Can Corporate Debt Foster Innovation and Growth?*

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Abstract

Recent empirical studies show that innovative firms heavily rely on debt financing. Debt overhang implies that debt hampers innovation by incumbents. A second effect of debt is that it stimulates innovation by entrants. Using a Schumpeterian growth model with endogenous R&D and financing choices, we demonstrate that this second effect always dominates, so that debt fosters innovation and growth at the aggregate level. Our analysis suggests that the relation between debt and investment is more complex than previously acknowledged and highlights potential limitations of empirical work that focuses solely on incumbents when measuring the effects of debt on investment.

Keywords: corporate debt, innovation, industry dynamics, growth.

JEL Classification: G32, O30.

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Over the last few decades, the US economy has become innovation driven. Public firms now spend twice as much on research and development than on capital expenditures, and fixed assets have fallen from 34% to less than 20% of total assets between 1975 and 2016 (see for example Corrado and Hulten (2010) or Doidge, Kahle, Karolyi, and Stulz (2018)). Creative destruction has been a driving force of this transition to a knowledge-based economy. A good example of this phenomenon is the swift rise to power of Apple and Samsung in the mobile phone industry, replacing Nokia as the market leader. This example of creative destruction was driven by the innovative success of Apple and Samsung, even though all three firms devoted large amounts of resources to R&D.[1]

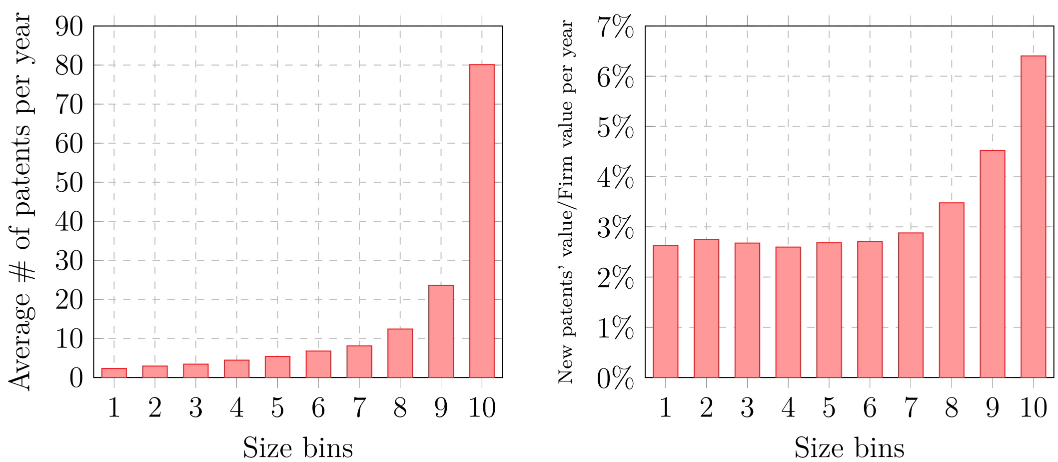

Figure 1: **Innovation quality and intensity.** The innovation data is based on Kogan, Papanikolaou, Seru, and Stoffman (2017) and the firm size data is from Compustat. The averages are conditional on issuing a new patent.

As shown in Figure 1, large firms play an important role for aggregate levels of innovation. Decades of empirical research have shown that debt is a key source of financing for these firms; see e.g. Graham, Leary, and Roberts (2015). In addition, even though debt is widely cast as an unlikely way to fund young and risky ventures, recent empirical studies show that small and young firms also heavily rely on debt financing. For example, Robb and Robinson (2014) find that formal debt financing (business bank loans, credit lines, and owner-backed

[1]See https://thenextweb.com/plugged/2019/03/29/24-years-global-phone-sales-graph-visualization/ for an impressive visualization of this change in market share.

bank loans) provides about 40% of firms' initial startup capital. Looking only at those firms that access equity sources, such as venture capital or angel financing, the average firm still has around 25% of its capital structure in the form of debt. A recent study by Hochberg, Serrano, and Ziedonis (2018) further documents a widespread use of loans to finance technology startups, even in early stages of development. Relatedly, Davis, Morse, and Wang (2018) find that venture debt is often a complement to equity financing, with over 40% of all financing rounds including some amount of debt.[2]

Given the change to an innovation-based economy and the heavy reliance of innovative firms on debt financing, a number of questions naturally arise. First, how does debt financing influence innovation at the firm level? Second, how do innovation and creative destruction in turn feed back into firms' financing policies? Third, what are the implications of debt financing in innovative firms for aggregate levels of innovation and growth?

This paper attempts to answer these questions by developing a Schumpeterian growth model in which firms' innovation and financing policies are endogenously determined. In this model, each incumbent has a portfolio of products and invests in innovative effort by spending resources on R&D, thereby determining the frequency of arrival of new innovations. Innovations improve the quality of produced goods. Firms therefore expand into new product lines when R&D is successful, which allows them to profit from their own innovations. These profits are however compromised when competitors develop better products. The force of creative destruction therefore affects firms R&D policies, as each product remains profitable until it is overtaken by another firm's innovation. Investment in innovation and creative destruction determine firms' cash flow risk, which feeds back into their financing decisions. In the model, R&D and financing policies maximize shareholder wealth. As a result, financing choices reflect not only market frictions such as taxes, default costs, and refinancing costs, but also conflicts of interest between shareholders and debtholders.[3]

[2] While start-ups cannot typically obtain debt financing from traditional banks, major U.S. banking institutions, public firms, and private firms specialize in providing loans to the very start-ups that banks turn away. In related research, Mann (2018) shows that patents are pledged as collateral to raise significant debt financing, and that the pledgeability of patents contributes to the financing of innovation. Suh (2019) finds that firm ownership of patents increases firms' total debt-to-assets ratio by 18%. Xu (2019) shows that firms use trademarks as collateral for debt financing.

[3] A simplified version of this model has been shown to capture the main stylized facts about corporate

After solving for individual R&D and financing choices, we embed the single-firm model into an industry equilibrium in which the rate of creative destruction is endogenously determined. We derive a steady state equilibrium in which new product lines replace existing ones and entrants replace incumbents that exit the industry. In this equilibrium, firms exhibit a wide variation in leverage, size, and innovation rates. Furthermore, all industry-wide equilibrium variables are constant over time, although individual firms are continually adjusting, with some of them expanding into new product lines, others contracting, some starting up, and others defaulting on their debt or exiting.

In equilibrium, capital structure and R&D influence each other through three main channels. First, R&D policy influences firms' risk profile and the aggregate level of creative destruction, which in turn affects their capital structure decisions. Second, levered firms are subject to debt overhang, which alters their incentives to innovate and, thus, the level of competition. Third, debt financing changes the surplus from entering the industry, which again influences the aggregate level of creative destruction and competition.

Starting with firm-level policies, we find that there is significant interaction between leverage and innovation. High levels of debt lead to less innovation by *incumbents* due to debt overhang, in that shareholders endogenously cut investment when its benefits mostly accrue to debtholders by rendering debt less risky. The effect of debt on innovation by incumbents is sizeable and larger for firms closer to financial distress. We also show that R&D and creative destruction play a key role in determining financing choices by affecting cash flow risk and the probability of default. Our model predicts substantial intra-industry variation in leverage and innovation, in line with the evidence in Xu (2012) and Kogan et al. (2017) and large effects of debt financing on firm turnover and industry structure.

Underinvestment by incumbents suggests that debt may hamper innovation and growth. A novel and surprising result of the paper is to demonstrate that debt financing does in fact foster investment and growth at the aggregate level. This is the outcome of two opposing forces. First, as discussed above, innovation by incumbents is negatively associated with debt at the individual firm level. Second, for a given rate of creative destruction, debt increases

leverage (Strebulaev (2007), Morellec, Nikolov, and Schürhoff (2012), and Danis, Rettl, and Whited (2014)).

the surplus from entering the industry (by providing a tax shield in the model), leading to a higher entry rate, to more innovation by entrants due to the higher mass (number) of entrants, and to a higher mass of incumbents (thereby muting the effects of the firm-level debt overhang). We demonstrate that the second effect dominates in equilibrium, so that debt financing fosters creative destruction and growth.

Importantly, the economic mechanism underlying this result suggests that measuring the effects of debt on innovation (or investment) using shock-based causal inference can potentially be problematic. According to our model, any exogenous policy shock that would make debt more valuable—e.g., an increase in the corporate tax rate—would result in higher leverage ratios and in lower innovation rates for incumbents. This could lead to a negative relation between innovation and debt at the firm level in the data. Yet, at the aggregate level, the increase in debt benefits would foster entry, increase the mass of entrants and incumbents, and thereby spur innovation and growth. Shock responses would therefore not recover the theory-implied causal effects as they would capture neither the influence of debt financing on entry nor that on the mass of incumbents. Our analysis suggests that the relation between debt and investment is more complex than acknowledged by prior work. It therefore highlights potential limitations of empirical work that focuses solely on firm-level investment when measuring the effects of debt on corporate investment (see, e.g., Giroud, Mueller, Stomper, and Westerkamp (2012) or Favara, Morellec, Schroth, and Valta (2017)).

Remarkably, our result that debt fosters creative destruction and growth does not hinge upon the specific trade-off used to determine firms' financing decisions but on the fact that debt increases the surplus from entering the industry. This result would also hold for example if debt reduced the cost of informational asymmetries between insiders and outsiders (Myers (1984)) or the cost of free cash flow and managerial flexibility (Jensen (1986)), as in both cases debt financing would increase the surplus of entrants (thereby stimulating entry), and reduce innovation by and facilitate exit of incumbents.[4] Importantly, while debt subsi-

[4]Because informational asymmetries or managerial agency conflicts are more difficult to calibrate than the corporate tax rate, this paper looks at whether the tax advantage provided by debt financing is sufficient to overturn the negative effects of debt financing on investment. We also do not use a mechanism based on exogenous financing constraints because these models generally require debt to be fully collateralized and lead to the counterfactual prediction that all debt is risk-free. In addition, the empirical studies of Robb

dizes entry, it also leads to underinvestment for incumbents and, therefore, does not work like government policies targeted at inducing innovation (e.g. patent policy) which would stimulate innovation by both entrants and incumbents.

Our result also does not hinge upon a specific sharing of the surplus associated with debt issuance among the various firm stakeholders. That is, our result does not preclude debtholders or consumers from extracting some or most of these benefits via a higher cost of debt or lower prices.[5] The sharing of the surplus will only affect the magnitude of the effect of debt on innovation and growth by determining which share accrues to equity holders. In the end, because equity holders have the ultimate decision-making authority on financing decisions, they will only issue debt if they can capture a part of the surplus associated with debt financing, which will inevitably yield our result that debt fosters innovation.

Lastly, our result that debt increases entry is consistent with the evidence in Black and Strahan (2002) and Kerr and Nanda (2009), who examine entrepreneurship and creative destruction following US banking deregulation. Their empirical analysis shows that US banking reforms, that made bank debt widely available and cheaper by increasing competition, brought growth in both entrepreneurship and business closures (see also Cetorelli and Strahan (2006), Amore, Schneider, and Žaldokas (2013), and Chava et al. (2013)).

We examine the robustness of our result that debt stimulates innovation by introducing an inelastic supply of entrants, subsidies to entrants, and debt renegotiation. As expected, the inelastic supply of entrants limits entry and weakens the effect of debt on innovation. In contrast, debt renegotiation and subsidies strengthen it. We also examine the welfare effects of debt financing in a general equilibrium version of our model. We show that welfare is non-monotonic in the rate of creative destruction and that debt financing only improves welfare if the technological improvements associated with innovations are sufficiently large.

and Robinson (2014), Hochberg et al. (2018), and Davis et al. (2018) show that debt financing is used by innovative firms that have access to and use both debt and equity financing. Lastly, consistent with the mechanism in the paper, Kerr and Nanda (2009) and Chava, Oettl, Subramanian, and Subramanian (2013) find that debt affects the entry incentives of young firms.

[5]It would be easy to integrate Nash bargaining over the surplus created by debt financing between shareholders and debtholders as in, e.g., Hugonnier, Malamud, and Morellec (2015). This would have no bearing on our result that debt fosters innovation and growth. Our paper follows most of the literature by assuming that debt markets are competitive so that creditors do not capture any of this surplus.

Our article contributes to several literatures. First, we contribute to the large literature studying the effects of debt on investment, following the seminal contribution of Myers (1977). This literature generally shows that debt financing hampers investment at the firm level, due to debt overhang.[6] A number of empirical studies have recently provided empirical support for this prediction; see for instance Chava and Roberts (2008), Giroud et al. (2012), or Favara et al. (2017). This literature generally emphasizes the negative effects of debt on investment at the firm level. By contrast, we show that debt fosters investment and growth at the aggregate level by stimulating entry. This result goes against standard economic intuition and highlights an important and overlooked effect of debt on investment.

Second, our paper relates to the literature on dynamic capital structure choice initiated by Fischer, Heinkel, and Zechner (1989) and Leland (1994). Models in this literature generally maintain the Modigliani and Miller (1958) assumption that investment and financing decisions are independent by assuming that the assets of the firm are exogenously given. This allows them to focus solely on the liability side of the balance sheet (see for example Fan and Sundaresan (2000), Duffie and Lando (2001), Hackbarth, Miao, and Morellec (2006), Gorbenko and Strebulaev (2010), Glover (2016), or DeMarzo and He (2021)). Our paper advances this literature by endogenizing not only firms' capital structure choices but also their investment policy. A key contribution with respect to this literature is that we embed the individual firm choices into a Schumpeterian industry equilibrium in which the rate of creative destruction is endogenous. We show that while debt leads to underinvestment by incumbents, it increases creative destruction and growth by stimulating entry.

Third, our paper relates to the literature on debt in industry equilibrium. In a closely related paper, Miao (2005) builds a competitive equilibrium model in which firms face idiosyncratic technology shocks and can issue debt at the time of entry before observing their profitability. In this model, all firms have the same debt level. An important assumption in Miao (2005) is that there are no costs of adjusting capital. As a result, there is no debt overhang in the sense of Myers (1977) because the absence of adjustment costs or frictions

[6] Mello and Parsons (1992) and Parrino and Weisbach (1999) examine the effects of debt financing on corporate investment in dynamic models of the firm. Our study departs from prior work by endogenizing capital structure choices and by embedding the single-firm model in an industry equilibrium.

make investment independent of financing (Manso (2008)).[7] By contrast, firms have different (endogenous) debt levels in our model and investment and financing decisions interact, leading to debt overhang and underinvestment by incumbents.[8] Other important contributions to this literature include Fries, Miller, and Perraudin (1997) and Zhdanov (2007), which respectively study static and dynamic capital structure choices in the Leahy (1993) model. In these models, incumbent firms are exposed to a single industry shock. They all have the same assets and the same debt level and there is no investment.

Lastly, our paper contributes to the literature on Schumpeterian growth models. Schumpeterian growth theory has been widely used in the literature on innovation and industry structure and evolution; see for example Klette and Kortum (2004), Lentz and Mortensen (2008), Aghion, Akcigit, and Howitt (2014), Akcigit and Kerr (2018), and Acemoglu, Akcigit, Alp, Bloom, and Kerr (2018). However, to the best of our knowledge, this literature has not studied the effects of debt financing on innovation, Schumpeterian competition, and industry dynamics. This is relatively surprising given that innovative firms heavily rely on debt financing. Our paper fills this gap by extending the model proposed by Klette and Kortum (2004) to incorporate debt financing. In our model, firms hold debt and default, which influences their R&D policies and the industry level of creative destruction.

Section 1 describes individual firm choices and embeds the single-firm model into an industry equilibrium. Section 2 analyzes the model implications. Section 3 examines the robustness of our results. Section 4 closes the model in general equilibrium.

1 Model

We present the model in steps, starting with the investment and financing decisions of an individual firm. We then embed the single-firm model into an industry equilibrium.

[7] In Miao (2005), firms underinvest in that levered firms exit the industry at a higher rate than unlevered firms would. This feature is also present in our model.

[8] In related research, Malamud and Zucchi (2019) develop a model of cash holdings, innovation, and growth in the presence of Schumpeterian competition. Firms are all equity financed in their model. Maksimovic and Zechner (1991) develop a three-period model in which investment decisions reflect debt choices in industry equilibrium. They do not study entry and exit decisions, which are central to our analysis.

1.1 Assumptions

Throughout the paper, time is continuous and shareholders and creditors are risk neutral and discount cash flows at the constant rate $r > 0$. The economy consists of a unit mass of differentiated products (or product lines) that are produced by incumbent firms.

A firm is defined by its portfolio of products. The discrete number of different products supplied by any given firm at time $t \geq 0$, denoted by P_t, is defined on the integers and is bounded from above by $\bar{p}$. As a result of competition between firms, each product is produced by a single firm and yields a profit flow of one. The profit flow of the firm evolves through time as a birth-death process that reflects product creation and destruction.

To increase its portfolio of products, a firm invests in innovative effort, i.e. spends resources on R&D. Investment in innovative effort determines the frequency of arrival of new innovations $\lambda_t \in [0, \bar{\lambda}]$, i.e. the Poisson rate at which new innovations arrive. The number of new products generated by an innovation is given by

$$X_t = \min\left(Y_t, \bar{p} - P_{t^-}\right) \;\; \textit{with } Y_t \sim Bin(n, \theta),$$

where n is an exogenous upper bound on the number of new products that can be developed following an innovation and $Bin(n, \theta)$ is the binomial distribution with innovation quality θ.[9] This specification implies that the expected number of new products is approximately $n\theta$. Therefore, a higher quality θ leads to a higher expected number of new product lines. Bounding the number of new products X_t from above by $\bar{p} - P_{t^-}$ ensures that P_t never exceeds $\bar{p}$. These assumptions imply that the total number of products developed by the firm up to time t, denoted by I_t, evolves as

$$dI_t = X_t dN_t^I,$$

where dN_t^I is a Poisson process with intensity λ_t.

[9] In an earlier version of the model, we considered that entrants and incumbents could additionally invest in innovation quality, thereby determining endogenously the number of products generated by entry or by incumbents' innovations. The current setup generates the very same implications but is less cumbersome. We thank the editor for encouraging us to simplify the model in this direction.

A firm's existing products can become obsolete because some other firm innovates on a good the firm is currently producing. In this case, the incumbent producer loses the product from its portfolio due to creative destruction. Since any firm is infinitesimal, we can ignore the possibility that it innovates on a good it is currently producing. Because of creative destruction, each product becomes obsolete at an exponentially distributed time with intensity f. We call f the rate of creative destruction, that each firm takes as given. Subsection 1.3 embeds the single-firm model into an industry equilibrium and endogenizes the rate f of creative destruction. The total number O_t of products lost by the firm up to time $t \geq 0$ because of creative destruction evolves as

$$dO_t = dN_t^O,$$

where dN_t^O is a Poisson process with intensity fP_{t^-}. The total number products in a firm's portfolio P_t is therefore given by

$$P_t = I_t - O_t.$$

A firm performing R&D with intensity λ_t incurs flow costs $q(P_t, \lambda_t)$. To make sure that shareholders are better off with more products, we impose that the R&D cost function does not increase too fast in the number of products in that

$$q(p+1, \lambda) - q(p, \lambda) < 1. \tag{1}$$

In addition to the R&D costs, production requires a fixed cost of operation $\eta > 0$, where we assume that $\eta < 1$ so that a single product firm can be profitable. An incumbent firm's operating profit is the profit that comes from the operation of the product lines net of the fixed operating cost and the endogenous costs of performing R&D:

$$P_t - \eta - q(P_t, \lambda_t).$$

Profits are taxed at the constant rate $\pi > 0$. As a result, firms have an incentive to issue debt to reduce corporate taxes. To stay in a simple time-homogeneous setting, we follow the literature (e.g. Leland (1994), Duffie and Lando (2001), and Manso (2008)) and consider debt contracts that are characterized by a perpetual flow of coupon payments c. When making leverage choices, shareholders balance the benefits of debt against its costs. On the one hand, debt provides tax benefits.[10] On the other hand, firms incur a proportional cost ϕ when issuing debt and may default on their debt obligations at the endogenous time τ_D. In our model, default risk leads to endogenous distortions in R&D decisions when close to distress, reflecting debt overhang. In addition, a fraction $\alpha \in (0, 1)$ of assets in place and growth options is lost as a frictional cost in default. As in e.g. Mauer and Triantis (1994), debtholders may choose a new capital structure that maximizes the value of the firm after default before selling it to new owners. (We could alternatively assume that creditors do not relever the firm with no bearing on our results.) The Internet Appendix allows firms to dynamically optimize their capital structure. Lastly, due to the fixed cost of operation and the function that maps R&D to innovation (described in Section 2 below), a firm with zero product lines endogenously exits the economy at time $\tau_0 \equiv \inf\{t > 0 : P_t = 0\}$.[11]

As in Klette and Kortum (2004), a mass of entrants invests in R&D to become producers upon a successful innovation. An entrant that generates an innovation becomes an incumbent. Similarly to an incumbent, the entrant chooses its R&D intensity λ_t. Upon a successful innovation, the entrant generates $p_0 \in \{1, ..., n^e\}$ products, where we assume with no loss in generality that p_0 is distributed according to a (scaled) Binomial distribution with n^e draws that each succeed with probability θ^e.[12] The entrant has an R&D cost function

[10] Our result that debt fosters innovation and growth at the aggregate level hinges upon the fact that debt financing increases firm value, thereby raising the surplus from entering the industry. That is, as long as it is optimal for the firm to issue debt, this result obtains. In our model, the increase in firm value due to debt financing results from the tax benefits of debt. We could alternatively assume that firms obtain better financing terms with debt, for example due to lower sensitivity to informational asymmetries, or that debt increases firm value by reducing the agency cost of free cash flow. Because these alternative frictions are more difficult to calibrate, we focus in this paper on the tax benefits of debt.

