

$$\hat{V}'(\theta) = \frac{1}{\rho + \Lambda} \left(\mu - \pi + \Lambda V^D + \frac{\Delta^2 \Lambda^2 \left[\kappa^3(1-s) + 2\,\kappa^2\,\phi\theta(1-s)^2 + 4\,\kappa\,\phi^2\,\theta^2(1-s) + 2\,\phi^3\,\theta^3 \right]}{2\,\kappa\,\phi(\kappa(1-s) + \phi\,\theta)^2} \right).$$

The trading rate within $(\underline{\theta}, \overline{\theta})$ solves as in the baseline the equation (18) and is given by

$$\dot{\theta} = \frac{\kappa\,(\Lambda + \rho)\,(\kappa(1-s) + \phi\,\theta)\left[\theta(\theta\phi - \kappa s) - \frac{\pi}{\Delta^2\Lambda^2}\,(\kappa(1-s) + \phi\,\theta)^2 \right]}{\kappa^3\,s(3s - s^2 - 2) + 3\kappa^2\phi\theta(1-s)^2 + 3\,\kappa\,\phi^2\,\theta^2(1-s) + \phi^3\,\theta^3}. \tag{E.9}$$

Stock price under perpetual and full active ownership becomes $P^1 = \hat{V}(1) + \frac{\pi + \frac{1}{2}\kappa a(1)^2}{\Lambda + \rho}$, i.e.,

$$P^1 = \frac{1}{\rho + \Lambda} \left[\mu + \Lambda V^D + \frac{\Delta^2 \Lambda^2 \left(\kappa\,\phi(1-s) + \kappa^2 + \phi^2 \right)}{2\,\kappa\,\phi\,(\kappa(1-s) + \phi)} + \frac{1}{2\kappa}\left(\frac{\Delta\,\Lambda\,(\phi\,\theta - \kappa\,s)}{(\kappa(1-s) + \phi)} \right)^2 \right].$$

Note that $\dot{\theta} = 0$ at $\theta = \theta^C$ with

$$\theta^C = \frac{\kappa\left(2\pi\phi(1-s) + \Delta^2\Lambda^2 s + \Delta\Lambda\sqrt{\Delta^2\,\Lambda^2 s^2 + 4\pi\phi(1-s)} \right)}{2(\Delta^2\Lambda^2\phi - \pi\phi^2)}.$$

The lower boundary solves $\hat{V}(\underline{\theta}) = \underline{\theta}P^0$ and, provided that $\underline{\theta} \in (0,1)$, is given by

$$\underline{\theta} = \frac{\kappa\sqrt{8\pi\Delta^2\Lambda^2\phi(1-s)^3 + (\Delta^2\Lambda^2(2-s)s + 2\pi\phi(1-s))^2} + \Delta^2\Lambda^2\kappa(2-s)s + 2\pi\kappa\phi(1-s)}{2\Delta^2\Lambda^2\phi(1-s)}.$$

The randomization rate $\underline{\xi}$ satisfies (with $P(\underline{\theta}) = \hat{V}'(\theta)$):

$$(\rho + \Lambda)P(\underline{\theta}) = \mu + \frac{\phi m(\theta)^2 s}{2} - c(\underline{\theta}) + \Lambda V^D + \Lambda[a(\underline{\theta}) + m(\underline{\theta})](\Delta - B) + \underline{\xi}(P^0 - P(\underline{\theta}))$$

$$= \mu - \frac{\phi m(\theta)^2(1-s)}{2} + \Lambda V^D + \Lambda[a(\underline{\theta}) + m(\underline{\theta})]\Delta + \underline{\xi}(P^0 - P(\underline{\theta}))$$

$$= (\rho + \Lambda)\hat{P}(\theta) + \underline{\xi}(P^0 - P(\underline{\theta}))$$

with

$$\hat{P}(\theta) = \mu - \frac{\phi m(\theta)^2(1-s)}{2} + \Lambda V^D + \Lambda[a(\underline{\theta}) + m(\underline{\theta})]\Delta.$$

Thus, as in (A.20), we get

$$\underline{\xi} = \frac{(\Lambda + \rho)\big[\hat{P}(\underline{\theta}) - P(\underline{\theta})\big]}{P(\underline{\theta}) - P^0}. \tag{E.10}$$

