Financing Cycles*

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Abstract

Capital ages and must eventually be replaced. We propose a theory of financing in which firms finance new capital with debt and optimally deleverage to free up debt capacity as their capital ages, thereby generating debt cycles. Concurrently, firms shorten the maturity of their debt to match the remaining life of their capital, generating maturity cycles. These firm-level financing cycles drive aggregate leverage and maturity dynamics when capital age is correlated across firms. We provide time series and cross-sectional evidence that strongly supports these independent predictions and highlights the key roles of capital age and asset life in financing cycles.

Keywords: capital structure, leverage, debt maturity, capital age, financing cycles.

JEL Classification: E32, G31, G32.

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Capital ages and must eventually be replaced (Feldstein and Rothschild, 1974). As an example, in 2011 American Airlines ordered 460 airplanes to replace its ageing fleet.[1] Large, planned replacement investments are not exclusive to airlines, but are a hallmark of real-world business operations. For instance, the aggregate replacement investments of U.S. public firms amounted to \$1.27tn in 2019—representing around 21% of their capital stock. In this paper, we argue that planned replacement investments are an important driver of financing choices that lead to debt and maturity cycles at the firm level and spill over to aggregate debt dynamics when capital age is correlated across firms.

To demonstrate how planned replacement investments fundamentally affects firm financing, we proceed in two steps. We first develop a dynamic model of investment and financing in which capital ages and firms can choose not only how much debt to issue but also the maturity of this debt. In this model, firms borrow to finance investment and optimally deleverage to free up debt capacity as capital ages, allowing them to issue new debt when old capital needs to be replaced. To achieve these dynamics, firms issue debt with a maturity that matches the useful life of new assets and a repayment schedule that reflects the need to free up debt capacity as capital ages. These dynamics lead to firm-level debt cycles (Denis and McKeon, 2012; DeAngelo, Gonçalves, and Stulz, 2018) and to a matching between debt maturity and asset life (Stohs and Mauer, 1996) and spill over to aggregate debt dynamics when capital age is correlated across firms. They also imply that both leverage and debt maturity should be negatively related to capital age while debt maturity and the length of debt cycles should be positively related to the useful life of assets. We then test these independent predictions on a large sample of listed U.S. firms over the 1975–2018 period and, as hinted by Figure 1, find strong support for all these predictions in the data.

Our model builds on prior dynamic models of firm investment and financing (Gomes, 2001; Hennessy and Whited, 2005; DeAngelo, DeAngelo, and Whited, 2011). But it differs in that capital has a finite useful life, as in e.g. Arrow (1964), Rogerson (2008), Rampini (2019), or Livdan and Nezlobin (2021), instead of being geometrically depreciated. Just

[1]See the Financial Times of July 7 2012, Procurement: Dependent on vision and strategy.

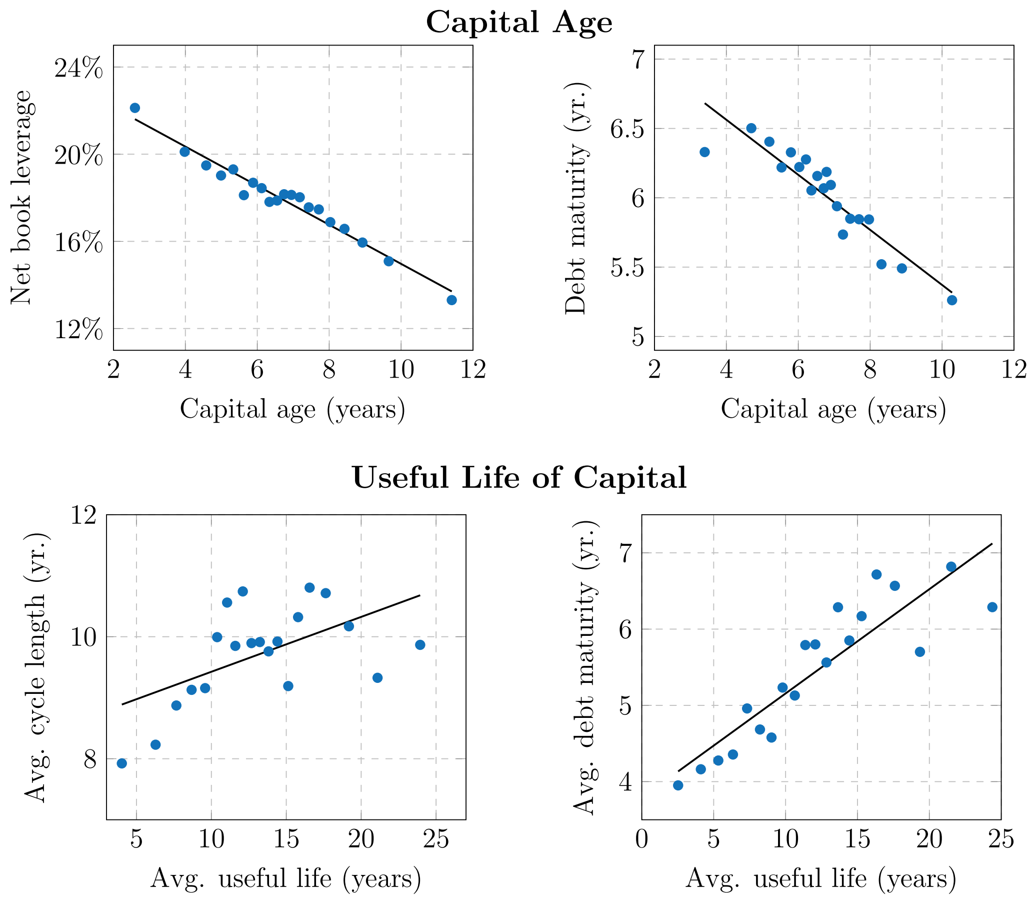

Figure 1: **Debt financing, capital age, and capital's useful life**. The top panels control for firm fixed effects. Each dot corresponds to $1/20^{th}$ of the sample firms. The sample period is from 1975 to 2018. Variables are defined in Table A.1.

as any non-geometric form of depreciation, a finite useful life makes capital age relevant for investment and financing decisions.[2] A finite useful life means that the productivity of capital (but not its value) remains constant over its lifespan after which it needs to be replaced—a good approximation for many forms of capital. As an example, consider two airlines with the same number of airplanes. One airline utilizes airplanes which are, on average, older than

[2]The standard assumption of geometric depreciation makes capital age irrelevant for the firm's problem since a capital's future productivity (and value) can be perfectly described by its current productivity. Subsection I.D shows that our results are robust to alternative forms of depreciation. The key force underlying our results and predictions is that the firm replaces ageing capital via large, planned investments. As a result, similar financing dynamics would arise in a model with fixed investment costs; see Subsection I.E.

the airplanes of the other airline. Geometric depreciation of the airplanes would imply that the airline with younger airplanes should fly more passengers, as its capital is younger and therefore more productive. However, since the airlines have the same number of airplanes, they roughly fly the same number of passengers. In our model, as in the airline example, the firm knows it needs to make replacement investments in the future as its capital ages due to the finite life of assets (airplanes). That is, the firm faces large, planned investments.

In the model, the firm has an incentive to finance investment with debt because creditors are more patient than shareholders, which is equivalent to debt providing tax benefits. But since the firm faces a collateral constraint (Holmstrom and Tirole, 1997; Lian and Ma, 2021), it manages its leverage keeping in mind future funding needs. Therefore, the firm initially levers up when buying new capital. However, as capital ages, it progressively reduces its net debt to free up debt capacity that will be used to finance future replacement investments. These net debt dynamics generate firm-level debt cycles, imply that firms have inherently unstable leverage, consistent with the findings of DeAngelo and Roll (2015), and rationalize the *pro-active* leverage declines documented in Denis and McKeon (2012) and DeAngelo et al. (2018).[3] They also imply a negative relation between capital age and leverage and a positive relation between the length of debt cycles and the useful life of capital, in line with the patterns highlighted in Figure 1. Leverage dynamics in our model arise from the fact that capital ageing leads the firm to *predictably* replace existing capital in *lumps*. As we show in the paper, these leverage dynamics arise with any form of capital depreciation that leads to investment spikes (without requiring, e.g., fixed adjustment costs).

In our baseline model, debt issuance is costless and the firm issues and rolls-over one-period debt. With debt issuance costs (Altınkılıç and Hansen, 2000; Yasuda, 2005), the firm implements the same net debt dynamics as in the baseline model but only issues debt when buying capital in order to minimize issuance costs. To do so, the firm issues debt with a maturity that approximately matches the useful life of new assets and with a repayment schedule that progressively frees up debt capacity. By doing so, the firm ensures that the

[3] Notably, DeAngelo et al. (2018) find that this deleveraging reflects decisions to repay debt and retain earnings as opposed to exogenous shocks that drive stock-market prices up and leverage ratios down.

repayment of maturing debt provides enough financial slack to finance replacement investments. Therefore, capital ageing leads to a matching theory of debt maturity (Stohs and Mauer, 1996) and to firm-level maturity cycles. Notably, our model predicts that debt maturity should increase with the useful life of assets but decrease with capital age, in line with the empirical patterns highlighted by the right panels of Figure 1.

To examine whether firm-level debt and maturity cycles spillover into aggregate dynamics, we embed our single-firm model into an industry equilibrium with debt. We show that the correlation of capital age across firms determines whether debt and maturity cycles spillover into the aggregate. That is, if capital age is uncorrelated across firms, then firm-level debt and maturity cycles get smoothed out and, therefore, do not spillover into the aggregate. Instead, if capital age is correlated across firms, then so is investment, leading to aggregate debt and maturity cycles. Thus our model predicts that industries with greater (lower) within industry heterogeneity of capital age will have reduced (greater) aggregate cycles. It is well known that shocks to firm level investment can spillover into aggregate investment (Cooper and Haltiwanger, 1993; Caballero and Engel, 1999; Cooper, Haltiwanger, and Power, 1999; Winberry, 2021). Our results show that these investment shocks—due to capital ageing in the context of our study—can also impact aggregate financing.

The mechanism in our model produces three independent set of predictions, i.e. firm-level time-series predictions, cross-sectional predictions, and aggregate predictions. We test these predictions using data on U.S. public firms and various measures of capital age and useful life of assets. Our empirical analysis delivers three main results. First, we find that capital age is a significant predictor of both leverage and debt maturity at the firm-level, even after conditioning on a standard set of leverage and maturity controls, including firm age. In addition, when examining the importance of different factors in explaining leverage ratios as in Frank and Goyal (2009), we find that capital age is the factor with the most explanatory power. In separate tests aimed at exploring the mechanism, we show that the effects of capital age on leverage and debt maturity are weaker for R&D intensive firms or firms with greater intangible capital and stronger for firms in which investment is more lumpy, as measured by the firm-specific investment skewness or kurtosis. We also show that the effect of capital age on leverage is stronger in sub-samples of firms that rely less on leasing, and can become insignificant when firms rely almost exclusively on leasing to finance investment, in line with economic intuition.

Second, we find in cross-sectional tests that the useful life of assets is a significant predictor of both the length of debt cycles and average debt maturity. Notably, firms with longer-lived assets follow longer debt cycles and have a higher average debt maturity, in line with our predictions. Third, we test the model's aggregate predictions by examining whether industries with greater cross-sectional dispersion in capital age have lower debt and maturity cyclicality, as measured by the standard deviation of the annual industry observations. Consistent with the model predictions, we find that industries with higher capital age dispersion have lower leverage and maturity cyclicality. We also perform various robustness checks to confirm the validity of our results, including using alternative proxies for capital age and the useful life of assets, alternative measures of debt maturity, and alternative industry definitions. All these robustness tests confirm our findings.

Our paper makes several contributions. First, we develop a framework in which investment cycles lead to endogenous debt and maturity cycles. From a modeling perspective, this framework brings together the literature on vintage capital (Arrow, 1964; Ramey and Shapiro, 2001; Rogerson, 2008; Rampini, 2019; Livdan and Nezlobin, 2021; Ma, Murfin, and Pratt, 2021) and the literature on lumpy investment (Cooper and Haltiwanger, 1993; Caballero and Engel, 1999; Cooper et al., 1999; Winberry, 2021). While existing papers focus on investment dynamics, our paper instead articulates the effects of vintage capital and lumpy investment on financing decisions. Notably, our paper is the first to outline the consequences of the investment cycles associated with lumpy investment for financing cycles and to shed light on the implications of capital age for firm-level and aggregate debt dynamics.

Second, our paper contributes to the literature studying dynamic financing and investment decisions (Gomes, 2001; Hennessy and Whited, 2005; Clementi and Hopenhayn, 2006; Nikolov, Schmid, and Steri, 2019) by highlighting the role of capital age and asset life in determining not only leverage dynamics but also debt maturity choices. In this literature, our model shares several features with DeAngelo et al. (2011) in that investment spikes are accompanied by leverage spikes and firms deleverage progressively to free up debt capacity. However, our analysis is distinctive for *i*) the roles it assigns to the useful life of assets and capital age, *ii*) the associated implications it derives for firm-level and aggregate debt cycles, and *iii*) its analysis of debt maturity. Our model is also closely related to Eisfeldt and Rampini (2007) and Rampini (2019). In these studies, the market for physical capital is frictionless so that capital only affects the firm's future through its residual value. Our paper instead allows for frictions in the market for physical capital. As a result, firms retain and eventually replace their capital, which drives their financing decisions. In our model, capital ageing leads to firm-level debt cycles, consistent with the leverage dynamics documented by Denis and McKeon (2012) and DeAngelo et al. (2018), and has important implications for firm-level debt maturity choices and aggregate debt dynamics.

Third, our paper also contributes to the literature on debt maturity choice by proposing a theory in which firms match the maturity of their assets and debt liabilities.[4] We show that the maturity structure linkage emerges naturally in worlds in which *i*) firms borrow to meet funding needs for immediate investment and *ii*) subsequently deleverage to have debt capacity when assets in place reach the end of their useful life. In this literature, our paper is most closely related to Myers (1977) due to our focus on leverage, debt maturity, and investment. While Myers (1977) argues that firms with more growth options should shorten their debt maturity to reduce debt overhang,[5] our theory instead ties the choice of debt maturity to the useful life of assets in place. This allows us to show that optimal financing is characterized by cycles and to generate unique predictions relating capital age and the useful life of assets to leverage and debt maturity choices.

[4]See e.g. Cheng and Milbradt (2012), Diamond and He (2014), He and Milbradt (2016), Huang, Oehmke, and Zhong (2019), or Chen, Xu, and Yang (2021) for recent contributions.

[5]Myers (1977)'s conjecture has been recently challenged by Diamond and He (2014) who show that debt overhang may increase or decrease with debt maturity. Consistently, empirical work on debt maturity based on the hypothesis of reduced overhang of shorter term debt has had mixed success. Barclay and Smith (1995) and Guedes and Opler (1996) document a negative relation between maturity and growth opportunities, while Stohs and Mauer (1996) and Johnson (2003) find a positive relation after controlling for leverage.

Lastly, we leverage our theoretical analysis to contribute to the large empirical literatures on leverage (see e.g., Leary and Roberts, 2005; Lemmon, Roberts, and Zender, 2008; Frank and Goyal, 2009) and debt maturity (see e.g., Custódio, Ferreira, and Laureano, 2013; Choi, Hackbarth, and Zechner, 2018). We do so by showing that our mechanism for the formation of debt cycles (DeAngelo et al., 2018) is consistent with the dynamics of leverage around investment peaks (Bargeron, Denis, and Lehn, 2018) and the incidence of large, proactive increases in leverage (Denis and McKeon, 2012; DeAngelo and Roll, 2015). Our analysis additionally brings out the key roles of capital age and asset life in the dynamics of leverage and debt maturity and provides cross-sectional and time series evidence that is strongly supportive of the proposed mechanism. Finally, we find empirical evidence consistent with our mechanism generating not only debt cycles but also maturity cycles and show that the aggregate dynamics of both leverage and debt maturity can be better understood when taking into account capital age.

I Single-Firm Model

A Assumptions

Time is discrete and indexed by $t \in \{0, 1, 2, ...\}$. We consider a representative firm owned by a risk-neutral entrepreneur who discounts cash flows at a rate $r > 0$. At time 0, the entrepreneur creates the firm with an endowment of C_0 in cash.

Each period, the firm can use one unit of capital to produce one unit of the final good in the next period, which yields a profit of $\pi > 0$. The firm can acquire a unit of new capital, which is delivered immediately, for a price K. Capital cannot be sold—i.e. investment is irreversible—and has a finite useful life. Notably, we consider that capital has a constant productive capacity over a finite number n of periods after which it needs to be replaced.[6]

[6] In this respect, we depart from most existing work, which relies on geometric depreciation of capital following Hayashi (1982). There exists ample empirical evidence that geometric depreciation does not fully reflect reality (Feldstein and Rothschild, 1974; Harper, 1982; Ramey and Shapiro, 2001; Rogerson, 2008) and that depreciation is backloaded (Giandrea, Kornfeld, Meyer, and Powers, 2021). See also Subsection I.D.

That is, capital has a constant productivity over its lifespan but a declining value. This type of capital depreciation is also known as one-hoss-shay depreciation (see Arrow, 1964; Laffont and Tirole, 2001; Rampini, 2019; Livdan and Nezlobin, 2021) and is largely used in practice. Livdan and Nezlobin (2021) note for example that firm-level data on capital goods, such as property, plant, and equipment (PP&E), is prepared in practice almost exclusively under the assumption that the efficiency of capital goods is constant over a finite useful life.

As an example, consider two airlines with the same number of airplanes. One airline utilizes airplanes which are, on average, older than the airplanes of the other airline. Geometric depreciation of the airplanes (as in e.g. Hayashi, 1982) would imply that the airline with younger airplanes should fly more passengers, as its capital is younger and therefore more productive. However, since the airlines have the same number of airplanes, they roughly fly the same number of passengers. In this case, using a finite useful life better reflects their productivity.[7] While the use of one-hoss-shay depreciation depreciation makes our results and empirical predictions particularly crisp, Subsection I.D shows that they are robust to other forms of depreciation. As will become clear, our results also go through if profits π depend on capital age, for example due to increasing maintenance costs.

Figure 2 shows the cash flows of a firm that produces each period and replaces capital at the end of its useful life. Capital replacement leads to investment spikes, as observed in the data (Doms and Dunne, 1998; Cooper and Haltiwanger, 2006; Whited, 2006). In addition, the likelihood of observing an investment spike is increasing in the time since the previous spike (i.e. capital age), in line with the empirical evidence (see Cooper et al., 1999).

We assume that the purchase price of capital K is sufficiently small that investment is positive net present value (NPV). (Appendix A provides the exact parameter restriction.)

We also note that in our setting depreciation of capital can take the form of physical depreciation and/or (expected) technological obsolescence.

[7]One could argue that firms purchase many different types of capital and therefore geometric depreciation is a good approximation of their actual productive capacity. But as in the example given, there exists substantial within-firm variation in capital age in the data, and therefore depreciation of capital productivity$\neq$depreciation of capital value inside the firm, which is required to use geometric depreciation.

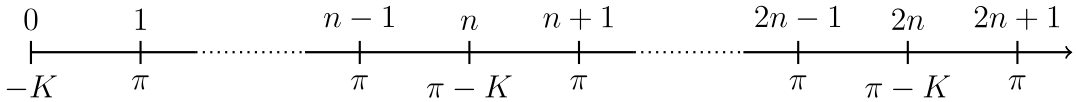

Figure 2: **Firm cash flows**. Each period, the firm produces and capital generates a profit of π the next period. Each n periods, new capital is bought at a price K.

The present value of the cash flows of a firm that always produces goods is given by

$$\sum_{t=1}^{\infty} \frac{1}{(1+r)^t}\pi - \sum_{i=0}^{\infty} \frac{1}{(1+r)^{i*n}}K = \frac{\pi}{r} - \frac{(1+r_n)K}{r_n},$$

where $r_n = (1+r)^n - 1$ is the n-period discount rate.

As in Rampini and Viswanathan (2010) and Rampini (2019), the firm finances investment with cash (retained earnings) or one-period debt.[8] Creditors are more patient than the entrepreneur and discount cash flows at a rate $\rho_D < r$, which generates an incentive for the firm to issue debt. This assumption is standard in discrete time dynamic financing and investment models (e.g., DeAngelo et al., 2011), and is equivalent to the existence of tax benefits of debt $\rho_D = (1-\tau)r < r$, where $\tau \in (0, 1)$ is the corporate tax rate.

When the firm produces the final good at time t, we consider that it can issue debt up to a cash flow-based collateral constraint (Stiglitz and Weiss, 1981; Holmstrom and Tirole, 1997; Clementi and Hopenhayn, 2006):

$$D_t \leq \phi \times \pi,$$

where D_t is total debt at time t and ϕ is the multiple of per period profits that can be pledged. This assumption reflects the finding in Lian and Ma (2021) that 80% of debt contracts are associated with cash-flow-based collateral constraints. We assume that $\phi \in [\underline{\phi}, \bar{\phi})$ is bounded.

[8]Section II introduces proportional debt issuance costs, allows the firm to issue multi-period debt, and derives the optimal debt maturity structure. The model can also be extended to incorporate costly equity issuance. With proportional or convex equity issuance costs, leverage and debt maturity will follow the same patterns as in the current model. For large enough equity issuance costs, the firm will finance investment exclusively with debt and financing dynamics will be exactly as in the baseline model. For low enough issuance costs, the firm will partly rely on equity to finance investment, leading to dampened debt cycles.

The upper bound $\bar{\phi}$ ensures that debt is risk-free. The lower bound $\underline{\phi}$ ensures that the firm can initially purchase the asset. Appendix A provides the exact parameter restrictions. Subsection I.F shows that an asset-based collateral constraint only strengthens our result that firms lower net debt as capital ages since the collateral value declines as capital ages.