[11] The R&D cost function used in our numerical analysis is derived from a function that maps investments in R&D q into an innovation intensity λ as in Acemoglu et al. (2018): $\lambda = \beta^{-\gamma} q^{\gamma} p^{1-\gamma}$, where $\beta > 0$ and $\gamma \in (0, 1)$ are parameters and p is the current number of product lines. A zero product line firm $p = 0$ can therefore not generate any innovations and endogenously decides to exit. Our theoretical results hold for any R&D cost function for which zero product line firms have no incentive to innovate.

[12] This implies that for $p_0 \in \{1, ..., n^e\}$ the probability density function is given by $\mathbb{P}(p_0 = p) =$

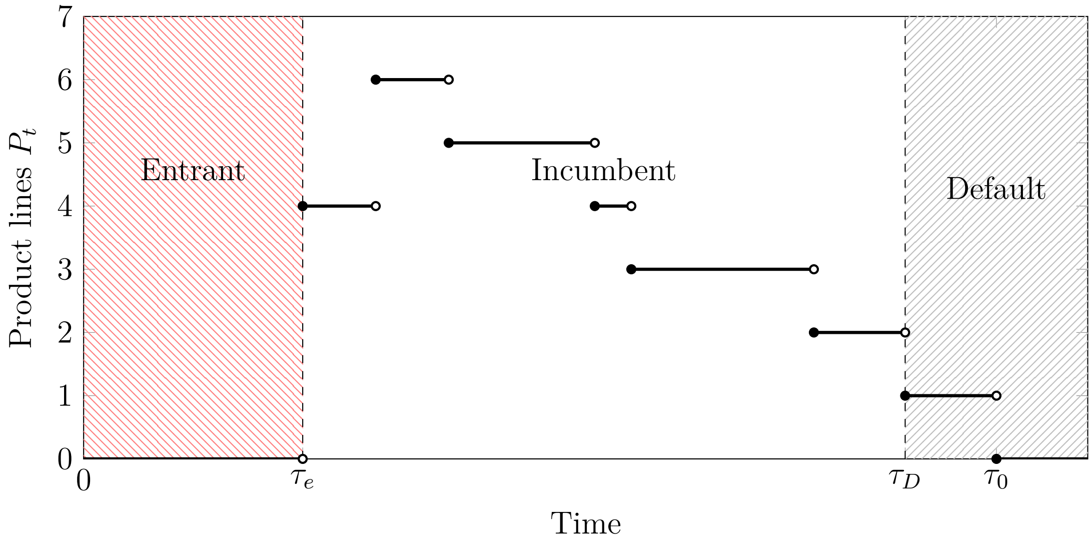

Figure 2: **Life cycle of a firm.** The firm starts as an entrant and becomes an incumbent with four product lines at time τ_e. The number of product lines then evolves as a birth-death process over time until the firm defaults at time τ_D, after which creditors sell the firm to new owners who operate it until the product becomes obsolete at time τ_0.

$q_e(\lambda)$. Because an entrant has no product lines before becoming an incumbent, it optimally has no debt and its optimal innovation strategy is time-homogeneous: $\lambda_t = \lambda^e$. As soon as an entrant has an innovative breakthrough and knows how many products this breakthrough generates, it chooses how much debt to issue. The cost of becoming an entrant is denoted by $H > 0$. We consider that entry costs are tax deductible, e.g. through depreciation. This assumption ensures that taxes have no bearing on innovation when firms are unlevered so that our results are directly comparable to those in prior work.

Figure 2 illustrates the life cycle of a firm that defaults at time τ_D with a strictly positive number of product lines (i.e. $\tau_D > \tau_0$). An entrepreneur first pays the entry cost and becomes an all-equity financed entrant, which incurs R&D expenses until it innovates for the first time. At time τ_e, the entrant experiences a breakthrough resulting in new product lines, decides how much debt to issue, and becomes an incumbent. Once the firm becomes an incumbent, it generates profits from its portfolio of products and continues to make R&D decisions,

$\left(1-(1-\theta^e)^{n^e}\right)^{-1}\binom{n^e}{p}(\theta^e)^p(1-\theta^e)^{n^e-p}$. Otherwise, the probability is zero.

which influences the intensity at which new innovations arrive. This process continues until the firm defaults at time τ_D with one product line remaining (i.e. the firm defaults when moving from two to one product line). After default, creditors sell the remaining product line to new owners who run the firm until its product becomes obsolete at time τ_0. When $\tau_D \wedge \tau_0 = \tau_0$ where $x \wedge y = \inf\{x, y\}$ (case not represented on Figure 2), the firm only defaults when losing its last product line to competitors (i.e. we have that $P_{\tau_D} = 0$).

1.2 Optimal Financing and Investment

We solve the model recursively, starting with the value of levered equity for a given financing policy. Since each product generates the same flow of profits, we only need to keep track of the size of the portfolio of products and the coupon when describing the state of the firm.

After debt has been issued, shareholders maximize equity value by choosing the firm's default and R&D policy. As a result, equity value for a given coupon c satisfies

$$E(p,c) = \sup_{\{\lambda_t\}_{t\geq 0}, \tau_D} \mathbb{E}_p \left[\int_0^{\tau_D \wedge \tau_0} e^{-rt} (1-\pi) \left(P_t - c - \eta - q(P_t, \lambda_t) \right) dt \right], \tag{2}$$

where $\mathbb{E}_p[\cdot] = \mathbb{E}_0[\cdot | P_0 = p]$. As shown by equation (2), shareholders receive the after-tax profits from P_t products minus the coupon payments c, the fixed operating cost η, and R&D expenses $q(P_t, \lambda_t)$ until they decide to default at time τ_D or the firm exits with zero products at time τ_0. Shareholders select the R&D strategy $\{\lambda_t\}_{t\geq 0}$ and default time τ_D to maximize equity value.

From equation (2), we have that equity value solves:

$$\begin{aligned} rE(p,c) = \sup \Bigg\{ 0, (1-\pi)(p - c - \eta) + \underbrace{fp \left(E(p-1, c) - E(p,c) \right)}_{\textit{Creative destruction}} \\ + \underbrace{\sup_{\lambda} \left\{ \lambda \left(\mathbb{E}\left[E(\min\{p + x, \bar{p}\}, c) \right] - E(p,c) \right) - (1-\pi) q(p,\lambda) \right\}}_{\textit{R\&D}} \Bigg\}, \end{aligned} \tag{3}$$

where $x \sim Bin(n, \theta)$ and $E(0,c) = 0$. We then have the following result.

Theorem 1 (Equity Value)**.** *A unique solution to the equity value defined in equation* (2) *exists. Equity value is non-decreasing in p and therefore the optimal default strategy is a barrier strategy $\tau_D = \inf\{t > 0 | P_t \le p_D\}$. If the R&D cost function is differentiable in the innovation intensity λ, the optimal level of R&D, if interior (i.e. $\lambda \in (0, \bar{\lambda})$), solves*

$$\mathbb{E}\left[E(\min\{p + x, \bar{p}\}, c)\right] - E(p, c) = (1 - \pi)\frac{\partial q(p, \lambda)}{\partial \lambda}.$$

The optimal R&D strategy, if interior, equates the marginal benefits and the marginal costs of R&D.[13] The marginal cost depends on the R&D cost function $q(p, \lambda)$. If an innovation arrives, the expected increase in equity value is

$$\underbrace{\mathbb{E}\left[E(\min\{p + x, \bar{p}\}, c)\right]}_{\textit{Post innovation}} - \underbrace{E(p, c)}_{\textit{Pre innovation}},$$

which is the marginal gain from increasing the arrival rate of innovations λ. Debt financing implies that shareholders do not fully capture the benefits of investment in that cash flows to shareholders are truncated at $\tau_D \wedge \tau_0$. As a result, the level of R&D that maximizes shareholder value is lower in a levered firm, notably when close to distress (see Section 2).

We also perform a comparative statics analysis with respect to the model's parameters:

Proposition 1 (Equity Value and Default Threshold)**.** *The following holds:*

1. *If $E(p, c) > 0$, equity value is strictly decreasing in the corporate tax rate π, the coupon payment c, the fixed operating cost η, the rate of creative destruction f, and the cost $q(p, \lambda)$ of performing R&D. If $\lambda > 0$, equity value is also strictly increasing in the innovation quality θ and the maximum number of product lines per innovation n.*

2. *The default threshold p_D is weakly increasing in the coupon payment c. In addition, for any given c, the default threshold p_D is independent of the corporate tax rate π and is weakly increasing in the fixed operating cost η, the rate of creative destruction f,*

[13] If there exists a $\hat{\lambda}$ such that $\frac{\partial q(p,\lambda)}{\partial \lambda} \ge \frac{\theta n}{r}$ for any $\lambda > \hat{\lambda}$, then in equilibrium $\lambda < \hat{\lambda}$ and imposing the bound on λ becomes void.

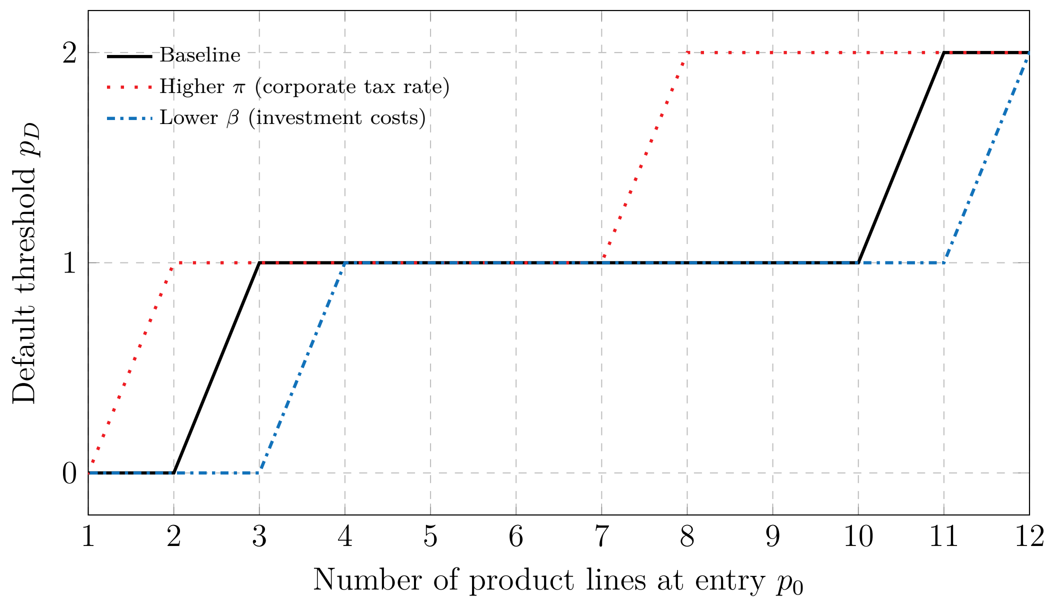

Figure 3: **Default threshold p_D in industry equilibrium.** Firms starting with a higher number of products initially choose a higher debt level and default with a higher number of products. Firms with $p_0 \leq 2$ default and exit when losing their last product to competitors in our baseline environment.

and the cost $q(p, \lambda)$ of performing R&D. If $\lambda > 0$, the default threshold p_D is weakly decreasing in the innovation quality θ and the maximum number of innovations n.

An increase in the corporate tax rate π, the coupon payment c, the fixed operating cost η, the rate of creative destruction f, or the cost $q(p, \lambda)$ of performing R&D decreases cash flows to shareholders or the expected lifetime of the firm and, therefore, reduces equity value. An increase in the innovation quality θ or the maximum number of products per innovation n has the opposite effect. By reducing the continuation value of equity, an increase in c, η, f, or $q(p, \lambda)$ or a decrease in θ or n increases incentives to default and therefore the default threshold selected by shareholders. Figure 3 shows the number of products in default as a function of the number of products at entry p_0 in industry equilibrium, which is described in the next subsection, for various economic environments. The baseline calibration is presented in Section 2. The figure shows that firms starting with a higher number of products initially choose a higher debt level and therefore default with a higher number of products. Firms with $p_0 \leq 2$ default and exit when losing their last product to competitors in our baseline environment. An increase in the corporate tax rate implies a higher debt level and therefore a weakly larger default threshold. A decrease in the cost of innovating (β) improves the firm's investment opportunity set and implies a weakly smaller default threshold.

Given the rate of creative destruction f and shareholders' R&D $\{\lambda_t\}_{t\geq 0}$ and default τ_D policies, the debt value $D(p,c)$ is the discounted value of the coupon payments until the time of default plus the recovery value. That is, we have

$$D(p,c) = \mathbb{E}_p \left[\int_0^{\tau_D \wedge \tau_0} e^{-rt} c dt + e^{-r(\tau_D \wedge \tau_0)} (1-\alpha) V(P_{\tau_D \wedge \tau_0}) dt \right],$$

where $V(p) = \sup_c E(p,c) + (1-\phi) D(p,c)$ is the optimally levered firm value.

Finally, we can also determine the value of an entrant given the rate of creative destruction f. Let τ_e be the time at which the entrant has a breakthrough and can develop its first product lines, which happens with intensity λ^e. The entrant's shareholders choose the R&D intensity that maximize their equity value, which consists of the proceeds once there is a breakthrough minus the tax-deductible R&D costs. That is, we have

$$\begin{aligned} E^e(f) &= \sup_{\lambda^e} \mathbb{E}_0 \left[e^{-r\tau_e} V_0(f) - \int_0^{\tau_e} e^{-rt} (1-\pi) q_e(\lambda^e) dt \right] \\ &= \sup_{\lambda^e} \left(\frac{\lambda^e V_0(f) - (1-\pi) q_e(\lambda^e)}{r + \lambda^e} \right), \end{aligned} \tag{4}$$

where

$$V_0(f) = \mathbb{E}_0 \left[\sup_{c \geq 0} \left\{ E(p_0, c) + (1-\phi)\, D(p_0, c) \right\} \right] = \mathbb{E}_0 \left[V(p_0) \right]. \tag{5}$$

As shown by equation (5), shareholders select the coupon that maximizes the value of their claim once they know how many products their innovation generates. Because the debt choice is affected by the number of product lines, the heterogeneity in entrants' R&D outcomes naturally leads to cross-sectional variation in the amount of debt issued. Entrants with no taxable income (i.e. with no products) endogenously refrain from issuing debt.

Lastly, free entry implies that

$$E^e(f) \le H(1-\pi),$$

which becomes an equality when there is entry. In equilibrium, competition implies that the value of becoming an entrant can never exceed the cost of entry.

1.3 Industry Equilibrium

This section incorporates the single-firm model into a Schumpeterian industry equilibrium. We look for a Markovian steady state industry equilibrium in which the number of firms and products is constant over time. In this industry equilibrium, both incumbents and entrants maximize their equity value taking as given the rate of creative destruction. That is, incumbents optimally choose their R&D and default decisions and entrants optimally choose their R&D and capital structure decisions. Given that we look for a Markovian steady state equilibrium, incumbents' optimal policies are a function of the number of product lines they own and the coupon payment on their debt. Entrants' optimal policies are time-homogeneous. Finally, free entry ensures that prospective entrants enter the industry as long as entry is profitable.

Definition 1 (Industry Equilibrium)**.** *The parameters and policies*

$$\Psi^* = \{f^*, c^*(p), \lambda^*(p,c), p_D^*(c), \lambda^{e*}\}$$

are an industry equilibrium if:

1. ***Incumbents:*** *Given the rate of creative destruction f^* and coupon payment c, incumbents' R&D $\lambda^*(p,c)$ and default $p_D^*(c)$ decisions maximize shareholder value. Given the rate of creative destruction f^*, creditors optimally relever a defaulted firm $c^*(p_D)$ before reselling it to new owners.*

2. ***Entrants:*** *Given the rate of creative destruction f^*, entrants level of R&D λ^{e*} and capital structure upon becoming an incumbent $c^*(p_0)$ maximize shareholder value.*

3. ***Entry:*** *The free entry condition holds:*

$$E^e(f^*) \leq H(1-\pi), \tag{6}$$

and the inequality binds when there is creative destruction $f^* > 0$*.*

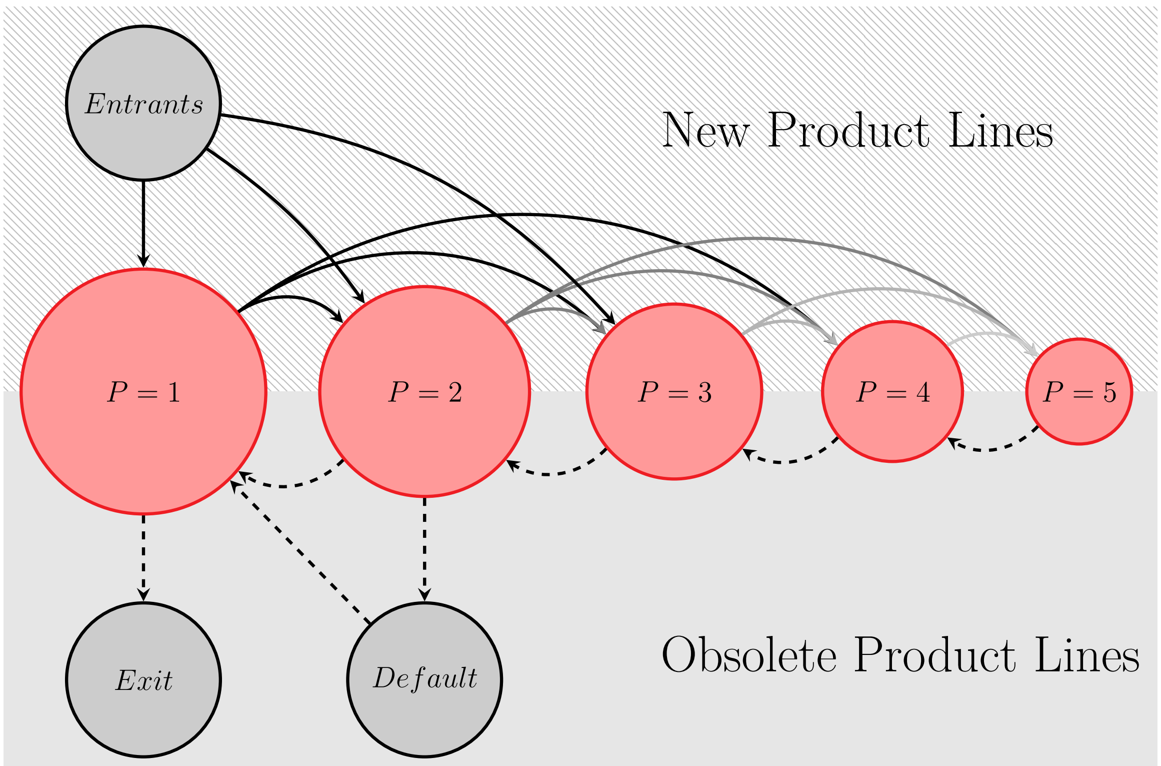

Figure 4: **Steady state equilibrium.** Example of a steady state distribution for an industry equilibrium in which $\bar{p}=5$ and $n=n^e=3$, and there is default, exit, and entry. Appendix 3 derives the steady state firm size distribution for any equilibrium.