The upper boundary, when interior, solves $\hat{V}(\overline{\theta}) = \hat{V}(1) - (1 - \overline{\theta})P^1$, with $P^1 = \hat{P}(1)$. Provided that $\overline{\theta} \in (0, 1)$, the closed-form expression is $\overline{\theta} = N/D$ with

$$
\begin{aligned}
N :=& \kappa s \left(\Delta^2 \Lambda^2 \left(2\kappa^2 + 4\kappa\phi - \phi^2\right) - 4\pi\kappa\phi(\kappa + \phi)\right) + \phi \left(2\pi\kappa(\kappa + \phi)^2 - \Delta^2\Lambda^2 \left(\kappa^2 + \kappa\phi - \phi^2\right)\right) \\
&+ \Delta^2\kappa^3\Lambda^2 s^3 - \kappa^2 s^2 \left(\Delta^2\Lambda^2(3\kappa + \phi) - 2\pi\kappa\phi\right) - \bigg[\Big(\phi \left(\Delta^2\Lambda^2 \left(\kappa^2 + \kappa\phi - \phi^2\right) - 2\pi\kappa(\kappa + \phi)^2\right) \\
&- \Delta^2\kappa^3\Lambda^2 s^3 + \kappa^2 s^2 \left(\Delta^2\Lambda^2(3\kappa + \phi) - 2\pi\kappa\phi\right) - \kappa s \left(\Delta^2\Lambda^2 \left(2\kappa^2 + 4\kappa\phi - \phi^2\right) + 4\pi\kappa\phi(\kappa + \phi)\right)\Big)^2 \\
&- 4\Delta^2\kappa\Lambda^2(s - 1)\phi(\kappa - \kappa s + \phi)^2 \left(\Delta^2\Lambda^2(\phi - \kappa s)^2 + 2\pi\kappa(\kappa - \kappa - s + \phi)^2\right)\bigg]^{-1/2}
\end{aligned}
$$

and

$$D := 2\,\Delta^2\,\Lambda^2\,\phi\left(\kappa(1 - s) + \phi\right)^2.$$

The randomization rate $\overline{\xi}$ satisfies as in (A.21):

$$\overline{\xi} = \frac{(\Lambda + \rho)\big[\hat{P}(\overline{\theta}) - P(\overline{\theta})\big]}{P(\overline{\theta}) - P^1}. \tag{E.11}$$

Finally, as in Proposition 2, the activist's value function after time T^β satisfies $V(\theta) = \theta P^0$ for $\theta \in [0, \underline{\theta})$, $V(\theta) = \hat{V}(\theta)$ for $\theta \in [\underline{\theta}, \overline{\theta}]$, and $V(\theta) = V(1) - (1 - \theta)P^1$ for $\theta \in (\overline{\theta}, 1]$.

Prior to time T^β, the value function satisfies (20). Likewise, as in Proposition 2 the stock price after time T^β satisfies $P(\theta) = V'(\theta)$ in all states θ where $V(\theta)$ is differentiable. Thus, $P(\theta) = \hat{V}'(\theta)$ for $\theta \in [\underline{\theta}, \overline{\theta}]$, $P(\theta) = P^0$ for $\theta \in [0, \underline{\theta})$, and $P(\theta) = P^1$ for $\theta \in (\overline{\theta}, 1]$. Before time T^β, stock price satisfies (21). And, before time T^β, efforts satisfy (30).

Last, fix $\theta_0 = \theta$. Then, the activist enters as long as (6) holds, that is, $V^\beta(\theta_0) - (1 - \eta)\tilde{\theta}_0 P^\beta(0) + \big[\theta_0 - (1 - \eta)\tilde{\theta}_0\big]P^\beta(\theta_0) \geq 0$ or equivalently

$$V^\beta(\theta_0) - (1 - \tilde{\eta})\theta_0 P^\beta(0) + \big[\theta_0 - (1 - \tilde{\eta})\theta_0\big]P^\beta(\theta_0) \geq 0,$$

with $\tilde{\eta} = \frac{\theta_0 - \tilde{\theta}_0 + \eta\tilde{\theta}_0}{\theta_0}$. Given our closed-form expressions, (tedious) calculations analogous to the ones underlying Proposition 4 yield the stipulated entry condition.

E.3   Proof of Corollary 4

Consider that $\tilde{\kappa} < \phi$ and that $F + U \geq 0$ when $s = 0$. Given the activist's initial stake $\theta_0 = \theta$, it follows (due to $\beta = 0$) that $\theta_t = \theta$ for all $t \in [0, T]$, so the average transition rate becomes $\overline{\lambda}_0 = \lambda = \lambda(\theta) = \Lambda(a(\theta) + m(\theta))$. Proposition 11 establishes that activism increases the transition rate, i.e., $\lambda(\theta) > \lambda^P$, when $\phi(1 - s) > \tilde{\kappa}(1 + s - s^2)$. This condition is satisfied

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(for any θ) in case $\overline{s} \geq s$ is sufficiently small. That is, when $\overline{s}$ is sufficiently small, activism increases transition rate relative to passive ownership, so that $\overline{\lambda}_0 > \overline{\lambda}_0^P = \lim_{\tilde{\kappa} \to \infty} \overline{\lambda}_0$.

According to Proposition 11, we have $\frac{\partial \lambda}{\partial s} < 0$ conditional on activist entry. Therefore, conditional on activist entry, $s = s^* = 0$ maximizes (average) transition rate $\overline{\lambda}_0$. As $s = 0$ implies by assumption $F + U \geq 0$ and induces activist entry (which implies a higher transition rate than passive ownership), we have $s^* = 0$.