The firm earns a return $\rho_C \in (0, \rho_D)$ on its cash holdings, implying that the firm never holds both cash and debt (as in Hennessy and Whited (2005) or DeAngelo et al. (2011)). In addition, since this return is below the discount rate r, the firm has no incentives to retain more cash than is needed to fund investment.

B Equity Value

At time t, the firm has cash reserves C_t and invests I_t in new capital (if at all). Dividends are then given by the budget constraint

$$
\begin{aligned}
Div_t &= \pi \mathbb{I}_{\{firm\ produces\}} - I_t + C_{t-1}(1+\rho_C) - C_t + D_t - D_{t-1}(1+\rho_D) \\
&= \pi \mathbb{I}_{\{firm\ produces\}} - I_t + ND_t - ND_{t-1}\left(1 + \rho_D \mathbb{I}_{\{ND_{t-1} \geq 0\}} + \mathbb{I}_{\{ND_{t-1} < 0\}} \rho_C\right) \\
&\geq 0,
\end{aligned} \tag{1}
$$

where $ND_t = D_t - C_t$ is the firm's net debt, which summarizes its financing policy, and $\mathbb{I}_{\{x \geq y\}}$ is the indicator function of the event $x \geq y$.

The problem of management is to maximize the present value of future dividends by choosing the firm's investment I_t and financing ND_t policies. That is, equity value solves

$$
E_0 = \sup_{\{I_t, ND_t\}_{t \in \{0,1,2,\ldots\}}} \sum_{t \geq 0}^{\infty} \frac{Div_t}{(1+r)^t}, \tag{2}
$$

where dividends follow from the budget constraint in equation (1) and are non-negative, and net debt satisfies the collateral constraint $ND_t \leq \phi \times \pi$.

C Financing and Investment

We solve management's optimization problem in steps, starting with investment policy. A first result, summarized in Proposition 1, is that the firm optimally invests when capital reaches the end of its useful life and never before:

Proposition 1 (Firm Investment)**.** *The firm replaces capital when it reaches the end of its useful life and never before.*

The intuition for the result in Proposition 1 is that capital has constant productivity as long as it has not reached the end of its useful life. Postponing replacement allows the firm to earn a return ρ_C on any capital it holds, rendering early replacement suboptimal. In addition, capital investment is positive NPV. Therefore, the firm always invests in new capital once existing capital has reached the end of its useful life.

Let us next turn to financing policy. Let $a \in \{0, 1, ..., n-1\}$ be the age of the firm's current capital. With a slight abuse of notation, we also use a as a time index. ND_a will therefore refer to net debt given that the firm has capital with age a. As we show next, the firm optimally retains earnings to lower its net debt and create financial slack as its capital ages. This financial slack allows the firm to invest in new capital by issuing new debt when reaching the useful life of old capital. Notably, we have the following result:

Theorem 1 (Debt Cycles)**.** *As capital ages, the firm frees up debt capacity to finance replacement investments, in that*

$$ND_{a+1} \leq ND_a.$$

Figure 3 shows the optimal dynamics of investment and financing. The firm finances investment by increasing net debt. It then optimally retains earnings to lower net debt. The firm thereby frees up debt capacity to be able to finance its replacement investment. These dynamics generate debt cycles that are driven by the firm's ageing capital.

The debt cycles depicted in Figure 3 are consistent with several empirical findings: *i*) Denis and McKeon (2012) find that firms lever up to finance investment, which occurs in

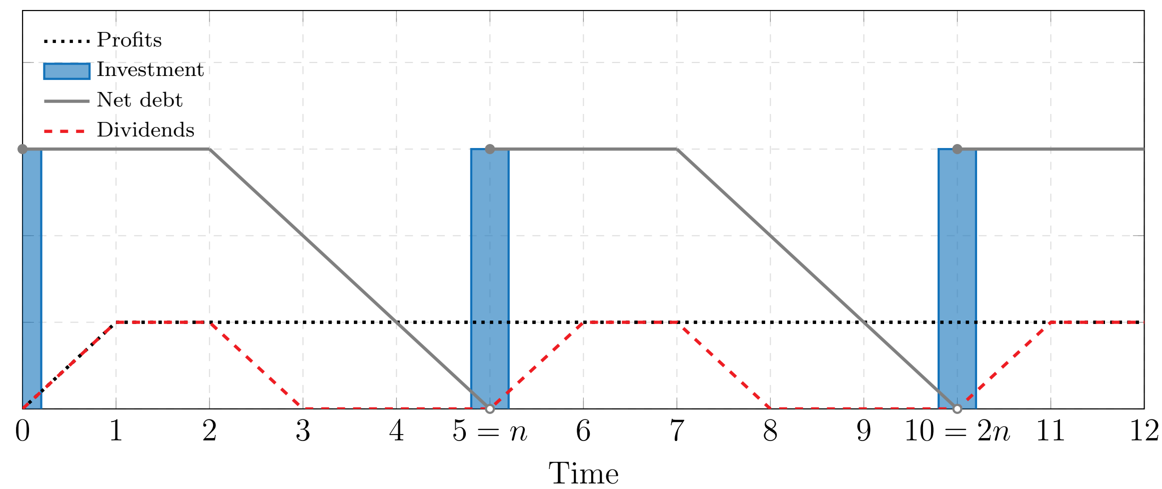

Figure 3: **Profits, investment, financing, and dividends.** This figure assumes that $C_0 = K$ and therefore the firm does not pay a dividend at time zero.

our model due to firms financing the replacement of ageing capital with debt; *ii*) Denis and McKeon (2012) and DeAngelo et al. (2018) find that firms significantly decrease leverage after reaching a peak, which occurs in our model because firms retain earnings and lower leverage to finance the eventual replacement of ageing capital; *iii*) DeAngelo and Roll (2015) find that corporate capital structure is inherently unstable, which is consistent with our debt cycles leading to inherently unstable firm leverage even in the absence of uncertainty; and *iv*) Strebulaev and Yang (2013) show that a large fraction of U.S. public firms has zero-leverage, which occurs in the model when $ND_a < 0$.

In addition to rationalizing prior findings, the model generates unique cross-sectional and time-series predictions for leverage. Within a firm, the model predicts that

Prediction 1. *Capital age and leverage are negatively related.*

This negative relation arises because of the need to free up debt capacity as capital ages (Theorem 1). While across firms, the model predicts that

Prediction 2. *The duration of debt cycles is positively related to the useful life of assets.*

This positive relation arises because the length of debt cycle is driven by the length of investment cycles; see Figure 3.

Our model also allows us to study the impact of lumpiness in investment and asset (in)tangibility on debt cycles. Notably, for a given level of cash flows π, a greater cost of physical capital K implies that investment is more lumpy and that the firm uses more physical capital (i.e. less intangible capital) to generate the same cash flows. The following proposition formalizes the effects of investment lumpiness and asset tangibility on debt cycles.

Proposition 2 (Debt Cycles, Lumpy Investment, and Tangible Capital)**.** *As capital becomes more expensive $K' > K$ the effects of capital age on leverage become more pronounced:*

$$|ND_{t+1} - ND_t| \leq |ND'_{t+1} - ND'_t|.$$

The more expensive capital K becomes the more financial slack the firm needs to finance the replacement investment. As a result, as shown by Proposition 2, the leverage cycles become more pronounced as the cost of physical capital K increases. This leads to the following prediction:

Prediction 3. *The effects of capital age on leverage are more pronounced in firms with more lumpy investment and less pronounced in firms with more intangible assets.*

D Other Forms of Capital Depreciation

Our model assumes that the efficiency of capital goods follows a one-hoss shay pattern, as in e.g. Arrow (1964), Rogerson (2008), Rampini (2019), or Livdan and Nezlobin (2021). This form of capital efficiency keeps the model tractable since capital age a is a sufficient statistic for the state of the firm when $t > 0$. This in turn allows us to generate crisp empirical predictions on financing decisions and debt maturity choices.

An important question is whether this form of capital efficiency or depreciation is necessary for our results. *The short answer is no.* Debt cycles are generated by large replacement investments financed with debt. Thus, any form of depreciation that leads to large replacement investments in the future suffices (see Proposition 3 below). But what forms of depreciation have this feature?

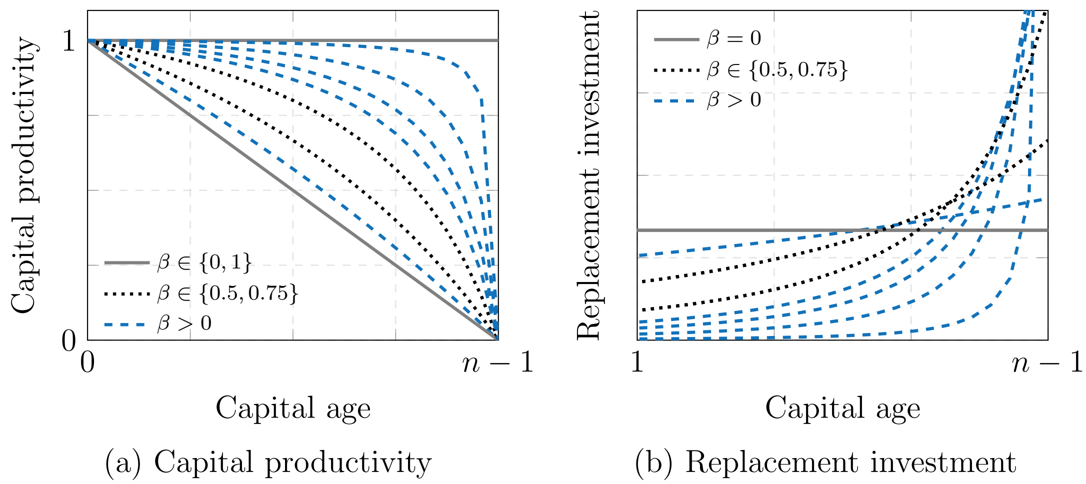

Figure 4: **Depreciation and replacement investments.** The figure shows the productivity of capital $S(a|\beta)$ as it ages and the replacement investment $S(a-1|\beta) - S(a|\beta)$ necessary due to the depreciation of the original capital.

The U.S. Bureau for Labor Statistics (BLS) estimates the productivity of capital in place relative to the productivity of new capital (or, equivalently, the productivity of capital a years after it has been installed) using the function

$$S(a|\beta) = \mathbb{I}_{\{a \leq n-1\}} \frac{n-a}{n-\beta a},$$

where $\beta \in [0, 1]$; see Giandrea et al. (2021). Our model with capital that has a finite useful life represents the case in which $\beta = 1$. The case $\beta = 0$ corresponds to a linear decrease in asset productivity. Figure 4a shows intermediate cases $\beta \in (0, 1)$. A linear decrease in productivity implies that the replacement investment needed to compensate for the lost productivity of the original capital is constant, in that $S(a-1|0) - S(a|0) = \frac{1}{n}\mathbb{I}_{\{a \leq n\}}$. By contrast, any form of depreciation with $\beta > 0$ back loads the replacement investment leading to large planned replacement investments in capital, as shown by Figure 4b. The U.S. Bureau for Labor Statistics uses $\beta = 0.75$ for structures and $\beta = 0.5$ for equipment (see Giandrea et al., 2021). With $\beta = 0.75$ and $n = 4$ (respectively $n = 5$), the firm makes 57.1% (respectively 50%) of its replacement investments in the last useful year of the asset.

Under additional restrictions given in Appendix B, we can establish that given an arbitrary form of capital depreciation and an arbitrary distribution of the firm's capital age:

Proposition 3 (Ageing Capital and Leverage with Arbitrary Capital Depreciation)**.** *Let time $T > t$ be the next time that the firm invests. Then for $t' \in \{t, ..., T-2\}$, capital ages while net debt weakly declines*

$$ND_{t'+1} \leq ND_{t'}.$$

E Non-Geometric Depreciation Versus Fixed Investment Costs

In our model, financing cycles are driven by the predictable "lumps and bumps" in investment created by non-geometric depreciation. In practice, other mechanisms/frictions could lead to predictable lumps and bumps in investment at the firm level, fixed investment costs being one of them. Indeed, in a standard model with decreasing returns to scale and geometric depreciation, the firm will postpone investment until the associated benefits are large enough to offset the fixed investment costs. This will happen when capital becomes sufficiently less productive (due to depreciation) that it becomes optimal to replace it (Cooper and Haltiwanger, 1993; Cooper et al., 1999). This alternative mechanism would lead to predictable investment cycles, as in our model with non-geometric depreciation. In addition, and as in our model, the firm would need to free up debt capacity as capital ages to be able to finance replacement investments, thereby generating financing cycles. Financing cycles thus arise more generally in the presence of (predictable) investment cycles, independently of the nature of the technology or friction that drives these cycles.

F Cash-Flow Versus Asset-Based Collateral Constraints

In recent research, Lian and Ma (2021) document that 20% of debt by value is based on constraints related to the liquidation value of physical assets ("asset-based lending" in creditor parlance) in US non-financial firms, whereas 80% is based predominantly on cash flows from firms' operations (as assumed in our baseline model). As we now show, debt cycles would mechanically be stronger with an asset-based collateral constraint.

Let V_a be residual value of capital, which we define as the present value of future cash flows that capital with age a generates:

$$V_a = \frac{\pi}{1+r} + ... + \frac{\pi}{(1+r)^{n-a}} = \sum_{t=a+1}^{n} \frac{\pi}{(1+r)^{t-a}}.$$

Assuming the firm is producing, an asset-based collateral constraint would restrict debt to be less than some fraction $\chi \in \left[0, \frac{1}{1+\rho_D}\right]$ of the capital's residual value[9]

$$D_a < \chi V_a.$$

Since the residual value of assets V_a decreases with capital age, such a constraint can only strengthen the debt cycles identified in Theorem 1. The reason is that firms are forced to deleverage because the collateral constraint becomes tighter as capital ages, which does not happen with a cash-flow based collateral constraint.

II Debt Maturity

A Assumptions

In the baseline model, there is no cost of issuing debt so that there is no cost for the firm of issuing and rolling over one-period debt. In practice, issuing debt is costly (Altınkılıç and Hansen, 2000; Yasuda, 2005). In this section, we introduce proportional debt issuance costs $\epsilon > 0$ and allow the firm to have multiple debt issues outstanding at the same time with (possibly) different maturities. Interest on debt is paid each period. We study the situation in which debt issuance costs become small $\epsilon \to 0$. To make sure that the firm does not have permanent debt in its capital structure, we assume that capital investment cannot be fully

[9] The repayment due next period is $D_a(1 + \rho_D)$.

financed by debt and current period profits:

$$K > \phi\pi + \pi. \tag{3}$$

All the results presented below hold for the non-permanent part of the firm's debt in case this assumption is not verified.

B Optimal Financing

With debt issuance costs, the firm implements the same net debt dynamics as in Section I but only issues debt when buying capital in order to minimize issuance costs. As a result, the debt maturity choice has no bearing on the debt cycles. To achieve these debt dynamics, the firm issues debt with a maturity that approximately matches the useful life of new assets and with a repayment schedule that progressively frees up debt capacity. This way, the firm makes sure that by repaying maturing debt it creates enough financial slack to finance replacement investments. The following theorem formalizes this result.

Theorem 2 (Long-Term Debt Financing). *With debt issuance costs, the firm optimally issues long-term debt with a repayment schedule such that net debt follows the same cycles as in Theorem 1. Furthermore, the firm only issues (long-term) debt when buying new capital in order to minimize issuance costs.*

Let M_a be the average maturity of outstanding debt given that capital age is a. When $ND_a \leq 0$, the firm has no debt outstanding and $M_a = 0$. When $ND_a > 0$, we have that

$$M_a = \sum_{i=a}^{n-1} \mathbb{I}_{\{ND_i > 0\}} (i + 1 - a) \frac{ND_i - \max\{ND_{i+1}, 0\}}{ND_a}.$$

We can then show that capital age and average debt maturity are negatively related.

Proposition 4 (Debt Maturity Cycles). *Average debt maturity is decreasing in capital age:*

$$M_{a+1} \leq M_a.$$

Figure 5 shows how average debt maturity evolves through time when assets have a useful life of 6 years and the firm implements the optimal debt maturity structure at issuance. The firm only issues debt when buying new capital. Debt issuance leads to an increase in the average debt maturity which then decreases as capital ages until the firm invests again. Therefore, capital ageing not only leads to debt cycles but also to maturity cycles.

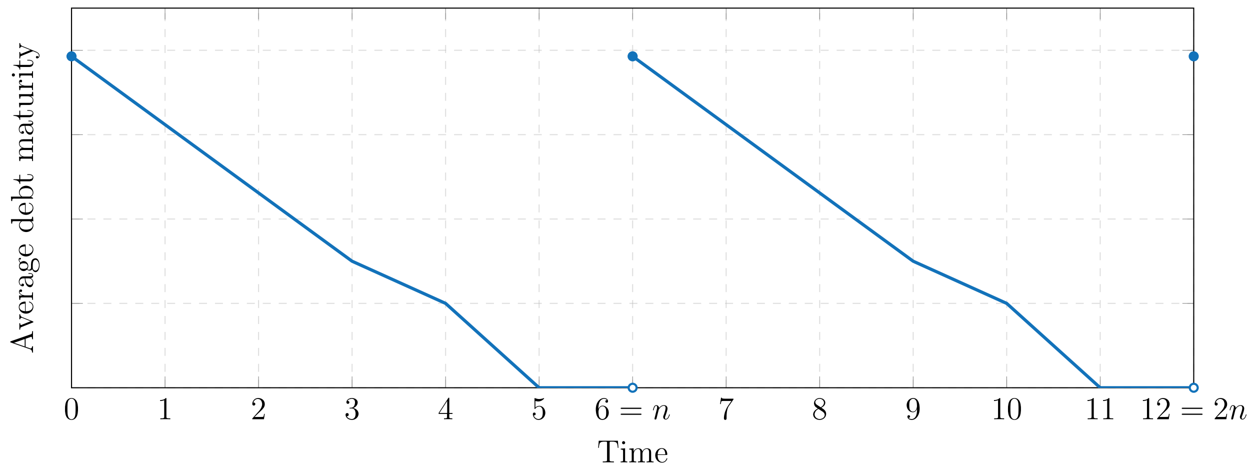

Figure 5: **Average debt maturity.** This figure considers the case of a firm with assets that have a useful life of 6 years and shows that average debt maturity given the optimal debt maturity structure at issuance.

An implication from the optimal financing policy is that the firm can postpone deleveraging when assets have a greater useful life and does so by issuing debt with a longer maturity. Notably we have that:

Theorem 3 (Maturity Matching)**.** *Increasing the useful life of assets increases average debt maturity in that* $\frac{\Delta M_a}{\Delta n} \geq 0$.

The model generates both cross-sectional and time-series predictions for debt maturity. Within a firm, the model predicts that (see Proposition 4)

Prediction 4. *Capital age and debt maturity are negatively related.*

While cross-sectionally, the model predicts that (see Theorem 3)

Prediction 5. *Average debt maturity is positively related to the useful life of a firm's assets.*

III Industry Equilibrium

This section embeds our single-firm model into a steady state industry equilibrium and studies the implications of ageing capital for aggregate levels of investment and corporate debt. To simplify notation, we only use time indices for time-varying quantities.

A Assumptions

There are three types of players: consumers which have demand for the final good, incumbent firms which produce the final good, and entrepreneurs which can create new firms.

Consumers' demand for the final good is given by their inverse demand function $P(Q) \geq 0$, with $P'(Q) < 0$ and where Q is the aggregate supply of the final good. The mass of incumbent firms in the industry is given by S. Each period, each of these firms produces one unit of the final good using their installed capital and makes financing choices as in the single-firm model of Section I. Finally, entrepreneurs can pay an entry cost $C_0 + H$ with $H > 0$ to create a new firm that is endowed with C_0 in cash.

B Equilibrium Quantities and Existence

The aggregate supply of the final good is $Q = S$ since a mass S of firms produce the final good. Given consumers' inverse demand function, the price for the final good and, thus, the profits of the firm (ignoring the purchase price of capital) in each period are given by

$$\pi = P(S).$$

Let $E_0(\pi)$ be the value of a new firm when the per period profits are π, as given by equation (2). In a steady state equilibrium, the free entry condition must hold

$$E_0(\pi) - (H + C_0) = 0, \tag{4}$$

ensuring that entrepreneurs are indifferent between entering or staying out of the industry.[10]

Definition 1 (Steady State Industry Equilibrium)**.** *An equilibrium*

$$\Psi^* = (S^*, Q^*, P^*, \pi^*, I_a^*, ND_a^*)$$

consists of a mass of incumbent firms S^*, an aggregate supply of the final good Q^*, a price for the final good P^*, incumbent firms per period profits π^*, and firms' investment and financing policies given their capital age (I_a^*, ND_a^*) such that:

1. ***Incumbents**: Given a price P^*, firms' profits are $\pi^* = P^*$ and incumbent firms choose their investment and financing strategies (I_a^*, ND_a^*) to maximise their equity value.*

2. ***Entrepreneurs**: create new firms until they are indifferent between entering or staying out of the industry:*

$$E_0(\pi^*) - (H + C_0) = 0.$$

3. ***Consumers**: pay the equilibrium price P^* for the good given their inverse demand function and the aggregate supply $Q^* = S^*$ of the final good, with*

$$P^* = P(S^*).$$

We assume that consumers' inverse demand function $P(Q)$ satisfies two conditions. First, we assume that $P(0) \geq r\left(K\left(1 + \frac{1}{r}\frac{\rho_C}{(1+\rho_C)^n - 1}\right) + H\right)$, so that entry is profitable if there are no competitors. Second, we assume that $\lim_{Q\to\infty} P(Q) = 0$, which implies that there is finite demand for the final good. Additionally, we assume that $H > \underline{H}$ where $\underline{H}$ is defined in Appendix D so that debt is risk-free in equilibrium. These assumptions ensure that

Proposition 5. *A unique industry equilibrium Ψ^* exists.*

[10] Equilibria with $E_0(\pi) - (H + C_0) < 0$ could also exist since no firm exits or enters in our equilibrium. We focus on the case where the free entry condition binds, which would be a situation in which firms sequentially decide whether to enter or not and so the marginal firm is exactly indifferent between these two choices.