Figure 4 shows an industry equilibrium with $\bar{p}=5$ and $n=n^e=3$ in which new product lines replace existing ones and entrants replace incumbents that exit the industry. The size of the circles indicates the mass of firms of each type. In a steady state equilibrium, the size of these circles is constant over time. Incumbents can move up due to innovations, which generate new product lines, and move down due to creative destruction. Because an innovation can generate up to three product lines and the number of new product lines is random, there are multiple upward flows. In this equilibrium, firms exit when they have zero product lines (i.e. when losing their last product line to a competitor) and can default when moving from two to one product line due to creative destruction. Therefore, there is a positive mass of entrants. All industry-wide variables are constant over time, even though individual firms can create new product lines, can lose product lines to competitors, and can default and exit. Debt financing affects industry structure and dynamics by changing firms' R&D policies and the rate of creative destruction.

We start by analyzing the existence and uniqueness of a stationary equilibrium with debt. To do so, we make the following assumption:

Assumption 1. *Debt value is non-increasing in the rate of creative destruction f.*

In our model, an increase in the rate of creative destruction f leads to an increase in default risk (and to a decrease in investment and collateral). Assumption 1 therefore states that the value of corporate debt does not increase as default risk increases. Together with Proposition 1, Assumption 1 ensures that the value of an entrant $E^e(f)$ is monotonically decreasing in the rate of creative destruction f, so that there exists only one level of creative destruction for which the free entry condition (6) binds and therefore a unique equilibrium exists. This assumption trivially holds when the firm is unlevered so that a unique equilibrium always exists. Figure 5 verifies this conjecture for the economic environments considered in Section 2. The following theorem formalizes the equilibrium existence and uniqueness result:

Theorem 2 (Equilibrium Existence and Uniqueness)**.** *There exists a unique stationary industry equilibrium with debt Ψ^*.*

In industry equilibrium, creative destruction arises because of innovations by incumbents and entrants. The level of innovation by incumbents depends on the mass of incumbents in the economy M^i and the average number of innovations per incumbent $\mathbb{E}[\lambda X] \approx \mathbb{E}[\lambda n \theta]$,[14] where an innovation generates X new product lines and the expectation is taken over the steady state distribution of incumbent firms. The level of innovation by entrants depends

[14]The equality is approximate since a firm can never have more than $\bar{p}$ product lines.

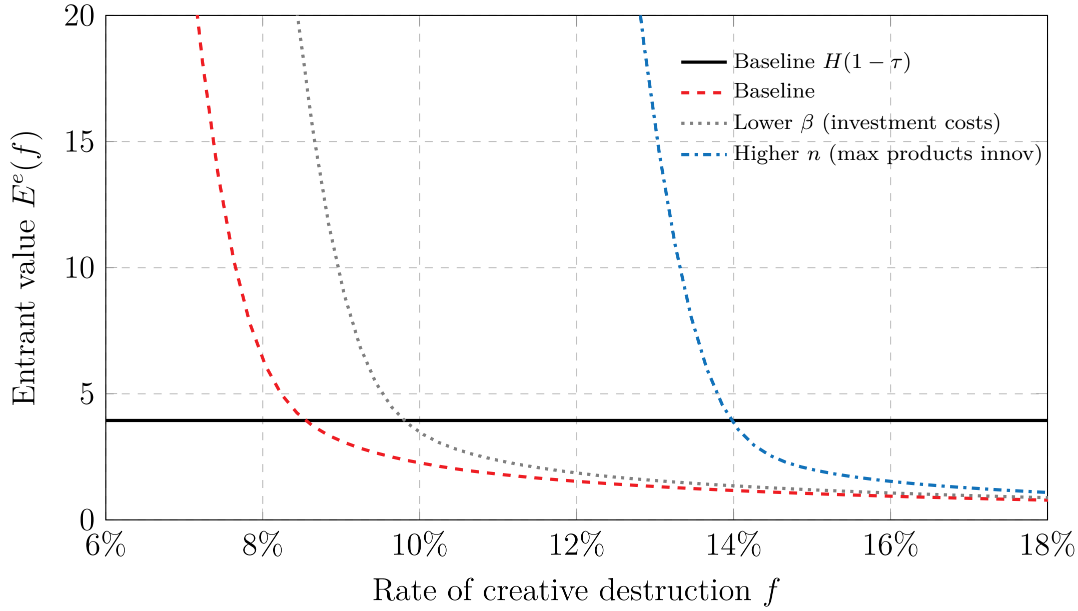

Figure 5: **Entrant value $E^e(f)$ as a function of the rate of creative destruction f.** The horizontal black line shows the baseline value for the entry cost, which should be equal to the entrant value in equilibrium $E^e(f^*) = H(1-\pi)$. The dashed red line shows the value of an entrant as a function of the rate of creative destruction in our baseline calibration.

on the mass of entrants M^e and the average number of innovations per entrant $\lambda^e \mathbb{E}[p_0]$. Appendix 3 shows how to calculate M^e and M^i. Altogether, this implies that

$$f^* = f^i + f^e = \underbrace{M^i \mathbb{E}[\lambda X]}_{Incumbents} + \underbrace{M^e \lambda^e \mathbb{E}[p_0]}_{Entrants}, \tag{7}$$

where we denote by f^i and f^e the respective contributions of incumbents and entrants to the rate of creative destruction. As equation (7) makes clear, innovation and investment are driven by both incumbents and entrants in industry equilibrium. As a result, they depend on firm financing. In particular, we show below that $\mathbb{E}[\lambda X]$ goes down due to debt financing (leveraged incumbents innovate less). That is, debt overhang pushes f^i down. This partial effect implies that debt financing lowers innovation. In our model, debt financing also impacts entry $M^e \lambda^e \mathbb{E}[p_0]$ (and thus the mass of incumbents M^i), which positively impacts the aggregate innovation intensity f^*. A priori, it is not obvious which of the two effects is more important. Proposition 2 shows that in equilibrium the second effect always dominates:

Proposition 2 (Debt Financing and Creative Destruction)**.** *Let $f^*_{No\ Debt}$ be the unique equilibrium rate of creative destruction when firms are restricted to have no debt. The rate of creative destruction f^* in the unique stationary industry equilibrium with debt Ψ^* satisfies:*

$$f^* \geq f^*_{No\ Debt}.$$

Following the seminal contribution by Myers (1977), a large literature in corporate finance has emphasized the negative effects of debt on investment. In contrast with this literature, Proposition 2 demonstrates that debt financing fosters innovation and creative destruction at the aggregate level. As we show below, this is the outcome of two opposing forces. First, debt hampers firm-level innovation by incumbents due to debt overhang, as recognized in prior work. Second, debt increases the surplus from entering the industry, thereby stimulating entry and increasing the mass of incumbents. Proposition 2 shows that the latter effect dominates in equilibrium so that debt spurs innovation. Since firms remain optimally debt-free when interest payments are not tax deductible, one interpretation of Proposition 2 is that the policy of tax deductibility of interest payments stimulates innovation. The intuition for this result is that debt financing increases the value of innovating firms by providing tax benefits. By increasing the value of incumbents, debt increases the surplus from entering the industry. In equilibrium, the free entry condition needs to bind so that the surplus generated by debt financing gets translated into a higher rate of creative destruction.

Importantly, our result that debt stimulates entry is consistent with the evidence in Kerr and Nanda (2009), who examine entrepreneurship and creative destruction following U.S. banking deregulation. Their empirical analysis shows that U.S. banking reforms—that made bank debt widely available and cheaper by increasing competition—brought growth in both entrepreneurship and business closures (see also Black and Strahan (2002), Cetorelli and Strahan (2006), Amore et al. (2013), or Chava et al. (2013)).

Lastly, empirical research concerned with measuring the effects of debt on investment tends to solely focus on firm-level investment by incumbents (see, e.g., Chava and Roberts

(2008), Giroud et al. (2012) or Favara et al. (2017)). Proposition 2 suggests that the relation between debt and investment is indeed more complex than acknowledged by prior work and that measuring the full effects of debt on innovation (or investment)—e.g. using shock-based causal inference—requires capturing the effects of debt financing not only on investment by incumbents but also on entry.

1.4 Refinancing

The Internet Appendix extends the model by allowing incumbent firms to dynamically optimize their capital structure as their portfolio of products evolves (i.e. as they acquire new products or lose products to competitors). We show in this appendix that the results derived in this section go through when we allow firms to restructure and demonstrate that there exists an industry equilibrium with debt. In this equilibrium, debt financing fosters innovation and growth.[15] In our dynamic capital structure version of the model, it is costly for firms to issue debt but there is no pre-commitment to a specific debt policy except that firms must repurchase existing debt (i.e., eliminate a debt covenant) before they can issue new debt with a higher face value. In recent research, DeMarzo and He (2021) use a model in which firms can continuously roll-over their debt to show that if debt financing increases firm value, it is because firms face fixed issuance costs of debt—which make it suboptimal to continuously adjust capital structure—or can commit to specific financing policies—which makes it impossible to freely adjust capital structure.[16]

[15] In effect, when firms have the ability to adjust their leverage ratio this further increases their (equity) value, which increases the quantitative impact of debt financing on innovation (Proposition 2).

[16] Specifically, DeMarzo and He (2021) show that if there are no issuance costs of debt and if firms are unable to commit to a specific financing policy, then there exists a Markov perfect equilibrium in barrier strategies in which firms "give up" the entire value of the tax shield. The reason is that future debt issuance decisions are value destroying from an ex-ante perspective. In general, mechanisms/frictions that discipline leverage choices—such as fixed issuance costs of debt (Dangl and Zechner (2021) or Benzoni, Garlappi, Goldstein, Hugonnier, and Ying (2021)), managerial agency conflicts (Morellec, Nikolov, and Schürhoff (2018) or Wong (2021)), lending relationships (Malenko and Tsoy (2021)), or covenants (Leland (1998))—lead to more conservative policies and re-establish a net benefit of debt, even without commitment. In our model, refinancing costs comprise both a fixed and a variable component (in line with the evidence in Altınkılıç and Hansen (2000) and Yasuda (2005)), ruling out equilibria à la DeMarzo and He (2021).

2 Model Analysis

2.1 Parameter Values

This section examines the implications of the model for innovation and financing policies. Our model has twelve structural parameters as listed in Table 1. We identify these parameters in two ways. First, we rely on the literature on innovation and financing decisions for estimates of the curvature of the R&D cost function, the scale of R&D, the risk-free rate, the corporate tax rate, bankruptcy costs, and debt issuance costs. Second, we calibrate the remaining model parameters by matching observed characteristics of innovation and capital structure policies of an average U.S. public firm, using firms' financial data from Compustat and the data on firms' innovation activity from Kogan et al. (2017).

We fix the six parameters (r, π, α, ϕ, γ, β) using values from prior literature. We first set the interest rate r at 4.2%, as in Morellec et al. (2012). We choose a corporate tax rate π of 15%, consistent with the estimates of Graham (1996). The bankruptcy cost α is set to 45%, as estimated by Glover (2016). The proportional cost of debt issuance ϕ is set to 1.09%, consistent with the evidence on debt underwriting fees in Altınkılıç and Hansen (2000). Following Acemoglu et al. (2018), we assume that the R&D cost function has the following form:

$$q\left(p,\lambda\right)=p\beta\left(\frac{\lambda}{p}\right)^{\frac{1}{\gamma}}\quad and\quad q_{e}\left(\lambda\right)=\beta\lambda^{\frac{1}{\gamma}},$$

where $\beta > 0$, $\gamma < 1$, and $\lambda \leq \bar{\lambda} = 15$. As in Acemoglu et al. (2018), we set the curvature of the R&D cost function at $\gamma = 0.5$ and the scale of R&D at $\beta = 4.066$. Similarly, we set innovation quality equal across entrants and incumbents, in that $\theta = \theta^e$. Lastly, we allow an entrant to have at most 7 product lines in that $n^e = 7$.

To obtain the remaining four parameter values (η, θ, n, H), we match four moments of interest in the data: the mean and variance of the leverage ratio, the mean innovation incidence (with a value of 50% indicating that a firm develops a new product on average every two years), and the turnover rate. The mean and variance of leverage are tightly linked to the parameters governing the fixed cost of operation η and the maximum number of new products per innovation n. Innovation incidence is directly linked to innovation quality θ. Lastly, the entry cost H pins down the turnover rate.

Panel A: Baseline parameter values		
Parameter	**Symbol**	**Value**
Max # products per firm	$\bar{p}$	80
Interest rate	r	4.2%
Tax rate	π	15%
Bankruptcy cost	α	45%
Debt issuance cost	ϕ	1.09%
After-tax entry cost	$H(1-\pi)$	3.35
Innovation curvature	γ	0.5
Scale of R&D	β	4.066
Fixed operating cost	η	0.973
Max # new products per innovation	n	2
Innovation quality parameter	$\theta = \theta^e$	0.125
Max # products at entry	n^e	7
Panel B: Variable definitions		
Moment	**Model**	**Data**
Leverage	$\frac{D(P_{it},c_{it})}{D(P_{it},c_{it})+E(P_{it},c_{it})}$	$\frac{dltt_{it}+dlc_{it}}{dltt_{it}+dlc_{it}+prcc\_f_{it}*csho_{it}}$
Innovation incidence	$\frac{\sum_i^N \sum_t^T \mathbb{I}_{\{P_{t+1}-P_t>0\}}}{NT}$	$\frac{\sum_i^N \sum_t^T \mathbb{I}_{\{fnpats_{it}>0\}}}{NT}$
Debt benefit	$\frac{\mathbb{E}[\pi D(P_{it},c_{it})]}{\mathbb{E}[V(P_{it},c_{it})]}$	

Table 1: **Baseline parameter values and definitions of moments.**

To compute the data counterparts of the model-implied variables, we use the Kogan, Papanikolaou, Seru, and Stoffman (2017) data on patent quantity merged with accounting variables from Compustat. We use the sample period 1975–2010. We exclude regulated and financial industries and apply standard Compustat filters: we remove observations with negative book equity and market-to-book larger than 20 as well as those with missing values for total assets and sales. All variables are winsorized at 2% and 98% in each fiscal year.

Panel B of Table 1 presents the definitions of the moments of interest in the data as well as their model counterparts. We compute the model-implied moments by simulating a balanced panel firms, similar to the ones observed in the data. We start the simulations from the steady state distribution of firms (see Appendix 3). Firms that exit are replaced with entrants to keep the panel balanced. We compute all variances by removing firm fixed effects.

Table 2 reports the model-implied moments and their data counterparts. The numbers in the table suggest that the model succeeds in replicating observed financing and innovation policies. The average (market) leverage ratio is equal to 22.74%, which is close to the empirical value of 22.44%. As we show below, the relatively low value of leverage in the model is the result of the endogenous rate of creative destruction that disciplines firms' financing policy and the endogenous R&D policy that feeds back into financing decisions. The model also closely matches the variance of leverage, which equals 1.96% in the data and 1.78% in the model, thus generating sizeable variation in financing policy. The innovation incidence equals 28.73% and is close to the observed value of 26.40%. Lastly, the model generates a turnover rate of 1.59%, which is between the turnover rate of 1.1% reported by Corbae and D'Erasmo (2021) and the turnover rate of 1.7% reported by Begenau and Salomao (2019).

Baseline calibration. All values are in %.

	Leverage Mean	**Leverage Variance**	**Innovation incidence**	**Turnover rate**	**Debt benefit**
No debt model	0.00	0.00	56.25	0.74	0.00
Debt model	22.74	1.78	28.73	1.59	3.41
Data	22.44	1.96	26.40	1.10-1.70	3.50

Table 2: **Baseline calibration of the model.** The definitions of all moments are provided in Panel B of Table 1.

A comparison between outcome variables in the debt and no debt models indicates that debt facilitates firm exit by increasing the turnover rate. Debt reduces the innovation incidence due to debt overhang and the higher rate of creative destruction. The results also show that despite the debt induced distortions in R&D, firms benefit substantially from debt financing. The implied net benefit of debt is around 3.41% of firm value, which is consistent with the estimates of Korteweg (2010) and van Binsbergen, Graham, and Yang (2010).

How do leverage and innovation vary with firm characteristics? In our model, interest expenses are tax deductible, providing firms with an incentive to issue debt. Debt financing gives shareholders an option to default, which is costly. Debt financing also reduces the benefits of innovation to shareholders because part of the benefits of investment accrue to creditors (as debt becomes less risky). Therefore, debt distorts innovation incentives. These distortions in innovation policy feed back into firms' cash flow dynamics, which influences the leverage choice. Table 3 illustrates that frictions (i.e. the corporate tax rate π or the cost of issuing debt ϕ) and the quality of the firm's investment opportunity set (as captured by the cost of investment β and the benefits of innovation θ) have important effects on financing decisions and the turnover rate. For example, higher costs of innovation lead to lower incentives to innovate by reducing the value of growth opportunities. In response, firms increase financial leverage. Table 3 also shows that when each innovation has the potential of creating more product lines (as θ gets larger), the potential costs of debt overhang are larger and firms opt for a lower leverage ratio. Overall, these results are consistent with the evidence in Smith and Watts (1992) and Barclay and Smith (1995) that firms with better growth opportunities adopt lower leverage ratios. Table 3 also shows that a decrease in entry costs (due, e.g., to government subsidies to innovative firms) increases the intensity of competition, thereby reducing the value of growth opportunities and leading incumbents to choose higher leverage ratios.

2.2 Debt and Innovation

As shown by Proposition 2, a surprising and novel prediction of our model is that debt financing fosters innovation and creative destruction at the aggregate level by increasing the surplus from entering the industry and, therefore, innovations by entrants f^e and the mass of incumbents M^i. To better understand this mechanism, we turn to analyzing the effects of debt financing on R&D investment by incumbents and entrants.

Comparative statics. All values are in %.					
	Leverage Mean	**Leverage Variance**	**Innovation Incidence**	**Turnover rate**	**Debt benefit**
Entry cost					
$H = 3.744$	26.12	2.10	18.09	1.90	3.92
$H = 3.941$	22.74	1.78	28.73	1.59	3.41
$H = 4.138$	16.61	1.42	45.33	1.04	2.49
Innovation quality					
$\theta = 0.105$	28.49	2.11	7.79	1.83	4.27
$\theta = 0.125$	22.74	1.78	28.73	1.59	3.41
$\theta = 0.145$	8.54	0.63	74.84	0.55	1.28
Innovation cost – level					
$\beta = 3.629$	18.68	1.65	39.44	1.27	2.80
$\beta = 4.066$	22.74	1.78	28.73	1.59	3.41
$\beta = 4.503$	24.97	1.93	21.16	1.69	3.75
Innovation cost – curvature					
$\gamma = 0.475$	13.13	0.99	56.93	0.78	1.97
$\gamma = 0.500$	22.74	1.78	28.73	1.59	3.41
$\gamma = 0.525$	26.10	2.36	17.72	1.91	3.92
Fixed operating cost					
$\eta = 0.946$	26.11	2.21	16.25	1.82	3.92
$\eta = 0.973$	22.74	1.78	28.73	1.59	3.41
$\eta = 1.000$	14.79	1.21	52.37	0.95	2.22
Tax benefit of debt					
$\pi = 0.10$	17.95	1.66	39.22	1.15	1.79
$\pi = 0.15$	22.74	1.78	28.73	1.59	3.41
$\pi = 0.20$	26.15	1.98	19.30	1.88	5.23
Debt issuance cost					
$\phi = 1.09\%$	22.74	1.78	28.73	1.59	3.41
$\phi = 2.18\%$	20.81	1.84	31.52	1.35	3.12
$\phi = 3.27\%$	19.99	1.76	33.72	1.29	3.00

Table 3: **Comparative statics of selected moments.**

2.2.1 R&D Investment and Debt Overhang

We first examine the effects of debt financing on *firm-level* investment by incumbents. To do so, we first show how debt financing affects investment in innovation intensity λ. The following proposition shows that, conditional on the number of product lines p, innovation is lower in a levered firm:[17]

Proposition 3. *If* $\left|\frac{\partial E(p+1,c)}{\partial c}\right| \geq \left|\frac{\partial E(p,c)}{\partial c}\right|$ *and* $\left|\frac{\partial E(p+1,c)}{\partial f}\right| \geq \left|\frac{\partial E(p,c)}{\partial f}\right|$*, then debt financing leads firms to innovate less in that* $\lambda(p,c) \leq \lambda(p)_{No\ Debt}$.