Next, consider $\tilde{\kappa} > \phi$. Then, according to Proposition 11, activism leads to a lower (average) transition rate $\overline{\lambda}_0$ than would prevail under passive ownership for any $s \in [0, \overline{s}]$. An increase in s raises the transition rate both under active and passive ownership; see Proposition 11. Thus, the optimal firm-level subsidy s^* maximizing transition rate is (weakly) positive. Provided that the entry condition does not bind for $s = 0$, it follows that $s > 0$ sufficiently small does not change the activist's entry decision relative to $s = 0$. Then, $s^* > 0$.

F Auxiliary Results and Extensions

F.1 Calculating $\overline{\lambda}_t$

We show how to calculate the average transition rate

$$\overline{\lambda}_t = \mathbb{E}_t^{T^\beta} \left[\int_t^\infty e^{-\Lambda(u-t)} \Lambda \lambda_u du \right]$$

for $t < T$ with $\theta_t = \theta$. Since $\Lambda e^{-\Lambda x}$ is the density of an exponential distribution with intensity Λ, we have $\overline{\lambda}_t = \mathbb{E}_t[\lambda_T | \theta_t = \theta]$, where the expectation is with respect to the (exponentially distributed) random time T as well as the random time T^β.

After time T^β, we can express $\overline{\lambda}_t$ as a function of $\theta_t = \theta$ only, i.e., $\overline{\lambda}_t = \overline{\lambda}(\theta)$. By standard arguments, $\overline{\lambda}(\theta)$ solves on $(\underline{\theta}, \overline{\theta})$ the ODE

$$\Lambda \overline{\lambda}(\theta) = \Lambda \lambda(\theta) + \overline{\lambda}'(\theta) \dot{\theta} \tag{F.1}$$

subject to appropriate boundary conditions to be characterized.

First, when the starting value satisfies $\theta > \theta^C$, i.e., θ drifts up, ODE (F.1) is solved on $(\theta, \overline{\theta})$ subject to

$$\overline{\lambda}(\overline{\theta}) = \frac{\Lambda \lambda(\overline{\theta}) + \overline{\xi} \lambda(1)}{\Lambda + \overline{\xi}}.$$

Second, when the starting value satisfies $\theta \in (\underline{\theta}, \theta^C)$, i.e., θ drifts down, (F.1) is solved on $(\underline{\theta}, \theta)$ subject to $\overline{\lambda}(\underline{\theta}) = \frac{\Lambda \lambda(\underline{\theta}) + \underline{\xi} \lambda(0)}{\Lambda + \underline{\xi}}$. For $\theta > \overline{\theta}$, we have $\overline{\lambda}(\theta) = \lambda(1) = \Lambda(a(1) + m(1))$ and, for $\theta < \underline{\theta}$, $\overline{\lambda}(\theta) = \lambda(0) = \Lambda(a(0) + m(0))$.

For $t < T^\beta$, we have that $\overline{\lambda}_t$ is constant over time with $\overline{\lambda}_t = \overline{\lambda}_0$. We obtain that $\overline{\lambda}_0 = \overline{\lambda}_0(\theta_0) := \frac{\Lambda \lambda(\theta_0) + \beta \overline{\lambda}(\theta)}{\Lambda + \beta}$, with $\lambda(\theta) = \Lambda(a(\theta) + m(\theta))$.

F.2 Calculating $\mathcal{G}$

We start by calculating (for $t < T$):

$$\mathcal{G}_t = \mathbb{E}_t \left[\int_t^\infty e^{-\rho(u-t)} \left(\pi^S \rho \mathcal{I}_u + \frac{\kappa a_u^2 + \phi m_u^2}{2} \right) du \right]$$

$$= \pi^S - \mathbb{E}_t^{T^\beta} \left[\int_t^\infty e^{-(\rho+\Lambda)(u-t)} \left(\lambda_u \pi^S - \frac{\kappa a_u^2 + \phi m_u^2}{2} \right) du \right].$$

Next, define $\mathcal{E}_t := \pi^S - \mathcal{G}_t$. We integrated out the random time T; the expectation $\mathbb{E}_t^{T^\beta}$ is taken (at time t) with respect to the random time T^β. The value of T^β becomes deterministic for $t > T^\beta$.