C Capital Age Distribution and Aggregate Financing

So far, we have not discussed the capital age distribution within our industry. It turns out that the equilibrium Ψ^* is independent of the capital age distribution. However, as we show below, aggregate investment and financing do depend on it. In the Internet Appendix, we argue that the nature of the shocks that firms face—aggregate versus idiosyncratic—determines whether financing cycles spillover to the aggregate. Notably, we show that aggregate shocks—such as shocks to credit supply—can increase the correlation across firms' capital age, leading to aggregate financing cycles.[11]

Let the distribution of capital age at time t be given by $q_t = (q_t^0, ..., q_t^{n-1})$, where q_t^a is the fraction of firms that have capital that is a years old. In the steady state equilibrium, q_t evolves according to

$$q_{t+1}^a = \begin{cases} q_t^{a-1} & \textit{if } a > 0, \\ q_t^{n-1} & \textit{otherwise}, \end{cases}$$

since capital ages until the end of its useful life (which occurs after n years) at which point the firm buys new capital. Proposition 6 characterizes aggregate production, financing, net debt, and maturity dynamics in our steady state industry equilibrium.

Proposition 6. *In steady state industry equilibrium:*

1. *Aggregate production is constant through time $Q^* = S^*$.*
2. *Aggregate investment is $S^* * q_t^0$.*
3. *Aggregate net debt is $S^* * \sum_{a=0}^{n-1} q_t^a * ND_a^*$.*
4. *Aggregate average debt maturity is $\frac{1}{\sum_{a=0}^{n-1} q_t^a * ND_a^*} \sum_{a=0}^{n-1} q_t^a * ND_a^* * M_a^*$, where M_a^* is the average debt maturity of a firm with capital age a.*

[11] One way in which can we arrive at steady state equilibria with different capital age distributions in the model without shocks is if firm entry is restricted in the first $n-1$ periods. Let $q_{n-1} = (q_{n-1}^1, ..., q_{n-1}^{n-1})$ be a probability distribution. At time $t \in \{0, ..., n-1\}$ only $S^* q_{n-1}^{n-1-t}$ firms are allowed to enter the industry. At time $n-1$ the capital age distribution would then be q_{n-1}.

Figure 6 shows two different types of capital age distributions and their evolution over time. Both distributions lead to the same steady state equilibrium Ψ^* and, therefore, to the same investment and financing choices by firms given their capital age and aggregate output. In the top panel, capital age has a smooth distribution and therefore there is no correlation in capital age across firms. In the bottom panel, all capital has the same age. Therefore, the distribution is lumpy and there is perfect correlation in capital age across firms.

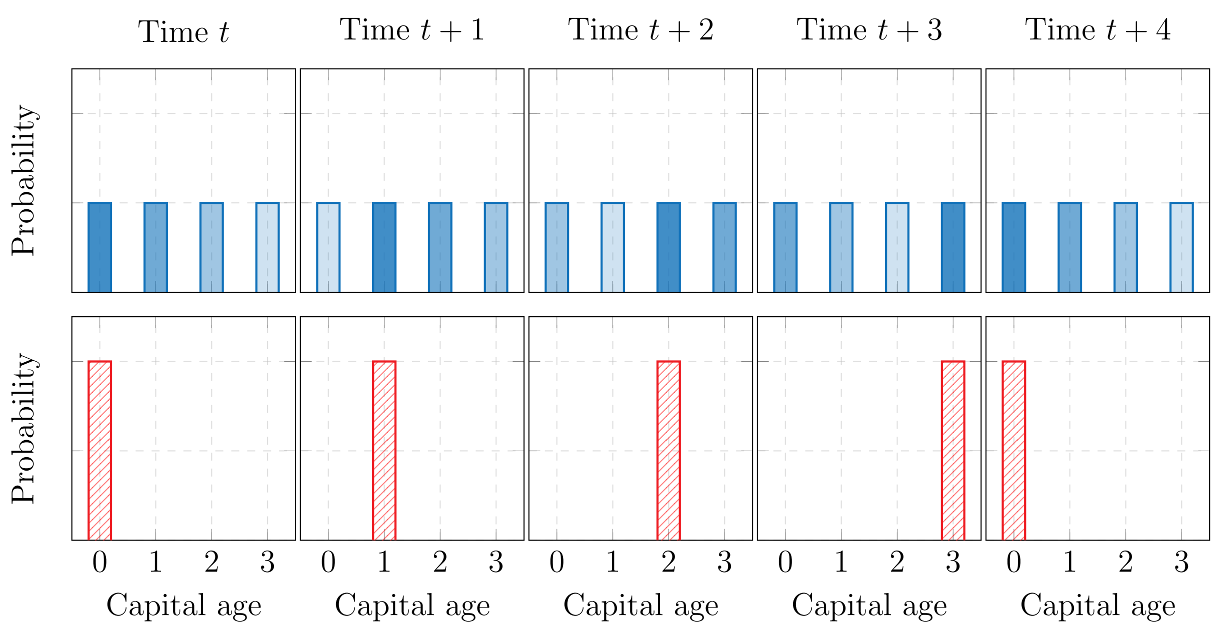

Figure 6: **Capital age distribution over time.** The top panels show a smooth capital age distribution (no correlation across firms' capital age) while the bottom panels show a lumpy capital age distribution (perfect correlation across firms' capital age).

These patterns have first order implications for aggregate investment and financing. Figure 7 shows how the smooth and lumpy capital age distributions of Figure 6 translate into aggregate financing and investment.[12] While the firm level financing policies (first panel of Figure 7) are independent of the capital age distribution, they lead to markedly different aggregate financing and investment dynamics (second, third, and fourth panel). Given the

[12] While we have not explicitly discussed the debt maturity choice in the industry equilibrium section, Theorem 2 shows how to determine optimal debt maturity given the net debt dynamics.

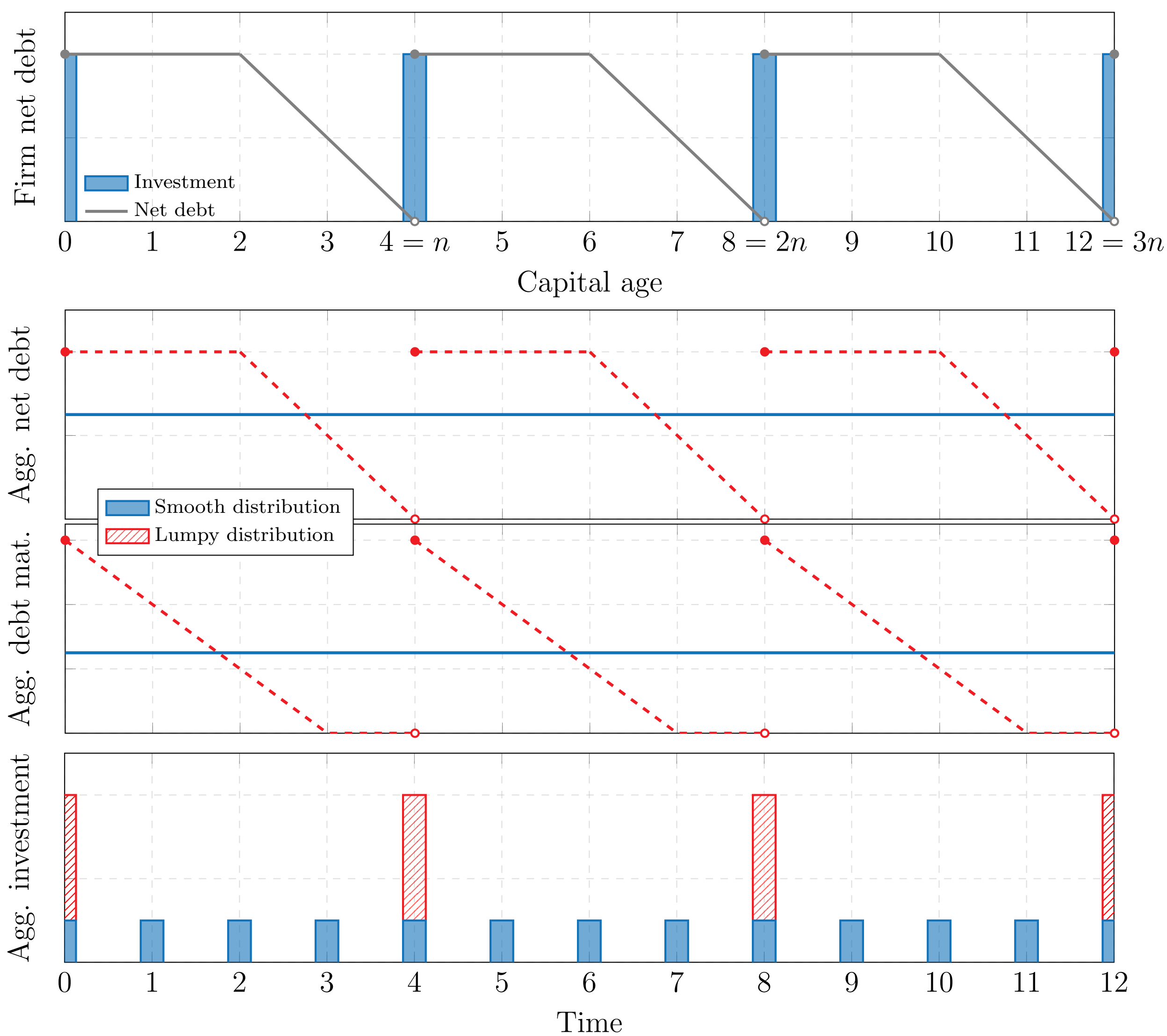

Figure 7: **Capital age distribution and aggregate investment, net debt, and debt maturity.** The first panel shows the firm level investment and financing policies. The second panel shows aggregate net debt using the capital age distribution from Figure 6. The third panel shows aggregate debt maturity using the capital age distribution from Figure 6. The fourth panel shows aggregate investment using the capital age distribution from Figure 6.

smooth capital age distribution (no correlation in capital age across firms), investment is constant through time and, as a result, so is financing. The individual firm-level debt cycles get smoothed out when aggregated. The lumpy capital age distribution (perfect correlation in capital age across capital firms) yields very different results. Investment is highly cyclical and financing reflects these investment cycles leading to leverage and maturity cycles at the aggregate level. The model thus implies that

Prediction 6. *A higher dispersion of capital age across firms within an industry implies a lower dispersion of aggregate leverage and debt maturity in that industry.*

These results show that whether firm-level cyclicality in investment and financing translates into aggregate financing and investment cycles depends on the capital age distribution. Correlation across firms' capital age is necessary for the firm level investment, leverage, and maturity cycles to spillover into the aggregate. It is well known that shocks to firm level investment can spillover into aggregate investment (Cooper and Haltiwanger, 1993; Caballero and Engel, 1999; Cooper et al., 1999; Bachmann, Caballero, and Engel, 2013; Winberry, 2021). Our results show that these investment shocks (due to the replacement of ageing capital) can also impact aggregate financing and lead to leverage and maturity cycles.

IV Empirical Analysis

A Data

Our empirical analysis is based on a sample of listed U.S. firms from annual Compustat over the period of 1975–2018. We use a sample selection procedure that is similar to that in Peters and Taylor (2017) and Lin, Palazzo, and Yang (2020). In particular, we exclude firms whose SIC code is between 4900 and 4999 (utility or regulated firms), between 6000 and 6999 (financial firms), or greater than 9000 (government agencies etc.). We also exclude firms operating in R&D–intensive sectors (SIC codes 737, 384, 382, 367, 366, 357, and 283). We winsorize all variables at 1% and 99% levels to mitigate the impact of potential outliers.

We drop all observations with missing values on one or more variables of interest. We then remove observations with a market-to-book ratio larger than 20 or with negative book equity. Our final sample consists of 77,877 firm-year observations with 6,620 unique firms.

B Measuring Capital Age

Our model predicts that debt and maturity should decrease with capital age (a), while the length of debt cycles and average debt maturity should increase with the useful life of assets (n). To test these predictions, we need measures of a firm's capital age and the useful life of its assets. We construct our measure of capital age as in Lin et al. (2020). In particular, we first calculate net and gross investment for firm i at time t, respectively, as:

$$I_{i,t}^{net} = ppent_{i,t+1} - ppent_{i,t} \text{ and } I_{i,t}^{gross} = \delta_{i,t+1} ppent_{i,t} + I_{i,t}^{net},$$

where $ppent_{i,t}$ refers to property, plant, and equipment and $\delta_{i,t}$ is the BEA-inferred industry depreciation rate of firm i at time t. We then define the capital age measure $CA_{i,t}$ as:

$$CA_{i,t} = \begin{cases} (CA_{i,t-1} + 1) \times \frac{(1-\delta_{i,t}) ppent_{i,t-1}}{ppent_{i,t}} + \frac{I_{i,t-1}^{gross}}{ppent_{i,t}} & \text{if } I_{i,t-1}^{gross} > 0, \\ CA_{i,t-1} + 1 & \text{otherwise.} \end{cases}$$

When the firm has positive gross investment in the previous period, capital age is a weighted average of the old capital, which ages one year, and new capital, which is one year old. The respective weights of old and new capital, $\frac{(1-\delta_{i,t}) ppent_{i,t-1}}{ppent_{i,t}}$ and $\frac{I_{i,t-1}^{gross}}{ppent_{i,t}}$, reflect the respective shares of the old and new capital in this period's total capital. When gross investment is negative, we assume that all of capital vintages are disposed of in an equal way so that capital ages by one year. We initialize the firm–level measure of capital age by calculating the ratio of accumulated depreciation and amortization (`dpact`) to current depreciation and amortization (`dp`). Subsection IV.F considers alternative measures of capital age.

To measure the useful life of assets, we follow the empirical literature which relies on deflating gross property, plant and equipment by current depreciation (Stohs and Mauer, 1996; Custódio et al., 2013; Livdan and Nezlobin, 2021). We proxy for the useful life of firm i's assets at time t by

$$UL_{i,t} = \left\| \frac{ppegt_{i,t} + ppegt_{i,t-1}}{2dpc_{i,t}} \right\|,$$

where $\| \cdot \|$ rounds the useful life to the nearest integer. The measure can be interpreted as the number of years needed to fully depreciate the capital stock, which is time invariant in the model. As in Livdan and Nezlobin (2021), we cap the measure at 25 years. Subsection IV.F shows that our results are robust to using alternative measures of useful life.

We test the model predictions on financing using three measures of leverage: net book leverage, net market leverage, and net lease-adjusted leverage (defined as in Rampini and Viswanathan (2013)). We test the predictions on debt maturity using the ratios of debt maturing in more than 3 and 5 years to total debt (as in Custódio et al., 2013) and debt maturity from Capital IQ (as in Choi et al., 2018), which we refer to as debt maturity in our analysis. Summary statistics for our measures of capital age and useful life of assets and for the dependent variables are presented in Table 1. Appendix E provides the definitions and summary statistics of all the variables used in our empirical analysis.

Panel A of Table 1 shows that average capital age in our sample equals 6.72 years, which is close to the value of 5.7 years in Lin et al. (2020). Moreover, capital age exhibits substantial variation across firms with a standard deviation of 3.3 years. The average useful life of assets is 13 years, similar to the value of 12.6 years in Livdan and Nezlobin (2021), and suggests that average capital age equals half of the useful life of assets, as in the model. Sample firms have an average net book leverage ratio of 17.9% (net market leverage ratio of 21.3% and net lease-adjusted leverage ratio of 31.9%) and, on average, 49.2% (30.8%) of their debt is maturing in more than 3 (5) years. Debt maturity from Capital IQ is 6.0 years, in line with prior studies (e.g., Choi et al., 2018) and close to average capital age.

Panel B of Table 1 shows the within-firm correlations between the variables of interest.

Panel A: Summary statistics								
	Capital age	Useful life	Net book leverage	Net market leverage	Net lease-adj. leverage	% debt mat.$> 3y$	% debt mat.$> 5y$	Debt mat. (yr.)
Mean	6.724	13.004	0.179	0.213	0.319	6.013	0.308	0.492
Standard deviation	3.289	5.862	0.240	0.276	0.240	4.646	0.303	0.338
Q1	4.271	9.000	0.037	0.029	0.174	3.000	0.000	0.152
Median	6.284	13.000	0.194	0.190	0.336	4.930	0.250	0.550
Q3	8.665	17.000	0.339	0.396	0.486	7.455	0.549	0.784
N	77877	70707	77877	77877	77877	18832	77877	77877

Panel B: Within-firm pairwise correlations								
	Capital age	Useful life	Net book leverage	Net market leverage	Net lease-adj. leverage	% debt mat.$> 3y$	% debt mat.$> 5y$	Debt mat. (yr.)
Capital age	1							
Useful life	0.243	1						
Net book lev.	-0.119	-0.0722	1					
Net market lev.	-0.103	-0.0562	0.831	1				
Net lease-adj. lev.	-0.0519	-0.0884	0.891	0.743	1			
% debt mat.$> 3y$	-0.112	0.00258	0.131	0.0737	0.0896	1		
% debt mat.$> 5y$	-0.126	0.0122	0.0818	0.0468	0.0492	0.645	1	
Debt mat. (yr.)	-0.0722	0.00221	0.00917	-0.00214	0.00173	0.183	0.227	1

Table 1: **Summary statistics: capital age and financing**. The table contains the summary statistics of capital age, the useful life of assets, and the financing variables. These include net book leverage, net market leverage, net lease-adjusted leverage and three measures of debt maturity: the ratios of debt maturing in more than 3 or 5 years to total debt as well as the debt maturity from Capital IQ. Panel A contains the summary statistics and Panel B contains the within-firm pairwise correlations between the respective variables. The sample period is from 1975 to 2018. All variables are defined in Table A.1.

As hinted by Figure 1, both net leverage and debt maturity are negatively correlated with capital age while debt maturity is positively correlated with the useful life of assets.

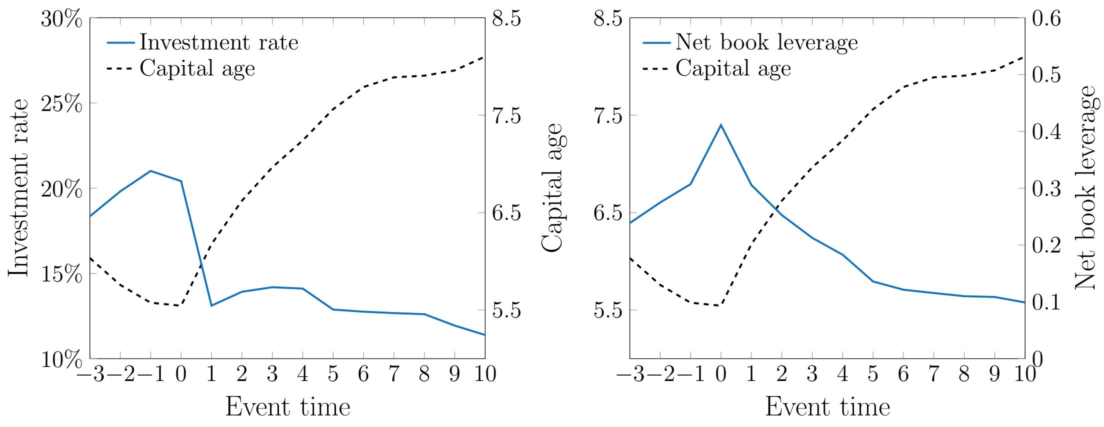

Figure 8: **Debt cycles: peak to trough**. The figure presents the evolution of capital age, net book leverage, and investment around leverage peaks. Event time $t = 0$ indicates the leverage peak. We only include debt cycles which span at least 10 years from peak to trough. The sample period is from 1975 to 2018. All variables are defined in Table A.1.

Before formally testing the model predictions, we illustrate our mechanism with Figure 8, which shows the evolution of capital age, net book leverage, and investment around leverage peaks. Event time $t = 0$ indicates the peak of the debt cycle, i.e. the time when each firm attains its highest net book leverage ratio (DeAngelo et al., 2018). The figure shows that capital age is the lowest after a peak in leverage, indicating that firms have replaced old capital. Over time, capital age increases while net book leverage decreases. Leverage peaks occur after investment peaks have led to the replacement of old capital. Figure IA.1 in the Internet Appendix focuses instead on investment spikes and shows that i) capital age drops sharply when a firm's investment rate spikes, ii) investment spikes are mostly financed with cash and debt, leading to a sharp increase in net book leverage, and iii) following the spike, capital age increases while net leverage decreases over time, in line with our predictions.

C Financing Cycles: Within-Firm Evidence

We first test the model's predictions regarding the within-firm relationship between capital age and financing. Predictions 1 and 4 suggest that leverage and debt maturity should both be negatively related to the within-firm evolution of capital age. To formally test these two predictions, we first estimate fixed-effect leverage regression models while controlling for standard determinants of leverage. Notably, we run regressions of the form

$$Lev_{i,j,t+1} = \phi CA_{i,t} + \beta X_{i,t} + \eta_i + \gamma_{t+1} + \kappa_{j,t+1} + \varepsilon_{i,t+1},$$

where $Lev_{i,j,t+1}$ is the net leverage of firm i in industry j, and the vector of controls X includes profitability, size, market-to-book, tangibility, cash flow volatility, R&D, and firm age (Lemmon et al., 2008). All specifications include firm fixed effects η_i and year fixed effects γ_t to account for time-invariant firm heterogeneity and time-varying factors common to all firms, respectively. Some specifications additionally include industry-year fixed effects $\kappa_{j,t}$ to control for industry-level shocks that can drive investment and leverage, where we use the Hoberg-Phillips fixed industry classification with 100 industries (Hoberg and Phillips, 2010, 2016). (Subsection IV.F shows that our results are robust to changing the industry definition.) Finally, we cluster standard errors at the firm level.