Figure 6 shows the quantitative effects of debt overhang by plotting the difference between R&D investment in the first-best case, in which R&D and default policy maximize firm value, and the baseline debt case, in which R&D policy maximizes shareholder value and is therefore subject to debt overhang. We first compute the decrease in the innovation intensity in the base case compared to the first-best case, conditional on the number of product lines p and coupon c. We then calculate the average of this number over c using the steady state distribution of the baseline case.

Figure 6 shows that when investment decisions maximize shareholder value and firms have debt outstanding, firms spend less on R&D. The effects of debt overhang are substantial in the model. Depending on firm size p, firms invest up to 22% more in innovation intensity in the first-best case compared to the baseline case. This distortion, that is solely due to debt, is especially strong for small firms, which are closer to financial distress. These effects tend to become smaller when firm size p increases as debt becomes less risky. As a result, wealth transfers to debtholders due to new investment are limited and so are the distortions in investment policy due to debt overhang. Figure 6 also demonstrates that distortions in investment are greater when the entry cost is smaller or when the quality of the investment opportunity set worsens, as firms adopt higher leverage ratios (see Table 3).

[17] The conditions in Proposition 3 ensure that shareholders in firms with more product lines, which are less likely to default, are more impacted by an increase in the rate of creative destruction or in the debt coupon. Indeed, a firm that is further away from default has debt that is less risky (so that the present value of coupon payments is larger for any c), implying a larger impact of an increase in c on equity value. Similarly, a firm with more product lines (i.e. further away from default) will be more affected by an increase in the rate of creative destruction, which reduces the value of its assets in place and growth options.

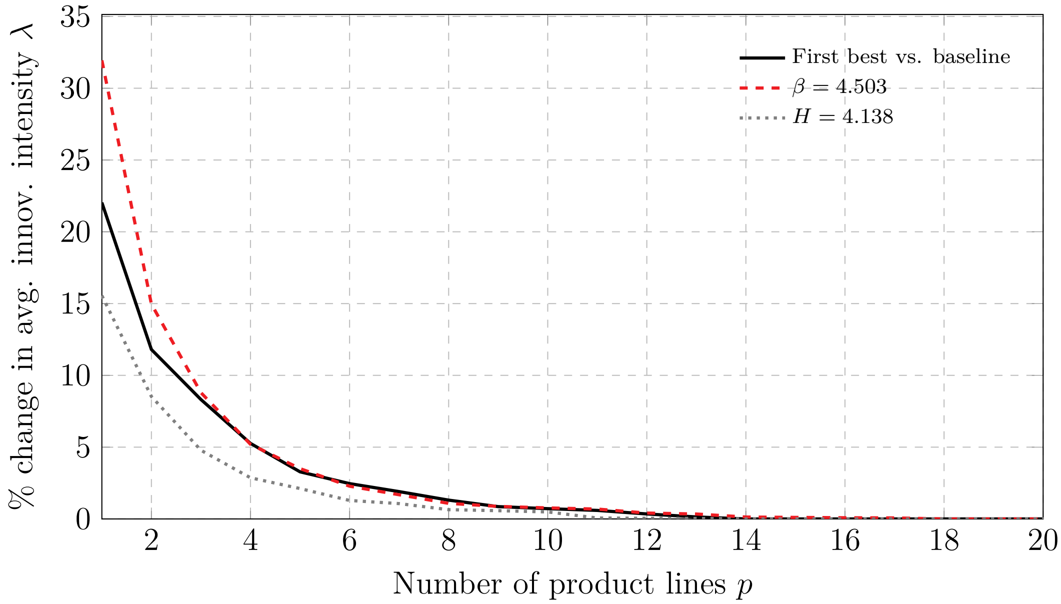

Figure 6: **Debt overhang.** The figure plots the change in innovation intensity by incumbents due to debt overhang as a function of firm size p for $p \leq 20$.

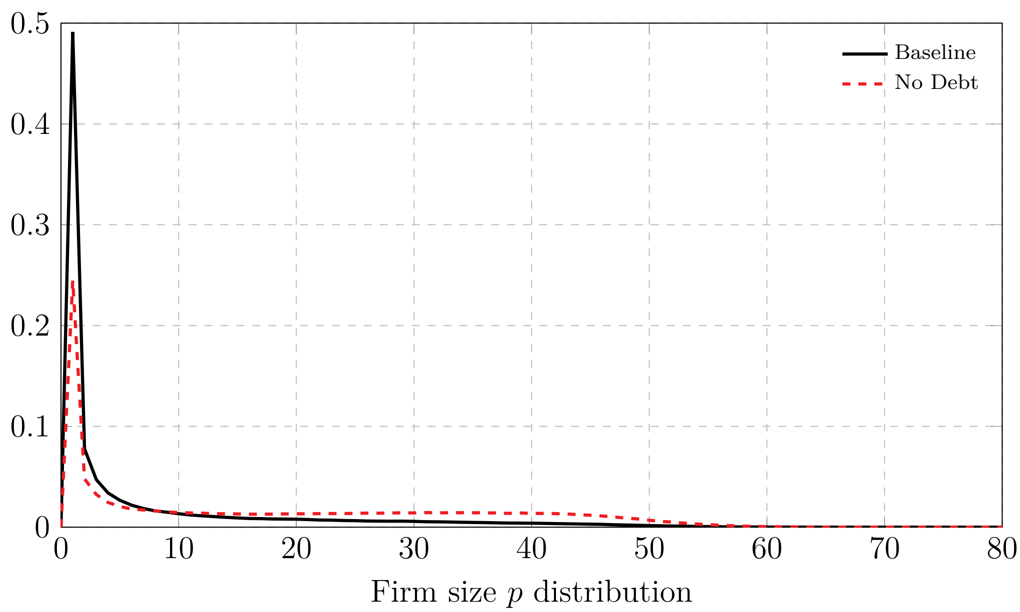

Figure 7: **Firm size distribution.** The graph shows the distribution of the number of products p in the no debt and baseline case.

Because of its effects on investment, debt financing also has important implications for the size distribution of firms. Notably, Figure 7 shows that the distribution is more skewed when firms can issue debt. This change can be attributed to the higher entry and turnover rates and to debt overhang, which reduces incumbents' incentives to innovate and grow.

Lastly, Figures 6 and 7 together show that debt financing leads to underinvestment by incumbents and to a decrease in the average firm size. Both effects combine to drastically reduce innovation by an average incumbent, as captured by $\mathbb{E}[\lambda X]$ in equation (7) (see Figure 8 below). Notably, it is possible to show that if $\bar{p} \to \infty$ and $\eta = 0$, then investment becomes linear in the number of product lines p for an all equity firm in that $\lambda(p) = \hat{\lambda}p$.[18] This in turn implies that innovation by the average incumbent is given by $\mathbb{E}[\lambda X] \approx \hat{\lambda} n\theta \mathbb{E}[p]$ for an all equity firm. That is, both underinvestment (lower $\hat{\lambda}$) and a smaller firm size (lower $\mathbb{E}[p]$) negatively impact innovation by an average incumbent when firms are financed with debt.

2.2.2 Debt Financing, Entry, and Aggregate Innovation

In industry equilibrium, aggregate innovation depends on innovation by incumbents and by entrants. As shown by equation (7), innovation by incumbents depends on the mass of incumbents in the economy M^i and the average number of innovations per incumbent $\mathbb{E}[\lambda X]$ (which, we just show, is negatively affected by debt). Innovations by entrants depends on the mass of entrants M^e and the average number of innovations per entrant $\lambda^e \mathbb{E}[p_0]$. The following proposition shows how M^i, M^e, λ^e, and f^e are affected by debt financing.

Proposition 4. *The average number of innovations per entrant $\lambda^e \mathbb{E}[p_0]$ does not depend on financing decisions in that $\lambda^e = \lambda^e_{No\ Debt}$. If debt overhang and higher levels of creative destruction lead incumbents to be on average smaller in an economy with debt financing, then there must be more incumbents $M^i > M^i_{No\ Debt}$. In addition, if incumbents exit more frequently in an economy with debt financing, then there must be more entrants $M^e > M^e_{No\ Debt}$ and entrants must contribute more to aggregate innovation $f^e > f^e_{No\ Debt}$.*

[18] In the no debt model with $\bar{p}, \bar{\lambda} \to \infty$ and $\eta = 0$ conjecture that the equity (firm) value is linear in p in that $e(p) = \hat{e}p$. In that case, optimal investment (as in Theorem 1) solves $\hat{e}(p+n\theta) - \hat{e}p = (1-\pi)\gamma\beta\lambda^{\gamma-1}p^{1-\gamma}$ and is given by $\lambda = \left(\frac{\hat{e}n\theta}{(1-\pi)\gamma\beta}\right)^{\frac{1}{\gamma-1}} p = \hat{\lambda}p$. As a result, the HJB equation for equity value becomes $r\hat{e}p = (1-\pi)p - fp\hat{e} + \hat{\lambda}p\hat{e}n\theta - (1-\pi)\beta p\hat{\lambda}^{\gamma}$, thereby verifying the conjecture that equity value is linear in p.

Figure 8 shows the quantitative effects of debt financing on innovation by entrants $f^e = M^e \lambda^e \mathbb{E}[p_0]$, innovation by incumbents $f^i = M^i \mathbb{E}[\lambda X]$, and their individual components when varying the cost of innovation β. The left column of Figure 8 shows that debt financing increases innovations by entrants and that this effect arises exclusively from the increase in the mass of entrants M^e (extensive margin). The innovation intensity of an individual entrant $\lambda^e \mathbb{E}[p_0]$ (intensive margin) is unaffected by debt financing because the free-entry condition must hold in equilibrium (see Proposition 4). Because an increase in β worsens the firm's investment opportunity set and leads to higher leverage ratios (as shown in Table 3), the magnitude of the effect increases as β increases. The bottom panel of the right column shows that debt financing reduces innovation of an average incumbent. The effect is large, as it reflects both debt overhang (see Figure 6) and the decrease in average firm size (see Figure 7). At the same time, the higher entry rate translates into a larger mass of incumbents (middle right panel) so that the level of innovation by incumbents f^i is only marginally negatively impacted by debt financing (top right panel).

As illustrated by Figure 8, the effects of debt financing on aggregate innovation are essentially driven by the increase in the mass of entrants M^e. A second effect of debt financing is that it increases the mass of incumbents M^i that innovate by stimulating entry. To assess the relative magnitudes of these effects, we can compare the change in f^e to the change in the aggregate rate of creative destruction f due to debt financing:

$$\frac{f^e - f^e_{No\ Debt}}{f - f_{No\ Debt}} \approx 1.44.$$

In our baseline environment, about 144% of the increase in the rate of creative destruction f due to debt can be attributed to the increase in the entry rate M^e, implying that debt financing reduces investment by incumbents. As illustrated by Figure 9, the magnitude of this effect varies with input parameter values. For example, the top left panel of Figure 9 shows that this effect becomes weaker as the cost of innovation decreases can fall below 100% when β is sufficiently low. In this case, firms adopt lower leverage ratios (as shown in Table 3) and the rest of the increase in the aggregate level of innovation is due to the larger mass of incumbents M^i (as shown by the top right panel of 8 where debt increases f^i

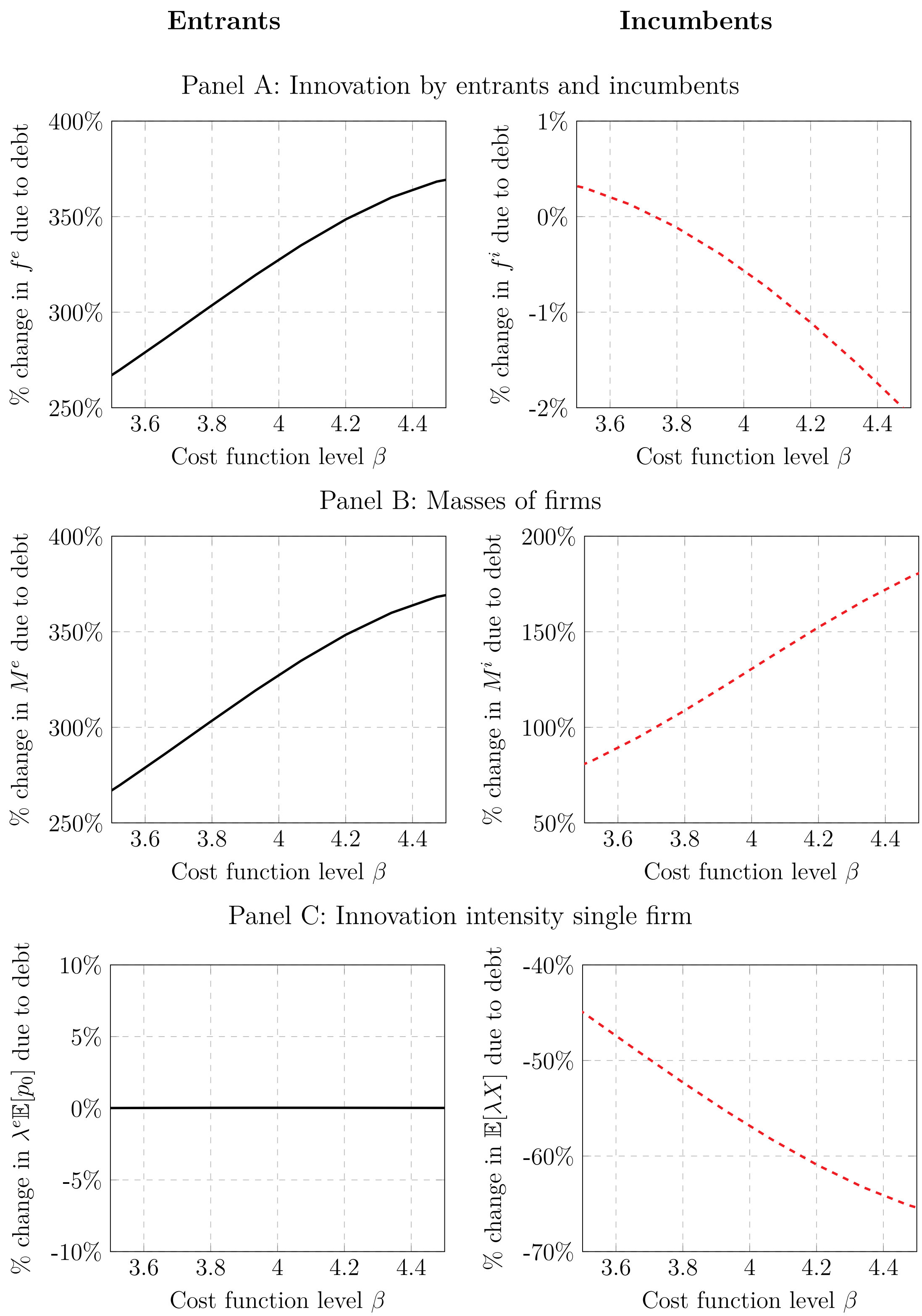

Figure 8: **Debt financing and innovation.** The figures show the effects of debt financing on innovation by incumbents and entrants. The comparative statics are smoothed using a third-order polynomial.

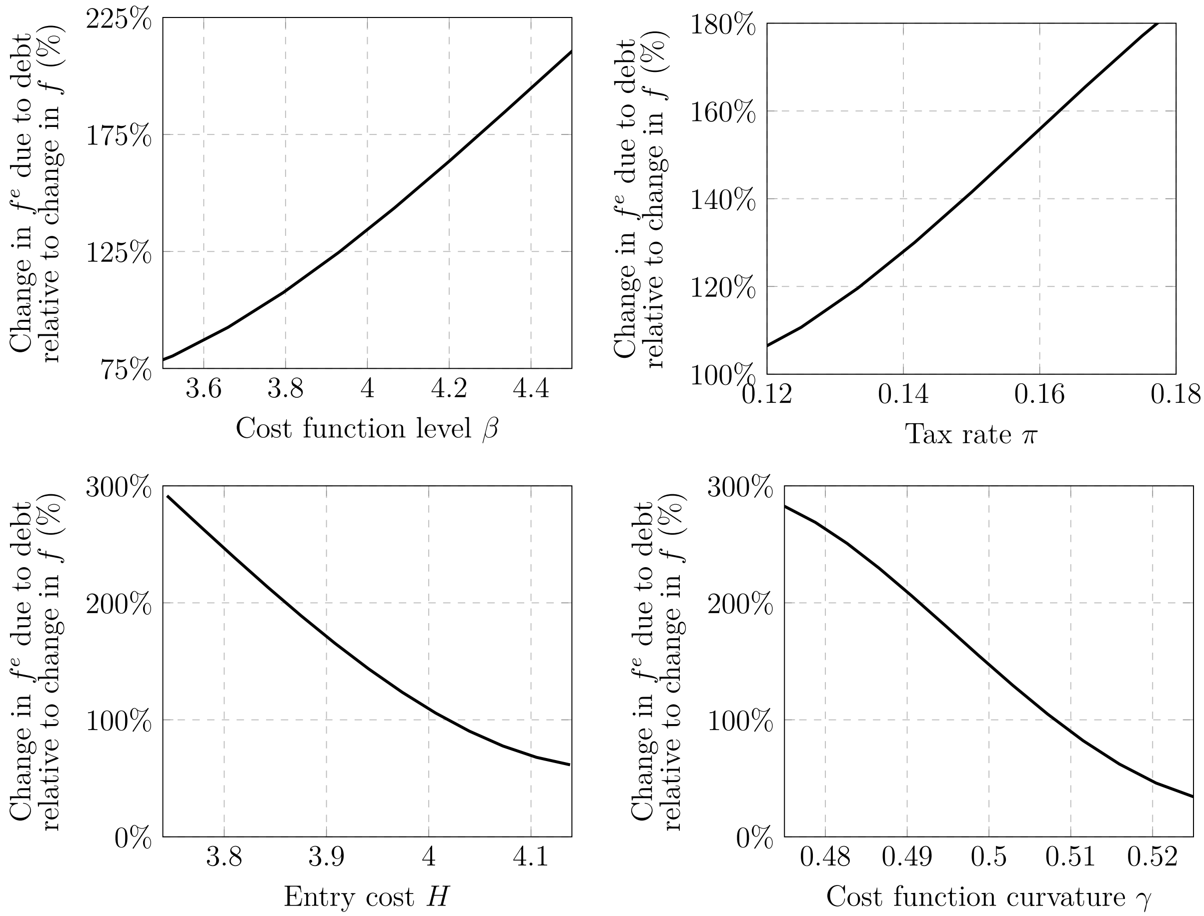

Figure 9: **The effects of debt financing on innovation by entrants.** The figures show entrants' contribution to the aggregate increase in creative destruction due to debt financing. The comparative statics are smoothed using a third-order polynomial.

for low values of β). A similar result arises for low values of the corporate tax rate or high values of the entry cost, which also lead to conservative debt policies. In most environments however, the effects of debt financing on entry account for more than 100% of the increase in aggregate innovation. Underinvestment by incumbents acts as a balancing force and has a negative effect on the aggregate rate of creative destruction. That is, the net effect of debt financing on innovation results from two very large and opposing forces, that partially offset each other in equilibrium.