After time T^β, we can express $\mathcal{E}_t$ as a function of $\theta_t = \theta$ only, i.e., $\mathcal{E}_t = \mathcal{E}(\theta)$. By standard arguments, $\mathcal{E}(\theta)$ solves on $(\underline{\theta}, \overline{\theta})$ the ODE

$$(\Lambda + \rho)\mathcal{E}(\theta) = \lambda(\theta)\pi^S - \frac{\kappa a(\theta)^2 + \phi m(\theta)^2}{2} + \mathcal{E}'(\theta)\dot{\theta} \tag{F.2}$$

subject to appropriate boundary conditions to be characterized. Define

$$\Omega(\theta) := \lambda(\theta)\pi^S - \frac{\kappa a(\theta)^2 + \phi m(\theta)^2}{2}.$$

When $\theta > \theta^C$, i.e., θ drifts up, ODE (F.2) is solved on $(\theta, \overline{\theta})$ subject to

$$\mathcal{E}(\overline{\theta}) = \frac{\Omega(\overline{\theta}) + \overline{\xi}\left(\frac{\Omega(1)}{\Lambda+\rho}\right)}{\rho + \Lambda + \overline{\xi}}.$$

When $\theta \in (\underline{\theta}, \theta^C)$, i.e., θ drifts down, (F.2) is solved on $(\underline{\theta}, \theta)$ subject to

$$\mathcal{E}(\underline{\theta}) = \frac{\Omega(\underline{\theta}) + \underline{\xi}\left(\frac{\Omega(0)}{\Lambda+\rho}\right)}{\Lambda + \rho + \underline{\xi}}.$$

For $\theta > \overline{\theta}$, we have $\mathcal{E}(\theta) = \frac{\Omega(1)}{\Lambda+\rho}$ and, for $\theta < \underline{\theta}$, $\mathcal{E}(\theta) = \frac{\Omega(0)}{\Lambda+\rho}$.

For $t < T^\beta$, we have that $\mathcal{E}_t$ is constant over time with $\mathcal{E}_t = \mathcal{E}_0$. We obtain that

$$\mathcal{E}_0 := \frac{\Omega(\theta_0) + \beta\mathcal{E}(\theta)}{\Lambda + \rho + \beta}.$$

One then obtains $\mathcal{G} = \mathcal{G}_0 = \pi^S - \mathcal{E}_0$. Minimizing $\mathcal{G}$ is equivalent to maximizing $\mathcal{E} = \mathcal{E}_0$.

F.3 Robustness: Empty Smooth Trading Region

When the smooth trading region is empty, the activist's optimal trading after time T^β involves an immediate lumpy trade either toward zero or one. In particular, in state θ, the activist either immediately buys or sells the entire firm (i.e., $d\theta \in \{-\theta, 1 - \theta\}$). Then, there

exists a cutoff point $\theta^{BS} \in [0,1]$ such that for $\theta > \theta^{BS}$ the activist optimally buys and $\theta < \theta^{BS}$ sells the entire firm, with indifference at θ^{BS} when $\theta^{BS} \in (0,1)$.

When $\theta P^0 > \hat{V}(1) - (1-\theta)P^1$ for all $\theta \in [0,1]$, then $\theta^{BS} = 1$. When $\theta P^0 < \hat{V}(1) - (1-\theta)P^1$ for all $\theta \in [0,1]$, then $\theta^{BS} = 0$. Otherwise, θ^{BS} is defined as the unique point in $[0,1]$ at which

$$\theta^{BS} P^0 = \hat{V}(1) - (1 - \theta^{BS})P^1.$$

The value function satisfies $V(\theta) = \theta P^0$ for $\theta < \theta^{BS}$ and $V(\theta) = \hat{V}(1) - (1-\theta)P^1$ for $\theta > \theta^{BS}$. When $\theta^{BS} = 1$, then $V(\theta^{BS}) = P^0$ and $P(\theta^{BS}) = P^0$. When $\theta^{BS} = 0$, then $V(\theta^{BS}) = \hat{V}(1) - P^1$ and $V(\theta^{BS}) = P^1$. When $\theta^{BS} \in (0,1)$, then $V(\theta^{BS}) = \hat{V}(1) - (1 - \theta^{BS})P^1$; without loss of generality, we may assume that in case of indifference the activist buys the entire firm, which pins down $P(\theta^{BS}) = P^1$.

F.4 Minimum Required Ownership

We now sketch the model solution when the activist's stake θ_t must at any point in time $t \geq 0$ exceed a minimum ownership level $\hat{\theta} \leq \tilde{\theta}_0$ for activist effort to have an impact. Formally, we stipulate that the probability of success at time T equals $m_t + a_t \mathbb{I}\{\theta_t \geq \hat{\theta}\}$. As such, when $\theta_t < \hat{\theta}$, the activist effort has no impact and the activist rationally does not exert any effort. As the activist values (due to disutility flow π) the firm less than passive investors, holding a stake $\theta_t \in (0, \hat{\theta})$ is clearly suboptimal, so that at no point in time the activist's stake θ_t lies in the interval $(0, \hat{\theta})$.

Note that imposing a minimum ownership requirement $\theta \geq \hat{\theta}$ has no effects on the solution to the baseline model and is entirely inconsequential in the following instances. First, when $\theta_0 \geq \min\{\theta^C, \overline{\theta}\}$ — which is certainly the case when $\tilde{\theta}_0 \geq \min\{\theta^C, \overline{\theta}\}$) — the activist starts out with initial stake $\theta_0 \geq \hat{\theta}$ and gradually acquires a larger ownership stake. Therefore, within the baseline equilibrium, we have $\theta_t \geq \hat{\theta}$.