Table 2 presents the resulting estimates for net book leverage (columns 1 to 3), net market leverage (columns 4 to 6), and net lease-adjusted leverage (columns 7 to 9). The results confirm the sign of the univariate correlations from Table 1 and suggest that capital age and leverage are negatively related, even when including standard explanatory variables and fixed effects. In particular, a one standard deviation increase in capital age is associated with a 3.4 percentage point lower net book leverage ratio (3.6 percentage point lower net market leverage ratio and 1.7 percentage point lower net lease-adjusted leverage ratio); a 18.9% reduction in net leverage relative to the mean. This effect is economically significant and comparable to that of profitability. The inclusion of capital age does not change the signs of the leverage factors, but does impact the magnitudes of several variables. In unreported results, we find that capital age provides economically meaningful incremental explanatory power, as the adjusted within R^2 increases between 5% and 24% when including capital age in the specification (columns 2, 5 and 8), depending on the leverage measure. In Table IA.2 in the Internet Appendix we carry out an analysis of the importance of different determinants of leverage similar to Frank and Goyal (2009). We document that capital age is by and large the most important factor in terms of its explanatory power.

To investigate the relation between debt maturity and capital age, we follow the approach of Custódio et al. (2013) and Choi et al. (2018) and estimate maturity regressions of the form

$$Mat_{i,j,t+1} = \phi CA_{i,t} + \beta X_{i,t} + \eta_i + \gamma_{t+1} + \kappa_{j,t+1} + \varepsilon_{i,t+1},$$

where $Mat_{i,j,t+1}$ is maturity of debt of firm i in industry j, and the vector of controls X include size, size squared, market-to-book, asset maturity, abnormal earnings, cash flow volatility, R&D, net book leverage, and firm age. As in the leverage regressions, η_i, γ_t, $\kappa_{j,t}$ are firm, year, and industry-year fixed effects.

Table 3 presents the resulting estimates for the share of debt maturing in more than 3 years (columns 1 to 3), the share of debt maturing in more than 5 years (columns 4 to 6), and debt maturity from Capital IQ (columns 7 to 9). The results show that capital age and debt maturity are negatively related, in line with Prediction 4. The negative correlation is robust to controlling for typical determinants of debt maturity. More specifically, a one standard deviation increase in capital age is associated with a 0.433 year lower debt maturity and with a 3.3 (respectively 2.3) percentage point lower share of debt maturing in 3 (respectively 5) years. Furthermore, the economic effect is significant, as capital age also provides additional explanatory power: the adjusted within R^2 respectively increases by 42%, 57%, and 42% for debt maturing in more than 3 years, 5 years, and for debt maturity.[13] Finally, the fact that capital age is significant while asset maturity (i.e. the useful life of new capital) is not, is consistent with our model prediction that asset maturity is mainly a time-invariant firm characteristic while capital age can predict financing decisions.

[13] We compare the adjusted R^2 when adding capital age to the models in columns 2, 5, and 8 in Table 3.

	Net book leverage			Net market leverage			Net lease-adjusted lev.		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Capital age	-0.041***	-0.033***	-0.034***	-0.042***	-0.034***	-0.036***	-0.024***	-0.018***	-0.017***
	(-17.15)	(-11.61)	(-9.69)	(-14.81)	(-10.19)	(-9.32)	(-10.71)	(-6.41)	(-5.03)
Profitability		-0.037***	-0.029***		-0.054***	-0.046***		-0.038***	-0.031***
		(-16.14)	(-10.72)		(-20.46)	(-14.83)		(-17.27)	(-11.80)
Size		0.065***	0.084***		0.098***	0.125***		0.065***	0.084***
		(7.99)	(8.68)		(10.88)	(11.40)		(7.80)	(8.66)
Market-to-book		-0.015***	-0.018***		-0.033***	-0.031***		-0.014***	-0.017***
		(-6.42)	(-7.10)		(-14.27)	(-12.52)		(-6.13)	(-6.98)
Tangibility		0.042***	0.041***		0.053***	0.045***		0.038***	0.037***
		(8.56)	(7.37)		(9.63)	(7.31)		(8.07)	(6.97)
Cash flow volatility		-0.007***	-0.004		-0.010***	-0.005*		-0.004**	-0.001
		(-3.81)	(-1.55)		(-4.58)	(-1.78)		(-2.11)	(-0.47)
R&D		-0.008**	-0.007*		-0.012***	-0.008*		-0.009***	-0.008*
		(-2.40)	(-1.72)		(-3.73)	(-1.81)		(-2.66)	(-1.90)
Firm age		-0.048	0.013		-0.034	0.001		-0.027	0.027
		(-1.42)	(0.46)		(-1.01)	(0.03)		(-0.84)	(0.94)
Year FE	Yes	Yes	No	Yes	Yes	No	Yes	Yes	No
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Ind.-Year FE	No	No	Yes	No	No	Yes	No	No	Yes
Observations	65464	49882	33643	65464	49882	33643	65464	49882	33643
Adj. within R^2	0.0252	0.0798	0.0789	0.0176	0.0996	0.0984	0.0102	0.0693	0.0677

Table 2: **Capital age and leverage**. This table presents estimates from regressions of net leverage ratios on lagged capital age. The dependent variable is *Net book leverage* in columns 1 to 3; *Net market leverage* in columns 4 to 6 and *Net lease-adjusted leverage* in columns 7 to 9. Each explanatory variable is standardized by its full-sample standard deviation. The models in columns 3, 6 and 9 include industry-year fixed effects created using Hoberg-Phillips fixed industry classification with 100 industries. The sample period is from 1975 to 2018. All variables are defined in Table A.1. t-statistics are reported in parentheses. Standard errors are clustered at the firm level. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

	% debt maturing > 3y			% debt maturing > 5y			Debt maturity (yr.)		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Capital age	-0.043***	-0.035***	-0.033***	-0.035***	-0.029***	-0.023***	-0.245**	-0.343***	-0.433***
	(-15.69)	(-10.18)	(-7.12)	(-12.12)	(-7.98)	(-4.89)	(-2.10)	(-2.59)	(-2.86)
Size		0.068***	0.111***		0.008	0.026		1.898**	2.245**
		(2.95)	(3.75)		(0.39)	(1.07)		(2.45)	(2.51)
Size squared		-0.009	-0.052*		0.038**	0.028		-1.311	-1.590*
		(-0.46)	(-1.93)		(2.05)	(1.17)		(-1.64)	(-1.69)
Market-to-book		0.011***	0.008**		0.006**	0.000		0.074	0.071
		(3.56)	(2.19)		(2.21)	(0.03)		(0.76)	(0.61)
Asset maturity		0.003	0.000		0.008**	0.004		0.207*	0.208
		(0.85)	(0.00)		(2.10)	(0.77)		(1.79)	(1.50)
Abnormal earnings		0.006***	0.005***		0.005***	0.006***		0.005	0.026
		(5.32)	(3.29)		(5.97)	(4.86)		(0.24)	(0.78)
Cash flow volatility		-0.004	-0.004		-0.004	-0.001		-0.044	0.001
		(-1.45)	(-1.06)		(-1.51)	(-0.46)		(-0.71)	(0.02)
R&D		0.004	0.002		0.001	-0.002		0.016	-0.013
		(0.84)	(0.28)		(0.22)	(-0.34)		(0.09)	(-0.08)
Net book leverage		0.027***	0.032***		0.014***	0.013***		0.015	-0.005
		(7.89)	(6.99)		(4.15)	(3.03)		(0.16)	(-0.04)
Firm age		-0.073	-0.019		-0.102**	-0.032		3.218**	2.785*
		(-1.58)	(-0.42)		(-1.96)	(-0.59)		(2.15)	(1.73)
Year FE	Yes	Yes	No	Yes	Yes	No	Yes	Yes	No
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Ind.-Year FE	No	No	Yes	No	No	Yes	No	No	Yes
Observations	65464	48654	32608	65464	48654	32608	15923	13001	11478
Adj. within R^2	0.0108	0.0199	0.0161	0.0082	0.0127	0.0079	0.0010	0.0054	0.0055

Table 3: **Capital age and debt maturity – within-firm regressions**. The table presents estimates from regressions of debt maturity on lagged capital age. The dependent variable is *% of debt maturing in > 3 years* in columns 1 to 3; *% of debt maturing in > 5 years* in columns 4 to 6; and *Debt maturity (yr.)* in columns 7 to 9. Each explanatory variable is standardized by its full-sample standard deviation. Models in columns 3, 6 and 9 include industry-year fixed effects created using Hoberg-Phillips fixed industry classification with 100 industries. The sample period is from 1975 to 2018. All variables are defined in Table A.1. t-statistics are reported in parentheses. Standard errors are clustered at the firm level. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

In Table IA.3 in the Internet Appendix, we carry out an analysis of the importance of the factors that we use in our debt maturity regressions. Similar to leverage, we show that capital age has the most explanatory power.

D Financing Cycles: Exploring the Mechanism

Having established that capital age plays an important role in explaining within-firm variation in net leverage and debt maturity, we now go deeper in the analysis of the mechanism by investigating various economic channels which are bound to influence our results. We examine three such channels: the intensity of intangible capital use, the lumpiness of investment, and the reliance on leasing. According to Prediction 3, the first two channels are expected to respectively weaken and strengthen the effects of capital age on leverage and debt maturity. In addition, despite not being explicitly accounted for in the model, we expect leasing to weaken the mechanism in the paper, as firms relying more on leasing will finance fewer of their assets with debt.

First, we investigate how the extent of intangible capital use affects the strength of our mechanism. To this end, each fiscal year we split the firms into terciles based on their capital intangiblity, proxied by the ratio of intangible assets to total assets, and on the R&D to sales ratio, if R&D is positive. We expect that firms with a larger share of intangibles on their balance sheet and firms spending more on R&D will exhibit a less pervasive influence of capital age on financing. We test our hypothesis by running net leverage and debt maturity regressions in each subsample. Table 4 presents the results of our tests.

The results in Panel A indicate that firms which rely more heavily on intangible capital and firms with larger R&D expenses have a lower sensitivity of net leverage to ageing capital. For example, when using capital age as the only explanatory variable, a one standard deviation increase in capital age is associated with a 7.8 percentage point lower net book leverage when firms spend less on R&D, but only a 4.4 percentage point decrease when their R&D expenses are high. When controlling for other leverage determinants, the difference between the two terciles remains large. Importantly, the adjusted R^2 is dramatically larger in the lowest terciles of intangibility and R&D, further indicating that our mechanism is able to explain more variation in net leverage when intangible capital is less important. Panel B of 4 presents the results of our test for debt maturity. The effect of capital age on debt maturity is stronger when firm rely less on intangible capital and spend less on R&D investment. The quantitative difference between the first and third terciles in terms of the explanatory power and size of the effect of capital age on debt maturity are also meaningful.

Second, we analyze the role of investment lumpiness. Our model implies that when the spells between investment spikes become longer, our mechanism becomes more important. As such, we expect that financing policy of firms with more lumpy investment is more sensitive to capital age. Given the maturity matching in Prediction 5, we expect investment lumpiness to be particularly important for debt maturity. To test the hypothesis, we split firms into terciles based on two proxies of investment lumpiness—the firm-level skewness and kurtosis of investment. We then run the leverage and debt maturity regressions with and without control variables in each sub-sample. Table 5 presents the resulting estimates.

The results in Panel A indicate that firms for which investment is more lumpy have a higher sensitivity of net leverage to ageing capital. For example, when using capital age as the only explanatory variable, a one standard deviation increase in capital age is associated with a 5.3 percentage point lower net book leverage when investment is more lumpy, but only a 3.1 percentage point decrease when their investment is less lumpy. Importantly, the adjusted R^2 is again larger in the highest terciles of investment lumpiness, further indicating that our mechanism is able to explain more variation in net leverage when investment is more lumpy. Panel B of 5 presents the results of our test for debt maturity. As expected, the effect of capital age on debt maturity is stronger when investment is more lumpy. The quantitative difference between the first and third terciles in terms of the explanatory power and size of the effect of capital age on debt maturity are also substantial.

Panel A: Net leverage

	Intangibility				R&D			
	Low	High	Low	High	Low	High	Low	High
Capital age	-0.045***	-0.028***	-0.022***	-0.017***	-0.078***	-0.044***	-0.049***	-0.028**
	(-8.18)	(-5.22)	(-3.63)	(-3.02)	(-7.01)	(-3.33)	(-6.03)	(-2.02)
Controls	No	No	Yes	Yes	No	No	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Ind.-Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	9770	9538	9770	9538	3148	3212	3148	3212
Adj. within R^2	0.0293	0.0126	0.0973	0.0343	0.1108	0.0198	0.1732	0.0792

Panel B: Debt maturity

	Intangibility				R&D			
	Low	High	Low	High	Low	High	Low	High
Capital age	-0.046***	-0.039***	-0.037***	-0.030***	-0.048***	-0.038**	-0.037**	-0.022
	(-5.10)	(-4.40)	(-4.00)	(-3.14)	(-3.46)	(-2.29)	(-2.40)	(-1.24)
Controls	No	No	Yes	Yes	No	No	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Ind.-Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	9770	9538	9207	9410	3148	3212	3074	3134
Adj. within R^2	0.0075	0.0044	0.0170	0.0106	0.0086	0.0037	0.0081	0.0123

Table 4: **Capital age and financing – the role of intangible capital intensity**. This table presents estimates from regressions of net leverage (Panel A) and debt maturity (Panel B) on lagged capital age for subsamples of firms split each fiscal year into terciles by the degree of intangible capital intensity (the ratio of intangible to total assets and the ratio of R&D to sales, when R&D is non-zero). The dependent variable is *Net book leverage* in Panel A and *% of debt maturing in > 3 years* in Panel B. Each explanatory variable is standardized by its full-sample standard deviation. We control for all independent variables from Tables 2 in Panel A and 3 in Panel B. All models include industry-year fixed effects created using the Hoberg-Phillips fixed industry classification with 100 industries. The sample period is from 1975 to 2018. All variables are defined in Table A.1. t-statistics are reported in parentheses. Standard errors are clustered at the firm level. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

Panel A: Net leverage								
	Skewness				Kurtosis			
	Low	High	Low	High	Low	High	Low	High
Capital age	-0.034***	-0.053***	-0.025***	-0.044***	-0.041***	-0.052***	-0.029***	-0.041***
	(-6.05)	(-10.63)	(-3.83)	(-7.83)	(-5.54)	(-10.11)	(-3.42)	(-7.25)
Controls	No	No	Yes	Yes	No	No	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Ind.-Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	11558	13819	9124	12269	11211	14066	8373	12733
Adj. within R^2	0.0174	0.0414	0.0485	0.0987	0.0203	0.0395	0.0517	0.1034

Panel B: Debt maturity								
	Skewness				Kurtosis			
	Low	High	Low	High	Low	High	Low	High
Capital age	-0.029***	-0.053***	-0.019**	-0.038***	-0.038***	-0.051***	-0.031***	-0.037***
	(-4.09)	(-8.28)	(-2.18)	(-4.75)	(-4.64)	(-8.39)	(-2.86)	(-4.95)
Controls	No	No	Yes	Yes	No	No	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Ind.-Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	11558	13819	8692	11935	11211	14066	7912	12418
Adj. within R^2	0.0033	0.0124	0.0136	0.0238	0.0047	0.0118	0.0122	0.0224

Table 5: **Capital age and financing – the role of investment lumpiness**. This table presents estimates from regressions of net leverage (Panel A) and debt maturity (Panel B) on lagged capital age for subsamples of firms split into terciles by the proxy of investment lumpiness (firm-level skewness and kurtosis). The dependent variable is *Net book leverage* in Panel A and *% of debt maturing in > 3 years* in Panel B. Each explanatory variable is standardized by its full-sample standard deviation. We control for all independent variables from Tables 2 in Panel A and 3 in Panel B. All models include industry-year fixed effects created using the Hoberg-Phillips fixed industry classification with 100 industries. The sample period is from 1975 to 2018. All variables are defined in Table A.1. t-statistics are reported in parentheses. Standard errors are clustered at the firm level. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

Third, we focus on leasing, with the expectation that firms which predominantly lease their assets should be less affected by our mechanism. Table IA.1 in the Internet Appendix splits firms into terciles, quintiles and deciles based on the ratio of capitalized rental expenses to total lease-adjusted assets. The effect of capital age on net leverage is always stronger in subsamples of firms that rely less on leasing, and largely insignificant when firms rely almost exclusively on leasing to finance their investment, confirming the intuition that our mechanism is less effective when firms rely more on leasing to finance assets.

E Financing Cycles: Cross-Sectional Evidence

We next turn to the cross-sectional predictions of the model, namely that firms with longer-lived assets should follow longer debt cycles (Prediction 2) and have a higher average debt maturity (Prediction 5). We proxy for the useful life of the firm assets using an accounting based measure—the ratio of the firm's book value of its physical assets to its depreciation costs. This measure is a proxy for the economic life of the firm's assets, n in the model, and does not directly depend on capital adjustment costs. Indeed, the measure captures the number of years to fully depreciate the capital stock and does not rely on when the firm actively chooses to replace it. As a consequence, the cross-sectional tests allow us to further examine the importance of our mechanism for debt and maturity cycles.

To test the first prediction, we need to obtain a measure of the length of a firm's debt cycle. To do so, we define a leverage spike as an instance in which the firm's net book leverage exceeds its firm-specific median by one standard deviation. The length of the cycle is then the number of years between consecutive leverage spikes, where we require a minimum of five years between spikes. We then average the useful life of assets and the length of the cycles for each firm in our sample. To test the second prediction, we examine the relationship between average debt maturity and the average useful life of assets. The bottom panels in Figure 1 show that both the debt cycle length and average debt maturity are increasing in the firm's average useful life.

To formally test the model predictions, we regress both the maximum length of the debt cycle and the average length of the debt cycle on the average useful life of the assets. Formally, we run cross-sectional regressions of the form

$$Cycle_i = \alpha + \phi UL_i + \beta X_i + \varepsilon_i,$$

where $Cycle_i$ is either the maximum or the average length of the cycle of firm i, and X is a vector of averaged firm-level controls that includes market-to-book, tangibility, profitability, size, cash flow volatility, R&D, and firm age. Given that observations are at the firm-level, we cluster standard errors at the industry level using the Hoberg-Phillips fixed industry classification with 100 industries (Hoberg and Phillips, 2010, 2016).

Table 6 presents the resulting estimates for the maximum debt cycle lengths (columns 1 to 2) and the average debt cycle length (columns 3 to 4). The results suggest a strong positive association between the cycle length and the firm's average asset life, consistent with Prediction 2, and are robust to controlling for common determinants of leverage. A one-year increase in asset life is associated with a one- to two-month increase in average debt cycle length, depending on the specification. Moreover, consistent with the model, Table IA.8 in the Internet Appendix shows that the relation between asset life and investment cycle length has the same sign and magnitude as that between asset life and debt cycles. Thus, consistent with Prediction 2, firms with longer lived assets have longer debt cycles.

To test Prediction 5, namely that the average useful life is positively associated with the average debt maturity, we regress the firm-level averages of the debt maturity measures on the average useful life of the assets. Formally, we run cross-sectional regressions of the form

$$Mat_i = \alpha + \phi UL_i + \beta X_i + \varepsilon_i,$$

where Mat_i is the average of the debt maturity for firm i, and X is a vector of average firm-level controls that includes market-to-book, size, size squared, abnormal earnings, cash flow volatility, R&D, net book leverage, and firm age.

Table 7 presents the resulting estimates for average % debt maturing in more than 3

	Max. debt cycle		Avg. debt cycle	
	(1)	(2)	(3)	(4)
Useful life	0.189***	0.104***	0.143***	0.079***
	(5.56)	(3.65)	(5.01)	(3.22)
Market-to-book		0.152		0.042
		(0.83)		(0.22)
Tangibility		-0.976		-0.715
		(-1.50)		(-1.21)
Profitability		6.381***		5.584***
		(4.22)		(4.16)
Size		0.228***		0.145*
		(2.65)		(1.85)
Cash flow volatility		-16.967***		-14.888***
		(-4.89)		(-4.90)
R&D		11.952**		9.594**
		(2.40)		(2.35)
Firm age		0.119***		0.084***
		(9.71)		(8.81)
Observations	1937	1932	1937	1932
Adjusted R^2	0.03	0.21	0.02	0.15

Table 6: **Useful life and debt cycles – cross-sectional regressions**. This table presents estimates from regressions of maximum and average debt cycle length on average useful life. The dependent variable is *Maximum debt cycle length* in columns 1 and 2, and *Average debt cycle length* in columns 3 and 4. The sample period is from 1975 to 2018. Firms with no leverage spike have a cycle length that is undefined and are dropped from the sample. All variables are defined in Table A.1. t-statistics are reported in parentheses. Standard errors are clustered at the industry level using Hoberg-Phillips fixed industry classification with 100 industries. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

years (columns 1 and 2) and 5 years (columns 3 and 4), and average debt maturity (columns 5 and 6). The results document a positive and significant relationship between the average debt maturity and the average useful life. This relationship is also similar in magnitude as the relationship between useful life and debt cycle lengths. A one-year increase in average useful life is associated with a roughly one- to two-month increase in average debt maturity, depending on the specification. The results are consistent with Prediction 5 that firms with longer-lived assets will have longer debt maturities, matching the duration of their debt with the duration of their assets.