3 Exploring The Mechanism

In our model, debt financing has two separate effects on innovation. First, it stimulates entry which fosters innovation. Second, it leads to debt overhang by incumbents which hampers innovation. The first effect is reinforced by the perfectly elastic supply of new entrants and by entry subsidies. The second is reinforced by the absence of debt renegotiation, which limits the benefits of investment to shareholders. This section discusses the effects of relaxing these assumptions on our results.[19]

3.1 Inelastic Supply of Entrants

Our paper follows classic models of firm entry dynamics and of endogenous growth and innovation (see, e.g., Hopenhayn (1992), Melitz (2003), Klette and Kortum (2004), Lentz and Mortensen (2008), or Akcigit and Kerr (2018)) by assuming that the cost of entry is fixed and the supply of entrants is perfectly elastic. This assumption is important for our result that debt fosters innovation because this result relies on increased innovation by entrants. An inelastic supply of entrants hampers entry and therefore innovation by entrants. The more constrained the supply of entrants, the less they innovate. As a result, debt financing fosters innovation if and only if the supply of entrants is sufficiently elastic.

To characterize the effects of debt financing of innovation when the supply of entrants is inelastic, we start from the free entry condition, which must hold in equilibrium:

$$E^e(f) = (1-\tau)\mathcal{H}(m).$$

Recall that in this equation, m is the flow entrants and $\mathcal{H}(m)$ the entry cost function. This free entry condition implies that in the no debt model

$$E^e_{No\ Debt}(f^*_{No\ Debt}) = (1-\tau)\mathcal{H}(m_{No\ Debt}(f^*_{No\ Debt})),$$

where $m_{No\ Debt}(f^*_{No\ Debt})$ is the flow of entrants in the model without debt. Keeping $f^*_{No\ Debt}$

[19] We thank an anonymous referee for suggesting this section.

fixed, debt financing impacts this free-entry condition in two ways. First, the option to issue debt increases the value of entering the industry $E^e(f^*_{No\ Debt})\,(\uparrow)$. Second, the flow of firms that exit (and therefore enter) increases $m(f^*_{No\ Debt})\,(\uparrow)$, due to the increase in the rate of creative destruction and debt overhang. Another consequence of debt financing is that firms are on average smaller so that more firms produce all the goods $M^i\,(\uparrow)$, which also pushes up the flow of firms that exit/enter $m(f^*_{No\ Debt})\,(\uparrow)$. This higher flow of firms that exit/enter then raises the entry cost $\mathcal{H}(m(f^*_{No\ Debt}))\,(\uparrow)$. Therefore, given $f^*_{No\ Debt}$, it is unclear whether

$$E^e(f^*_{No\ Debt}) > (1-\tau)\mathcal{H}(m(f^*_{No\ Debt}))\ \ or\ E^e(f^*_{No\ Debt}) < (1-\tau)\mathcal{H}(m(f^*_{No\ Debt})).$$

as both the left and right hand side of these relations are affected by debt financing. If entry costs are not very sensitive to the mass of entrants (as in our baseline model where $\mathcal{H}'(m) = 0$), then the first case occurs and debt stimulates innovation (as f needs to increase to so that the free entry condition holds). If entry costs are very sensitive to the mass of entrants ($\mathcal{H}'(m) \gg 0$), then the second case occurs and debt hampers innovation ($f^* < f^*_{No\ Debt}$).

To quantitatively examine the robustness of the result in Proposition 2, we assume that

$$\mathcal{H}(m) = \hat{H} m^{\frac{1}{\xi}},$$

where $\hat{H} > 0$ and $\xi > 0$ is the elasticity of the supply curve for entrants. A lower value for ξ implies that the cost of entry is more sensitive to the flow m of entrants. As a result, the effect of debt on entry and innovation increases with the elasticity ξ. In our baseline specification, we have $\xi = \infty$ and $\hat{H} = H$. Using a setup similar to Clementi and Palazzo (2016), we can micro-found this functional form of the entry cost.[20]

To clearly identify the effects of debt financing on innovation, we set the constant $\hat{H}$ in the entry cost function $\mathcal{H}(m)$ such that the solution to the model without debt does not

[20]Indeed, assume there is a unit flow of entrepreneurs that each receive an independent signal s, which is drawn from a Pareto distribution with scale parameter of 1 and a shape parameter ξ. Given a signal s, the entrepreneur faces an entry cost $\hat{H}/s$. A higher signal s implies a lower entry cost and an entrepreneur enters if and only if $(1-\pi)\frac{\hat{H}}{s} \leq E^e(f)$. In this framework, prospective entrants are heterogeneous. If we want a flow $m < 1$ of entrepreneurs to enter then the marginal entrepreneur's signal is $m = \mathbb{P}(s \geq s_m) = s_m^{-\xi}$ and therefore the entry cost (of the marginal entrepreneur) is given by $\mathcal{H}(m) = \hat{H} m^{\frac{1}{\xi}}$.

depend on ξ.[21] That is, we set $\hat{H} = H\left(m_{No\ Debt}(f^*_{No\ Debt})\right)^{-\frac{1}{\xi}}$. Accordingly, the equilibrium rate of creative destruction in the model without debt does not depend on the elasticity of the supply curve for entrants in that $f^*_{No\ Debt}(\xi) = f^*_{No\ Debt}$ for all ξ. By contrast, with $\hat{H} = H\left(m_{No\ Debt}(f^*_{No\ Debt})\right)^{-\frac{1}{\xi}}$, the entry cost in the model with debt is now given by

$$\mathcal{H}(m) = H\left(\frac{m}{m_{No\ Debt}(f^*_{No\ Debt})}\right)^{1/\bar{\xi}} > \mathcal{H}(m_{No\ Debt}(f^*_{No\ Debt})) = H,$$

where the inequality follows from the fact that $m > m_{No\ Debt}(f^*_{No\ Debt})$ as debt financing stimulates exit and therefore entry. This equation shows that increasing ξ leads to lower entry costs in the model with debt by making the entry cost less sensitive to the flow of entrants m. As a result, entry and innovation increase with ξ in the model with debt. Figure 10 examines how changing the elasticity of the supply of entrants ξ impacts innovation by entrants and incumbents. The results are similar to those in Figure 8 and Proposition 4. Notably, Figure 10 shows that increasing the elasticity ξ increases the impact of debt financing on the rate of creative destruction by fostering the effects of debt on entry and hence on M^e and M^i. Since the entry cost depend on the flow of firms that exit/enter in this figure (unlike in the baseline model), the innovation intensity of a single entrant $\lambda^e\mathbb{E}[p_0]$ (intensive margin) does change with debt. As a result, changes in innovation by entrants f^e come from both changes in the mass of entrants M^e (extensive margin) and changes in $\lambda^e\mathbb{E}[p_0]$ (intensive margin).

In our model, debt financing increases the entrant value (conditional on f) by providing tax benefits. Since the effect of debt on entry increases with ξ, we have that debt stimulates innovation whenever the elasticity of the supply curve for entrants is above the break-even elasticity $\bar{\xi}$ defined by:

$$\bar{\xi} = \{\xi | f^*(\xi) = f^*_{No\ Debt}\},$$

where $f^*(\xi)$ is the equilibrium rate of creative destruction in the model with debt given ξ.

[21] We could instead set $\hat{H}$ such that the model with debt is unaffected by the elasticity ξ of the supply curve for entrants with similar results.

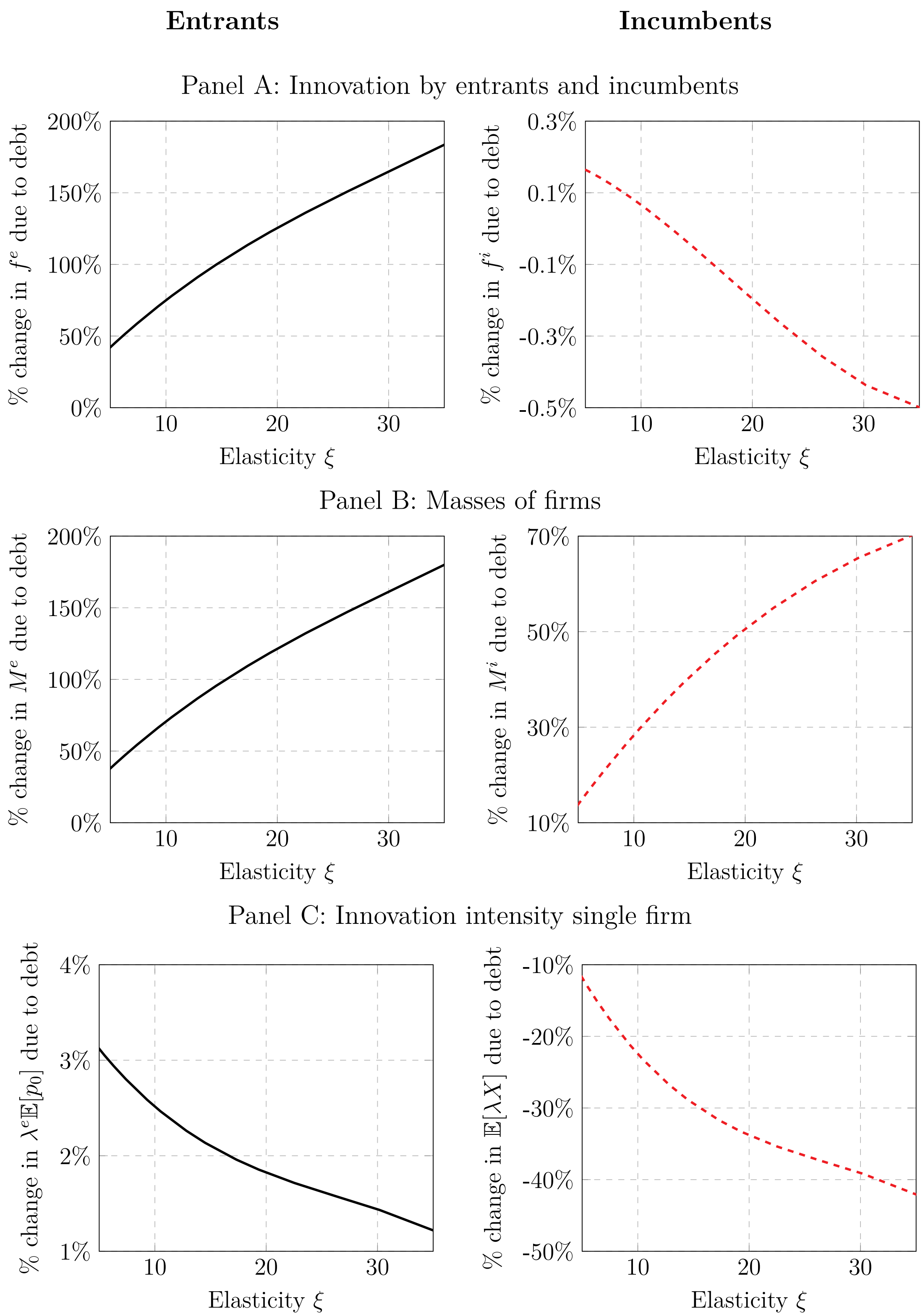

Figure 10: **Debt financing and innovation.** The figures show the effects of debt financing on innovation by incumbents and entrants. We make use of the entry cost function $\mathcal{H}(m) = H\left(m/m_{No\ Debt}(f^*_{No\ Debt})\right)^{\frac{1}{\xi}}$. The comparative statics are smoothed using a third-order polynomial.

The break-even elasticity $\bar{\xi}$ is implicitly defined by

$$E^e(f^*(\bar{\xi})) = E^e(f^*_{No\ Debt}) = (1-\pi)\mathcal{H}(m(f^*_{No\ Debt})) = (1-\pi)H\left(\frac{m(f^*_{No\ Debt})}{m_{No\ Debt}(f^*_{No\ Debt})}\right)^{1/\bar{\xi}}, \quad (8)$$

where $E^e(f)$ is the entrant value defined in equation (4) and $m(f)$ is the flow of entrants given f and where the second equality follows from the free entry condition.[22] Solving equation (8) for $\bar{\xi}$ yields:

$$\bar{\xi} = \frac{\log\left(\frac{m(f^*_{No\ Debt})}{m_{No\ Debt}(f^*_{No\ Debt})}\right)}{\log\left(\frac{E^e(f^*_{No\ Debt})}{(1-\pi)H}\right)}.$$

In our baseline model, the break-even elasticity $\bar{\xi}$ equals 0.57, which implies that $f^*(\xi) > f^*_{No\ Debt}$ and debt stimulates innovation whenever $\xi \in [0.57, +\infty)$. This value for the break-even elasticity is well below the elasticity of 2.69 used in Clementi and Palazzo (2016) to match entry and exit dynamics. This suggests that our result that debt stimulates innovation is robust even when the supply of entrants is not perfectly elastic.

3.2 Strengthening the mechanism

3.2.1 Debt Renegotiation

While an inelastic supply of entrants makes our result weaker, introducing debt renegotiation in the model would strengthen it for at least three reasons. First, debt renegotiation implies that shareholders recover part of their investments in successful innovations in the event of default. This first effect reduces debt overhang and therefore stimulates firm-level investment (keeping leverage fixed), as argued theoretically and shown empirically by Acharya and Subramanian (2009) and Favara et al. (2017). Second, by reducing default costs (keeping leverage fixed), debt renegotiation increases firm (shareholder) value (Fan and Sundaresan (2000) and Antill and Grenadier (2019)) and, as a result, innovation by entrants. Third, a

[22] Conditional on being in equilibrium, we do not need the equilibrium entry cost to calculate the equilibrium flow of entrants $m(f)$.

higher recovery value increases leverage since default costs are lower (Leland (1994)), which increase the net benefits of debt and, thus, innovation by entrants. These three effects imply that allowing for debt renegotiation in default would positively impact innovation and strengthen our result that debt financing stimulates innovation.

3.2.2 Subsidies

Many governments set up programs to incentivize companies to innovate. Introducing subsidies to innovative firms in our model is equivalent to reducing the cost of entry. As shown by Table 3, a decrease in entry costs increases the intensity of competition by fostering entry and therefore reduces the value of investment opportunities for incumbents. This leads incumbent firms to adopt higher leverage ratios, thereby increasing the net benefit of debt financing and increasing the turnover rate. The bottom left panel of Figure 9 shows indeed that the effect of debt on entry becomes stronger as the entry cost decreases. That is both the mass of entrants M^e and the mass of incumbents M^i increase as subsidies increase, leading to a strengthening of the effects of debt on creative destruction.

4 General Equilibrium

This section closes the model in general equilibrium to endogenize the growth rate, labor demand, and the interest rate in the economy. The general equilibrium setup builds on Klette and Kortum (2004). We study a stationary equilibrium with a balanced growth path. This subsection describes the key features of the general equilibrium framework. Appendix 5 provides a detailed and formal description.

There is a representative household with logarithmic preferences who perfectly elastically supplies labor at a fixed wage w. Entrants and incumbents use labor to perform R&D and produce output. All costs in the model come in the form of labor costs, and therefore aggregate production equals aggregate consumption $\mathcal{C}_t$.

For each type of product there is a leading producer, as in the industry equilibrium model. The production technology of good i's leading producer is ν_t^i and determines the number of products that one unit of labor produces. A firm that innovates on product i improves the production technology and becomes the leading producer. Each innovation is a quality improvement applying to a product drawn at random. The innovation increases the production technology proportionally. That is, when an innovation arrives at time t for product i, the production technology increases from $\nu^i_{t^-}$ to $\nu^i_t = (1+\delta)\nu^i_{t^-}$ where $\delta > 0$ represents the (technological) improvement brought by an innovation. Each firm uses one unit of labor for each of its product lines. Given the representative agent's preferences, this setup implies that a firm's profits per product line only depend on the wage rate, which allow us to use the industry equilibrium framework of Section 1. As a consequence, all the results derived in industry equilibrium still hold in general equilibrium.

This also implies that Proposition 2, which shows that creative destruction is higher in an industry equilibrium with debt, still holds true in general equilibrium. The growth rate of (consumption in) the economy is given by

$$g^* = f^* * \delta = \textit{rate of creative destruction} \times \textit{increase in output due to an innovation.}$$

Therefore, this higher rate of creative destruction implies that the growth rate is also higher in the presence of debt. The following proposition formalizes this result.

Proposition 5 (Debt Financing and Growth)**.** *Let $g^*_{No\ Debt}$ be the unique equilibrium growth rate when firms are restricted to have no debt. The growth rate g^* in the unique stationary industry equilibrium with debt Ψ^* satisfies:*

$$g^* \geq g^*_{No\ Debt}.$$

This result follows directly from Proposition 2 and the fact that, as we show in Appendix 5, the equilibrium growth rate of consumption g^* is proportional to the rate of creative destruction f^*. When firms are allowed to issue debt, levered incumbents face debt overhang which lowers investment. But the possibility to issue debt also increases firm value, which spurs entry and therefore innovation and growth.

In this economy, welfare is given by the utility of the representative household:

$$U_0 = \int_0^\infty e^{-rt}(\ln(\mathcal{C}_t) - wL^S)dt = \frac{r\ln(\mathcal{C}_0) + g - rwL^S}{r^2}, \tag{9}$$

where L^S is the total amount of labor supplied. This equation shows that welfare is increasing in the growth rate of the economy g but decreasing in the amount of labor supplied L^S. Debt financing impacts both quantities leading to ambiguous welfare effects.

Proposition 6 shows that even though debt always increases economic growth, its impact on welfare depends on the size of technological improvements due to innovations.

Proposition 6 (Debt Financing, Technological Improvements, and Welfare)**.** *Debt financing improves welfare U_0 if and only if the improvement from an innovation δ is sufficiently large*

$$U_0 > U_0^{No\ Debt} \;\Leftrightarrow\; \delta > \bar{\bar{\delta}}.$$

Proposition 6 shows that the impact of debt financing on welfare depends on the innovation step size δ, with larger technological improvements leading to a higher welfare with debt financing. In our model, innovation benefits society (increases welfare) by increasing the efficiency of the production technology. On the other hand, innovation is costly for society as it requires additional resources (labor); see equation (9). Larger improvements in technology δ make innovations more valuable from a societal perspective and therefore increase the benefits of debt financing. As a result, and as shown by Proposition 6, there exists a level of technological improvements $\bar{\bar{\delta}}$ above which debt financing improves welfare. This result complements our result on the positive impact of debt financing on growth (Proposition 2) by showing that debt financing also positively impacts welfare if the technological improvements generated by innovation are sufficiently large. Importantly, while debt subsidizes entry, it also leads to underinvestment for incumbents and, therefore, does not work like government policies targeted at inducing innovation (e.g. patent policy) which would stimulate innovation by both entrants and incumbents. Therefore, while government subsidies could also increase the aggregate level of innovation, the implications for incumbents and for the size distribution of firms would be different. In addition, deadweight losses of default and debt overhang (as well as other debt induced distortions not modelled in the paper such as risk-taking) could be avoided with non-debt subsidies, leading to potential improvements in welfare.

5 Conclusion

This paper investigates the relation between debt financing, innovation, and growth in a Schumpeterian growth model with endogenous R&D and financing choices. In the model, each firm's R&D policy influences its risk profile, which feeds back in its capital structure decisions. In addition, a levered firm's R&D policy can be altered by its financing decisions, due to conflicts of interest between shareholders and debtholders. As a result, financing and investment are intertwined at the firm level.

We embed the single-firm model into an industry equilibrium that endogenizes the rate of creative destruction and derive a steady state equilibrium in which innovating firms introduce new products that replace existing ones, and entrants replace exiting incumbents. In this equilibrium, firms' R&D and capital structure decisions affect the aggregate level of creative destruction, which in turn feeds back in their policy choices. Debt financing has large effects on firm entry, firm turnover, and industry structure and evolution.

Based on the resulting equilibrium, the paper shows that while debt hampers innovation by incumbents due to debt overhang, it also stimulates entry. The paper also demonstrates that this second effect always dominates in equilibrium, so that debt fosters innovation and growth at the aggregate level. Our analysis suggests that the relation between debt and investment is more complex than acknowledged by prior work. It therefore highlights potential limitations of empirical work that focuses solely on firm-level investment when measuring the effects of debt on corporate investment.