Second, when $\hat{\theta} \leq \underline{\theta}$, then the activist's stake in the baseline never takes values in $(0, \hat{\theta})$. To see why this is the case, suppose that $\theta_0 \geq \underline{\theta}$. In this case, we have that the activist's stake never takes values in $(0, \underline{\theta})$. When, on the other hand, $\tilde{\theta}_0 \leq \theta_0 < \underline{\theta}$, the activist sells its entire stake at time T^β, while its stake is constant beforehand so that θ never takes values in $(0, \hat{\theta})$.

Otherwise, when $\hat{\theta} > \underline{\theta}$ and $\theta_0 \geq \tilde{\theta}_0 \geq \hat{\theta}$ satisfies $\theta_0 < \theta^C$, the activist gradually divests its stake after entry until its stake reaches the minimum required stake $\hat{\theta}$. Because $\hat{\theta} > \underline{\theta}$ and therefore $\hat{V}(\hat{\theta}) > \hat{\theta}P^0$, the activist is strictly better off not trading at all than selling off its entire stake at once. Then, the activist stops trading at $\hat{\theta}$ and its stake remains constant up to time T. Note that while the activist does not exit the firm in this case, the qualitative model outcomes are similar to the baseline, in that the activist gradually divests and sells its stake whenever $\theta_0 < \theta^C$, even if we impose $\theta \geq \hat{\theta}$.

We conclude that including a minimum stake requirement for the activist does not change our key findings in a qualitative sense and, in many instances of our model, has no effects at all. Indeed, as we show in Proposition 2, there exists a strictly positive (endogenous) level $\theta_{\min} > 0$ so that the activist's stake never takes any value in $(0, \theta_{\min})$ and the activist always holds non-trivial ownership of the firm. Imposing that the activist's stake must exceed $\hat{\theta} \leq \theta_{\min}$ would in fact not change our results.

F.5   Micro-foundation of Negative Reservation Utility

Suppose that the activist derives negative flow disutility $\pi^B \geq 0$ until the firm is transformed irrespective of its investment decision. This captures the activist's broad mandate as in Oehmke and Opp (2022) and implies a negative reservation utility as we now show.

To begin, note that for all times $t \geq T$, the activist derives a utility flow of $-\pi^B$ if the firm fails at transitioning at time T. Total disutility upon failure, therefore, equals $\frac{\pi^B}{\rho}$. The activist's terminal payoff upon success at time T then equals $\theta_T(V^D + \Delta)$. The activist's terminal payoff upon failure at time T equals $\theta_T V^D - \frac{\pi^B}{\rho}$.

The activist's payoff at $t < T$ can be written as[32]

$$
V_t = \max_{(a_u, c_u, B_u, d\theta_u)_{u \geq t}} \mathbb{E}_t \left[\int_t^T e^{-\rho(u-t)} \left(\theta_u(\mu - \pi - c_u)du - \pi^B du - \frac{\kappa a_u^2}{2}du - (P_u + dP_u)d\theta_u \right) \right.
$$
$$
\left. + e^{-\rho T}\theta_T \left[V^D - \frac{\pi^B}{\rho \theta_T} + (a_T + m_T)\left(\Delta - B_T + \frac{\pi^B}{\rho \theta_T} \right) \right] \right].
$$

That is, total time-t dis-utility flow before successful transition is $\theta_t \pi + \pi^B$, which captures value-alignment preference via π and broad mandate via π^B.

If the activist does not invest, then passive investors implement managerial effort $m^P = \frac{\Lambda \Delta}{\phi}$, and the activist's lifetime utility in that scenario becomes

$$
\overline{R} = \mathbb{E}_0 \left[\int_0^{T^S} e^{-\rho t}(-\pi^B)dt \right] = \frac{1}{\Lambda + \rho}\left(-\pi^B + \frac{-\pi^B \Lambda(1 - m^P)}{\rho} \right) \leq 0, \qquad \text{(F.3)}
$$

with the inequality being strict if $\pi^B > 0$. Here, T^S is the first time at which the firm becomes clean; if the firm fails to transform, then $T^S = \infty$ and otherwise $T^S = T$ whereby T arrives at exogenous intensity Λ. We can rewrite $\overline{R} = -\frac{\pi^B}{\rho} + \frac{\pi^B}{\rho}\left(\frac{\Lambda m^P}{\rho + \Lambda} \right)$. In case we do not assume a separate outside option component, the activist enters now if and only if (with $K(\theta_0)$ from (5)):

$$
E(\theta_0) := \max_{\theta_0 \in [\tilde{\theta}_0, 1]} \left[V^\beta(\theta_0) - K(\theta_0) \right] \geq \overline{R}, \qquad \text{(F.4)}
$$

with $R := \overline{R} < 0$ when $\pi^B > 0$. Therefore, assuming $\pi^B > 0$ leads effectively to a negative outside option or reservation utility for the activist, possibly incentivizing the activist to invest in the firm despite potential financial losses, i.e., $\left[V^\beta(\theta_0) - K(\theta_0) \right] < 0$.