	% debt maturing $> 3y$		% debt maturing $> 5y$		Debt maturity (yr.)	
	(1)	(2)	(3)	(4)	(5)	(6)
Useful life	0.015***	0.010***	0.013***	0.009***	0.146***	0.099***
	(7.67)	(9.48)	(7.03)	(8.19)	(5.28)	(4.63)
Market-to-book		0.017***		0.002		0.066
		(2.78)		(0.45)		(0.82)
Size		0.066***		0.028***		-0.062
		(8.01)		(4.12)		(-0.29)
Size squared		-0.000		0.002***		0.065**
		(-0.51)		(3.48)		(2.58)
Abnormal earnings		0.041		0.022		0.654
		(1.35)		(0.99)		(1.56)
Cash flow volatility		-0.338**		-0.102		-5.313***
		(-2.27)		(-1.31)		(-3.49)
R&D		0.450***		0.424***		1.942
		(3.73)		(3.97)		(1.08)
Net book leverage		0.359***		0.238***		1.053**
		(10.88)		(8.44)		(2.57)
Firm age		-0.001*		0.001**		0.016**
		(-1.82)		(2.52)		(2.25)
Observations	4704	4382	4704	4382	2684	2559
Adjusted R^2	0.09	0.46	0.10	0.38	0.05	0.21

Table 7: **Useful life and debt maturity – cross-sectional regressions**. The table presents estimates from regressions of debt maturity on average useful life. The dependent variable is the average of each firm's *% of debt maturing in > 3 years* in columns 1 to 2; *% of debt maturing in > 5 years* in columns 3 to 4; and *Debt maturity (yr.)* in columns 5 to 6. The sample period is from 1975 to 2018. All variables are defined in Table A.1. t-statistics are reported in parentheses. Standard errors are clustered at the industry level using the Hoberg-Phillips fixed industry classification with 100 industries. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

F Aggregate Financing Cycles

Our model also has predictions for aggregate leverage and debt maturity cycles. Notably, the model's key implication is that industries with a wider cross-sectional dispersion in capital age have reduced leverage and maturity dispersion; see Prediction 6. To test this prediction, we need proxies for the cross-sectional dispersion in capital age and for financing cycles at the industry level. We first group firms into industries using the Hoberg-Phillips fixed industry classification with 100 industries (Hoberg and Phillips, 2010, 2016), and apply the same industry filters as before. To ensure a cross-section of firms within an industry, we drop industry-year observations with less than 3 firms. Likewise, to ensure a sufficient time-series to examine the cyclicality of the industry, we drop industries with less than ten consecutive years of data. These filters result in a final sample of 80 industries.

To generate a proxy for industry-level dispersion in capital age, we take the average of each industry's annual standard deviation of firm-level capital age. We refer to this average as the industry's *capital age dispersion*. To generate a proxy for the financing cyclicality of the industry, we first calculate aggregate industry-level annual observations for net book leverage and share of debt maturing in more than 3- or 5-years using the same method as their firm-level counterparts.[14] We then take the standard deviation of these aggregates over the time series and denote these variables *aggregate leverage dispersion* and *aggregate debt maturity dispersion*.

As an example of the intuition, Figure 9 plots the annual aggregated net book leverage, linearly detrended for comparison, for the industries with the 25$^{\text{th}}$ percentile and 75$^{\text{th}}$ percentile of the capital age dispersion distribution over the years that both industries were in the sample (1991-2008). Both industries exhibit variation in net book leverage and show some degree of cyclicality. The industry with more capital age dispersion, primarily related to industrial machinery manufacturing, however, has a smoother profile of net leverage and is less cyclical relative to the industry with the 25$^{\text{th}}$ percentile of capital age dispersion,

[14]We do not calculate a measure for the debt maturity from Capital IQ because it only is defined for a subsample of firms and the time period is limited to 2002-2018.

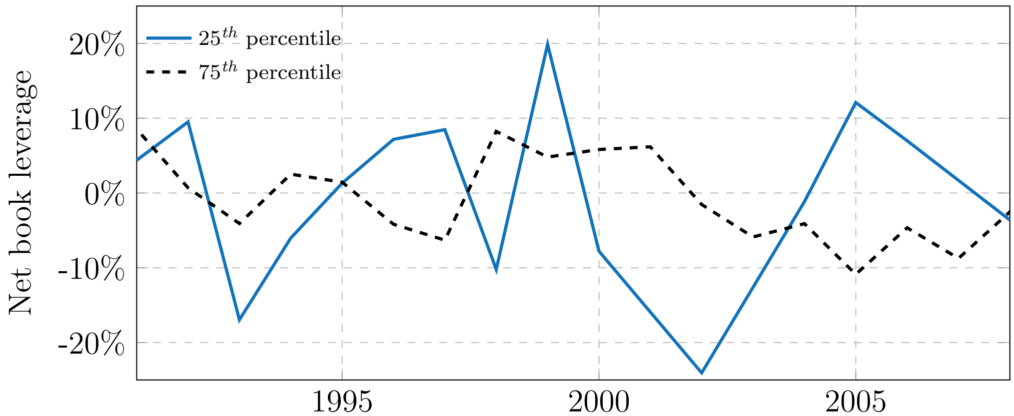

Figure 9: **Industry debt cycles and capital age dispersion**. The figure presents the linearly detrended aggregate industry net book leverage over time for the Hoberg-Phillips fixed 100 industries with the 25^{th} and 75^{th} percentile of capital age dispersion distribution. The sample period is from 1991 to 2008, the years in which both industries were in the sample. All variables are defined in Table A.1.

mainly related to toys and recreation. This matches our model's prediction that capital age dispersion mitigates the cyclicality of leverage in an industry.

To further explore the relation between industry cyclicality and capital age dispersion, we regress the aggregate leverage and debt maturity dispersion on the aggregate capital age dispersion. The former is a proxy for the cyclicality of net book leverage and debt maturity, with a higher standard deviation proxying for industries with a higher cyclicality. Table 8 presents the resulting estimates.[15] For robustness, we also report results from linearly detrending the industry levels to account for any general industry trends that might drive dispersion. Consistent with Prediction 6, the table shows that there is a negative and statistically significant relation between capital age dispersion and both aggregate leverage dispersion (columns 1 and 2) and aggregate debt maturity dispersion (columns 3 to 6).

[15]We adjust standard errors for heteroskedasticity using the HC3 method, which is the more conservative method when dealing with small sample sizes (Long and Ervin, 2000).

	Dispersion							
	Book lev.		Lease-adj. lev.		Mat. ($> 3y$)		Mat. ($> 5y$)	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Capital age dispersion	-0.020**	-0.018**	-0.016**	-0.014*	-0.050***	-0.044***	-0.037***	-0.033***
	(-2.245)	(-2.113)	(-2.015)	(-1.979)	(-3.288)	(-3.000)	(-3.036)	(-2.993)
Detrended:	No	Yes	No	Yes	No	Yes	No	Yes
Observations	80	80	80	80	80	80	80	80
Adj. R^2	0.03	0.03	0.02	0.02	0.10	0.08	0.07	0.07

Table 8: **Capital age dispersion and financing cyclicality**. This table presents estimates from regressions of aggregate industry leverage and debt maturity dispersion on capital age dispersion. The dependent variable is *Book leverage dispersion* in column 1 and 2, *Lease-adjusted leverage dispersion* in columns 3 and 4, *Maturity dispersion (> 3 years)* in columns 5 and 6 and *Maturity dispersion (> 5 years)* in columns 7 and 8. Industry definitions are based on the Hoberg-Phillips fixed industry classification with 100 industries. In columns 1, 3, 5 and 7 the dispersion is based on the raw levels of the industry. In columns 2, 4, 6 and 8, the dispersion is based on the linearly detrended levels. The sample period is from 1988 to 2018. All variables are defined in Table A.1. t-statistics are reported in parentheses. Standard errors are adjusted for heteroskedasticity using the robust HC3 method. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

G Robustness

We examine the robustness of our results in the Internet Appendix. First, we show that our results are largely robust to employing a different measure of capital age. We create this measure by assuming that when the firm disinvests, it disposes of the oldest capital vintages first, unlike in our main measure, where *all* vintages are equally affected (Lin et al., 2020). Tables IA.4 and IA.5 show that the effect of the alternative proxy of capital age on net leverage and debt maturity remains economically similar to the main specification.

Second, we show that our results are robust to changing the industry definition. While our main specifications use the Hoberg-Phillips fixed industry classification with 100 industries, Table IA.6 and IA.7 use the Hoberg-Phillips fixed industry classification with 50 industries and the Fama-French industry classification with 49 industries. Third, we show that our cross-sectional results for debt cycles carry over to investment cycles in Table IA.8 and are robust to alternative measures of the useful life of assets in Tables IA.9, IA.10, and IA.11.

Finally, in Table IA.12 we examine the relation between the aggregate leverage and debt maturity dispersion and capital age dispersion for the Hoberg-Phillips fixed industry classification with 50 industries and Fama-French industry classification with 49 industries. Results for leverage dispersion are similar to the Hoberg-Phillips fixed 100 industry classification. Results are not statistically significant for maturity dispersion (potentially due to the smaller sample size), although the direction and magnitudes are similar.

V Conclusion

Capital ages and must eventually be replaced. This paper develops a dynamic investment and financing model to study how ageing capital generates variation in financing decisions. In this model, firms issue debt to finance investment. As capital ages, they deleverage to free up debt capacity, which allows them to replace old capital by issuing new debt. To achieve these dynamics, firms issue debt with a maturity that matches the useful life of new assets and an amortization schedule that reflects the need to free up debt capacity as capital ages. These debt dynamics lead to debt cycles consistent with leverage being fundamentally unstable (Denis and McKeon, 2012; DeAngelo et al., 2018) and to a maturity matching theory of debt (Stohs and Mauer, 1996). They also imply that both leverage and debt maturity should be negatively related to capital age while both the duration of debt cycles and debt maturity should be positively related to the useful life of assets. We embed this single-firm model into an industry equilibrium and show that debt and maturity cycles spill over into the aggregate when capital age is correlated across firms.

We take the model predictions to the data and find that all our measures of leverage and debt maturity are negatively related to capital age while all measures of the duration of debt cycles or debt maturity are positively related to the useful life of assets. The effects we document are stronger in firms with more lumpy investment, with a smaller fraction of intangible assets, and relying less on leasing. We also find that capital age dispersion is negatively related to leverage and maturity cyclicality, consistent with the model predictions.

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Appendix

The first part of the appendix derives the single firm results (Proposition 1, Theorem 1, and Proposition 2). The second part derives the financing results under arbitrary capital depreciation schedules (Proposition 3). The third part derives the debt maturity results (Theorem 2, Proposition 4, and Theorem 3). The fourth part derives the industry equilibrium results (Proposition 5, Proposition 6, and Proposition 7). The final part defines the variables used in the empirical analysis.

A Single Firm

We impose the following parameter restrictions. First we assume that

$$K < \frac{\pi}{r}\left(1 + \frac{1}{r}\frac{\rho_C}{(1+\rho_C)^n - 1}\right)^{-1}, \tag{A.1}$$

which ensures that investing is positive NPV for an unlevered firm. Second, we assume that

$$\phi \geq \underline{\phi} = \frac{\max\{K - C_0, 0\}}{\pi}, \tag{A.2}$$

$$\phi < \bar{\phi} = \min\left\{\frac{1}{r} - \frac{K}{\pi}\left(1 + \frac{1}{r}\frac{\rho_C}{(1+\rho_C)^n - 1}\right), \frac{1}{\rho_D}\left(1 - \frac{K}{\pi}\frac{\rho_C(1+r)^n + r(1+\rho_C)^n - r}{(1+\rho_C)^n - 1}\right)\right\}. \tag{A.3}$$

As we show below, the upper bound on ϕ ensures that debt is risk-free. The lower bound on ϕ ensures that the firm can initially purchase the asset.

The single-firm results are organised as follows. First, we show that investing is positive NPV when investment is internally financed (Lemma 1). Second, we show that this is also true when the firm can issue debt and that the firm has no incentive to default (Lemma 2 and Proposition 1). Having established that the firm invests and does not default, we derive the firm's optimal financing policy (Theorem 1). We then establish that the firm pays dividends in period $t+1$ only if the collateral constraint binds in period t (Lemma 3) and that the collateral constraints binds when the firm invests (Lemma 4).

Lemma 1 (Benchmark Firm Value)**.** *The value of a firm that retains profits to finance investment internally is given by*

$$C_0 + \frac{\pi}{r} - K\left(1 + \frac{1}{r}\frac{\rho_C}{(1+\rho_C)^n - 1}\right).$$

Proof. If the firm saves s today and for the next $n-1$ periods and earns a rate ρ_C on its cash balances, then the future value of its savings in $n-1$ periods is

$$\sum_{i=0}^{n-1} s(1+\rho_C)^i = s\frac{(\rho_C+1)^n - 1}{\rho_C}.$$

As a result, the firm has enough savings to finance investment after n periods if

$$s = K\frac{\rho_C}{(\rho_C+1)^n - 1}.$$

The firm earns enough to save for investment if

$$\pi - s = \pi - K\frac{\rho_C}{(\rho_C+1)^n - 1} \geq 0,$$

This is guaranteed by restriction (A.1). The value of a firm that saves to finance investment is then given by

$$C_0 - K + \sum_{t=1}^{\infty} \frac{\pi - s}{(1+r)^t} = C_0 + \frac{\pi}{r} - K\left(1 + \frac{1}{r}\frac{\rho_C}{(1+\rho_C)^n - 1}\right),$$

which is bigger than C_0 given the restriction on K. □

Lemma 2 (Firm Investment and Default)**.** *The firm replaces capital when it has reached the end of its useful life and never before. Furthermore, the firm never defaults on its debt obligations.*

Proof. We want to show that the firm always invests when assets reach the end of their useful life and has no incentive to default. To do so, we assume that creditors always believe that the firm will not default and therefore charge an interest rate ρ_D on debt. We then show that, given this belief, the firm has no incentive to default and always invests so that the belief is consistent and constitutes an equilibrium.

Since the firm holds cash $C_0 > 0$ and there is no debt payment due, the firm never defaults at time $t = 0$. Furthermore, the firm never defaults when it holds a positive amount of cash as net debt is negative. Therefore, we assume in this lemma that net debt is positive, in that $ND_t > 0$. Assume now that the firm does not invest at time $t = 0$ and defaults at $t = 1$. This is suboptimal since

$$\underbrace{C_0 + D_0}_{\textit{Value of firm that defaults at } t=1} \leq C_0 + \phi\pi < \underbrace{C_0 + \frac{\pi}{r} - K\left(1 + \frac{1}{r}\frac{\rho_C}{(1+\rho_C)^n - 1}\right)}_{\textit{Value of an internally financed firm}} \leq E_0$$

where the first inequality follows from the cash flow based collateral constraint and the second inequality follows from the restrictions on ϕ; see equations (A.2) and (A.3).[16] As a result, default can only happen for $t > 1$.

Assume that the firm has net debt $ND_t > 0$ at time $t > 0$ and defaults at time $t+1 > 1$. If the firm has capital installed at time t and therefore produces the final good at time $t+1$, we have that $\rho_D ND_t \leq \rho_D \phi\pi < \pi$ (see equation (A.3)). Therefore, the firm can make the interest payment $\rho_D ND_t$ and a positive dividend payment

$$Div_{t+1} \geq \pi - \rho_D \phi\pi > 0$$

if it chooses $ND_{t+1} = ND_t$ and defaults at $t+2$. As a result, the firm will not default if it produces the good at $t+1$.

Assume next that the firm has no (more) installed capital at time t and does not invest so that it does not produce the good at $t+1 > 1$ and therefore defaults at $t+1$. Clearly, each period since the last time it invested $t' \geq t-n$ it must be that leverage is $ND_{t'} = \phi\pi$. Otherwise, the firm would benefit from increasing leverage and bringing dividend payments forward in time since $\rho_C < \rho_D < r$ and $\rho_D\phi\pi < \pi$. This also implies that the firm pays a dividend of $Div_{t'} = \pi - \rho_D\phi\pi$ for the n-periods $t' \in [t-n+1, t]$.

Our objective is now to show that there is a profitable deviation for the firm's shareholders, namely to save for the n-periods $t' \in [t-n+1, t]$ and invest at time t and thereby avoid default at $t+1$. If instead of paying dividends, the firm saves $s < \pi - \rho_D\phi\pi$ each period after the last time it invested ($t' \in [t-n+1, t]$) and puts this money in a savings account, then its savings at time t amount to:

$$\sum_{a=0}^{n-1} s(1+\rho_C)^{n-1-a} = s\frac{(1+\rho_C)^n - 1}{\rho_C}.$$

Instead, paying out s each period generates a value at time t of

$$\sum_{a=0}^{n-1} s(1+r)^{(n-1-a)} = s\frac{(1+r)^n - 1}{r}.$$

The firm saves enough to finance investment if

$$s = K\frac{\rho_C}{(1+\rho_C)^n - 1}$$

We need that the firm generates enough profits to save this amount. That is, we need

$$\pi(1-\rho_D\phi) > K\frac{\rho_C}{(1+\rho_C)^n - 1}, \tag{A.4}$$

[16] We need (A.2) to hold since it ensures that the firm has enough resources to invest at time zero.

which holds under restriction (A.3). The firm prefers saving over paying dividends if

$$\underbrace{s\frac{(1+r)^n-1}{r}}_{\textit{Pay dividends}} = K\frac{\rho_C}{(1+\rho_C)^n-1}\frac{(1+r)^n-1}{r} < \underbrace{\frac{\pi-\rho_D\phi\pi}{r} - K\left(1+\frac{1}{r}\frac{\rho_C}{(1+\rho_C)^n-1}\right)}_{\textit{Internally financed firm with debt obligations } \phi\pi}.$$

The firm that would save for investment is worth at least as much as the internally financed firm that makes coupon payments on its debt forever.[17] This condition can be written as

$$\phi < \frac{1}{\rho_D}\left(1-\frac{K}{\pi}\frac{\rho_C(1+r)^n+r(1+\rho_C)^n-r}{(1+\rho_C)^n-1}\right),$$

which holds under restriction (A.3).

A direct implication of the fact that it never defaults is that the firm always replaces capital at the end of its useful life. The firm also never replaces capital early. If it would do so, then it could increase its firm value by delaying replacement and yield a return of $\rho_C K > 0$ on the cost of capital, which could be paid out as a dividend while leaving all other policies and cash flows unchanged. □

Proof of Proposition 1. It follows directly from Lemma 2. □

Proof of Theorem 1. We want to show that the firm's net debt is weakly decreasing in capital age. To establish this result, we first need to show that the firm only pays dividends when the collateral constraint binds in the previous period.

We show below that it is suboptimal for the firm to pay dividends at time $t+1$ if the collateral constraint does not bind at time t. Therefore, the collateral constraint must bind at time t if the firm pays dividends at time $t+1$.

We know from Proposition 1 that the firm always replaces capital when it reaches the end of its useful life. We know from Lemma 2 that the debt is risk-free. Assume that for some t, $Div_{t+1} > 0$ while $ND_t < \phi\pi$. Define ΔDiv_t as

$$\Delta Div_t = \min\left\{\frac{Div_{t+1}}{1+\rho_D}, \phi\pi - ND_t\right\}.$$

Increasing dividends at time t to $Div'_t = Div_t + \Delta Div_t$ by using debt financing would imply that $Div'_{t+1} \geq Div_{t+1} - (1+\rho_D)\Delta Div_t$. The inequality follows from the fact that the interest rate is lower if net debt was negative before $ND_t < 0$.[18] This change in policy would increase

[17] Observe that the value of the internally financed firm is actually a lower bound since some of the savings can be used to temporarily lower net debt, which yields a rate of return $\rho_D > \rho_C$.

[18] Indeed, if $ND_t < 0$ and $ND_t + \Delta Div_t \leq 0$ then the discount rate is ρ_C and the change in the amount

shareholder value since its effect on equity value (at time t) is at least

$$\Delta Div_t - \frac{(1+\rho_D)\Delta Div_t}{1+r} > 0.$$

As a result, if $ND_t < \phi\pi$, then $Div_{t+1} = 0$ and therefore if $Div_{t+1} > 0$ then $ND_t = \phi\pi$.

Assume $a > 0$ and $ND_{a-1} < ND_a \leq \phi\pi$. If $ND_{a-1} > 0$ then

$$Div_a = \pi + ND_a - ND_{a-1}(1+\rho_D) > \rho_D\phi\pi - \rho_D ND_{a-1} + (ND_a - ND_{a-1}) > 0$$

because $\phi < \frac{1}{\rho_D}$, see equation (A.3). While if $ND_{a-1} < 0$

$$Div_a = \pi + ND_a - ND_{a-1}(1+\rho_C) > 0.$$

But this contradicts the previous result and therefore $ND_{a-1} \geq ND_a$.

□

Lemma 3. *If $Div_{t+1} > 0$ then $ND_t = \phi\pi$.*

Proof. This result follows directly from the proof of Theorem 1. □

Lemma 4. $ND_{a=0} = \phi\pi$.