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Appendix

The appendix consists of five parts. We solve the debt case (Theorem 1 and Proposition 1) in Section 1. Section 2 embeds the model into an industry equilibrium (Theorem 2 and Proposition 2). Section 3 derives the steady state firm size distribution and masses of incumbents and entrants. Section 4 derives the results related to the impact of debt financing on investment and the masses of incumbents and entrants (Proposition 3 and Proposition 4). Section 5 closes the model in general equilibrium (Proposition 5 and Proposition 6).

1 Debt Financing

We start by establishing the single firm results (Theorem 1) and intermediate results that show that equity value is continuous and decreasing in f and c (Lemma 1). We then prove the comparative statics results in Proposition 1.

In the baseline debt model an incumbent's coupon is constant. Therefore, we write the equity value as

$$E(p) = E(p, c)$$

and use this notation when it does not lead to confusion. Furthermore, equity value indirectly depends on f and c. When necessary, we make this dependence explicit by writing $E(p|f, c)$.

Proof of Theorem 1. The proof has several steps. First, we establish existence of the equity value. Then we show that it is increasing in the number of product lines p. Finally, we derive the first-order condition for the internal optimal level of R&D.

1. Equation (3) shows that the equity value for $p \in \{1, ..., \bar{p}\}$ can be rewritten as

$$E(p) = \sup_{\lambda,\tau_D} \Bigg\{ \mathbb{E}_p \left[\int_0^{\tau_D} e^{-(r+\lambda+pf)t}(1-\pi)(p-c-\eta-q(p,\lambda))dt \right] + \mathbb{E}_p \left[\int_0^{\tau_D} e^{-(r+\lambda+pf)t} \left(\lambda \mathbb{E}\left[E(\min\{p+x,\bar{p}\})\right] + pfE(p-1) \right) dt \right] \Bigg\}.$$

with $E(0) = 0$. Define $\mathcal{M}(E)$ as the mapping

$$\mathcal{M}(E) = \sup_{\lambda,\tau_D} \Bigg\{ \mathbb{E}_p \left[\int_0^{\tau_D} e^{-(r+\lambda+pf)t}(1-\pi)(p-c-\eta-q(p,\lambda))dt \right] + \mathbb{E}_p \left[\int_0^{\tau_D} e^{-(r+\lambda+pf)t} \left(\lambda \mathbb{E}\left[E(\min\{p+x,\bar{p}\})\right] + pfE(p-1) \right) dt \right] \Bigg\}.$$

Any fixed point of this mapping is bounded from above by $\bar{p}/r$ and from below by zero. Furthermore, the mapping is monotone in E and finally,

$$\begin{aligned}\mathcal{M}(E+L) = \sup_{\lambda,\tau_D} \Bigg\{ & \mathbb{E}_p \left[\int_0^{\tau_D} e^{-(r+\lambda+pf)t}(1-\pi)(p-c-\eta-q(p,\lambda))dt \right] \\ & + \mathbb{E}_p \left[\int_0^{\tau_D} e^{-(r+\lambda+pf)t}\lambda \left(\mathbb{E}\left[E(\min\{p+x,\bar{p}\}) \right] + L \right) dt \right] \\ & + \mathbb{E}_p \left[\int_0^{\tau_D} e^{-(r+\lambda+pf)t} pf \left(E(p-1) + L \right) dt \right] \Bigg\}, \\ \mathcal{M}(E+L) \leq \mathcal{M}(E) + & \frac{\bar{\lambda}+\bar{p}f}{r+\bar{\lambda}+\bar{p}f} L,\end{aligned}$$

because $\lambda \leq \bar{\lambda}$ by assumption. Therefore, the mapping $\mathcal{M}(E)$ satisfies Blackwell's sufficient conditions for a contraction (see Theorem 3.3 on page 54 in Stokey, Lucas, and Prescott (1989)) and it is a contraction mapping, which implies that a fixed point exists and is unique. The equity value is the fixed point of this mapping.

2. The next step is to show that equity value is non-decreasing in p. We do this by showing that having one extra product line improves a firm's cash flows even if shareholders run the firm as if it does not have this extra product line. Assume that the firm currently has $p+1$ product lines and that it separates one product line and runs the firm as if it has only p product lines. The firm receives cash flows from this extra product line at least until the product line becomes obsolete, the firm's non-separated number of product lines reaches $\bar{p}$ or zero, or the firm defaults. The firm receives the extra (gross) profits from this separated product line but it also incurs possibly higher R&D costs (since they depend on P_t). The equity value of this $p+1$ firm with a separated product line is given by

$$E(p) + \mathbb{E}_p \left[\int_0^{\tau_D(p)\wedge\tau_0(p)\wedge\tau_{\bar{p}}(p)} e^{-(r+f)t}(1-\pi)\left(1 - q(P_t+1,\lambda_t) + q(P_t,\lambda_t)\right) dt \right],$$

where $\tau_D(p)$ is the optimal default time of a firm that starts with p product lines, $\tau_0(p)$ is the first time the firm has zero product lines if it starts with p product lines, and $\tau_{\bar{p}}(p)$ is the first time a firm with p product lines has $\bar{p}$ product lines. The first term is the cash flows from the p product line firm, and the second term is the cash flow from the separated product line minus the changes in R&D costs. The conditions on the R&D cost function ensure that the second term is non-negative. Furthermore, the optimal R&D and default strategy followed by a $p+1$ product line firm (weakly) dominates the one chosen by a firm that separates one product line and uses the strategy from a

p product line firm. Therefore,

$$\begin{aligned} E(p) \leq & E(p) \\ & + \mathbb{E}_p \left[\int_0^{\tau_D(p)\wedge\tau_0(p)\wedge\tau_{\bar{p}}(p)} e^{-(r+f)t}(1-\pi)\left(1 - q(P_t+1,\lambda_t) + q(P_t,\lambda_t)\right) dt \right] \\ \leq & E(p+1), \end{aligned}$$

which shows that the equity value $E(p)$ is non-decreasing in p. This also implies that a barrier default strategy is the optimal default strategy.

3. Finally, if the cost function $q(p, \lambda)$ is differentiable the internal optimal level of R&D should satisfy the first-order condition that follows from equation (3).

□

Lemma 1. *The equity value $E(p|f, c)$ is continuous and non-increasing in f and c. If $E(p|f, c) > 0$ then the equity value is decreasing in f and c.*

Proof. We first show that equity value decreases with the rate of creative destruction f.

1. Fix $f_2 < f_1$. Let P_t^1 be the number of product lines of a firm facing a rate of creative destruction f_1. We know that

$$E(p|f_1) = \mathbb{E}_p \left[\int_0^{\tau_D^1 \wedge \tau_0^1} e^{-rt}(1-\pi)\left(P_t^1 - c - \eta - q(P_t^1, \lambda_t^1)\right) dt \right],$$

where $\{\lambda_t^1\}, \tau_D^1$ are shareholders optimal strategy given f_1. The dynamics of P_t^1 are

$$dP_t^1 = dI_t^1 - dO_t^1 = \max\left(Y_t^1, \bar{p} - P_{t^-}^1\right) dN_t^1 - dO_t^1$$

with

$$\begin{aligned} &\mathbb{E}\left[dP_t^1\right] = \lambda_t^1 \mathbb{E}\left[\max\left(Y_t^1, \bar{p} - P_{t^-}^1\right)\right] dt - f_1 P_{t^-}^1 dt, \\ &Y_t^1 \sim Bin(\theta, n). \end{aligned}$$

2. Define $\tilde{P}_t^2$ as,

$$d\tilde{P}_t^2 = d\tilde{I}_t^1 - Z_t dO_t^1 - dH_t,$$

where

$$
\begin{aligned}
\tilde{I}_t^1 &= \max\left(Y_t^1, \bar{p} - \tilde{P}_{t^-}^2\right) dN_t^1, \\
Z_t &\sim Bin\left(1, \frac{f_2}{f_1}\right), \\
H_t &\sim Poisson\left(f_2\left(\tilde{P}_{t^-}^2 - P_{t^-}^1\right)\right).
\end{aligned}
$$

The construction of Z_t and H_t implies that,

$$
\mathbb{E}_t\left[Z_t dO_t^1 + dH_t\right] = \frac{f_2}{f_1} f_1 P_{t^-}^1 dt + f_2\left(\tilde{P}_{t^-}^2 - P_{t^-}^1\right) dt = f_2 \tilde{P}_{t^-}^2 dt.
$$

These dynamics imply that $\tilde{P}_t^2$ evolves according to the R&D strategy $\{\lambda_t^1\}$ given a failure intensity of f_2. The construction $\tilde{P}_t^2$ ensures that

$$
P_t^1 \leq \tilde{P}_t^2.
$$

If $\tilde{P}_{t^-}^2 = P_{t^-}^1$ then innovation dynamics are the same $dI_t^1 = d\tilde{I}_t^2$. Furthermore, product line failure is higher for $P_{t^-}^1$ since $f_2/f_1 < 1$ and if a product line fails for $\tilde{P}_{t^-}^2$ then it fails for $P_{t^-}^1$. Therefore, if $\tilde{P}_{t^-}^2 = P_{t^-}^1$ then $\tilde{P}_t^2 \geq P_t^1$. If $\tilde{P}_{t^-}^2 > P_{t^-}^1$ then product line failure can never imply $\tilde{P}_t^2 < P_t^1$ since product lines drop by only one. Furthermore, by construction innovation happens at the same time and the number of product lines created for both is either Y_t or $\bar{p}$ is reached. This implies that if at time t product lines are created and $\tilde{P}_{t^-}^2 > P_{t^-}^1$ then $\tilde{P}_t^2 = \min\left(\tilde{P}_{t^-}^2 + Y_t, \bar{p}\right) \geq \min\left(P_{t^-}^1 + Y_t, \bar{p}\right) = P_t^1$. Therefore, if $\tilde{P}_{t^-}^2 > P_{t^-}^1$ then $\tilde{P}_t^2 \geq P_t^1$.

3. Given the assumptions on the cost function the equity value satisfies

$$
\begin{aligned}
E(p|f_1) &= \mathbb{E}_p\left[\int_0^{\tau_D^1 \wedge \tau_0^1} e^{-rt}(1-\pi)\left(P_t^1 - c - \eta - q(P_t^1, \lambda_t^1)\right) dt\right] \\
&\leq \mathbb{E}_p\left[\int_0^{\tau_D^1 \wedge \tau_0^1} e^{-rt}(1-\pi)\left(\tilde{P}_t^2 - c - \eta - q(\tilde{P}_t^2, \lambda_t^1)\right) dt\right] \\
&\leq E(p|f_2).
\end{aligned}
$$

If the equity value is positive then $\tau_D^1 \wedge \tau_0^1 > 0$, and the second inequality becomes a strict inequality. This shows that $E(p|f)$ is non-increasing in f and strictly decreasing in f when $E(p|f) > 0$.

4. The next step is showing that the equity value is continuous in f. The mapping $\mathcal{M}(E|f)$ is continuous in f. Therefore, for every $\epsilon > 0$ there exists a $\delta > 0$ such that

for $f' \in (f-\delta, f+\delta)$,

$$\|\mathcal{M}(E(p|f)|f') - E(p|f)\| = \|\mathcal{M}(E(p|f)|f') - \mathcal{M}(E(p|f)|f)\| < \epsilon.$$

Fix one such ϵ. Define $\mathcal{M}^m(E|f)$ as applying the mapping $\mathcal{M}(\cdot|f)$ m times to E. Applying the mapping $\mathcal{M}$ again leads to,

$$\|\mathcal{M}^2(E(p|f)|f') - \mathcal{M}(E(p|f)|f')\| < U\|\mathcal{M}(E(p|f)|f') - E(p|f)\| < U\epsilon.$$

where

$$U = \frac{\bar{\lambda} + \bar{p}f'}{r + \bar{\lambda} + \bar{p}f'}.$$

This process can be repeated and leads to

$$\|\mathcal{M}^{m+1}(E(p|f)|f') - \mathcal{M}^m(E(p|f)|f')\| < U^m\epsilon.$$

Therefore, the distance between $E(p|f)$ and $E(p|f')$ is bounded by

$$\begin{aligned}
\|E(p|f) - E(p|f')\| &= \|E(p|f) - \mathcal{M}^\infty(E(p|f)|f')\| \\
&\le \sum_{i=0}^{\infty} \|\mathcal{M}^{i+1}(E(p|f)|f') - \mathcal{M}^i(E(p|f)|f')\| \\
&< \epsilon \sum_{i=0}^{\infty} U^i \\
&= \epsilon \frac{1}{1-U} = \epsilon \frac{r + \bar{\lambda} + \bar{p}\,(f + (f'-f))}{r} = \epsilon \frac{r + \bar{\lambda} + \bar{p}\,(f+\delta)}{r}.
\end{aligned}$$

Take an $\tilde{\epsilon} > 0$ and set

$$\epsilon = \tilde{\epsilon}\frac{r}{r + \bar{\lambda} + \bar{p}(f+1)}.$$

Then define $\tilde{\delta} = \min\{\delta, 1\}$. We get that for $f' \in (f-\tilde{\delta}, f+\tilde{\delta})$

$$\frac{r + \bar{\lambda} + \bar{p}\,(f + (f'-f))}{r} \le \frac{r + \bar{\lambda} + \bar{p}\,(f+1)}{r} = \tilde{\epsilon}.$$

This implies that for every $\tilde{\epsilon} > 0$ there exists a $\tilde{\delta} > 0$ such that for $f' \in (f-\tilde{\delta}, f+\tilde{\delta})$,

$$\|E(p|f) - E(p|f')\| < \tilde{\epsilon}$$

Therefore, $E(p|f)$ is continuous in f. The same argument shows that $E(p|c)$ is continuous in c.

5. The final step is showing that the equity value is non-increasing in c and decreasing if $E(p|c) > 0$. The mapping $\mathcal{M}(E|c)$ is non-increasing in c and non-decreasing in E. Therefore, for a $c < c'$ we have that

$$\begin{aligned} E(p|c) &= \mathcal{M}(E(p|c)|c) \geq \mathcal{M}(E(p|c)|c') \geq \mathcal{M}^2(E(p|c)|c') \\ &\geq \mathcal{M}^{n>2}(E(p|c)|c') \geq \mathcal{M}^{\infty}(E(p|c)|c') = E(p|c'), \end{aligned}$$

which proves the result. The first inequality becomes a strict inequality when $E(p|c) > 0$, which shows the decreasing result.

□

Proof of Proposition 1. The result for c and f follows from Lemma 1. Take any other parameter (or the function $q(p, \lambda)$) and call it Ξ. If $E(p|\Xi) > 0$ then the mapping $\mathcal{M}(E|\Xi)$ is decreasing in Ξ and increasing E. Therefore, we have

$$\begin{aligned} E(p|\Xi) &= \mathcal{M}(E(p|\Xi)|\Xi) > \mathcal{M}(E(p|\Xi)|\Xi') \geq \mathcal{M}^2(E(p|\Xi)|\Xi') \\ &\geq \mathcal{M}^{n>2}(E(p|\Xi)|\Xi') \geq \mathcal{M}^{\infty}(E(p|\Xi)|\Xi') = E(p|\Xi'), \end{aligned}$$

which proves the result.

The results on the default threshold follow from the results on the equity value. The result that the tax rate π does not impact the default threshold follows from the fact that all cash flows in the equity value scale linearly in $(1 - \pi)$. □

2 Industry Equilibrium

This section establishes the existence and uniqueness of an industry equilibrium (Theorem 2) assuming that debt value is decreasing in f (i.e. debt value decreases with default risk). We first need to establish that a unique firm value exists

Lemma 2. *For a given f, firm value $V(p|f)$ exists and is unique.*

Proof. Define the mapping

$$\mathcal{V}(p|V, f) = \sup_{c} \left\{ E(p, c|f) + (1 - \phi) D(p, c|V, f) \right\}.$$

First, observe that if $V \in [0, \bar{p}/r]^{\bar{p}+1}$ then $\mathcal{V} \in [0, \bar{p}/r]^{\bar{p}+1}$. If $c = 0$ then $E(p, 0) \geq 0$ and as a result $\mathcal{V}(p|V, f) \geq 0$. Furthermore, if $V(p) \leq \bar{p}/r$ then given any $c \leq \bar{p}$ $\mathcal{V}(p|V, f) \leq \bar{p}/r$

while if $c > \bar{p}$ the firm would directly default and as a result $\mathcal{V}(p|V,f) = (1-\phi)(1-\alpha)V(p) < \bar{p}/r$. Therefore, $\mathcal{V}(p|V,f)$ maps a function from $[0,\bar{p}/r]^{\bar{p}+1}$ into $[0,\bar{p}/r]^{\bar{p}+1}$.

Second, given the L_∞ norm observe that

$$\begin{aligned}
&\|\mathcal{V}(p|V,f) - \mathcal{V}(p|V',f)\| \\
&\leq \max\left\{\|\mathcal{V}(p|V,f,c(p)) - \mathcal{V}(p|V',f,c(p))\|, \|\mathcal{V}(p|V,f,c'(p)) - \mathcal{V}(p|V',f,c'(p))\|\right\} \\
&< (1-\phi)(1-\alpha)\|V - V'\|
\end{aligned}$$

where $c(p)$ is the optimal coupon given p, V, and f and $c'(p)$ is the optimal coupon given p, V', and f. The first inequality follows from the fact that the firm value maximizing coupon is chosen. The second inequality follows from the fact that the investment and default strategy do not depend on V (or V') directly (only through the chosen coupon) since the equity value does not depend on it. Therefore, given c the equity value is the same and the only difference between the debt values is the payoff at default, which happens at the same time. $\mathcal{V}(p|V,f)$ is then contraction mapping because $\alpha > 0$. The contraction mapping theorem then ensures that a fixed point of $\mathcal{V}$ (in V) exists and is unique. □

Proof of Theorem 2. The proof has several steps:

1. The first step is showing that the equity value converges to zero when $f \to \infty$. Assume that this is not the case. Then for some p we have that $E(p|f) > 0$ when $f \to \infty$. From equation (3) it follows that for any $p > 0$ with $E(p|f) > 0$

$$\begin{aligned}
0 =& \frac{-rE(p|f) + (1-\pi)(p-c)}{f} \\
&+ \frac{\max_{(\lambda)}\left\{\lambda\left(\mathbb{E}\left[E(\min\{p+x,\bar{p}\})\right] - E(p|f)\right) - (1-\pi)q(p,\lambda)\right\}}{f} \\
&+ p\left\{E(p-1|f) - E(p|f)\right\}.
\end{aligned}$$

   Given that $E(p|f) \leq \bar{p}/r$ and $\lambda \leq \bar{\lambda}$, taking $f \to \infty$ implies that

$$0 = p\left\{E(p-1|f=\infty) - E(p|f=\infty)\right\}$$

   and therefore that

$$E(p|f=\infty) = E(p-1|f=\infty)$$

   for any p for which $E(p|f=\infty) > 0$. Given that $E(0|f=\infty) = 0$ this implies that

$$E(p|f=\infty) = 0,$$

   which is a contradiction. Therefore, equity value does converge to zero when $f \to \infty$.

2. The debt value also goes to zero when $f \to \infty$ since the default time and the recovery value in default go to zero. Therefore, both firm value $V_0(f)$ and the entrant value $E^e(f)$ go to zero as $f \to \infty$.

3. Define firm value as

$$F(p|V, f, c) = E(p|f, c) + (1 - \phi)D(p|V, f, c).$$

4. By Lemma 1, equity value is continuous in f and therefore

$$\lim_{f' \to f} \|E(p|f, c) - E(p|f', c)\| = 0.$$

As a result, the dynamics of P_t will also be the same under f and $f' \to f$. If in addition the default threshold is the same then

$$\lim_{f' \to f} \|D(p|V, f, c) - D(p|V, f', c)\| = 0$$

since the default times will converge. Therefore, $F(p|V, f, c)$ and $\sup_c F(p|V, f, c)$ are continuous in f. Assume now the default threshold is not the same and assume without loss of generality that $\lim_{f' \to f} P_D(f') = P_D(f) + 1$ then

$$\lim_{\epsilon \to 0} \lim_{f' \to f} \|D(p|V, f, c) - D(p|V, f', c - \epsilon)\| = 0.$$

Therefore, we know that there exists a function $c'(f)$ such that

$$\lim_{f' \to f} F(p|V, f', c'(f)) = F(p|V, f, c).$$

As a result,

$$\lim_{f' \to f} \sup_c F(p|V, f', c) = \sup_c F(p|V, f, c)$$

and therefore $\sup_c F(p|V, f, c)$ is continuous in f.