We can solve the model of this variant as in our baseline case. As the activist's payoff has a component that does not depend on its stake, we impose for regularity purposes that the activist can affect firm performance and design the manager's contract if and only if $\theta \geq \hat{\theta}$ for some exogenous constant $\hat{\theta} \in (0, \tilde{\theta}]$. For $\theta < \hat{\theta}$, passive investors are in control and optimally set the managerial effort level m^P, while the activist's effort becomes zero.

Although the analysis will become more complicated and involved, the key economic insights remain similar. To highlight the key economic trade-offs associated with activism in a tractable manner and to afford maximum theoretical clarity, we, therefore, take the

---

[32]When $\theta_T = 0$, then we set $\theta_T\left[V^D - \frac{\pi^B}{\rho \theta_T} + (a_T + m_T)\left(\Delta - B_T + \frac{\pi^B}{\rho \theta_T} \right) \right] = 0$.

reservation value (for simplicity) as exogenous.

F.5.1 Solution Details

Effort Incentives. Suppose $\theta \geq \hat{\theta}$. As in the baseline, the manager's effort is $m_t = m(\theta) = \frac{\Lambda B_t}{\phi}$, while the activist's effort is $a_t = a(\theta) = \frac{\pi^B/\rho + \theta(\Delta - B_t)}{\kappa}$, with endogenous payment B_t to the manager in case of a successful transition.

Optimal Efforts and Value Function. Consider $\theta \geq \hat{\theta} > 0$. After time T^β, the activist's value function solves in the smooth trading region (where $d\theta = \dot{\theta}dt$) the HJB equation

$$(\rho + \Lambda)V(\theta) = \max_{B \geq 0, \dot{\theta}} \left\{ \theta \left(\mu - \pi - c \right) - \frac{\kappa a^2}{2} - \pi^B \right.$$
$$\left. + \Lambda \theta \left[V^D - \frac{\pi^B}{\rho\theta} + (a + m)\left(\Delta - B + \frac{\pi^B}{\rho\theta} \right) \right] + \dot{\theta} \left[V'(\theta) - P(\theta) \right] \right\},$$

For $\dot{\theta} \in (-\infty, +\infty)$ to be optimal, we must have $V'(\theta) = P(\theta)$. For convenience, define $\Pi^B := \pi^B/\rho$.

We can solve the optimization in the HJB equation for $m(\theta) = \frac{\Lambda\kappa\left(\Pi^B + \Delta\theta\right)}{\phi\theta(\kappa + \phi\theta)}$ and $a(\theta) = \frac{\Lambda\phi\theta\left(\Pi^B + \Delta\theta\right)}{\kappa^2 + \phi\theta\kappa}$. That is, for $\tilde{\Delta} := \Delta + \Pi^B/\theta$ and $\tilde{\kappa} := \kappa/\theta$ (assuming these are well-defined), efforts satisfy

$$m = m(\theta) = \frac{\tilde{\kappa}}{\phi}\left(\frac{\Lambda\tilde{\Delta}}{\phi + \tilde{\kappa}} \right) \quad \text{and} \quad a = a(\theta) = \frac{\phi}{\tilde{\kappa}}\left(\frac{\Lambda\tilde{\Delta}}{\phi + \tilde{\kappa}} \right). \tag{F.5}$$

Note that the above expression for $m(\theta)$ tends to ∞ as $\theta \to 0$. The reason is that when $\theta = 0$, the activist does not own the firm anymore and does not incur the cost of compensating the manager for its effort (which is borne by the firm's passive owners). Because of its broad impact preferences, however, the activist still benefits from green transition despite $\theta = 0$. Hence, the activist would like to implement arbitrarily high managerial effort (and set effort to the upper boundary, i.e., $m(\theta) = \overline{m}$), if it could choose the managerial contract terms despite zero ownership. We preclude this possibility and ensure optimal interior efforts $(a, m) \in (0, \overline{a}) \times (0, \overline{m})$ by assuming that the activist must maintain a minimum stake of $\hat{\theta}$ to be able to affect firm performance and to set the managerial contract terms. When $\theta < \hat{\theta}$, the firm's passive owners set managerial contract terms and optimally implement $m(\theta) = m^P$, while the activist's effort has no more impact and therefore (optimally) equals $a(\theta) = 0$. Thus, above expression for $m(\theta)$ and $a(\theta)$ only applies for $\theta \geq \hat{\theta}$. The assumption of the minimum required stake is not material to our findings.