Proof. We want to show that $ND_{a=0} = \phi\pi$. We do so by showing that $ND_{a=0} < \phi\pi$ can never occur. Assume that for some $t' \geq 0$ with $a = 0$ we have $ND_{t'} < \phi\pi$. Let $t'' > t'$ be the next time that $ND_{t''} = \phi\pi$ and $a = 0$. Assume that t'' does not exist. In this case, and owing to Theorem 1 and Lemma 3, the firm never pays dividends for $t > t'$ since $ND_t < \phi\pi$. Therefore, equity value is zero. But this cannot be the optimal strategy since investment is positive NPV (Lemma 2) and therefore generates a surplus that can be distributed to shareholders, which would yield a positive equity value. As a result, t'' must exist. We know that $ND_{t''-n} < \phi\pi$ since $t' \leq t'' - n < t''$. Given that Theorem 1 implies that net debt is weakly decreasing within a cycle and $ND_{t''-n} < \phi\pi$, we have that $ND_t < \phi\pi$ for $t \in [t''-n, t''-1]$ because of the definition of t' and t''. From Lemma 3, it then follows that the firm does not pay any dividends over the interval $t \in [t''-n+1, t'']$ where $t''-n+1 > 0$.

that needs to be repaid at $t+1$ is

$$(1+\rho_C)(ND_t + \Delta Div_t) - (1+\rho_C)ND_t = (1+\rho_C)\Delta Div_t < (1+\rho_D)\Delta Div_t.$$

If $ND_t < 0$ and $ND_t + \Delta Div_t > 0$, this change is

$$(1+\rho_D)(ND_t + \Delta Div_t) - (1+\rho_C)ND_t = (1+\rho_D)\Delta Div_t + ND_t(\rho_D - \rho_C) < (1+\rho_D)\Delta Div_t.$$

Instead, if $ND_t > 0$ this change is $(1+\rho_D)\Delta Div_t$.

Each period t, the firm has a cash flow of π but needs to pay interest. The firm can save at least $s = K\frac{\rho_C}{(1+\rho_C)^n-1}$ since equation (A.4) holds. Therefore, the firm lowers net debt by at least s each period over this time interval and as a result net debt decreases by at least

$$\sum_{a=0}^{n-1} s(1+\rho_C)^a = s\frac{(1+\rho_C)^n - 1}{\rho_C} = K.$$

As a result, we have that

$$\pi - \left(1 + \rho_D \mathbb{I}_{\{ND_{t''-1}\geq 0\}} + \rho_C \mathbb{I}_{\{ND_{t''-1}<0\}}\right) ND_{t''-1} > \pi - \rho_D\phi\pi - ND_{t''-1} > K - ND_{t''-n+1}.$$

This implies that the dividend at time t'', which follows from the budget constraint, is

$$\begin{aligned} Div_{t''} &= \pi - K + ND_{t''} - \left(1 + \rho_D \mathbb{I}_{\{ND_{t''-1}\geq 0\}} + \rho_C \mathbb{I}_{\{ND_{t''-1}<0\}}\right) ND_{t''-1} \\ &> K - K + ND_{t''} - ND_{t''-n+1} = \phi\pi - ND_{t''-n+1} \\ &> 0. \end{aligned}$$

This makes it impossible that $ND_{t''-1} < \phi\pi$ owing to Lemma 3. This result in combination with Theorem 1 then implies that $ND_{t''-n} = \phi\pi$ but this contradicts the fact that $ND_t < \phi\pi$ for $t \in [t''-n, t-1'']$. This rules out that $ND_{a=0} < \phi\pi$ so that we must have $ND_{a=0} = \phi\pi$. □

Proof of Proposition 2. We show using backward induction that higher investment costs $K' > K$ lead to stronger leverage cycles.

Assume $K \leq \pi - \rho_D\phi\pi$. In that case, the firm always keep its net debt at $\phi\pi$ and invests using retained earnings. As a consequence,

$$|ND_a - ND_{a-1}| = 0 \leq |ND'_a - ND'_{a-1}|.$$

Assume next that $K > \pi - \rho_D\phi\pi$ so that $K' > \pi - \rho_D\phi\pi$. In that case, the firm needs debt capacity $ND_{a=n-1} < \phi\pi$ to finance investment and we know from Lemma 3 that $Div_{a=0} = 0$. Furthermore, Lemma 4 implies that $ND_{a=0} = \phi\pi$. From the budget constraint it then follows that

$$0 = \pi - K + \phi\pi - \left(1 + \rho_D \mathbb{I}_{\{ND_{a=n-1}\geq 0\}} + \rho_C \mathbb{I}_{\{ND_{a=n-1}<0\}}\right) ND_{a=n-1}.$$

There is a unique $ND_{a=n-1}$ that solves this equation. Furthermore, this $ND_{a=n-1}$ is decreasing in K. These results also hold true for $ND'_{a=n-1}$ and imply that

$$0 \leq ND_{a=0} - ND_{a=n-1} = \phi\pi - ND_{a=n-1} < \phi\pi - ND'_{a=n-1} = ND'_{a=0} - ND'_{a=n-1}$$

and therefore

$$|ND_{a=0} - ND_{a=n-1}| \leq |ND'_{a=0} - ND'_{a=n-1}|.$$

We are going to show the result for $a > 0$ using backwards induction. We have just shown that $ND_{a=n-1} \geq ND'_{a=n-1}$. Assume now that $ND_a \geq ND'_a$ and $a > 0$. We want to show that $ND_{a-1} \geq ND'_{a-1}$ and the proposition's result. There are three cases.

1. Assume $ND_{a-1} < \phi\pi$ and $ND'_{a-1} < \phi\pi$ then we have that $Div_a = Div'_a = 0$, see Lemma 3. Assume $ND_{a-1} < ND'_{a-1}$ then the budget constraint implies that

$$\begin{aligned}
0 &= \pi + ND_a - \left(1 + \rho_D \mathbb{I}_{\{ND_{a-1} \geq 0\}} + \rho_C \mathbb{I}_{\{ND_{a-1} < 0\}}\right) ND_{a-1} \\
&= \pi + ND'_a - \left(1 + \rho_D \mathbb{I}_{\{ND'_{a-1} \geq 0\}} + \rho_C \mathbb{I}_{\{ND'_{a-1} < 0\}}\right) ND'_{a-1}, \\
ND_a - ND'_a &= \left(1 + \rho_D \mathbb{I}_{\{ND_{a-1} \geq 0\}} + \rho_C \mathbb{I}_{\{ND_{a-1} < 0\}}\right) ND_{a-1} \\
&\quad - \left(1 + \rho_D \mathbb{I}_{\{ND'_{a-1} \geq 0\}} + \rho_C \mathbb{I}_{\{ND'_{a-1} < 0\}}\right) ND'_{a-1} \\
&< 0,
\end{aligned}$$

This contradicts the fact that $ND_a \geq ND'_a$. Thus, we must have $ND_{a-1} \geq ND'_{a-1}$. We still need to show the proposition's result. We know that the budget constraint

$$0 = \pi + ND_a - \left(1 + \rho_D \mathbb{I}_{\{ND_{a-1} \geq 0\}} + \rho_C \mathbb{I}_{\{ND_{a-1} < 0\}}\right) ND_{a-1}$$

holds. From this budget constraint it directly follows that

$$\begin{aligned}
0 \leq ND_{a-1} - ND_a &= \pi - \left(\rho_D \mathbb{I}_{\{ND_{a-1} \geq 0\}} + \rho_C \mathbb{I}_{\{ND_{a-1} < 0\}}\right) ND_{a-1} \\
&\leq \pi - \left(\rho_D \mathbb{I}_{\{ND'_{a-1} \geq 0\}} + \rho_C \mathbb{I}_{\{ND'_{a-1} < 0\}}\right) ND'_{a-1} \\
&= ND'_{a-1} - ND'_a.
\end{aligned}$$

The inequality follows from the fact that $ND_{a-1} \geq ND'_{a-1}$. Therefore

$$|ND_{a-1} - ND_a| \leq |ND'_{a-1} - ND'_a|.$$

2. Assume $ND_{a-1} < \phi\pi$ and $ND'_{a-1} = \phi\pi$ then we have that $Div_a = 0$ from Lemma 3. The budget constraint then implies that

$$\begin{aligned}
0 &= -Div_a + \pi + ND_a - \left(1 + \rho_D \mathbb{I}_{\{ND_{a-1} \geq 0\}} + \rho_C \mathbb{I}_{\{ND_{a-1} < 0\}}\right) ND_{a-1} \\
&= -Div'_a + \pi + ND'_a - (1 + \rho_D) ND'_{a-1} \\
&\leq \pi + ND'_a - (1 + \rho_D) ND'_{a-1}.
\end{aligned}$$

As a consequence,

$$Div_a \geq (ND_a - ND'_a) - \left(1 + \rho_D \mathbb{I}_{\{ND_{a-1} \geq 0\}} + \rho_C \mathbb{I}_{\{ND_{a-1} < 0\}}\right) ND_{a-1} + (1 + \rho_D) ND'_{a-1} > 0,$$

which is a contradiction. Therefore, this case cannot arise.

3. Assume $ND_{a-1} = \phi\pi$ and $ND'_{a-1} \leq \phi\pi$. This case directly implies that $ND_{a-1} \geq ND'_{a-1}$. If $ND'_{a-1} = \phi\pi$ then

$$0 \leq ND_{a-1} - ND_a = \phi\pi - ND_a \leq \phi\pi - ND'_a = ND'_{a-1} - ND'_{a-1}.$$

If $ND'_{a-1} < \phi\pi$ then $Div'_a = 0$ by Lemma 3. From the budget constraint it then follows that

$$\begin{aligned} 0 \leq & ND_{a-1} - ND_a \\ = & - Div_a + \pi - \rho_D ND_{a-1} \\ \leq & \pi - \left(\rho_D \mathbb{I}_{\{ND'_{a-1} \geq 0\}} + \rho_C \mathbb{I}_{\{ND'_{a-1} < 0\}}\right) ND'_{a-1} \\ = & ND'_{a-1} - ND'_a. \end{aligned}$$

Therefore,

$$|ND_{a-1} - ND_a| \leq |ND'_{a-1} - ND'_a|.$$

These steps recursively establish our result.

□

B Arbitrary Capital Depreciation

We now allow for arbitrary depreciation schedules of capital assuming that capital fully depreciates in n periods, where n can be arbitrarily large. Let π_t be the firm profits at time t. To keep the analysis tractable, we make the following two assumptions:

1. The collateral constraint is time-invariant

$$D_t \leq \Phi.$$

2. Given the optimal policies, the firm generates enough profits to make interest payments

$$\pi_t > \rho_D \Phi > \rho_D D_t.$$

Proof of Proposition 3. We first establish that the firm never defaults and that the collateral constraint must bind at time t for the firm to pay dividends at time $t+1$ (the equivalent of Lemma 3). We then show that once net debt starts decreasing it does so (at least) until the firm invests. As a consequence, during this period, capital ages while net debt declines.

The second condition ensures that the firm never defaults since it can always make interest payments and therefore the rate of return on debt is ρ_D.

We want to show that the firm's net debt is weakly decreasing in capital age at least until the firm invests. To obtain this result, we first need to show that the firm only pays dividends when the collateral constraint binds in the previous period. We show below that it is suboptimal for the firm to pay dividends at time $t+1$ if the collateral constraint does not bind at time t. Therefore, the collateral constraint must bind at time t if the firm pays dividends at time $t+1$.

We first establish that if the firm has net debt $ND_t < \Phi$ then $Div_{t+1} = 0$. For this purpose, assume that we have $Div_{t+1} > 0$ while $ND_t < \Phi$ for some t. Define ΔDiv_t as

$$\Delta Div_t = \min\left\{\frac{Div_{t+1}}{1+\rho_D}, \Phi - ND_t\right\}.$$

Increasing dividends at time t to $Div'_t = Div_t + \Delta Div_t$ using debt financing implies that $Div'_{t+1} \geq Div_{t+1} - (1+\rho_D)\Delta Div_t$, where the inequality follows from the fact that the interest rate is lower if net debt is negative (i.e. if $ND_t < 0$); see footnote 18. This change in policy would increase shareholder value since its effect on equity value (at time t) is at least

$$\Delta Div_t - \frac{(1+\rho_D)\Delta Div_t}{1+r} > 0,$$

which contradicts optimality of the firm's policies. Therefore, if $Div_{t+1} > 0$ then $ND_t = \Phi$.

Next, we show that net debt weakly decreases over time at least until the firm invests. Let $t' \in \{t, T-2\}$ where T is the next time the firm invests. There are two cases. First, if $ND_{t'} = \Phi$ then $ND_{t'} \geq ND_{t'+1}$ because of the collateral constraint. Second, if $ND_{t'} < \Phi$ then the firm does not pay dividends at time $t'+1$ since $ND_{t'} < \Phi$. Furthermore, $ND_{t'} < ND_{t'+1}$ since $\pi_{t'+1} > \rho_D\Phi > \rho_D ND_{t'}$. Finally, from t' to $t'+1$ installed capital becomes a year older since there is no investment while net debt weakly decreases. □

C Debt Maturity

This appendix first establishes the optimal debt issuance strategy (Theorem 2). It then shows that average debt maturity is decreasing in capital age (Proposition 4) and increasing in asset maturity (Theorem 3).

Proof of Theorem 2. We first show that the net debt dynamics are the same when $\epsilon \to 0$

than when debt issuance is frictionless. These net debt dynamics allow us to show the absence of permanent debt and derive the optimal debt issuance strategy.

Let $E_0(\epsilon)$ be the equity value given issuance costs ϵ. Without issuance costs, debt maturity is irrelevant as any long-term debt contract can be implemented by a sequence of short-term contracts. Furthermore, $E_0(0) \geq E_0(\epsilon)$ since issuance cost depress firm value. As a result, the net debt and investment dynamics are the same as in the baseline model when $\epsilon \to 0$. If this was not the case, then we would have $\lim_{\epsilon \downarrow 0} E_0(\epsilon) < E_0(0)$ and using the one-period debt implementation from the baseline model would dominate for sufficiently small issuance costs $\epsilon \to 0$.

Given these net debt dynamics, the firm wants to issue debt that minimizes issuance costs. Observe that cash generates a lower return than debt $\rho_C < \rho_D$ and given that debt issuance costs are small $\epsilon \to 0$, the firm only has debt outstanding when $ND_t > 0$ and only cash in hand when $ND_t < 0$.

Because the firm always invests when assets reach the end of their useful life (Proposition 1), we have that $ND_{a=n-1} < 0$ since it needs both cash and debt to finance investment; see equation (3). As a result, the firm does not issue debt with a maturity longer than n-periods.

To minimize issuance costs the firm only issues debt when it invests with a maturity that matches the net debt dynamics during the capital's lifetime. □

Proof of Proposition 4. We first establish that average debt maturity has a recursive structure that depends on the ratio of this and next periods net debt. We then establish that the ratio of this and next periods net debt can be ordered, which allows us to show that average debt maturity declines as capital ages.

Define $\hat{a}$ as the largest capital age such that debt is positive

$$\hat{a} = \sup\{a | ND_a > 0\}.$$

Given that $K > \phi\pi + \pi$ (see equation (3)), we know that $ND_{n-1} < 0$ and therefore that $\hat{a} < n - 1$. Furthermore, from Theorem 1 we have that $ND_a \leq 0$ for $a > \hat{a}$. Therefore average debt maturity is $M_a = 0$ for $a > \hat{a}$.

We can write the average debt maturity as

$$
\begin{aligned}
M_a &= \sum_{i=a}^{n-1} \mathbb{I}_{\{ND_i>0\}}(i+1-a)\frac{ND_i - \max\{ND_{i+1},0\}}{ND_a} \\
&= \sum_{i=a}^{\hat{a}}(i+1-a)\frac{ND_i - \max\{ND_{i+1},0\}}{ND_a} \\
&= \frac{1*ND_a - 1*ND_{a+1} + 2*ND_{a+1} - ... - (\hat{a}-a)ND_{\hat{a}} + (\hat{a}+1-a)ND_{\hat{a}}}{ND_a} \\
&= \frac{ND_a + ... + ND_{\hat{a}}}{ND_a} = 1 + \frac{ND_{a+1} + ... + ND_{\hat{a}}}{ND_a} \\
&= 1 + \frac{ND_{a+1}}{ND_a}M_{a+1}.
\end{aligned}
$$

Define $B_a = \frac{ND_{a+1}}{ND_a}$ for $a < \hat{a}$. The above equation can be rewritten as

$$
M_a = 1 + B_a M_{a+1}.
$$

From Theorem 1 and the definition of $\hat{a}$ it follows that $B_a \in (0,1]$.

We want to show that $B_{a+1} \leq B_a$ for $a < \hat{a}-1$. Assume first that $ND_{a+1} = \phi\pi$. In this case, we have $B_{a+1} \leq 1 = \phi\pi/\phi\pi = ND_{a+1}/ND_a = B_a$ (Theorem 1). Assume next that $ND_{a+1} < \phi\pi$. Then we also have $ND_{a+2} \leq ND_{a+1} < \phi\pi$ (Theorem 1). From the budget constraint in equation (1), the fact that $ND_{a+2} \geq ND_{\hat{a}} > 0$ (Theorem 1), and the fact that the firm pays no dividends at $a+2$ since $ND_{a+1} < \phi\pi$ (Lemma 3), it then follows that

$$
ND_{a+2} = ND_{a+1}(1+\rho_D) - \pi
$$

and therefore

$$
B_{a+1} = (1+\rho_D) - \frac{\pi}{ND_{a+1}}.
$$

If $ND_a < \phi\pi$ then the same argument implies that

$$
B_a = (1+\rho_D) - \frac{\pi}{ND_a}.
$$

Since ND_a is weakly decreasing in a (Theorem 1), we then have that $B_{a+1} \leq B_a$.

If $ND_a = \phi\pi$ the same argument implies that

$$
ND_{a+1} = Div_{a+1} + ND_a(1+\rho_D) - \pi \geq ND_a(1+\rho_D) - \pi
$$

and therefore

$$B_a \geq (1+\rho_D) - \frac{\pi}{ND_a}.$$

and we get that $B_{a+1} \leq B_a$

As a consequence

$$1 \geq B_0 \geq B_1 \geq ... \geq B_{\hat{a}-1} > 0.$$

It is easy to see that $M_{\hat{a}} = 1$ and therefore

$$M_{\hat{a}-1} = 1 + B_{\hat{a}-1}M_{\hat{a}} \geq 1 = M_{\hat{a}}.$$

We can now establish our result using backward induction. Assume that $M_{\hat{a}-i-1} \geq M_{\hat{a}-i} \geq 0$. We then know that

$$M_{\hat{a}-i-2} = 1 + B_{\hat{a}-i-2}M_{\hat{a}-i-1} \geq 1 + B_{\hat{a}-i-1}M_{\hat{a}-i-1} \geq 1 + B_{\hat{a}-i-1}M_{\hat{a}-i} = M_{\hat{a}-i-1} \geq 0,$$

which recursively establishes that the debt maturity is decreasing in a. □

Proof of Theorem 3. We first show that increasing asset life by a year yields the same net debt dynamics just one year lagged. This result in combination with Proposition 4 allows us to show that average debt maturity weakly increases with asset life.

Define the function

$$d(ND_{a-1}, ND_a) = \pi - K\mathbb{I}_{\{a=0\}} + ND_a - \left(1 + \mathbb{I}_{\{ND_{a-1}\geq 0\}}\rho_D + \mathbb{I}_{\{ND_{a-1}<0\}}\rho_C\right) ND_{a-1},$$

which is the "*dividend*" the firm would pay when capital has age a and debt levels are ND_{a-1} and ND_a, see equation (1). Observe that

$$\frac{\partial d(ND_{a-1}, ND_a)}{\partial ND_{a-1}} < 0. \tag{A.5}$$

Given ND_a, if the firm pays no dividends then the net debt from the previous period ND_{a-1} solves

$$d(ND_{a-1}, ND_a) = 0,$$

which has a unique solution that we call $\hat{ND}(ND_a)$. Given ND_a, if the firm pays dividends $Div_a > 0$, then the net debt from the previous period ND_{a-1} solves

$$d(ND_{a-1}, ND_a) = Div_a,$$

which has a unique solution that we call $\tilde{ND}(ND_a, Div_a)$. Equation (A.5) implies that

$$\tilde{ND}(ND_a, Div_a) < \hat{ND}(ND_a). \tag{A.6}$$

Let $ND_a(n)$ be the net debt of a firm with asset maturity n and capital age a with other quantities made dependent on n in a similar way. We first want to establish that $ND_a(n) = ND_{a+1}(n+1)$ for $a \geq 0$. We do so using backward induction. Lemma 4 implies that $ND_0(n) = ND_0(n+1) = \phi\pi$. We additionally know that $ND_{a=n-1}(n) < 0 < \phi\pi$ and similarly that $ND_{a=n}(n+1) < 0 < \phi\pi$ as otherwise the firm cannot finance investment; see equation (3). This together with Lemma 3 implies that $Div_0(n) = Div_0(n+1) = 0$. Therefore,

$$ND_{a=n-1}(n) = ND_{a=n}(n+1) = \hat{ND}(\phi\pi).$$

We can now establish recursively that $ND_a(n) = ND_{a+1}(n+1)$. Indeed assume that $ND_a(n) = ND_{a+1}(n+1)$. There are two cases to consider.

Case 1: If $\phi\pi \geq \hat{ND}(ND_a(n))$ then $\phi\pi \geq \hat{ND}(ND_a(n)) > \tilde{ND}(ND_a(n), Div_a)$ for any $Div_a > 0$, see equation (A.6), and it cannot be the case that the firm pays dividends at time a because in that case the debt level at $a-1$ would have been $\phi\pi > \tilde{ND}(ND_a(n), Div_a)$, which violates Lemma 3. As a result, when $\phi\pi \geq \hat{ND}(ND_a(n))$ then $ND_{a-1}(n) = \hat{ND}(ND_a(n))$ and via the same reasoning $ND_a(n+1) = \hat{ND}(ND_{a+1}(n+1)) = \hat{ND}(ND_a(n))$. Therefore,

$$ND_{a-1}(n) = ND_a(n+1) = \hat{ND}(ND_a(n)).$$

Case 2: If $\phi\pi < \hat{ND}(ND_a(n))$ then it must be that the firm pays dividends since otherwise the debt level in the previous period would violate the collateral constraint. Given that the firm pays dividends and Lemma 3, we must have that

$$ND_{a-1}(n) = ND_a(n+1) = \phi\pi.$$

This recursively establishes that $ND_a(n) = ND_{a+1}(n+1)$ for $a \geq 0$. Furthermore, we have $ND_0(n+1) = \phi\pi = ND_0(n) = ND_1(n+1)$; see Lemma 4.