5. Combining this result with Lemma 2 implies that $V(p|f)$ is continuous in f.

6. The above also implies that

$$V_0(f) = \mathbb{E}_0 \left[V(p_0|f)\right]$$

where p_0 is the (random) number of product lines at entry.

7. If there exists an $E^e(f) \geq H(1-\pi)$ then the intermediate value theorem ensures existence of an f such that $E^e(f) = H(1-\pi)$, which is an industry equilibrium.

8. If for all f we have that $E^e(f) < H(1-\pi)$ then entry is never optimal. Given the fact that P_t is non-decreasing for $f = 0$, it follows that for $p > 0$ and $c = 0$ the equity value is positive $E(p|c = 0, f = 0) > 0$. Therefore, a steady state equilibrium exists in which all firms have $\bar{p}$ product lines and no one innovates.

This establishes that an equilibrium exists. The next step is to show that it is unique:

1. First, we show the entrant value is strictly decreasing in f. Since the equity value (for any positive value) and debt value are strictly decreasing in f, the optimal firm value $V_0(f)$ must be strictly decreasing in f as well. Take an $f_1 < f_2$ then

$$\begin{aligned} V_0(f_2) &= \mathbb{E}_0 \left[E(p_0|f_2, c_2) + (1-\phi) D(p_0|f_2, c_2) \right] \\ &< \mathbb{E}_0 \left[E(p_0|f_1, c_2) + (1-\phi) D(p_0|f_1, c_2) \right] \leq V_0(f_1), \end{aligned}$$

where c_2 is the firm value maximizing coupon given p_0 and f_2. Because the entrant value is

$$E^e(f) = \sup_{\lambda} \left(\frac{\lambda V_0(f) - (1-\pi) q_E(\lambda)}{r + \lambda} \right),$$

it is also strictly decreasing in f.

2. There are now two cases. If $E^e(0) \leq H(1-\pi)$ then $E^e(f) < H(1-\pi)$ for all $f > 0$ and the only equilibrium rate of creative destruction is $f^* = 0$. If $E^e(0) > H(1-\pi)$ then there exists a unique f^* such that

$$E^e(f^*) = H(1-\pi),$$

which is a condition that needs to be satisfied in equilibrium if $f^* > 0$. This proves that any equilibrium must have a rate of creative destruction f^*.

□

Proof of Propositon 2. Let $f^*_{No\ Debt}$ be the unique equilibrium rate of creative destruction when firms are restricted to have no debt. Let f^* be the rate of creative destruction in the unique stationary industry equilibrium with debt Ψ^*. The proof has several steps.

1. Because the option to issue debt increases shareholder value, we have that

$$E^e(f^*_{No\ Debt}) \geq E^e_{No\ Debt}(f^*_{No\ Debt}).$$

2. If $f^*_{No\ Debt} = 0$ then it directly follows from Theorem 2 that

$$f^* \geq f^*_{No\ Debt}.$$

3. If $f^*_{No\ Debt} > 0$ then

$$E^e(f^*_{No\ Debt}) \geq E^e_{No\ Debt}(f^*_{No\ Debt}) = H(1-\pi).$$

The proof of Theorem 2 shows that the entrant value is continuous in f and that $\lim_{f\to\infty} E^e(f) = 0$. Therefore, there exists an $f^* \geq f^*_{No\ Debt}$ such that

$$E^e(f^*) = H(1-\pi).$$

This f^* is the rate of creative destruction in the unique stationary industry equilibrium with debt.

□

3 Steady State Distribution and Masses of Agents

This appendix derives the steady state firm size distribution of incumbents and the masses of incumbents and entrants in our economy.

Incumbents

Let $S(p,c)$ be the steady state distribution of (incumbent) firms with p product lines and a debt coupon of c. If firms with p product lines and a coupon of c decide to default, then $S(p,c) = 0$. Assuming these firms do not default, the steady state distribution for p product lines $S(p,c)$ solves

$$\begin{aligned}
0 = &- \underbrace{\lambda(p,c) * (1-\psi(p,0)) * S(p,c)}_{\textit{Exit: Product line creation}} - \underbrace{f * p * S(p,c)}_{\textit{Exit: Product line becomes obsolete}} \\
&+ \underbrace{\sum_{i=1}^{\min(n,p)} \lambda(p-i,c) * \psi(p-i,i) * S(p-i,c)}_{\textit{Entry: Product line creation}} \\
&+ \underbrace{f * (p+1) * S(p+1,c)}_{\textit{Entry: Product line becomes obsolete}} + \underbrace{s * \mathbb{P}(p_0 = p) * \mathbb{I}_{\{c=c^*(p)\}}}_{\textit{Entry: Entrants}}, \\
&+ \underbrace{\sum_{c'\in\mathcal{C}} f * (p+1) * S(p+1,c') * \mathbb{I}_{\{S(p,c')=0\}} * \mathbb{I}_{\{c=c^*(p)\}}}_{\textit{Entry: Incumbents that default and then become a c-coupon firm}}
\end{aligned}$$

where $\psi(p, X)$ is the probability density function of $\min(X, \bar{p} - p)$ with $X \sim Bin(n, \theta)$, $\mathbb{P}(p_0 = p)$ is the product line distribution of entrants upon entry, and $\mathcal{C}$ is the set of coupons observed in equilibrium.

Firms can exit for two reasons. First, they can create new product lines (first term). Second, one of their product lines can become obsolete (second term). Firms can enter for four reasons. First, a firm with less than p product lines can create new product lines and become a p-product line firm (third term). Second, a product line of a firm with $p+1$ product lines can become obsolete (fourth term). Third, there is endogenous entry (fifth term). Fourth, a firm can default after which creditors relever and become a (p, c) firm (sixth term).

The term s determines the flow of entrants. In steady state, the constant s ensures that the outflow of firms is equal to the inflow of firms.

$$s = \sum_{c' \in \mathcal{C}} f * S(1, c').$$

Masses

Given the steady state distribution of incumbents, we next need to determine the masses of different types of firms. We know that flows in and out of the the mass of incumbents M^i and entrants M^e should equate so

$$\underbrace{M^e * \lambda^e}_{Entrants \rightarrow Incumbents} = \underbrace{M^i * exit\ rate\ incumbents}_{Incumbents \rightarrow Exit}. \tag{3.1}$$

We also know that there is a unit mass of products and therefore

$$M^i \mathbb{E}[P_t] = 1. \tag{3.2}$$

where $\mathbb{E}[P_t]$ is the average number of product lines of an incumbent firm. Using equations (3.1) and (3.2), we get that

$$M^i = \mathbb{E}[P_t]^{-1},$$
$$M^e = \frac{exit\ rate\ incumbents}{\lambda^e \mathbb{E}[P_t]}.$$

4 Equilibrium Investment and Masses of Agents

Proof of Proposition 3. We want to show that debt financing hampers innovation by incumbents. With debt financing, incumbents have a positive coupon payment $c > 0$ and face a higher rate of creative destruction $f^* > f^*_{No\ Debt}$. Proposition 1 in the paper shows that firms facing a higher coupon or a higher rate of creative destruction have a lower equity value:

$$\frac{\partial E(p,c)}{\partial c} \leq 0,$$
$$\frac{\partial E(p,c)}{\partial f} \leq 0.$$

These results in combination with the conditions $\left|\frac{\partial E(p+1,c)}{\partial c}\right| \geq \left|\frac{\partial E(p,c)}{\partial c}\right|$ and $\left|\frac{\partial E(p+1,c)}{\partial f}\right| \geq \left|\frac{\partial E(p,c)}{\partial f}\right|$ imply that

$$E(p+1,c|f^*) - E(p+1,0|f^*_{No\ Debt}) \leq E(p,c|f^*) - E(p,0|f^*_{No\ Debt}) \leq 0$$

and therefore that

$$\mathbb{E}\left[E(\min\{p+x,\bar{p}\},c|f^*)\right] - E(p,c|f^*)$$
$$\leq \mathbb{E}\left[E(\min\{p+x,\bar{p}\},0|f^*_{No\ Debt})\right] - E(p,0|f^*_{No\ Debt}).$$

Given the first-order condition for the optimal level of investment (given in Theorem 1 in the paper), this result implies that debt financed firms innovate less. □

Proof of Proposition 4. We know that in equilibrium the following equality must hold:

$$M^i = \mathbb{E}[P_t]^{-1}$$

The unit mass of products must be produced by incumbents, which directly implies the first result that $M^i > M^i_{No\ Debt}$. Second, the mass of firms entering the industry should equal the mass of incumbents leaving in equilibrium:

$$M^e \lambda^e = M^i * \textit{exit rate incumbents}. \tag{4.3}$$

An entrant value is given by

$$E^e(f) = \sup_{\lambda^e} \frac{\lambda^e V_0(f) - (1-\pi) q_E(\lambda^e)}{r + \lambda^e}.$$

Observe that for each $V_0(f)$ there is a unique value $E^e(f)$.[23] Therefore, $V_0(f^*) = V_0^{No\ Debt}(f^*_{No\ Debt})$ since $E^e(f^*) = E^e_{No\ Debt}(f^*_{No\ Debt})$ (due to the free entry condition). This implies that λ^e does not depend on whether firms are debt financed: $\lambda^e = \lambda^e_{No\ Debt}$. This result combined with

[23] Assume this is not the case. Then there must be a $V_0^+ > V_0^-$ such that

$$\sup_{\lambda^e} \frac{\lambda^e V_0^+ - (1-\pi) q_E(\lambda^e)}{r + \lambda^e} = \sup_{\lambda^e} \frac{\lambda^e V_0^- - (1-\pi) q_E(\lambda^e)}{r + \lambda^e}$$

equation (4.3) implies that M^e must be larger in an economy with debt financing.

The last result that $f^e > f^e_{No\ Debt}$ follows from the definition of f^e. □

5 General Equilibrium Setup

This appendix embeds our model into a general equilibrium setup to endogenize the growth rate of the economy, labor supply, and the interest rate. The general equilibrium setup is similar to Klette and Kortum (2004) and leads to a stationary equilibrium with a balanced growth path.

Production

There is a unit mass of differentiated goods in the economy, which are indexed by $i \in [0, 1]$. A measure L^P of labor is used for production, a measure L^F of labor is used for fixed operating cost, a measure $L^{R\&D}$ of labor performs R&D, and a measure L^E of labor is used to generate entrants. Labor supply L^S is perfectly elastic and receives a wage w per unit supplied in each of these activities.

Incumbent firms use labor and installed product lines to produce goods. An improvement in the production technology increases the amount of the consumption good that one unit of labor produces. For each type of product there is a leading producer, as in the industry equilibrium model. The production technology of good i's leading producer is ν_t^i and determines the number of products that one unit of labor produces.

A firm that innovates on product i improves the production technology and becomes the leading producer. Each innovation is a quality improvement applying to a good drawn at random. The innovation increases the production technology proportionally. That is, when an innovation arrives at time t, the production technology increases from ν_{t-}^i to $\nu_t^i = (1+\delta)\nu_{t-}^i$ with $\delta > 0$. A firm that is the leading producer for product i is a monopolist for that good and can choose to supply or not supply that good. If the firm supplies the good then it uses one unit of labor to generate ν_t^i units of the product. If the firm does not supply the good, output and profits for good i are zero.[24]

Let y_t^i be the amount of good i produced at time t. As in Klette and Kortum (2004) or Aghion, Bloom, Blundell, Griffith, and Howitt (2005), the aggregate consumption good is

This leads to a contradiction:

$$\sup_{\lambda^e} \frac{\lambda^e V_0^+ - (1-\pi)q_E(\lambda^e)}{r+\lambda^e} < \frac{\lambda^{e-} V_0^+ - (1-\pi)q_E(\lambda^{e-})}{r+\lambda^{e-}}.$$

where λ^{e-} is the optimal innovation intensity given V_0^-.

[24]We can obtain equivalent results when each production line has as production function $\nu_t^i(l - \mathbb{I}_{\{l\geq 1\}}k(l-1))$ where l is the amount of labor used, $k(0) = 0$, $k'(\cdot) > 0$, and the firm produces the maximum amount of the good among production quantities that maximize its profits.

produced using a logarithmic aggregator

$$\ln(Y_t) = \int_0^1 \ln\left(y_t^i\right) di,$$

with Y_t the aggregate production of the consumption good.[25]

Innovation

Firms can invest in R&D. Investment in R&D leads to product innovations. R&D investment costs come in the form of labor costs and are given by:

$$q(p, \lambda) = w * \tilde{q}(p, \lambda). \tag{5.1}$$

That is, a firm with p products that has an R&D policy λ requires $\tilde{q}(p, \lambda)$ units of labor.[26] We define the innovation cost function for an entrant in a similar way:

$$q_e(\lambda) = w * \tilde{q}_e(\lambda).$$

Incumbents also incur fixed operating cost, which come in the form of $\tilde{\eta}$ hours of labor:

$$\eta = w * \tilde{\eta} \tag{5.2}$$

with $\eta < 1 - w$.

Default and Entry

Debt distorts investment in R&D and can lead to default. Debt issuance cost ϕ represent the share of the debt value captured by the bank/underwriter. In addition, a fraction $\alpha \in (0, 1)$ of assets in place and growth options is lost as a frictional cost in default. As in e.g. Mauer and Triantis (1994), debtholders may choose a new capital structure that maximizes the value of the firm after default before selling it to new owners. We assume these default cost represents the share of firm value captured by the new owners.

[25] This is a limiting case of the Dixit and Stiglitz (1977) aggregator when the elasticity of substitution ϵ goes to 1

$$\lim_{\epsilon \to 1} \ln\left(\left(\int_0^1 \left(y_t^i\right)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}\right) = \lim_{\epsilon \to 1} \frac{\ln\left(\int_0^1 \left(y_t^i\right)^{\frac{\epsilon-1}{\epsilon}} di\right)}{\frac{\epsilon-1}{\epsilon}} = \lim_{\epsilon \to 1} \frac{\int_0^1 \ln\left(y_t^i\right)\left(y_t^i\right)^{\frac{\epsilon-1}{\epsilon}} di}{\int_0^1 \left(y_t^i\right)^{\frac{\epsilon-1}{\epsilon}} di} = \int_0^1 \ln\left(y_t^i\right) dt.$$

[26] The condition on the R&D cost that ensures that the equity value is non-decreasing in p (see (1)) boils down to $q(p+1, \lambda) - q(p, \lambda) \leq 1 - w$ in the general equilibrium framework.

Because firms exit, there must be entry in a stationary equilibrium. As in the industry equilibrium model, entrants have no product lines but perform R&D in the hope of developing innovations, so they can become the leading producer for at least one product. In the industry equilibrium model, entrants pay a fixed entry cost $H(1-\pi)$ to become an entrant. In our general equilibrium model, these fixed costs are replaced by labor costs (as in e.g. Klette and Kortum (2004) or Lentz and Mortensen (2008)). An entrepreneur can hire one unit of labor, which costs $w(1-\pi)$ after tax and generates an idea with Poisson intensity $\hat{H}^{-1}$. Once the entrepreneur has generated this idea, she can become an entrant. Since in equilibrium the cost and benefits should equate for an entrepreneur, the free entry condition becomes

$$E^e(f) = \hat{H}w(1-\pi).$$

Representative Household

There is a representative household with logarithmic preferences:

$$U_0 = \int_0^\infty e^{-rt} \left(\ln\left(\mathcal{C}_t\right) - wL_t^S\right) dt$$

where $\mathcal{C}_t$ is aggregate consumption and r is the discount rate. The representative household's labor supply L_t^S is perfectly elastic at a wage rate w.

Equilibrium Properties

Since our model is a closed economy and all costs come in the form of labor costs and collected taxes are paid out to the representative household, consumption equals production for each good i, and therefore aggregate consumption and production are also equal

$$\mathcal{C}_t = Y_t.$$

The logarithm of aggregate consumption $\ln(\mathcal{C}_t)$ is the numéraire in this economy. The representative household owns all (financial) assets in the economy and receives all labor income.

Using the logarithm of consumption $\ln(\mathcal{C}_t)$ as the numéraire, the representative household's optimal consumption across goods implies that the price of good i should be $\frac{1}{y_t^i} = p_t^i$, where the marginal benefit of good i is equal to its marginal cost. The average cost of production is $\frac{w}{\nu_t^i}$. Therefore, the profit on product i is given by

$$\nu_t^i \left(\frac{1}{\nu_t^i} - \frac{w}{\nu_t^i}\right) = 1 - w.$$

This result implies that the equity value is as in the industry equilibrium framework (see equation (2)), except that the profit flow from a product line is $1-w$ instead of 1 and the fixed operating cost and the R&D cost depend on the wage rate w (see (5.2) and (5.1)).

In equilibrium, the growth rate g, the interest rate $\tilde{r}$, and labor supply L^S are determined by market clearing. Since we use the logarithm of consumption as the numéraire, the agent is effectively risk-neutral in the numéraire and therefore,[27]

$$\tilde{r} = r.$$

Consumption grows at a rate of

$$d\ln(\mathcal{C}_t) = d\int_0^1 \ln(y_t^i)di = \ln(1+\delta)f dt = g dt$$

where f is the rate of creative destruction in the economy, which results from innovations by incumbents and entrants.

Finally, there is a labor supply L^S which is used for production L^P, for the fixed operating cost L^F, for research $L^{R\&D}$, and to generate entrants L^{Ent}:

$$\begin{aligned}
L^P &= 1, \\
L^F &= \int_{\mathcal{F}_t^i} dj, \\
L^{R\&D} &= \int_{\mathcal{F}_t^i} \tilde{q}(P_t^j, \lambda_t^j)dj + \tilde{q}_e(\lambda^e)\int_{\mathcal{F}_t^e} dj, \\
L^{Ent} &= \frac{\textit{turnover rate incumbents} * \int_{\mathcal{F}_t^i} dj}{h},
\end{aligned}$$

[27] The risk-free interest rate $\tilde{r}$ should be set such that a household is indifferent between consuming today or tomorrow. Given that there is no aggregate uncertainty, the Hamiltonian for the consumption smoothing problem, with $\hat{\mathcal{C}}_t = \ln(\mathcal{C}_t)$ logarithm of aggregate consumption, $\hat{Y}_t = \ln(Y_t)$ logarithm of aggregate production, S_t savings, and κ_t the co-state, is

$$H(\hat{\mathcal{C}}, \hat{Y}, S, \tilde{r}, \kappa, t) = e^{-rt}u(\hat{\mathcal{C}}) + \kappa[\tilde{r}S + \hat{Y} - \hat{\mathcal{C}}]$$

where $u(\hat{\mathcal{C}}) = \hat{\mathcal{C}} = \ln(\mathcal{C})$. The optimal solution satisfies the following conditions

$$\begin{aligned}
H_{\hat{\mathcal{C}}}(\hat{\mathcal{C}}_t, \hat{Y}_t, S_t, \tilde{r}_t, \kappa_t, t) &= e^{-rt}u'(\hat{\mathcal{C}}_t) - \kappa_t = 0, \\
H_S(\hat{\mathcal{C}}_t, \hat{Y}_t, S_t, \tilde{r}_t, \kappa_t, t) &= \kappa_t\tilde{r}_t = -\frac{d\kappa_t}{dt},
\end{aligned}$$

see Chapter 7 in Acemoglu (2009). Taking the total derivative yields

$$\begin{aligned}
0 &= -re^{-rt}u'(\hat{\mathcal{C}}_t)dt + e^{-rt}u''(\hat{\mathcal{C}}_t)d\hat{\mathcal{C}}_t - d\kappa_t \\
&= -r\kappa_t dt + 0 + \tilde{r}_t\kappa_t dt \\
\tilde{r}_t &= r,
\end{aligned}$$

which is the Euler equation that the interest rate $\tilde{r}_t$ solves.

where subscript j indicates firm j, $\mathcal{F}_t^i$ is the set of active incumbents and $\mathcal{F}_t^e$ is the set of active entrants. Labor supply is set such that the labor market clears at a wage w:

$$L^S = L^P + L^F + L^{R\&D} + L^{Ent}.$$

The utility of the representative household is

$$\begin{aligned}
U_0 &= \int_0^\infty e^{-rt}(\ln(\mathcal{C}_0) + gt - wL^S)dt \\
&= \frac{\ln(\mathcal{C}_0) - wL^S}{r} + \left[\frac{-1}{r}e^{-rt}gt\right]_0^\infty + \int_0^\infty \frac{1}{r}e^{-rt}gdt \\
&= \frac{r\ln(\mathcal{C}_0) + g - rwL^S}{r^2}.
\end{aligned}$$

This equation shows that welfare is increasing in the growth rate of the economy g but decreasing in the amount of labor supplied L^S.