Defining $\tilde{V}^D := V^D - \Pi^B/\theta$, $\tilde{\mu} = \mu - \pi^B/\theta$ (assuming these are well-defined), we obtain the following closed-form expression for the value function $V(\theta)$ in the smooth trading region, with $V(\theta) = \hat{V}(\theta)$ and

$$\hat{V}(\theta) = \theta \left(\frac{\tilde{\mu} - \pi + \Lambda\tilde{V}^D}{\rho + \Lambda} + \frac{\tilde{\Delta}^2\Lambda^2\left(\tilde{\kappa}^2 + \phi\tilde{\kappa} + \phi^2\right)}{2\phi\tilde{\kappa}\left(\tilde{\kappa} + \phi\right)(\rho + \Lambda)} \right). \tag{F.6}$$

Note that $\theta \geq \hat{\theta} > 0$, so that the above expressions are well-defined.

Dynamic Trading. As in the baseline, we can differentiate the HJB equation under the envelope theorem to obtain (with $P(\theta) = V'(\theta)$):

$$(\rho + \Lambda)P(\theta) = \mu - \pi - c + \Lambda\left[V^D + (a + m)(\Delta - B)\right] - \theta\left(\frac{\partial c}{\partial a}\frac{\partial a}{\partial \theta}\right). \qquad \text{(F.7)}$$

In addition, the price $P(\theta)$ satisfies the pricing equation (17). Combining, we obtain that $\dot{\theta}$ satisfies (18). A closed-form expression is obtained by

$$\dot{\theta} = -\frac{\kappa\,\phi\,\theta^2\,(\kappa + \phi\,\theta)\,(\Lambda + \rho)\,\left(-\Delta^2\,\Lambda^2\,\phi\,\theta^3 + \Pi^B\Delta\Lambda^2(\kappa\theta - \phi\theta^2) + \Lambda^2\,(\Pi^B)^2\,\kappa + \pi\,\kappa^2\,\theta + 2\,\pi\,\kappa\,\phi\,\theta^2 + \pi\,\phi^2\,\theta^3\right)}{\Lambda^2\,\left(3\,\Delta^2\,\kappa^2\,\phi^2\,\theta^4 + 3\,\Delta^2\,\kappa\,\phi^3\,\theta^5 + \Delta^2\,\phi^4\,\theta^6 + 2\,\Delta\,\Pi^B\,\kappa^2\,\phi^2\,\theta^3 + (\Pi^B)^2(\kappa^4 + 3\,\kappa^3\,\phi\,\theta + 3\,\kappa^2\,\phi^2\,\theta^2)\right)}.$$

Lumpy Trading. Define $\hat{P}(\theta)$ according to (A.15), with the appropriate values of $a(\theta), m(\theta)$ under this model variant. which is the hypothetical price of the firm under the scenario that θ remains constant up to T. Note $P^0 = \hat{P}(0)$ and $P^1 = \hat{P}(1)$. The activist trades smoothly within some endogenous interval $(\underline{\theta}, \overline{\theta})$. The activist buys the entire firm at price $P^1 = \hat{P}(1)$ for $\theta > \overline{\theta}$ and sells the entire firm at price $P^0 = \hat{P}(0)$ for $\theta < \underline{\theta}$. If $\hat{\theta} > \underline{\theta}$, the activist stops trading at $\hat{\theta}$ in case θ reaches $\hat{\theta}$ from above.

Provided $\overline{\theta} \in (0, 1)$, we have indifference between not trading and buying the entire firm at $\theta = \overline{\theta}$, i.e., $\hat{V}(\overline{\theta}) = \hat{V}(1) - (1 - \overline{\theta})P^1$. Likewise, provided $\underline{\theta} \in (0, 1)$, the activist is indifferent between not trading and selling the entire firm at $\theta = \underline{\theta}$, i.e., $\hat{V}(\underline{\theta}) = \underline{\theta}P^0 + \overline{R}$, where $\overline{R} < 0$ from (F.3) is the activist's expected discounted disutility when the firm is owned entirely by passive investors.

These two equations can be solved for the thresholds $\underline{\theta} \leq \overline{\theta}$. Akin to the baseline, we can also solve for the randomization rate, if applicable.

Thus, after time T^β, the activists value function for $\theta \in [\overline{\theta}, 1]$ is $V(\theta) = \hat{V}(1) - (1 - \theta)P^1$ and the activist's value function for $\theta \in [0, \underline{\theta}]$ is $V(\theta) = \theta P^0 + \overline{R}$. Prior to time T^β, the value function of the activist $V^\beta(\theta)$ satisfies (20), while the stock price $P^\beta(\theta)$ satisfies (21).