A firm with assets that have a useful life of $n+1$ periods that issues debt with a maturity that is one year longer than a firm with assets that have a useful life of n firm has net debt dynamics $ND_{a+1}(n+1) = ND_a(n)$ for $a \geq 0$ with $ND_0(n+1) = ND_1(n+1) = \phi\pi$, which we just showed is the optimal net debt level when the useful life of assets is $n+1$. This in turn implies that $M_{a+1}(n+1) = M_a(n)$ and, in combination Proposition 4, leads to the desired result. □

D Industry Equilibrium

We first establish the existence and uniqueness of an industry equilibrium (Proposition 5). We then relate aggregate quantities to the capital age distribution (Proposition 6). Finally, we solve the model where debt capital supply is uncertain (Proposition 7).

We start the proof by deriving a lower bound on the entry cost $\underline{H}$. Indeed, if the single-firm model parameter restrictions (equations (A.1), (A.2), and (A.3)) hold for some level of profitability π then they also hold for firms with a higher profitability $\pi' > \pi$. Therefore, there exists a lower bound for profitability $\underline{\pi}$ such that the parameters restrictions from the single-firm model hold for any $\pi > \underline{\pi}$. We then define $\underline{H}$ as

$$\underline{H} = E_0(\underline{\pi}) - C_0.$$

If $H > \underline{H}$ then $\pi > \underline{\pi}$ (since the $E_0(\pi)$ is strictly increasing in π as we show below) and a solution to our single-firm model with risk-free debt exists.

Proof of Proposition 5. We first show that no industry equilibrium with default can exist. We then establish existence and uniqueness of a solution to the free-entry condition (equation (4)), which guarantees equilibrium existence and uniqueness.

For $\pi > \underline{\pi}$, if equilibria (in the default game between shareholders and creditors) of the single-firm model with and without default exist for a given level of profitability, then we assume that the firm and creditors select the single-firm model equilibrium without default. This equilibrium maximizes shareholder value since investment is positive NPV so that

$$E_0(\pi) > E_0^D(\pi),$$

where $E_0^D(\pi)$ is the equilibrium (with the highest) firm value (among equilibria) with default.

The (no default) equity value is continuous in π since it solves a deterministic optimisation problem with constraints that are continuous in π. Furthermore, it is strictly increasing in π because for any $\epsilon > 0$ we have that

$$E_0(\pi + \epsilon) \geq E_0(\pi) + \frac{\epsilon}{r} > E_0(\pi)$$

since the firm can always pay out the extra profits ϵ by increasing dividends. Similarly, it can be be shown that $E_0^D(\pi)$ is weakly increasing in π. Assume that there exists a level of profitability π such that an industry equilibrium with default exists:

$$E_0^D(\pi) = C_0 + H.$$

Then for this π we must have that $\pi > \underline{\pi}$. Otherwise,

$$C_0 + \underline{H} = E_0(\underline{\pi}) \geq E_0^D(\underline{\pi}) \geq E_0^D(\pi) = C_0 + H > C_0 + \underline{H}.$$

Given that $\pi > \underline{\pi}$, equations (A.1), (A.2), and (A.3) are satisfied. Therefore, both a no default equilibrium and a default equilibrium to the single-firm model exist. But since the firm value is higher in the no default equilibrium $E_0(\pi) > E_0^D(\pi)$, shareholders select the single-firm equilibrium without default. As a result, we can focus our attention on industry equilibria without default.

Given that the (no default) equity value is strictly increasing in profitability, we have that if $S' > S$ then $E_0(P(S)) > E_0(P(S'))$. The conditions on $P(S)$ and H imply that

$$E_0(P(0)) - (C_0 + H) > 0,$$
$$E_0(\underline{\pi}) - (C_0 + H) \leq 0.$$

Continuity of $E_0(P(S))$ in S then guarantees that there exists a unique S^* such that

$$E_0(P(S^*)) - (C_0 + H) = 0$$

where $P(S^*) \geq \underline{\pi}$ satisfies the parameter restrictions of the single-firm model (equations (A.1), (A.2), and (A.3)). As a result, the mass of entrants must be S^* in any industry equilibrium. Given this unique mass of entrants, there is a unique aggregate supply $Q^* = S^*$, price $P^* = P(S^*)$, profits $\pi^* = P(S^*)$, and firm policies (I_a^*, ND_a^*), which proves our result. □

Proof of Proposition 6. The result follows from the fact that the distribution of capital age in the economy is given by q_t and therefore aggregate production is given by $Q^* = S^* * \sum_{a=0}^{n-1} q_t^a = S^*$, aggregate investment is given by $S^* * q_t^0$, aggregate net debt is given by $S^* * \sum_{a=0}^{n-1} q_t^a * ND_a^*$, and average debt maturity is given by $\sum_{a=0}^{n-1} q_t^a ND_a^* * M_a^* / \sum_{a=0}^{n-1} q_t^a * ND_a^*$. □

E Data Definitions and Summary Statistics

I Capital IQ Maturity Data

We supplement the firm-level debt maturity proxy derived from Compustat with a more detailed measure from Capital IQ security issuance data, which covers the period of 2002 to 2018. To merge the security- and firm-level data, we use the most recent filing dates and remove any observations with the same ID/date, description, maturity, and interest rate. We further remove all securities with missing `gvkey` and drop entries for credit lines that reflect the drawdown limit only, as opposed to actual utilisation. We drop all observations with missing or negative maturity values. We then compute the firm-level maturity as the weighted average of individual-security maturities weighted by their notional amounts. As the final data filter, we drop observations for which the total debt in Capital IQ is greater than Compustat by more than 10%, as in Colla, Ippolito, and Li (2013).

II Definitions of Variables

The variables used in the paper are defined in Table A.1.

Variable	Definition
Capital age	See Subsection IV.B
Useful life	See Subsection IV.B
Net book leverage	Ratio of total debt (`dltt+dlc`) less cash (`che`) over total assets (`at`)
Net market leverage	Ratio of total debt (`dltt+dlc`) less cash (`che`) over total debt plus market value of equity (`prcc_f*csho`)
Net lease-adjusted leverage	Ratio of total debt (`dltt+dlc`) plus capitalized rental expenses (`xrent*10` as in Rampini and Viswanathan, 2013) less cash (`che`) over total assets plus capitalized rental expenses (`at+xrent*10`)
Rental leverage	Ratio of capitalized rental expenses (`xrent*10`) over total assets plus capitalized rental expenses (`at+xrent*10`)
% debt maturing > 3y	Ratio of long-term debt (`dltt`) minus debt maturing in 2- and 3-years (`dd2+dd3`) over total debt (`dlc+dltt`)
% debt maturing > 5y	Ratio of long-term debt (`dltt`) minus debt maturing in 2-, 3-, 4-, and 5-years (`dd2+dd3+dd4+dd5`) over total debt (`dlc+dltt`)
Debt maturity (yr.)	Average maturity of outstanding instruments from Capital IQ, weighted by notional
Investment	Capital expenditures (`capx`) over lagged installed capital (`l.ppegt`)
Investment spike	Dummy equal to 1 when investment (`capx/l.ppegt`) exceeds firm-level median by one std. dev. (before applying the 5-year window filter)
Investment cycle length	Number of years to the first investment spike, between subsequent investment spikes, or after the last spike, conditional on the 5-year window filter and their being at least one spike
Leverage spike	Dummy equal to 1 when net book leverage exceeds firm-level median by one std. dev. (before applying the 5-year window filter)
Debt cycle length	Number of years to the first leverage spike, between subsequent leverage spikes, or after the last spike, conditional on the 5-year window filter and their being at least one spike
Change in cash	Change in cash (`chech`) scaled by total assets (`at`)

Table A.1: **Definitions of variables**. The table contains the definitions of all variables used throughout the paper (in order of appearance).

Variable	Definition
Change in debt	Change in debt (dltis-dltr-dlcch) scaled by total assets (at)
Debt issuer	Dummy equal to 1 when net debt issuance is positive
Equity issuer	Dummy equal to 1 when non-employee-specific net equity issuance is positive, following McKeon (2015)
Profitability	Operating income (oibdp) over total assets (at)
Size	Natural log of real sales (log(sale/defl)), where defl is the CPI deflator
Tangibility	Ratio of property, plant and equipment (ppent) to total assets (at)
Market-to-book	Ratio of the sum of market value of equity (prcc_f*csho) and book value of debt (at-ceq) to total assets (at)
Cash flow volatility	Moving 3-year standard deviation of profitability
R&D	Ratio of R&D expenditure (xrd) to sales (sale), missing values replaced with zero
Firm age	Time since listing, which is defined as the first appearance of firm i in CRSP
Asset maturity	Gross property, plant and equipment over depreciation and amortization (ppegt/dp) times the proportion of property, plant and equipment in total assets (ppegt/at), plus current assets over the cost of goods sold (act/cogs) times the proportion of current assets in total assets (act/at)
Abnormal earnings	The difference between the income before extraordinary items, adjusted for common stock equivalents (ibadj-l.ibadj) over the market value of equity used in calculating earnings per share (prcc_f*cshpri)
Investment skewness (firm)	The firm-level skewness of investment; we require at least 5 observations per firm
Investment kurtosis (firm)	The firm-level kurtosis of investment; we require at least 5 observations per firm
Intangibility	Intangible capital stock (intan) to total assets at
Aggregate capital age dispersion	Mean of annual within-industry standard deviation of firm-level capital age
Aggregate book leverage dispersion	Standard deviation of annual industry-level net book leverage
Aggregate lease-adjusted leverage dispersion	Standard deviation of annual industry-level net lease-adjusted leverage

Table A.1: **Definitions of variables**. The table contains the definitions of all variables used throughout the paper (in order of appearance).

Variable	Definition
Aggregate maturity dispersion	Standard deviation of annual industry-level proportion of debt maturing in more than 3-years

Table A.1: **Definitions of variables**. The table contains the definitions of all variables used throughout the paper (in order of appearance).

III Summary Statistics

Table A.2 contains the summary statistics of all the variables used in the leverage and maturity regression models which were not provided in Table 1.

	Mean	Std. dev.	Q1	Median	Q3	N
Investment spike	0.163	0.370	0.000	0.000	0.000	77877
Change in cash	0.005	0.067	-0.009	0.000	0.015	77877
Change in debt	0.011	0.102	-0.031	-0.001	0.044	77877
Debt issuer	0.669	0.471	0.000	1.000	1.000	77784
Equity issuer	0.115	0.319	0.000	0.000	0.000	77877
Profitability	0.111	0.125	0.073	0.125	0.176	77738
Size	5.169	2.201	3.743	5.240	6.684	77439
Market-to-book	1.515	0.946	0.976	1.226	1.685	77877
Tangibility	0.356	0.234	0.170	0.308	0.514	77815
Cash flow volatility	0.045	0.046	0.017	0.030	0.054	60482
R&D	0.011	0.030	0.000	0.000	0.007	77877
Firm age	18.440	16.917	6.252	13.337	24.592	75060
Asset maturity	11.002	10.160	4.171	8.188	14.540	75588
Abnormal earnings	-0.018	0.274	-0.032	0.007	0.033	71568
Leverage spike	0.145	0.352	0.000	0.000	0.000	77877
Intangibility	0.106	0.159	0.000	0.028	0.148	70238
Inv. skewness (firm)	0.998	0.875	0.393	0.920	1.498	4969
Inv. kurtosis (firm)	3.730	2.784	2.040	2.807	4.351	4969
Rental leverage	0.167	0.162	0.045	0.123	0.239	77877

Table A.2: **Summary statistics**. The table contains the summary statistics of the variables used in the regression models of net leverage and debt maturity. The sample period is from 1975 to 2018. All variables are defined in Table A.1.

Internet Appendix to: Financing Cycles

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This internet appendix presents an extension of the industry equilibrium model that allows for time-varying debt capital supply and additional robustness tests that support the model predictions.

Aggregate Shocks and Financing Cycles

A question that we have not answered so far is what capital age distribution arises in equilibrium. As we argue now, aggregate shocks—such as shocks to credit supply—can increase the correlation across firms' capital age, leading to aggregate financing cycles. Consider the same industry equilibrium model as before, including the endogenous maturity choice. Just as in the debt maturity section, assume that $K > \phi P(0) + P(0)$ so that the firm has no permanent debt. Furthermore, assume that a new firm requires debt financing to invest $K > C_0$.

The firm's ability to issue debt and invest depends on the aggregate conditions in the debt capital market. If debt capital is available A, then the firm can issue debt up to the collateral constraint $\phi\pi$. If debt is unavailable U, the firm cannot issue any new debt. We assume that the state of the debt capital market follows a Markov chain. With probability $\mathbb{P}_{A\to U} = q$, debt capital becomes unavailable for a single period $\mathbb{P}_{U\to A} = 1$. We assume that the probability of debt capital markets becoming unavailable q is sufficiently small.

How does time-varying debt capital supply affect firms investment and financing decisions? Since q is small, financing and investment choices are unaffected when debt capital is available. The firm maximizes its leverage conditional on being able to make replacement investments, which are partially financed by debt. If debt capital is unavailable when the firm needs to invest, then it delays investment for a single period, which it can, since it holds cash $ND_{n-1} < 0$ and invests in the next period. The following proposition shows that in this case aggregate financing cycles arise:

Proposition 7. *There exists a stationary equilibrium in which firms financing choices when debt capital is available follow from Theorem 2 and firms only make replacement investments*

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when debt capital is available. In this stationary equilibrium, there are aggregate financing cycles.

This result shows that aggregate shocks—in this case to debt capital supply—lead to aggregate financing cycles. Idiosyncratic firm-level shocks have the opposite effect and smooth out aggregate financing cycles. As a result, the nature of the shocks firms face, aggregate versus idiosyncratic, determines whether or not financing cycles spillover.

Proof of Proposition 7. We want to show that a stationary equilibrium exists in which financing dynamics are the same as in Theorem 2 when debt capital is available and firms only make replacement investments when debt capital is available. We can use this result to then show that the economy faces aggregate financing cycles.

We look for a stationary equilibrium in which the mass of firms in the economy is constant but the supply of the final good is not due to the stochastic availability of debt capital. Firms assume that their competitors make replacement investments when debt capital is available and otherwise they delay investment. Furthermore, firms assume that no new firms enter. Indeed, firms do not enter when debt capital is available because the free entry condition binds. Firms do not enter when debt capital is unavailable because they cannot finance investment as $C_0 < K$.

We want to show that given these beliefs, firms do not default and make replacement investments when debt capital is available. Note that if debt capital is unavailable today then firms believe this will weakly increase profits since firms cannot invest today. Therefore, if debt capital is unavailable at time t, then $S_{t+1} \leq S_t$ and

$$P(S_{t+1}) \geq P(S_t). \tag{1}$$

Assume that debt capital is available at time t and q is small. Then Lemma 2 goes through. Therefore, the firm has no incentive to default at time t and invests if its assets have reached the end of their useful life. Assume next that debt capital is unavailable at time t. There are now two cases to consider: i) the asset has reached the end of its useful life or ii) the asset has not reached the end of its useful life. If the firm's assets have not reached the end of their useful life then its profits are weakly higher, see equation (1). Furthermore, debt capital is available again next period because of the structure of the Markov chain. The same arguments as in Lemma 2 then imply that the firm has no incentive to default. If the firm's assets have reached the end of their useful life then it must be that the firm holds cash $ND_t < 0$ as otherwise it would not have been able to invest at time t if debt capital was available since $K > \phi P(0) + P(0)$. Given that the firm holds cash and debt capital is available next period, the firm has no incentive to default and can invest next period since $\rho_C > 0$. As a result, firms never default and have an incentive to invest when debt capital is available (i.e. they invest every n or $n + 1$ periods).

Furthermore, the firm wants to maximize the amount of debt it has outstanding because of the benefits to debt $\rho_D < r$. The firm therefore has two options. First, it issues the maximum amount of debt and only makes replacement investments when debt capital is available, in which case the financing dynamics follow from Theorem 2.[19] Second, it issues an amount of debt such that it can always invest even when debt capital is unavailable. That is, the firm holds less net debt if it wants to invest when debt capital is unavailable, which is costly. For q sufficiently small, the firm prefers the first solution. As a consequence, firms' financing dynamics when debt capital is available are the same as in Theorem 2 and firms only make replacement investments when debt capital is available.

Given that at random times debt capital is unavailable and therefore firms cannot invest, in a stationary equilibrium all firms have the same capital age. Otherwise, the random unavailability of debt capital would cause any set of firms with a different capital age today to eventually have the same capital age. Proposition 6 then implies that the firm-level financing cycles spillover into aggregate financing cycles.

□

Robustness and Additional Results

1. Figure IA.1 shows firms' financing, investment, and capital age around investment spikes.

2. Table IA.1 presents estimates from regressions of net lease-adjusted leverage for subsamples of firms split according to their reliance on leasing.

3. Tables IA.2 and IA.3 respectively document the importance of capital age and all the other factors used in the leverage and debt maturity regressions following the approach of Frank and Goyal (2009).

4. Tables IA.4 and IA.5 respectively present estimates from regressions of net leverage ratios and debt maturity on an alternative measure of capital age. We create this measure as in Lin et al. (2020) by assuming that when the firm disinvests, it disposes of the oldest capital vintages first, unlike in our main measure, where *all* vintages are equally affected. For the rest its defined in the same way as our main capital age measure.

5. Tables IA.6 and IA.7 respectively present estimates from regressions of net leverage ratios and debt maturity on lagged capital age for different definitions of industries.

[19]When the firm makes a larger profit because some competitors cannot invest since debt capital is unavailable it has two options: pay a (larger) dividend or reduce net debt. The firm prefers to pay a larger dividend leaving the net debt dynamics unchanged because (net) debt earns interest at a rate below r.

6. Table IA.8 presents estimates from regressions of firm-level maximum and average investment cycle lengths on firm-level average useful life.
7. Table IA.9 presents estimates from regressions of firm-level maximum and average debt cycle lengths on firm-level average alternative measures of asset useful life. Table IA.10 presents estimates from regressions of firm-level maximum and average investment cycle lengths on firm-level average alternative measures of asset useful life.
8. Table IA.11 presents estimates from regressions of firm-level averages of % of debt maturing in $> 3y$, % of debt maturing in $> 5y$, and average debt maturity on firm-level average alternative measures of asset useful life.
9. Table IA.12 present estimates from regressions of aggregate industry leverage and debt maturity dispersion on capital age dispersion using different industry classifications.

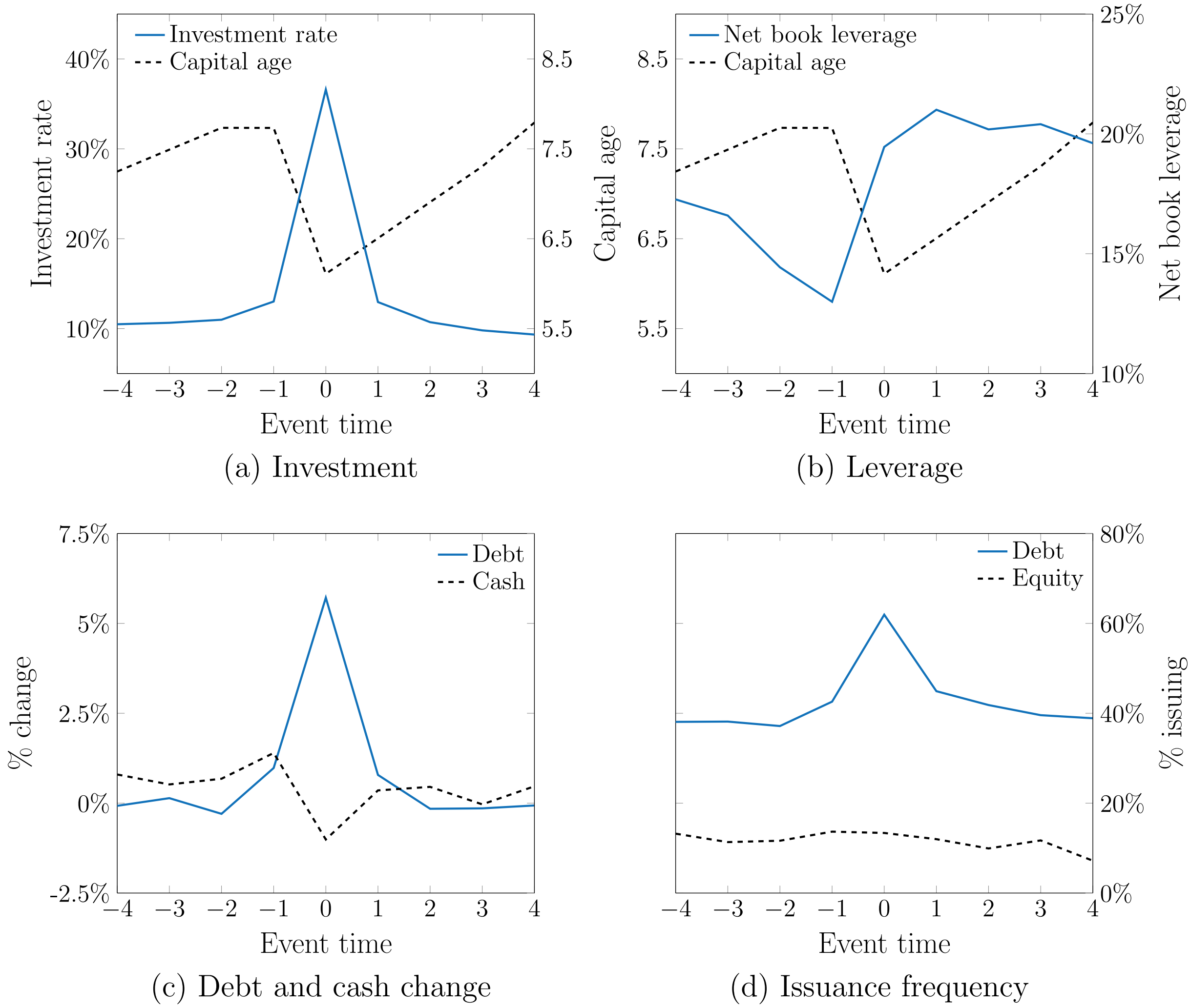

Figure IA.1: **Leverage and capital age cycles around investment spikes**. The figure presents the evolution of capital age, investment, net book leverage, changes in debt and cash to total assets, as well as the fraction of firms issuing net debt and net equity around investment spikes. Investment spikes are defined as cases in which firm investment exceeds its median by one standard deviation. The event time $t = 0$ indicates the investment spike. We only consider spikes which are not preceded or followed by another spike in a window of 5 years. The sample period is from 1975 to 2018. All variables are defined in Table A.1.