The formal equilibrium definition is

Definition 2 (General Equilibrium)**.** *The parameters and policies*

$$\Psi^* = \{g^*, L^{S*}, r^*, f^*, c^*(p), \lambda^*(p,c), p_D^*(c), \lambda^{e*}\}$$

are a general equilibrium if:

1. ***Incumbents:*** *Given the rate of creative destruction f^*, the interest rate r^*, and coupon c, incumbents production decision, level of R&D $\lambda^*(p,c)$, and default decision $p_D^*(c)$ maximize shareholder value. Given the rate of creative destruction f^*, creditors optimally relever a defaulted firm $c^*(p_D)$ before reselling it to new owners.*

2. ***Entrants:*** *Given the rate of creative destruction f^* and the interest rate r^*, entrants level of R&D λ^{e*} and capital structure upon becoming an incumbent $c^*(p_0)$ maximize their equity value.*

3. ***Entry:*** *The free entry condition holds:*

$$E^e(f^*) \leq \frac{w(1-\pi)}{h},$$

and the inequality binds when there is creative destruction $f^* > 0$.

4. ***Labor:*** *The labor supply L^{S*} ensures that the labor market clears:*

$$L^{S*} = L^P + L^F + L^{R\&D} + L^E$$

for a wage rate w.

5. ***Growth and interest rate:*** *The growth and interest rate follow from the Euler equation and the rate of creative destruction:*

$$\begin{aligned} d\ln\left(\mathcal{C}_t\right) = g^* dt &= \ln(1+\delta) f^* dt, \\ r^* &= r. \end{aligned}$$

Proofs for General Equilibrium Model

Proof of Proposition 5. This result follows directly from Proposition 2 and the fact that $g = f * \delta$. □

Proof of Proposition 6. Observe that f, $f^{No\ Debt}$, L^S, and $L^{S,No\ Debt}$ are independent of δ. Furthermore,

$$U_0 - U_0^{No\ Debt} = \frac{\delta\left(f - f^{No\ Debt}\right) - rw\left(L^S - L^{S,No\ Debt}\right)}{r^2},$$

which is a linear function in δ with a non-negative term multiplying δ. This proves the result. □

Internet Appendix (not for publication)

1 Debt Refinancing

This appendix extends the model by allowing firms to dynamically optimize their capital structure. Notably, firms that perform well may re-leverage to exploit the tax benefits of debt. For simplicity, we assume that firms can only reduce their indebtedness in default.[28] We consider that it is costly for firms to issue debt but there is no pre-commitment to a specific debt policy (in contrast with, e.g., Leland (1998)) except that firms must repurchase the existing debt (i.e., eliminate a debt covenant) before they can issue new debt with a higher face value. Firms can call their debt at price $\rho\left(p^I\right)c$ with $\rho(p^I) > 0$, where p^I is the number of product lines the firm had when it previously issued debt. The ability to buyback the debt for $\rho(p^I)$ implies that we have to keep track of the number of product lines the firm had the last time it issued debt p^I. We restrict the firm to refinance (in or outside default) at most K times and assume that $c \leq \bar{c}$. In this section, we present the solution for the stationary case when $K \to \infty$. Our results also hold for any finite K.

Define firm value as the equity value plus the debt value minus the issuance cost:

$$F\left(p, c, p^I\right) = E\left(p, c, p^I\right) + (1 - \phi)\, D\left(p, c, p^I\right).$$

The exact definition of the equity and debt value in case the firm can refinance its debt is given below. The payoff to shareholders of restructuring the firm's debt is given by the value of the firm after refinancing minus the cost of buying back the debt:

$$\sup_{c'>c} F(p, c', p) - \rho\left(p^I\right)c.$$

This implies that equity value, with the possibility to dynamically optimize the firm's capital structure, is given by

$$\begin{aligned} E\left(p, c, p^I\right) = \sup_{\{\lambda_t\}_{t\geq 0}, \tau_D, \tau_R} \Bigg\{ & \mathbb{E}_p\left[\int_0^{\tau_D\wedge\tau_0\wedge\tau_R} e^{-rt}(1-\pi)\left(P_t - c - \eta - q(P_t,\lambda_t)\right)dt\right] \\ & + \mathbb{E}_p\left[\mathbb{I}_{\{\tau_R<\tau_D\wedge\tau_0\}} e^{-r\tau_R}\left(\sup_{c'>c} F(P_{\tau_R}, c', P_{\tau_R}) - \rho\left(p^I\right)c\right)\right]\Bigg\}, \end{aligned}$$

where τ_R is the restructuring time chosen by shareholders. Shareholders receive the revenues generated by the portfolio of products minus the coupon payments, the fixed operating

[28] While in principle management can both increase and decrease debt levels, Gilson (1997) finds that transaction costs discourage debt reductions outside of renegotiation. Hugonnier et al. (2015) show in a Leland-type model that reducing debt is never optimal for shareholders if debt holders are dispersed and have rational expectations. That is, there is no deleveraging along the optimal path.

cost, the R&D cost, and corporate taxes until either the firm defaults or changes its capital structure. In default, equity value drops to zero. When refinancing, shareholders repurchase existing debt at price $\rho\left(p^I\right)c$ and obtain the (after issuance cost) optimal firm value with a larger coupon $F(P_{\tau_R}, c', P_{\tau_R})$.

Debt value also takes into account the possibility that the firm refinances and is given by:

$$D\left(p, c, p^I\right) = \mathbb{E}_p\left[\int_0^{\tau_D\wedge\tau_0\wedge\tau_R} e^{-rt}c dt + \mathbb{I}_{\{\tau_D\wedge\tau_0\le\tau_R\}}(1-\alpha)V(p) + \mathbb{I}_{\{\tau_R<\tau_D\wedge\tau_0\}}e^{-r\tau_R}\rho\left(p^I\right)c\right]$$

where $V(p) = \sup_{c\geq 0} F(p, c, p)$ is the optimally levered firm value. This equation shows that creditors receive coupon payments until either the firm defaults or refinances its debt. When the firm defaults ($\tau_D \wedge \tau_0 \le \tau_R$), creditors get the optimally levered firm value net of the proportional default costs α. When the firm refinances its debt ($\tau_R < \tau_D \wedge \tau_0$), creditors get $\rho\left(p^I\right)c$.

In the numerical analysis, we set $\rho\left(p^I\right)$ such that debt is called at a fraction κ of its risk-free value. The buyback price $\rho(p^I)$ therefore solves

$$\rho(p^I)c = \frac{\kappa c}{r}.$$

The entrant value is the same as in equation (4) with $V_0(f)$ defined as

$$V_0(f) = \mathbb{E}_0\left[\sup_{c\geq 0}\left\{E\left(p_0, c, p_0\right) + (1-\phi)D\left(p_0, c, p_0\right)\right\}\right],$$

An industry equilibrium is defined as before, except that firms' optimal policies additionally depend on the number of product lines the firm had the last time it issued debt p^I.

Proof of Equilibrium Existence

First, we establish existence of the equity and debt values (Theorem 3). Next, we establish the existence of an industry equilibrium (Theorem 4) and finally the effect of debt financing on creative destruction (Proposition 7).

In this appendix we denote by

$$E_K(p, c, p^I)$$

the equity value for a firm that can still restructure its debt K times. The debt value

$D_K(p,c,p^I)$ and firm value $F_K(p,c,p^I)$ are similarly defined. Furthermore, define

$$\begin{aligned}
E(p,c,p^I) &= \lim_{K\to\infty} E_K(p,c,p^I),\\
D(p,c,p^I) &= \lim_{K\to\infty} D_K(p,c,p^I),\\
F(p,c,p^I) &= \lim_{K\to\infty} F_K(p,c,p^I).
\end{aligned}$$

Theorem 3. *The equity and debt values exist. If the R&D cost function $q(p,\lambda)$ is differentiable in λ and the optimal level of R&D is internal $\lambda \in (0,\bar{\lambda})$ then it solves*

$$\mathbb{E}\left[E(\min\{p+x,\bar{p}\},c,p^I)\right] - E(p,c,p^I) = (1-\pi)\frac{\partial q(p,\lambda)}{\partial \lambda}.$$

Proof. We establish existence of the equity and debt value recursively.

1. From Theorem 1 it follows that the equity value for a firm that does not have the option to refinance exists. Therefore, also the debt value exists (where in the debt value we use the unlevered firm value as the recovery value). Let this equity and debt values define the firm value:

   $$F_0(p,c,p^I) = E_0(p,c,p^I) + (1-\phi)D_0(p,c,p^I).$$

   The state variable p^I plays no role if the firm cannot restructure.

2. Assume that $F_{K-1}(p,c,p^I)$ exists. First, observe the equity value $E_K(p,c,p^I)$ does not depend on $D_K(p,c,p^I)$ since the price at which the existing debt is bought back is $\rho(p^I)c$. The equity value for a firm that has K restructuring options is

   $$\begin{aligned}
   E_K(p,c,p^I) = \sup_{\{\lambda_t\}_{t\geq 0},\tau_D,\tau_R} &\left\{ \mathbb{E}_p\left[\int_0^{\tau_D\wedge\tau_0\wedge\tau_R} e^{-rt}(1-\pi)\left(P_t - c - \eta - q\left(P_t,\lambda_t\right)\right)dt\right]\right.\\
   &\left. + \mathbb{E}_p\left[\mathbb{I}_{\{\tau_R<\tau_D\wedge\tau_0\}} e^{-r\tau_R}\left(\sup_{c'>c} F_{K-1}(P_{\tau_R},c',P_{\tau_R}) - \rho(p^I)c\right)\right]\right\}.
   \end{aligned}$$

   Given $F_{K-1}(p,c,p^I)$, this implies that the equity value $E_K(p,c,p^I)$ is a fixed point of the mapping

$$\mathcal{M}_K(E) = \sup_{\lambda,\tau_D,\tau_R} \Bigg\{ \mathbb{E}_p \left[\int_0^{\tau_D \wedge \tau_R} e^{-(r+\lambda+pf)t} (1-\pi)\left(p - c - \eta - q\left(p,\lambda\right)\right) dt \right] \\ + \mathbb{E}_p \left[\int_0^{\tau_D \wedge \tau_R} e^{-(r+\lambda+pf)t} \lambda \mathbb{E}\left[E(\min(p+x,\bar{p}), c, p^I) \right] dt \right] \\ + \mathbb{E}_p \left[\int_0^{\tau_D \wedge \tau_R} e^{-(r+\lambda+pf)t} f p E(p-1, c, p^I) dt \right] \\ + \mathbb{E}_p \left[\mathbb{I}_{\{\tau_R < \tau_D\}} e^{-(r+\lambda+pf)\tau_R} \left(F_{K-1}(p, c', p) - \rho(p^I)c \right) \right] \Bigg\}$$

with $E_K(0, c, p^I) = 0$. The equity value is bounded from above by

$$\frac{(1-\pi)\bar{p} + \pi \bar{c}}{r}$$

and from below by zero, it is increasing in E, and

$$\mathcal{M}_K(E + L) \leq \mathcal{M}_K(E) + \frac{\bar{\lambda} + f\bar{p}}{r + \bar{\lambda} + f\bar{p}} L,$$

which holds even if the firm restructures its debt. Therefore, the mapping $\mathcal{M}_K(E)$ satisfies Blackwell's sufficient conditions for a contraction (see Theorem 3.3 on page 54 in Stokey et al. (1989)) and it is a contraction mapping, which implies that a fixed point exists and is unique. Let $E_K(p, c, p^I)$ be the fixed point of this mapping.

3. The debt value $D_K(p, c, p^I)$ follows from the optimal policies of the firm and therefore firm value $F_K(p, c, p^I)$ also exists. These steps recursively establish existence of the value functions.

4. Finally, if the cost function $q(p, \lambda)$ is differentiable the internal optimal level of R&D should satisfy the first-order condition that follows equity value's Hamilton-Jacobi-Bellman equation.

□

Lemma 3. *The entrant value $E^e(f)$ is continuous in f.*

Proof. Continuity is shown recursively.

1. From the proof of Theorem 2 it follows that $\sup_{c'>c} F_0(p, c, p^I|f)$ is continuous in f and c.

2. Assume that $\sup_{c'>c} F_{K-1}(p, c, p^I|f)$ is continuous in f and c. The mapping $\mathcal{M}_K(E|f)$ is continuous in f. Therefore, for every $\epsilon > 0$ there exists a $\delta > 0$ such that for $f' \in (f-\delta, f+\delta)$ we have

$$\begin{aligned}
&\|\mathcal{M}_K(E_K(p, c, p^I|f)|f') - E_K(p, c, p^I|f)\| \\
=&\|\mathcal{M}_K(E_K(p, c, p^I|f)|f') - \mathcal{M}_K(E_K(p, c, p^I|f)|f)\| \\
<&\epsilon.
\end{aligned}$$

Fix one such ϵ. Applying the mapping $\mathcal{M}_K$ again leads to,

$$\begin{aligned}
&\|\mathcal{M}_K^2(E_K(p, c, p^I|f)|f') - \mathcal{M}_K(E_K(p, c, p^I|f)|f')\| \\
\leq&U\|\mathcal{M}_K(E_K(p, c, p^I|f)|f') - E_K(p, c|f)\| \\
<&U\epsilon.
\end{aligned}$$

where,

$$U = \frac{\bar{\lambda} + \bar{p}f'}{r + \bar{\lambda} + \bar{p}f'}.$$

This process can be repeated and leads to

$$\|\mathcal{M}_K^{m+1}(E_K(p, c, p^I|f)|f') - \mathcal{M}_K^m(E_K(p, c, p^I|f)|f')\| < U^m\epsilon.$$

Therefore, the distance between $E_K(p, c, p^I|f)$ and $E_K(p, c, p^I|f')$ is bounded by

$$\begin{aligned}
&\|E_K(p, c, p^I|f) - E_K(p, c, p^I|f')\| \\
&= \|E_K(p, c, p^I|f) - \mathcal{M}_K^\infty(E_K(p, c, p^I|f)|f')\| \\
&\leq \sum_{i=0}^{\infty} \|\mathcal{M}_K^{i+1}(E_K(p, c, p^I|f)|f') - \mathcal{M}_K^i(E_K(p, c, p^I|f)|f')\| \\
&< \epsilon \sum_{i=0}^{\infty} U^i \\
&= \epsilon \frac{1}{1-U} \\
&\leq \epsilon \frac{r + \bar{\lambda} + \bar{p}\left(f + (f' - f)\right)}{r}.
\end{aligned}$$

Take an $\tilde{\epsilon} > 0$ and set

$$\epsilon = \tilde{\epsilon} \frac{r}{r + \bar{\lambda} + \bar{p}(f+1)}$$

then define $\tilde{\delta} = \min\{\delta, 1\}$. We get that for $f' \in (f - \tilde{\delta}, f + \tilde{\delta})$

$$\frac{r + \bar{\lambda} + \bar{p}\,(f + (f' - f))}{r} \leq \frac{r + \bar{\lambda} + \bar{p}\,(f + 1)}{r} = \tilde{\epsilon}.$$

This implies that for every $\tilde{\epsilon} > 0$ there exists a $\tilde{\delta} > 0$ such that for $f' \in (f - \tilde{\delta}, f + \tilde{\delta})$,

$$\|E_K(p, c, p^I|f) - E_K(p, c, p^I|f')\| < \tilde{\epsilon}.$$

Therefore, $E_K(p, c, p^I|f)$ is continuous in f. The same argument shows that $E_K(p, c, p^I)$ is continuous in c.

3. Since the equity value $E_K(p, c, p^I|f)$ is continuous in f, similar steps as in the proof of Theorem 2 show that for $\sup_c F_K(p_0, c, p_0|f)$ is continuous in f. This

4. Applying the previous steps recursively ensures that

$$V(f) = \mathbb{E}_0 \left[\sup_{c \geq 0} F\,(p_0, c, p_0) \right]$$

is continuous in f and therefore that the entrant value $E^e(f)$ is continuous in f.

□

Theorem 4 (Equilibrium Existence with Debt Refinancing)**.** *There exists an industry equilibrium ψ^* in the model with debt refinancing.*

Proof. The proof has several steps

1. It follows from Theorem 2 that $F_0(p, c, p^I)$ converges to zero as $f \to \infty$. Assume $F_{K-1}(p, c, p^I|f)$ converges to zero as $f \to \infty$. If $E_K(p, c, p^I|f)$ does not converge to zero as $f \to \infty$ then for some p we have that $E_K(p, c, p^I|f) > 0$ when $f \to \infty$. This directly implies that the firm does not restructure for this p. Furthermore, from equation (3) it follows that for any $p > 0$ with $E_K(p, c, p^I|f) > 0$

$$\begin{aligned}
0 = & \frac{-rE_K(p, c, p^I|f) + (1 - \pi)(p - c)}{f} \\
& + \frac{\max_{(\lambda)} \left\{ \lambda \left(\mathbb{E}\left[E_K(\min\{p + x, \bar{p}\}, c, p^I) \right] - E_K(p, c, p^I|f) \right) - (1 - \pi) q(p, \lambda) \right\}}{f} \\
& + p \left\{ E_K(p - 1, c, p^I|f) - E_K(p, c, p^I|f) \right\}.
\end{aligned}$$

Given that $E_K(p,c,p^I|f) \leq ((1-\pi)\bar{p}+\pi\bar{c})/r$ and $\lambda \leq \bar{\lambda}$, taking $f \to \infty$ implies that

$$0 = p\left\{E_K(p-1,c,p^I|f=\infty) - E_K(p,c,p^I|f=\infty)\right\}$$

and therefore that

$$E_K(p,c,p^I|f=\infty) = E_K(p-1,c,p^I|f=\infty)$$

for any p for which $E_K(p,c,p^I|f=\infty) > 0$. Given that $E_K(0,c,p^I|f=\infty) = 0$ this implies that

$$E_K(p,c,p^I|f=\infty) = 0$$

which is a contradiction. Therefore, the equity value goes to zero as $f \to \infty$. The debt value also goes to zero when $f \to \infty$ since the default time and the recovery value in default go to zero. This result implies that $F_K(p,c,p^I)$ goes to zero as $f \to \infty$. Recursively applying this argument ensures that the entrant value $E^e(f)$ goes to zero as $f \to \infty$.

2. If $\exists f$ such that $E^e(f) > H(1-\pi)$ then Lemma 3, the previous step, and the intermediate value theorem imply there exists an f^* such that

$$E^e(f^*) = H(1-\pi),$$

which is an industry equilibrium.

3. If $\nexists f$ such that $E^e(f) > H(1-\pi)$ then $f^* = 0$ is an industry equilibrium.

□

Proposition 7 (Debt Financing and Creative Destruction with Debt Refinancing)**.** *Let $f^*_{No\ Debt}$ be the unique equilibrium rate of creative destruction in case firms are restricted to have no debt. Then, there exists stationary industry equilibrium with debt Ψ^* that has a rate of creative destruction f^* and satisfies:*

$$f^* \geq f^*_{No\ Debt}.$$

Proof. The proof is the same as the one for Proposition 2. □

In a similar way as for the baseline model (Section 4), we can embed our model with debt refinancing into a general equilibrium framework. In this framework, we can show that debt financing stimulates growth as we show for the baseline model in Proposition 5.