Entry. The activist enters at $t = 0$ if and only if (F.4) holds. Suppose that the activist's initial stake satisfies $\theta_0 = \theta \in [\underline{\theta}, \overline{\theta}]$. In this case, one can show that the activist enters at $t = 0$ as long as the sum of financial and non-pecuniary payoff is positive, that is, as long as $F + U \geq 0$, where

$$F = \hat{F}(\theta; \beta) := \Delta^2\left(\frac{\phi\Lambda^2}{2(\rho + \Lambda)\tilde{\kappa}(\phi + \tilde{\kappa})^2}\right)\left(\tilde{\kappa}(1 - \tilde{\eta}) + \phi(1 - 2\tilde{\eta}) - \frac{2\beta\tilde{\kappa}\tilde{\eta}}{\Lambda + \rho + \beta}\right)$$

$$U = \hat{U}(\theta; \beta) := \frac{\mathcal{R}}{\theta} - \frac{\pi}{\Lambda + \rho}\left(1 - \frac{\beta\tilde{\eta}}{\Lambda + \rho + \beta}\right).$$

Here, $\mathcal{R}$ is a function of $\Pi^B = \pi^B/\rho$ with $\mathcal{R} := \frac{N}{D}$; the entry condition depends on Π^B and

π^B only via $\mathcal{R}$ and our baseline is obtained for $\mathcal{R} = 0$. The numerator N satisfies

$$
\begin{aligned}
N := \Lambda^2\,\Pi^B\,\Big(&\Lambda\,\Pi^B\,\kappa^3 + \Pi^B\,\beta\,\kappa^3 + \Pi^B\,\kappa^3\,\rho + \Lambda\,\Pi^B\,\eta\,\kappa^3 + \Pi^B\,\beta\,\eta\,\kappa^3 + \Pi^B\,\eta\,\kappa^3\,\rho + 2\,\Delta\,\Lambda\,\phi^3\,\theta^4 + \Lambda\,\Pi^B\,\phi^3\,\theta^3 \\
&+ 2\,\Delta\,\beta\,\phi^3\,\theta^4 + \Pi^B\,\beta\,\phi^3\,\theta^3 + 2\,\Delta\,\phi^3\,\rho\,\theta^4 + \Pi^B\,\phi^3\,\rho\,\theta^3 \\
&+ 2\,\Lambda\,\Pi^B\,\kappa\,\phi^2\,\theta^2 - 2\,\Delta\,\beta\,\eta\,\phi^3\,\theta^4 + 2\,\Delta\,\beta\,\kappa\,\phi^2\,\theta^3 - 2\,\Delta\,\eta\,\phi^3\,\rho\,\theta^4 \\
&+ 2\,\Pi^B\,\beta\,\kappa\,\phi^2\,\theta^2 + 2\,\Delta\,\kappa\,\phi^2\,\rho\,\theta^3 + 2\,\Pi^B\,\kappa\,\phi^2\,\rho\,\theta^2 \\
&+ 2\,\Lambda\,\Pi^B\,\kappa^2\,\phi\,\theta + 2\,\Pi^B\,\beta\,\kappa^2\,\phi\,\theta + 2\,\Pi^B\,\kappa^2\,\phi\,\rho\,\theta - 2\,\Delta\,\Lambda\,\eta\,\phi^3\,\theta^4 + 2\,\Delta\,\Lambda\,\kappa\,\phi^2\,\theta^3 \\
&+ 2\,\Pi^B\,\beta\,\eta\,\kappa^2\,\phi\,\theta - 2\,\Delta\,\Lambda\,\eta\,\kappa^2\,\phi\,\theta^2 - 2\,\Delta\,\Lambda\,\eta\,\kappa\,\phi^2\,\theta^3 - 4\,\Delta\,\beta\,\eta\,\kappa\,\phi^2\,\theta^3 - 2\,\Delta\,\eta\,\kappa^2\,\phi\,\rho\,\theta^2 - 2\,\Delta\,\eta\,\kappa\,\phi^2\,\rho\,\theta^3 \Big)
\end{aligned}
$$

and the denominator satisfies $D = 2\,\kappa\,\phi\,\theta\,(\kappa + \phi\,\theta)^2\,(\Lambda + \rho)\,(\Lambda + \beta + \rho)$. Thus, when $\mathcal{R} > 0$ is sufficiently large—which happens when Π^B, i.e., $\pi^B = \rho\Pi^B$, is sufficiently large (as $\lim_{\Pi^B \to \infty} \mathcal{R} = +\infty$)—then $U > 0$ and the activist enters the firm, even if it realizes financial losses from doing so and $F < 0$.

We could introduce an (additional) exogenous outside option component R (as in the baseline), in that the activist enters if and only if $E(\theta_0) := \max_{\theta_0 \in [\tilde{\theta}_0, 1]} \left[V^\beta(\theta_0) - K(\theta_0) \right] \geq \overline{R} + R$. Then, the entry condition (for $\theta_0 = \theta \in [\underline{\theta}, \overline{\theta}]$) becomes $F + U \geq 0$ with F from above and $U = \hat{U}(\theta; \beta) := \frac{\mathcal{R} - R}{\theta} - \frac{\pi}{\Lambda + \rho}\left(1 - \frac{\beta\tilde{\eta}}{\Lambda + \rho + \beta}\right)$ and the conclusions remain unchanged.