	Terciles		Quintiles		Deciles	
	1^{st}	3^{rd}	1^{st}	5^{th}	1^{st}	10^{th}
Capital age	-0.035***	0.004	-0.031***	0.010	-0.044***	0.013*
	(-6.54)	(0.79)	(-3.72)	(1.52)	(-3.64)	(1.83)
Profitability	-0.029***	-0.025***	-0.022***	-0.015***	-0.023***	-0.018***
	(-7.14)	(-7.40)	(-4.38)	(-3.99)	(-3.36)	(-3.66)
Size	0.063***	0.060***	0.058***	0.047**	0.018	0.019
	(4.47)	(3.89)	(2.94)	(2.45)	(0.54)	(0.82)
Market-to-book	-0.018***	-0.011***	-0.017***	-0.011***	-0.017*	-0.010**
	(-3.95)	(-3.79)	(-3.04)	(-3.16)	(-1.82)	(-2.21)
Tangibility	0.048***	0.043***	0.060***	0.041***	0.038**	0.049***
	(4.74)	(5.61)	(4.20)	(4.32)	(2.07)	(4.65)
Cash flow volatility	-0.006*	0.005*	-0.004	0.004	0.006	0.005
	(-1.80)	(1.75)	(-1.01)	(1.33)	(0.84)	(1.40)
R&D	0.003	0.007	0.006	-0.006	0.005	-0.054
	(0.48)	(1.07)	(0.64)	(-0.72)	(0.35)	(-1.26)
Firm age	0.099*	0.044	0.233***	0.118**	1.081**	0.071
	(1.95)	(0.73)	(3.41)	(2.30)	(2.40)	(1.39)
Year FE	No	No	No	No	No	No
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Ind.-Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	9382	10587	4865	5734	2818	2738
Adj. within R^2	0.0899	0.0583	0.0872	0.0576	0.0746	0.0946

Table IA.1: **Capital age and leverage – accounting for leasing**. This table presents estimates from regressions of leasing-adjusted leverage ratios on lagged capital age. The dependent variable is *Net lease-adjusted leverage*. In each fiscal year, the sample of firms is split according to rental leverage, i.e. the ratio between capitalized rental expenses and the lease-adjusted assets. Columns 1 and 2 split the sample into terciles, columns 3 and 4 into quintiles and columns 5 and 6 into deciles. Each explanatory variable is standardized by its full-sample standard deviation. All models include industry-year fixed effects created using the Hoberg-Phillips fixed industry classification with 100 industries. The sample period is from 1975 to 2018. All variables are defined in Table A.1. t-statistics are reported in parentheses. Standard errors are clustered at the firm level. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

Variable	Coef.	t-stat.	Adjusted within R^2 Individual	Cumulative
Capital age	-0.048***	-13.84	0.03437	0.03437
Size	0.093***	10.23	0.02048	0.04379
Tangibility	0.053***	9.12	0.01671	0.05277
Market-to-book	-0.027***	-10.73	0.01418	0.06688
Profitability	-0.026***	-9.46	0.01172	0.07837
Cash flow volatility	-0.004*	-1.89	0.00039	0.07859
R&D	-0.004	-0.93	0.00009	0.07892
Firm age	0.027	0.83	0.00005	0.07890

Table IA.2: **Capital age and leverage – importance of individual determinants**. This table presents estimates from regressions of net book leverage on lagged controls from Table 2. We obtain the coefficient estimates, t-statistic and the individual adjusted within R^2 by regressing net book leverage on each individual variable. We then sort the variables by their individual adjusted within R^2 and regress net book leverage by consecutively adding explanatory variables, which allows to obtain the cumulative adjusted within R^2. All regressions include firm and industry-year fixed effects, created using the Hoberg-Phillips fixed industry classification with 100 industries, and are run on the same sample as the regression model in column (3) in Table 2. All variables are defined in Table A.1. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

			Adjusted within R^2	
Variable	Coef.	t-stat.	Individual	Cumulative
Capital age	-0.046***	-10.6	0.00825	0.00825
Net book leverage	0.040***	9.07	0.00743	0.01313
Size	0.098***	8.00	0.00578	0.01519
Size squared	0.073***	6.24	0.00368	0.01552
Asset maturity	-0.012**	-2.40	0.00045	0.01548
Cash flow volatility	-0.008**	-2.38	0.00035	0.01552
Abnormal earnings	0.003**	2.10	0.00012	0.01588
Market-to-book	0.004	1.04	0.00004	0.01615
R&D	-0.003	-0.56	-0.00002	0.01612
Firm age	-0.010	-0.24	-0.00003	0.01609

Table IA.3: **Capital age and debt maturity – importance of individual determinants**. This table presents estimates from regressions of debt maturity (% of debt maturing in > 3 years) on lagged controls from Table 3. We obtain the coefficient estimates, t-statistic and the individual adjusted within R^2 by regressing net book leverage on each individual variable. We then sort the variables by their individual adjusted within R^2 and regress net book leverage by consecutively adding explanatory variables, which allows to obtain the cumulative adjusted within R^2. All regressions include firm and industry-year fixed effects, created using the Hoberg-Phillips fixed industry classification with 100 industries, and are run on the same sample as the regression model in column (3) in Table 3. All variables are defined in Table A.1. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

	Net book leverage			Net market leverage			Net lease-adjusted lev.		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Capital age (new)	-0.022***	-0.026***	-0.027***	-0.018***	-0.027***	-0.029***	-0.015***	-0.020***	-0.020***
	(-9.86)	(-10.88)	(-8.87)	(-6.76)	(-9.60)	(-9.10)	(-6.99)	(-8.63)	(-6.83)
Profitability		-0.040***	-0.033***		-0.058***	-0.051***		-0.041***	-0.033***
		(-16.38)	(-11.51)		(-21.10)	(-15.99)		(-17.82)	(-12.27)
Size		0.087***	0.109***		0.123***	0.154***		0.075***	0.095***
		(10.68)	(11.37)		(13.53)	(14.26)		(9.23)	(9.91)
Market-to-book		-0.013***	-0.016***		-0.032***	-0.030***		-0.013***	-0.017***
		(-5.54)	(-6.27)		(-13.75)	(-11.97)		(-5.74)	(-6.63)
Tangibility		0.052***	0.052***		0.063***	0.057***		0.043***	0.042***
		(10.66)	(9.19)		(11.53)	(9.23)		(9.13)	(7.93)
Cash flow volatility		-0.007***	-0.004*		-0.009***	-0.004		-0.004**	-0.001
		(-3.71)	(-1.67)		(-4.26)	(-1.63)		(-2.08)	(-0.58)
R&D		-0.011***	-0.008*		-0.014***	-0.008*		-0.011***	-0.009*
		(-2.91)	(-1.79)		(-3.89)	(-1.70)		(-3.06)	(-1.95)
Firm age		-0.065*	-0.000		-0.056*	-0.022		-0.030	0.028
		(-1.86)	(-0.01)		(-1.66)	(-0.66)		(-0.94)	(0.97)
Year FE	Yes	Yes	No	Yes	Yes	No	Yes	Yes	No
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Ind.-Year FE	No	No	Yes	No	No	Yes	No	No	Yes
Observations	62727	47851	31991	62727	47851	31991	62727	47851	31991
Adj. within R^2	0.0086	0.0758	0.0742	0.0039	0.0990	0.0982	0.0046	0.0718	0.0701

Table IA.4: **Capital age and leverage – alternative measure of capital age**. This table presents estimates from regressions of leverage on an alternative measure of lagged capital age, in which we assume that oldest capital vintages are disposed of first when firms disinvest. The dependent variables are *Net book leverage* in columns 1 to 3, *Net market leverage* in columns 4 to 6, and *Net lease-adjusted leverage* in columns 7 to 9. Each explanatory variable is standardized by its full-sample standard deviation. The models in columns 3, 6 and 9 include industry-year fixed effects created using Hoberg-Phillips fixed industry classification with 100 industries. The sample period is from 1975 to 2018. All variables are defined in Table A.1. t-statistics are reported in parentheses. Standard errors are clustered at the firm level. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

	% debt maturing $> 3y$			% debt maturing $> 5y$			Debt maturity (yr.)		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Capital age (new)	-0.023***	-0.022***	-0.023***	-0.019***	-0.019***	-0.019***	-0.043	-0.087	-0.158
	(-8.46)	(-6.65)	(-5.32)	(-6.75)	(-5.75)	(-4.48)	(-0.35)	(-0.63)	(-0.99)
Size		0.089***	0.140***		0.024	0.043*		1.925**	2.364**
		(3.74)	(4.52)		(1.16)	(1.67)		(2.46)	(2.47)
Size squared		-0.006	-0.049*		0.043**	0.036		-1.051	-1.297
		(-0.30)	(-1.78)		(2.20)	(1.46)		(-1.27)	(-1.31)
Market-to-book		0.011***	0.008**		0.007**	-0.000		0.066	0.073
		(3.71)	(2.16)		(2.31)	(-0.13)		(0.63)	(0.60)
Asset maturity		0.004	0.001		0.008**	0.006		0.153	0.165
		(0.96)	(0.27)		(1.99)	(1.10)		(1.28)	(1.14)
Abnormal earnings		0.006***	0.005***		0.006***	0.007***		0.013	0.040
		(4.61)	(2.69)		(5.93)	(4.77)		(0.54)	(1.20)
Cash flow volatility		-0.005*	-0.005		-0.005*	-0.003		-0.043	-0.037
		(-1.89)	(-1.44)		(-1.91)	(-0.86)		(-0.64)	(-0.44)
R&D		0.005	0.003		0.001	0.000		-0.057	-0.109
		(1.05)	(0.42)		(0.22)	(0.05)		(-0.38)	(-0.65)
Net book leverage		0.030***	0.034***		0.016***	0.014***		0.112	0.133
		(8.26)	(7.16)		(4.60)	(3.18)		(1.15)	(1.21)
Firm age		-0.081*	-0.023		-0.116**	-0.046		2.999*	2.481
		(-1.76)	(-0.49)		(-2.30)	(-0.84)		(1.96)	(1.51)
Year FE	Yes	Yes	No	Yes	Yes	No	Yes	Yes	No
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Ind.-Year FE	No	No	Yes	No	No	Yes	No	No	Yes
Observations	62727	46711	31030	62727	46711	31030	14999	12332	10858
Adj. within R^2	0.0036	0.0159	0.0147	0.0028	0.0105	0.0082	-0.0000	0.0039	0.0043

Table IA.5: **Capital age and debt maturity – alternative measure of capital age**. This table presents estimates from regressions of debt maturity on an alternative measure of lagged capital age, in which we assume that oldest capital vintages are disposed of first when firms disinvest. The dependent variables are *% of debt maturing in > 3 years* in columns 1 to 3; *% of debt maturing in > 5 years* in columns 4 to 6; and *Debt maturity (yr.)* columns 7 to 9. Each explanatory variable is standardized by its full-sample standard deviation. The models in columns 3, 6 and 9 include industry-year fixed effects created using Hoberg-Phillips fixed industry classification with 100 industries. The sample period is from 1975 to 2018. All variables are defined in Table A.1. t-statistics are reported in parentheses. Standard errors are clustered at the firm level. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

	Net book leverage		Net market leverage		Net lease-adj. lev.	
	(1)	(2)	(3)	(4)	(5)	(6)
Capital age	-0.034***	-0.034***	-0.035***	-0.035***	-0.017***	-0.019***
	(-9.64)	(-11.54)	(-8.93)	(-10.32)	(-4.95)	(-6.78)
Ind.-Yr. FE (HP50)	Yes	No	Yes	No	Yes	No
Ind.-Yr. FE (FF49)	No	Yes	No	Yes	No	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Observations	33881	49562	33881	49562	33881	49562
Adj. within R^2	0.0770	0.0802	0.0949	0.0950	0.0665	0.0684

Table IA.6: **Capital age and leverage – alternative industry definitions**. This table presents estimates from regressions of net leverage ratios on lagged capital age for different definitions of industries. The dependent variable is *Net book leverage* in columns 1 and 2, *Net market leverage* in columns 3 and 4 and *Net lease-adjusted leverage* in columns 5 and 6. We control for all independent variables from Table 2. Each explanatory variable is standardized by its full-sample standard deviation. All models include industry-year fixed effects created using the Hoberg-Phillips fixed industry classification with 50 industries (*HP50*) and the Fama-French 49 industries (*FF49*). The sample period is from 1975 to 2018. All variables are defined in Table A.1. t-statistics are reported in parentheses. Standard errors are clustered at the firm level. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

	% debt maturing $> 3y$		% debt maturing $> 5y$		Debt maturity (yr.)	
	(1)	(2)	(3)	(4)	(5)	(6)
Capital age	-0.033***	-0.035***	-0.023***	-0.027***	-0.353**	-0.269**
	(-7.37)	(-9.37)	(-5.15)	(-7.06)	(-2.43)	(-2.02)
Ind.-Yr. FE (HP50)	Yes	No	Yes	No	Yes	No
Ind.-Yr. FE (FF49)	No	Yes	No	Yes	No	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Observations	32850	48339	32850	48339	11643	12789
Adj. within R^2	0.0161	0.0187	0.0079	0.0112	0.0050	0.0038

Table IA.7: **Capital age and debt maturity – alternative industry definitions**. This table presents estimates from regressions of debt maturity on lagged capital age for different definitions of industries. The dependent variable is *% of debt maturing in > 3 years* in columns 1 and 2; *% of debt maturing in > 5 years* in columns 3 and 4; and *Debt maturity (yr.)* in columns 5 and 6. We control for all independent variables from Table 3. Each explanatory variable is standardized by its full-sample standard deviation. All models include industry-year fixed effects created using the Hoberg-Phillips fixed industry classification with 50 industries (*HP50*) and the Fama-French 49 industries (*FF49*). The sample period is from 1975 to 2018. All variables are defined in Table A.1. t-statistics are reported in parentheses. Standard errors are clustered at the firm level. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

	Max. Investment cycle		Avg. Investment cycle	
	(1)	(2)	(3)	(4)
Useful life	0.150***	0.085***	0.106***	0.060***
	(6.09)	(3.83)	(5.33)	(3.21)
Market-to-book		0.371**		0.262
		(2.21)		(1.60)
Tangibility		-0.516		-0.176
		(-0.90)		(-0.31)
Profitability		2.287*		1.476
		(1.81)		(1.33)
Size		0.327***		0.249***
		(5.43)		(4.32)
Cash flow volatility		-14.372***		-11.332***
		(-4.59)		(-4.33)
R&D		6.706*		5.655*
		(1.93)		(1.72)
Firm age		0.091***		0.055***
		(8.65)		(7.19)
Observations	2332	2327	2332	2327
Adjusted R^2	0.02	0.17	0.01	0.11

Table IA.8: **Useful life and investment cycles – cross-sectional regressions**. This table presents estimates from regressions of firms' maximum and average investment cycle length on average useful life. The dependent variable is *Maximum investment cycle length* in columns 1 and 2, and *Avg. investment cycle length* in columns 3 and 4. The sample period is from 1975 to 2018. Firms with no investment spike have a cycle length that is undefined and are dropped from the sample. All variables are defined in Table A.1. t-statistics are reported in parentheses. Standard errors are clustered at the industry level using Hoberg-Phillips fixed industry classification with 100 industries. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

	Max. debt cycle			Avg. debt cycle		
	(1)	(2)	(3)	(4)	(5)	(6)
Capital age	0.158**			0.141**		
	(2.49)			(2.43)		
Asset Mat.		0.051**			0.038*	
		(1.99)			(1.85)	
Asset Mat. (Cap)			0.121***			0.091***
			(2.99)			(2.87)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1933	1911	1911	1933	1911	1911
Adjusted R^2	0.21	0.21	0.21	0.15	0.15	0.15

Table IA.9: **Useful life and debt cycles – alternative measures of useful life**. This table presents estimates from regressions of firms' maximum and average debt cycle length on firms' average capital age, average asset maturity, and average asset maturity capped at 25 years. The dependent variable is *Maximum debt cycle length* in columns 1 to 3 and *Avg. debt cycle length* in columns 4 to 6. We control for all independent variables from Table 6. The sample period is from 1975 to 2018. Firms with no leverage spike have a cycle length that is undefined and are dropped from the sample. All variables are defined in Table A.1. t-statistics are reported in parentheses. Standard errors are clustered at the industry level using Hoberg-Phillips fixed industry classification with 100 industries. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

	Max. investment cycle			Avg. investment cycle		
	(1)	(2)	(3)	(4)	(5)	(6)
Capital age	0.131***			0.081**		
	(3.21)			(2.20)		
Asset Mat.		0.063***			0.057***	
		(4.08)			(4.39)	
Asset Mat. (Cap)			0.144***			0.121***
			(4.95)			(5.69)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Observations	2327	2301	2301	2327	2301	2301
Adjusted R^2	0.18	0.18	0.18	0.11	0.11	0.11

Table IA.10: **Useful life and investment cycles – alternative measures of useful life**. This table presents estimates from regressions of firms' maximum and average investment cycle length on firms' average capital age, average asset maturity, and average asset maturity capped at 25 years. The dependent variable is *Maximum investment cycle length* in columns 1 to 3 and *Avg. investment cycle length* in columns 4 to 6. We control for all independent variables from Table 6. The sample period is from 1975 to 2018. All variables are defined in Table A.1. t-statistics are reported in parentheses. Standard errors are clustered at the industry level using Hoberg-Phillips fixed industry classification with 100 industries. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

	% debt maturing $> 3y$			% debt maturing $> 5y$			Debt maturity (yr.)		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Capital age	0.007***			0.007***			0.032		
	(3.25)			(2.91)			(0.78)		
Asset Mat.		0.006***			0.005***			0.057***	
		(10.61)			(5.70)			(3.32)	
Asset Mat. (Cap)			0.009***			0.007***			0.095***
			(12.46)			(10.74)			(6.30)
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	4392	4323	4323	4392	4323	4323	2566	2519	2519
Adjusted R^2	0.43	0.47	0.47	0.35	0.38	0.39	0.19	0.21	0.22

Table IA.11: **Useful life and debt maturity – alternative measures of useful life**. The table presents estimates from regressions of debt maturity on firms' average capital age, average asset maturity, and average asset maturity capped at 25 years. The dependent variable is the average of each firm's *% of debt maturing in > 3 years* in columns 1 to 3; *% of debt maturing in > 5 years* in columns 4 to 6; and *Debt maturity (yr.)* in columns 7 to 9. We control for all independent variables from Table 7. The sample period is from 1975 to 2018. All variables are defined in Table A.1. t-statistics are reported in parentheses. Standard errors are clustered at the industry level using the Hoberg-Phillips fixed industry classification with 100 industries. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.

	Dispersion							
	Book lev.		Lease-adj. lev.		Mat. (> $3y$)		Mat. (> $5y$)	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Cap. age disp.	-0.029*	-0.033**	-0.019	-0.028*	-0.041	-0.023	-0.038	-0.022
	(-1.80)	(-2.22)	(-1.13)	(-1.76)	(-1.18)	(-0.93)	(-1.51)	(-1.04)
Industries	HP50	FF49	HP50	FF49	HP50	FF49	HP50	FF49
Observations	47	39	47	39	47	39	47	39
Adj. R^2	0.11	0.24	0.04	0.18	0.05	0.04	0.04	0.04

Table IA.12: **Capital age dispersion and industry-level financing dispersion – different industry classifications**. This table presents estimates from regressions of industry leverage and debt maturity dispersion, computed using aggregate net book leverage and % debt maturing $> 3y$ or $> 5y$, respectively, on capital age dispersion. The dependent variables are *Book leverage dispersion* in column 1 and 2, *Lease-adjusted leverage dispersion* in columns 3 and 4, *Maturity dispersion (> 3 years)* in columns 5 and 6, and *Maturity dispersion (> 5 years)* in columns 7 and 8. The industry definitions are created using the Hoberg-Phillips fixed industry classification with 50 industries (*HP50*) and the Fama-French 49 industries (*FF49*). The dispersion results use the linearly detrended levels. The sample period is from 1975 to 2018. All variables are defined in Table A.1. t-statistics are reported in parentheses. Standard errors are adjusted for heteroskedasticity using the robust HC3 method. We use * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ to indicate p-values.